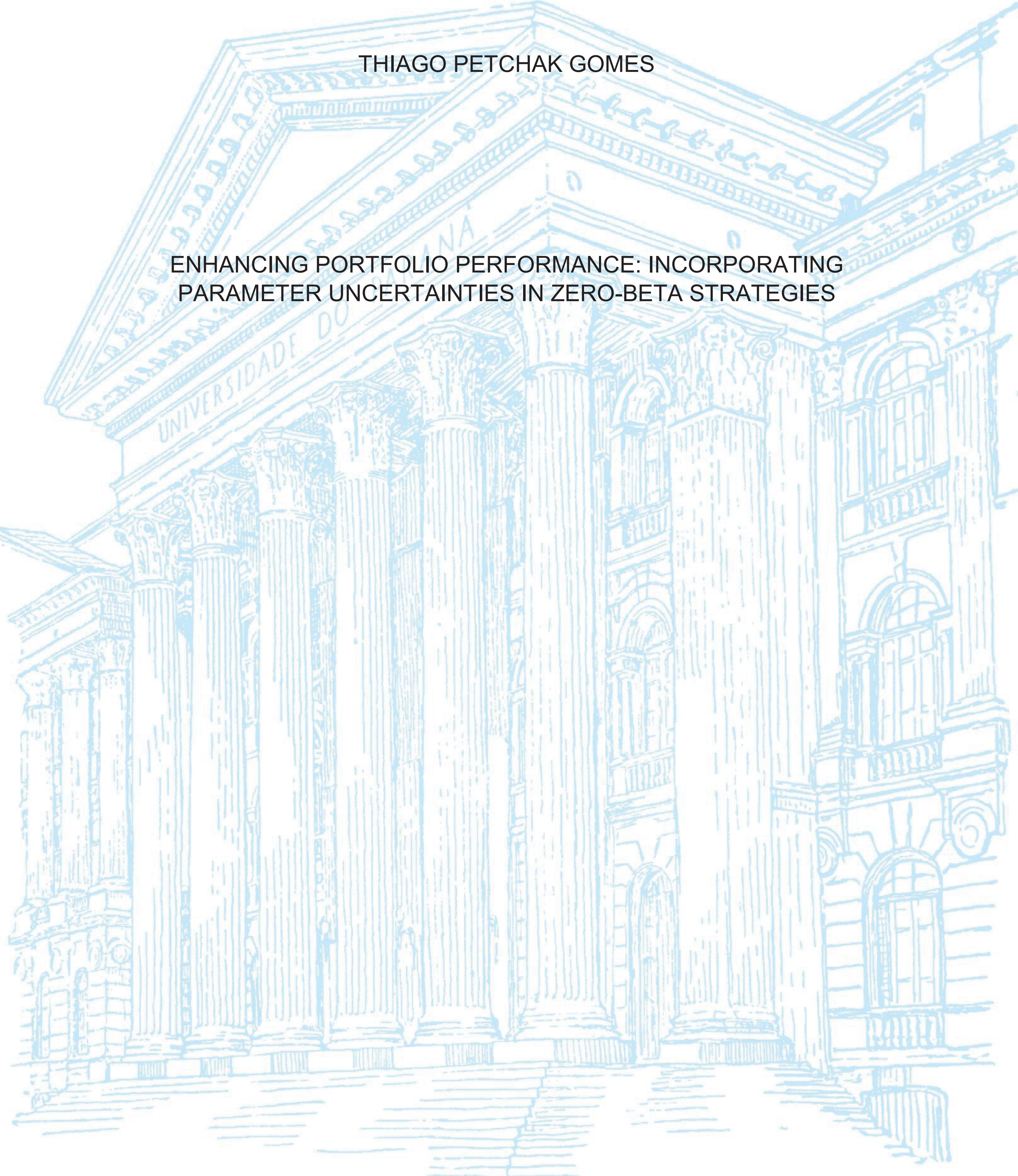
UNIVERSIDADE FEDERAL DO PARANÁ

THIAGO PETCHAK GOMES

ENHANCING PORTFOLIO PERFORMANCE: INCORPORATING PARAMETER UNCERTAINTIES IN ZERO-BETA STRATEGIES



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2023

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Dissertação apresentada ao curso de Pós-Graduação em Gestão de Organizações, Liderança e Decisão, Setor de Ciências Sociais Aplicadas, Universidade Federal do Paraná, como requisito à obtenção do título de Mestre em Gestão

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Os membros da Banca Examinadora designada pelo Colegiado do Programa de Pós-Graduação GESTÃO DE ORGANIZAÇÕES, LIDERANÇA E DECISÃO da Universidade Federal do Paraná foram convocados para realizar a arguição da Dissertação de Mestrado de **THIAGO PETCHAK GOMES** intitulada: **Enhancing Portfolio Performance: Incorporating Parameter Uncertainties in Zero-Beta Strategies**, sob orientação do Prof. Dr. JOSÉ ROBERTO FREGA, que após terem inquirido o aluno e realizada a avaliação do trabalho, são de parecer pela sua APROVAÇÃO no rito de defesa.
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RESUMO

O objetivo deste estudo é analisar uma estratégia de portfólio com beta zero que considera não apenas as estimativas pontuais dos parâmetros, mas também as incertezas dos retornos esperados e dos betas. Compreender os potenciais riscos e recompensas da arbitragem estatística à luz destes erros de estimativa pode ser crucial para o processo de tomada de decisões em finanças. Como forma de metodologia, estudo utiliza o Filtro de Kalman para calcular os betas das ações e suas respectivas incertezas – via modelo multifatorial de precificação de ativos de Chen, Roll e Ross (1986) –, bem como, estimativas de preço e dividendo dos analistas para calcular os retornos esperados e as incertezas das ações. A pesquisa empregou duas abordagens distintas para a construção de carteira beta zero: uma que maximiza a razão entre o retorno esperado e as incertezas dos parâmetros, chamada de "carteira estocástica *long-short*", e outra que maximiza apenas o retorno esperado, ignorando a incertezas dos parâmetros, denominada "portfólio normal *long-short*". Quanto aos resultados, no período de estudo, que vai de 2015 a 2022, uma comparação entre as carteiras estocásticas *long-short* e as carteiras normais *long-short* revelou diferenças notáveis em seus desempenhos. As carteiras estocásticas long-short exibiram desempenho superior em múltiplas dimensões-chave: demonstraram maior acurácia em suas previsões, proporcionaram maiores retornos realizados e exibiram uma variabilidade significativamente menor no seu desempenho. Estes resultados podem sugerir a superioridade da abordagem estocástica *long-short* ao longo do nosso período de análise quando comparada com a carteira normal *long-short.* Em relação às contribuições, investidores, gestores de fundos e profissionais do setor financeiro podem utilizar as abordagens do estudo para desenvolver e gerir melhor as suas carteiras de investimento, especialmente no processo de otimização. Contudo, os resultados também sugerem cautela ao confiar apenas nas estimativas dos analistas para abordagens de arbitragem estatística. Embora a carteira beta zero que considerou a incerteza dos parâmetros tenha gerado um retorno acumulado positivo durante o período analisado, houve alguns anos em o que parecia ser uma oportunidade de arbitragem resultou em um retorno negativo.

Palavras-chave: otimização estocástica carteira zero beta; carteira mercado neutro; Filtro de Kalman; modelo multifatorial de precificação de ativos

ABSTRACT

The aim of this study is to analyse a zero-beta portfolio strategy that considers not only the parameter point estimates but also the uncertainties of the expected returns and the betas. Understanding the potential risks and rewards of statistical arbitrage in light of these estimation errors can be crucial for the decision-making process in finance**.** The study uses the Kalman Filter to calculate the stocks' betas and their respective uncertainties – from Chen, Roll, and Ross' (1986) multifactor asset pricing model –, as well as analysts' price and dividend estimates to calculate the stocks' expected returns and uncertainties. The research employed two distinct approaches when constructing a zero-beta portfolio: one that maximizes the ratio between the expected return and the uncertainties of the parameters, called “long-short stochastic portfolio”, and the other that solely maximizes the expected return, ignoring the parameters' uncertainties, called “long-short normal portfolio”. During the study period spanning from 2015 to 2022, a comparison between the long-short stochastic portfolios and the long-short normal portfolio revealed notable differences in performance. The long-short stochastic portfolios exhibited superior performance across multiple key dimensions: they demonstrated a higher degree of prediction accuracy, delivered greater realized returns, and exhibited significantly lower variability in their performance, as evidenced by a reduced standard deviation. These findings may suggest the superiority of the long-short stochastic approach throughout our analysis period when contrasted with the long-short normal portfolio. The research findings offer practical implications for portfolio management, emphasizing the importance of considering parameter uncertainties in investment decision-making. Investors, fund managers, and practitioners in the financial industry can use the findings of the study to develop and better manage their investment portfolios, especially in the process of portfolio optimization. However, the results also suggest caution when relying solely on analysts' estimates for statistical arbitrage approaches. Even though the zero-beta portfolio that factored in parameter uncertainty generated a positive cumulative return during the analyzed period, there were some years where the apparent arbitrage opportunity resulted in a negative return.

Keywords: stochastic zero beta portfolio optimization; Market neutral; Kalman Filter; multi-factor asset pricing model.

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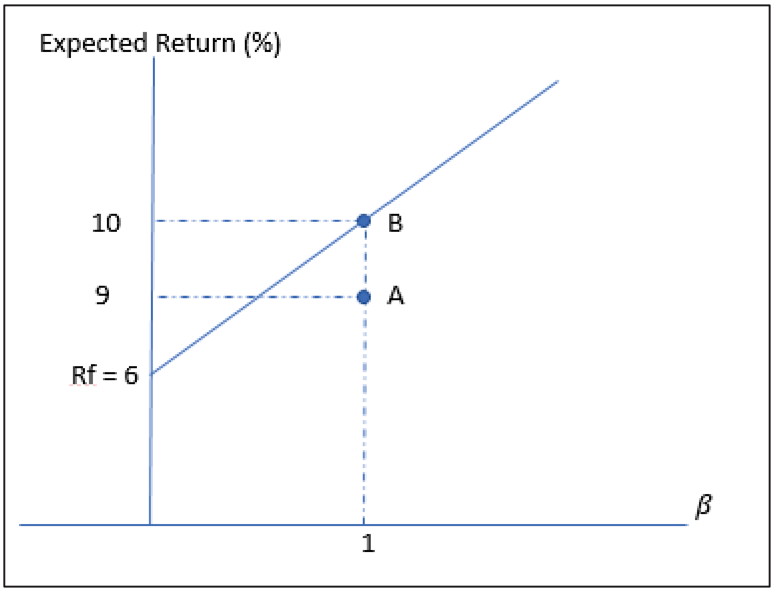
1 INTRODUCTION

In a situation where there are no restrictions on taking advantage of arbitrage opportunities, consider two well-diversified portfolios with identical betas but different expected returns, as illustrated in Figure 1. In this scenario, an investor can potentially make a risk-free profit by selling (shorting) the portfolio with the lowest expected return and buying the portfolio with the highest expected return. It is worth noting that a significant challenge lies in identifying and constructing such portfolios. For instance, the efficient market hypothesis (Fama E. F., 1970), even in its weak form, posits that analyzing past data may not be a reliable method for achieving abnormal profits.

Figure 1 - Arbitrage opportunity

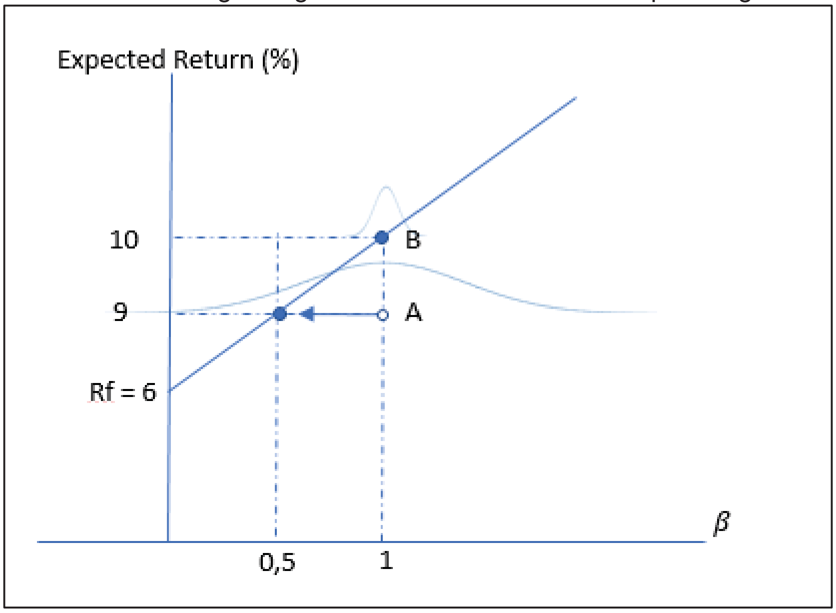


Source: Self Elaboration

It is important to recognize that Figure 1 provides information about the expected return and beta of the portfolio but does not include their confidence intervals. Overlooking the uncertainty associated with these parameters can have adverse consequences for investors who sell Portfolio A and buy Portfolio B. Black extensively explored this issue of neglecting estimation errors in 1993, suggesting that many anomalies identified in investment literature may be a result of poor data analysis practices. Furthermore, Morettin and Bussab (2017) emphasize that relying solely on single-point estimates fails to provide insight into the extent of potential errors. They recommend constructing a confidence interval based on the distribution of these point estimates, which allows for a more comprehensive assessment of the data. For example, the expected return point estimate of Portfolio A presented in Figure 1 may be lower than Portfolio B. However, if both estimations have large enough estimation errors it is reasonable that portfolio A could result in a higher realized return than portfolio B.

Considering the estimation error in beta values is also crucial for an investor pursuing statistical arbitrage. Ignoring the potential error in the beta estimates can lead to unfavorable outcomes for the investor. In Figure 1, if one or both of the portfolios have beta values with significant estimation errors, what might initially appear as a statistical arbitrage opportunity could actually result in losses. To illustrate this, Figure 2 provides an example where the actual beta of Portfolio A is lower than that of Portfolio B, despite both having the same point estimates. In this scenario depicted in Figure 2, if an investor shorts Portfolio A and buys Portfolio B, during a bear market, Portfolio B might incur higher losses than Portfolio A, resulting in a financial loss for the investor.

Figure 2 - The risk of neglecting the estimations errors when pursuing an arbitrage



Source: Self Elaboration

Thus, when looking at it from the angle of expected returns, if an investor intends to create a statistical arbitrage position at a specific beta value, it would make sense to construct two portfolios with the most significant divergence in their point estimates and minimal estimation errors. Conversely, when considering the beta values, it would be logical to select two portfolios with point estimates that are as close as possible and have low estimation errors. Therefore, when evaluating the viability of statistical arbitrage, it is advisable to factor in not only precise point estimates for the parameters but also account for the associated estimation errors.

By considering the uncertainties associated with expected returns and betas, this study aims to provide valuable insights into the effectiveness of the zero-beta portfolio strategy. Understanding the potential risks and rewards of statistical arbitrage in light of these estimation errors is crucial for informed decision-making in the realm of finance.

The general objective[1] of this study is to investigate the zero-beta portfolio strategy. Specifically, the study is interested in assessing a version of this strategy that considers not just the point estimates of certain parameters but also considers the associated uncertainties in expected returns and betas. Additionally, the study compares this approach with a zero-beta portfolio strategy that does not factor in these uncertainties. The stock expected return distribution will be predicted using analysts' estimations and their deviations, while the betas and their uncertainty will be calculated using Chen, Roll, and Ross' (1986)

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[1] The specific objectives are:
1.1) To calculate the betas of companies and their uncertainties, for the periods of 2015 to 2022, considering the macroeconomic factors proposed by Chen, Roll, and Ross (1986) by applying the Kalman Filter approach;
1.2) To calculate the securities expected return and their uncertainties using analysts estimations about price target and dividends;
1.3) To combine the parameters uncertainties of the securities using the Bayesian approach;
1.4) To model optimum zero beta portfolios, yearly, from 2015 to 2022, that aims to maximize the ratio of the expected return divided by the uncertainty;
1.5) To analyse the realized return of the portfolios

multifactor pricing models from the Kalman Filter, a Bayesian[2] approach to continuously estimate the state of a noisy system (Wells, 1996).

The study consists of five sections. Section 2 establishes the theoretical foundation for developing the model and the methodology presented in Section 3. Section 4 presents the outcomes derived from the data and methodology outlined in Section 3. Lastly, Section 5 addresses the findings and outlines potential future research directions.

2 THEORETICAL FOUNDATIONS[3]

2.1 OPTIMIZATION UNDER PARAMETERS UNCERTAINTY

According to Rockafellar and Wets (1991), many systems that require control or analysis involve uncertain parameters. They suggest that when faced with a probabilistic distribution of unknown parameters, it may be appropriate to consider stochastic models.

Robust optimization assumes a deterministic uncertainty model, while stochastic optimization assumes a probabilistic uncertainty model (Bertsimas, Brown, & Caramanis, 2011). According to the authors, in robust optimization, the decision-maker constructs a solution that is feasible for any realization of the uncertainty in a given set. On the other hand, the authors state that stochastic optimization assumes that the uncertainty has a probabilistic description.

Regardless of the method employed, both optimization approaches take uncertainty into account. Viewing it from the angle of portfolio optimization, Bertsimas, Brown, and Caramanis (2011) propose that mean-variance models,

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(Ribeiro Jr, 2022) states that one of the fundamental differences between classical and Bayesian statistics is that in Bayesian statistics the unknown parameters are treated as random variables.

[3] The articles used to support the methodology applied in this dissertation were searched using mainly the Web of Science Database by typing the following keywords: stochastic optimization; stochastic optimization finance; robust optimization; portfolio optimization; robust portfolio optimization; zero beta portfolio; long-short portfolio; statistical arbitrage; Market neutral; robust zero beta portfolio optimization; asset pricing models; beta; Kalman Filter; beta methodology Kalman Filter; Beta methodology moving window; portfolio optimization expected return; portfolio optimization analyst expected return; and mean-variance approach.

Then, the results from the previous keywords were ordered from the highest number of citations to the lowest one. The articles that have similarities to the object of this study were applied in this dissertation.

which solely rely on the expected return point estimate, could result in extreme allocations and prove sensitive to minor perturbations and parameter estimations. They further recommend that incorporating expected return uncertainty into optimization methods may serve to mitigate these challenges.

The notion of optimization in the presence of uncertainty finds applications across diverse domains, spanning engineering, finance, economics, management, natural sciences, and statistics. As an example, Saadouli et al. (2014) employed an optimization approach within the realm of medical scheduling, accounting for uncertain parameters, and achieving improved operational efficiency and cost reductions. Similarly, the domain of medical research, particularly in the context of brain image corrections, has also delved into optimization under uncertainty, as exemplified by Jenkinson et al. (2002). Furthermore, Candes et al. (2006) conducted research in the domain of signal reconstruction, specifically addressing the challenge of reconstructing signals from incomplete frequency data through optimization techniques.

Lu and Shen (2021) recently discussed the role of uncertainty in operations management optimization. They noted that estimation errors and data contamination could affect parameters. Thus, they assert that models with uncertain parameters require nontraditional approaches. In operations management, robust optimization has numerous applications in inventory management, production planning, pricing, and revenue management, scheduling and project management, transportation and vehicle routing, facility location, and network design (Lu & Shen, 2021). Wang (2014) and Calderano (2017) have indicated that, from the valuation perspective, pricing is only known to lie within an interval; therefore, robust optimization may be useful in modeling problems with multiple sources of uncertainty.

2.2 PORTFOLIO OPTIMIZATION UNDER PARAMETERS UNCERTAINTY

When it comes to finance, robust programming involves considering security prices, interest rates, exchange rates, and portfolio optimization (Xidonas, Steuer, & Hassapis, 2020). This can result in more stable outputs when compared to the classical approach. However, while there is a lot of literature about robust portfolios, there are few empirical studies indicating the effectiveness of this method with real results (Xidonas, Steuer, & Hassapis, 2020). The authors propose that additional empirical studies should be conducted to determine whether robust optimization yields better returns when tested in real-world scenarios.

As noted by Markowitz (1991), if an investor possesses precise knowledge about the returns of all stocks, they would logically choose to invest solely in the security offering the highest return, without any inclination toward diversification. However, a precise knowledge about the returns of all stock is not possible. Maenhout (2004) affirms that considering the uncertainty surrounding expected returns is a prudent approach when making portfolio decisions. Furthermore, Goldfarb and Iyengar (2003) have developed a model that takes into account uncertainties in parameters, applicable to both the classical mean-variance and value at risk approaches.

Fabozzi, Huang, and Zhou (2009) present robust portfolio optimization considering the mean-variance, the value-at-risk, and the conditional value at risk. They also note that when these models do not consider the parameter uncertainty, they can suffer from data inadequacy.

The original portfolio optimization concept introduced by Markowitz (1951) laid the foundation for reliable portfolio optimization. However, this field faced persistent challenges, including the sensitivity of portfolios to errors in parameter estimation, as well as the inherent difficulty in accurately forecasting future stock prices (Zhang, Li, & Guo, 2018; Kemaloglu, Inan, & Apaydin, 2018). Addressing the issue of parameter estimation errors, Zhang, Li, and Guo (2018) argue that portfolio optimization models typically treat parameter values as if they were unquestionably accurate, thereby overlooking the associated estimation errors. This oversight results in portfolios that are highly sensitive to asset selection. In response to this problem, Zhang, Li, and Guo (2018) propose the implementation of robust techniques that take into consideration both the portfolio optimization model and the uncertainties surrounding parameter values.

According to Kolm, Tütüncü, and Fabozzi (2013), estimating expected returns poses a practical challenge within the framework of modern portfolio theory, mainly because "risk-return optimization can be very sensitive to changes in the inputs." The authors argue that relying solely on point estimates for parameters, without considering their associated uncertainties, may not be a wise approach. They also highlight that "recent portfolio optimization approaches have begun to account for the uncertainty surrounding expected returns and risk." To mitigate this issue, the authors reference several techniques, including Bayesian methods, the Black-Litterman approach, and robust optimization.

Finally, Fabozzi, Huang, & Zhou (2009) present robust portfolio optimization considering the mean-variance, the value-at-risk, and the conditional value-at-risk. The authors also state that those three models, when they do not consider the parameters' uncertainties, suffer from data inadequacy.

2.3 STATISTICAL ARBITRAGE CONSIDERING THE UNCERTAINTIES OF THE PARAMETERS

Statistical arbitrage is a self-finance investment strategy with a positive expected return and zero or close to zero expected risk (Caneo & Kristjanpoller, 2020). This process involves long and short investment strategies for assets with similar and particular characteristics (Caneo & Kristjanpoller, 2020). Kwan (1999) states that a long-short strategy aims to benefit investors from potentially profiting from under and overvalues assets, which makes the investor expected profit to be higher than long-only investing.

Statistical arbitrage is a commonly employed strategy embraced by institutional investors, hedge funds, mutual funds, and proprietary trading firms (Ziping, Rui, & Palomar, 2019; Elliot, Van der Hoek, & Malcolm, 2005). The primary objective is to capitalize on perceived market imbalances, wherein traders strive to exploit these deviations to their fullest advantage, thereby driving prices toward a rational equilibrium (Göncü & Akyldirim, 2016; Do & Faff, 2010).

The profit in a statistical arbitrage process materializes when asset mispricing corrects itself in the future (Ziping, Rui, & Palomar, 2019; Bowen, 2016). The disparity between the returns on the long and short portfolios is commonly referred to as the "spread" (Elliot, Van der Hoek, & Malcom, 2005; Kwan, 1999). Statistical arbitrage is recognized as a market-neutral strategy because it can hedge against systematic risk, and its profitability remains largely independent of market movements (Ziping, Rui, & Palomar, 2019; Bowen, 2016;

Elliot, Van der Hoek, & Malcom, 2005; Göncü & Akyldirim, 2016; Kwan, 1999). Within the context of this dissertation, the expected return of the portfolio is determined by the difference between the expected return of long positions and that of short positions, also referred as the spread.

A crucial point to consider, as asserted by Kwan (1999), is that achieving market neutrality does not necessarily require equal aggregate weights for beta. This concept, as demonstrated by Bodie, Kane, and Marcus (2013), is illustrated in equation 1:

$$\beta_z = w_s\beta_s - (1 - w_s)\beta_l = 0 \qquad (1)$$

Where:

β_z is the zero-beta portfolio

w_s is the weight of the portfolio short

β_s is the beta of the portfolio short

β_s is the beta of a portfolio long

It is worth mentioning that Göncü & Akyldirim (2016) and Anish (2021) state that once there is uncertainty about portfolio mean and standard deviation, statistical arbitrage is no longer a guaranteed approach, due to “error in trader’s guess or forecast of the long-term mean levels”. The expected profit is also based on the mean reversion price behavior (Ziping, Rui, & Palomar, 2019). About this problem, Do and Faff (2010) claim that there is a continuing downward trend in statistical arbitrage profitability. Beyond forecast errors that may cause a loss on long and short portfolios, Do & Faff (2010) state that arbitrageurs face other risks, such as “noise trader risk” – when illogical trading caused by noise traders prevent arbitrage approaches.

From the parameters uncertainty perspective, Anish (2021) states that statistical arbitrage's covariance is a point estimate, and therefore, this approach is susceptible to estimation errors: for that reason, the author claims that there is a strong need to work with more than point-estimate, and, therefore, the author developed a statistical arbitrage model that takes into account uncertain covariance. According to Anish (2021), taking into account the uncertainty of covariance in portfolio optimization leads to robust weightings.

2.4 THE RISK-RETURN MODELS

Continuing with the statistical arbitrage strategy introduced in this study's introduction, there are two significant sets of parameters essential for forming both the long and short portfolios: expected return and risk factors.

2.4.1 The securities’ expected returns

This study determines the expected return of securities based on analyst estimations. Furthermore, the variation in estimations for specific securities will be regarded as the uncertainty associated with this parameter. These assertions find support in the literature that follows.

According to Xue, Di, & Zhang (2019), Qin, Kar, & Zheng (2016), Chen & Peng (2017), and Huang (2012) the security market is very complex and there are situations that historical data cannot be used to predict a security return and it is necessary to use expert’s estimation. Echterling, Eierle, & Ketterer (2015) affirm that a common method presented in financial literature to set the implied cost of capital is the usage of analyst forecasts. Bielstein & Hanauer (2019) states that one of the practical difficulties of Markowitz's mean-variance portfolio optimization is to estimate the stock’s expected return. For that parameter, the authors use analysts' forecasts to estimate the stock’s expected return. Balakrishnan, Shivakumar, and Taori's (2021) empirical study concludes that “analysts' cost of equity estimates are meaningful expected return proxies”. Fernandes, Ornelas, & Cusicanqui (2012) present a portfolio optimization technique that combines analysts’ expectations with estimations’ risk.

Zhai and Bai (2018) build a portfolio with experts’ opinions about the expected return, in which the returns distributions are considered as the securities’ expected return uncertainty. Xue, Di, and Zhang (2019) discuss portfolio selection under an environment of uncertainty in which the expected return is extracted by an expert’s estimation. Chen, Li, & Liu (2019), Chen & Peng (2017), and Huang (2012) portfolio selection articles consider experts’ estimations for the securities return and treat them as uncertain, with intervals instead of only a point estimate.

Fabozzi, Huang, & Zhou (2009) assert that the parameters estimate can be set by historical data or by expert prediction and, in the former case, instead of using the predictions of only one expert, it might be useful to combine the estimation from “several experts and consider each of their prediction as a likelihood distribution”. Goetzmann and Massa (2005) construct a portfolio considering the dispersion of stock return opinion. Rapach, Strauss, and Zhou recommend that the combination of numerous forecasts delivers better empirical out-of-sample equity premium predictions when compared to individual forecasts. Finally, Verardo (2009) measured the uncertainty about a firm fundamental by the dispersion in analyst forecasts.

2.4.2 The factor models

An important consideration is that achieving market neutrality may necessitate more than a single-factor model. Bowen (2016) employed a statistical arbitrage approach that utilized a multi-factor model, accounting for market, size, value, momentum, and reversal factors. This multifactor model approach has also been adopted by Caneo and Kristjanpoller (2020). It is essential to clarify that this study's primary objective is not to devise a novel risk-return model. Nevertheless, it is noteworthy that a well-constructed risk-return model can enhance the effectiveness of the statistical arbitrage process. Consequently, this section will offer a concise overview of this topic.

As stated by Fama and French (1997), a common problem in defining the cost of capital is choosing the model: once either the capital asset pricing model of Sharpe (1964) and Lintner (1965) or Fama-French (1995) faces parameter standard errors. Campbell (1996) states that knowing how to measure the risk of an asset and what economic forces drive the additional risk an investor get for bearing the risk are among the most fundamental question in finance.

As mentioned by Elton and Gruber (1997) the single-index model with the market as a factor was the “earliest index model that received wide attention” and it was first presented by Markowitz and later developed by Sharpe (1964) (Elton & Gruber, 1997). The authors state that the market model is as presented in equation 2:

$$R_{it} = \propto_i + \beta_i R_{mt} + \varepsilon_{it} \tag{2}$$

Where:

R_{it} is the return of stock I in the period t;

$\propto_i$ is the exclusive expected return of asset I;

β_i is the sensitivity of asset I to the market;

R_{mt} is the market return on the period t;

ε_{it} is the idiosyncratic risk of asset I in period t with zero mean and variance σ

Elton and Gruber (1997) indicate that the single index model with the market as a factor has the following benefits: a low number of required estimates; the inputs required were easy for analysts to understand; and there was an increase in the accuracy of portfolio optimization compared with prior estimates. Galagedera (2007), however, states that a high number of studies show that CAPM could not explain empirically the equity return, therefore other fundamental variables – such as size; book-to-market; macroeconomic; and price-to-earnings – were incorporated in different models trying to improve the single factor model. The CAPM, nevertheless, is the most widely pricing model used either by practitioners and in the classroom (Jagannathan, Schaumburg, & Zhou, 2010).

In addition, Elton and Gruber (1997) inform that after the market single index model was presented, many multi-index models were published, in which the prototype is presented in equation 3:

$$R_{it} = \propto_i + \sum_{j=1}^{J} \beta_i \, I_{jt} + \varepsilon_{it} \tag{3}$$

Where:

β_i is the sensitivity of asset i to index I;

I_{jt} is the jth index;

J is the number of indexes

Jagannathan, Schaumburg, & Zhou (2010) state that multifactor asset pricing models have a special appeal for practitioners and risk managers, once they presumably show with better detail the magnitude an asset is bearing different sources of risk. Elton and Gruber (1997) state that there are three types of multi-index structures: “1) market plus industry indexes; 2) surprises in basic economic indexes (e.g., production and inflation) (see Chen, Roll and Ross, 1986); and portfolio of traded securities (e.g., an index of small minus large securities) (see Fama and French, 1992)”.

Avanidhar (2010) reviews the literature on the cross-sectional risk-return model and indicates that there are more than fifty variables that have been used to explain asset return. Avanidhar (2010), in accordance with Elton and Gruber's (1997) statement, then states that the multifactor risk-return models are sorted in style by Fama-French (1993); factor based on macroeconomics influences, as Chen, Roll, and Ross (1986); Connor and Korajczyk (1988, 1993) model.

Fama and French (1992 and 1993) claim that the market beta by itself cannot explain the US common stock returns. In addition, the authors state that adding the firm size and the book-to-market to the market factor increases the explanatory power of the model. Fama and French (1996) assert that even though a large number of anomalies disappear when applying the three-factor model (Fama-French, 1993), it might be also considered that some irrational pricing can still exist. Galagedera (2007) claims that Fama-French's (1993) multifactor model was useful to explain cross-section equity return.

Beyond the three-factor model (Fama-French, 1993), Fama-French (2015) developed a five-factor model. According to Fama-French (2015), a five-factor model capturing size, value, profitability, and investment patterns performs better on average than the three-factor model of Fama-French (1993). Fama and French (2015) argue that the model with five factors is sustained not only by empirical evidence but also by fundamental issues, supported by Miller and Modigliani's (1961) market value equation of a company.

Another important pricing model is the Arbitrage Pricing Theory proposed by Ross (1976) that considers the sensibilities of a stock/portfolio to unexpected news. Brigham and Ehrhardt (2020) demonstrate the APT equation in a very didactic way, as shown in equation 4.

$$\overline{r_i} = \widehat{r_i} + \left(\overline{F_1} - \widehat{F_1}\right) b_{i1} + \cdots + \left(\overline{F_j} - \widehat{F_j}\right) b_{ij} + e_1 \qquad (4)$$

Where,

$\overline{r_i}$= Realized rate of return of stock i

$\widehat{r_i}$= Expected rate of return on Stock i

$\overline{F_j}$= Realized value of economic Factor j

F_j= Expected value of Factor j

b_{ij}= Sensitivity of Stock I to economic Factor j

e_1= Effect of unique events on the realized return of Stock i

As it can be seen from equation 4, the APT does not indicate the sensitivity of a stock to the announcement of an economic factor, but its sensitivity to unexpected news: realized value minus the expected value of a given factor. Chen, Roll, and Ross (1986), posteriorly, indicated the economic factors that could be consistent with the APT model: interest rate – long term minus short term –, inflation, industrial production, and the spread of high-grade bonds minus low-grade bonds

Nevertheless, Galagedera (2007) affirms that there is no consensus about what model is the best and that the quest for a robust asset pricing model continues. In any form, this dissertation will test empirically market-neutral portfolios under parameter uncertainties by Chen, Roll, and Ross (1986) model.

In addition to choosing the models for implementing statistical arbitrage, another critical consideration is the set of assumptions used to compute these models. Regardless of the selected risk-return model, a crucial assumption pertains to whether the factors are constant or subject to changes over time.

The empirical tests conducted in this study assume that the factors undergo variations over time. This assumption aligns with findings in finance research, as demonstrated by Jagannathan and Wang (1996) and Groenewold and Fraser (1999). Bramante and Gabbi (2006) also emphasize a substantial body of literature providing empirical evidence that asset betas are not static. To calculate these time-varying factors in this dissertation, the Kalman Filter technique will be employed.

Asafo-Adjei, Adam, Adu-Asare Idun, and Ametepi (2022) have utilized the Kalman Filter approach to estimate the systematic risk of twenty emerging markets. Asafo-Adjei, Adam, Adu-Asare Idun, & Ametepi (2022) inform that studies that compare the Kalman Filter with GARCH models indicate that the Kalman Filter is preferred and the most accurate, due to issues of forecast errors.

Mergner and Bulla (2008) investigate the time-varying beta for 18 sectors in Europe using six different models: a bivariate t-GARCH(1,1) model, two Kalman filter (KF)-based approaches, a bivariate stochastic volatility model estimated via the efficient Monte Carlo likelihood technique, as well as two Markov switching models. The authors then concluded that the Kalman Filter is the best model to describe the time-varying betas. Choudhry & Wu (2009) compared the weekly time-varying beta of UK firms using the Kalman Filter and bivariate GARCH, BEKK GARCH, and GARCH-GJR. The authors then conclude that, given the measurement of forecast errors, the Kalman Filter approach leads to a better result when compared with the three GARCH models.

Mamaysky, Spiegel, and Zhang (2008) compared the beta using the OLS model and the Kalman Filter model and concluded that the Kalman Filter produces better beta forecasts. Triloke, Zhang, & Wang (2013) concluded empirically the superiority of the Kalman Filter to the moving window beta estimate. Bramante and Gabbi (2006) modeled portfolio optimization in the European and American markets under changing risk via time-varying beta, in which the beta is calculated using the Kalman Filter approach.

The Kalman Filter can be used as a tool to reduce the lack of precision caused by noise or other variables not considered in the valuation models, by minimizing the quadratic function of estimator error (Grewal and Andrews, 2014).

> If one wants very precise estimates of their characteristics over time, the one has to consider their dynamics. The problem is that one those not always know their dynamics very precisely either. Given this state of partial ignorance, the best one can do is express our ignorance more precisely – using probabilities. The Kalman filter allows us to estimate the state of dynamic systems with certain types of random behavior by using statistical information. (Grewal and Andrews, 2014).

By analyzing the characteristics of the problem presented in this paper, the Kalman Filter may be a useful tool to resolve that problem for beta estimation. Applying the Kalman Filter, the beta estimation of a company for t can be calculated as shown in equation 5.

$$\widehat{\beta_{t+1}} = \beta_{t-1} + KG(\beta_t - \beta_{t-1}) \quad (5)$$

Where $\widehat{\beta_{t+1}}$ is the estimated beta for the next period, β_t is the realized beta at year t, β_{t-1} is the beta at t-1 and KG is the Kalman Gain, which can be calculated by applying the equation 6.

$$KG = \frac{\Delta_{\beta_{t-1}}}{\Delta_{\beta_{t-1}} + \Delta_{\beta_t}} \quad (6)$$

The term $\Delta_{\beta_{t-1}}$ is the uncertainty of β_{t-1} and Δ_{β_t} is the uncertainty of β_t. Equation 7 indicates the methodology used in this paper to calculate $\Delta_{\beta_{t-1}}$ for a given security j.

$$\Delta_{\beta_{t-1}} = \sqrt{(X'X)^{-1} \frac{\sum\left(\sum \beta_{I_{t-1}} * r_{I_t} - r_{j_t}\right)^2}{N-k}} \quad (7)$$

Where r_{I_t} is the predicted return at time t, r_{j_t} is the realized return of stock j at time t and N is the number of observations. β_t can be calculated from a linear regression of period t, according to equation 8.

$$\beta_t = (X'X)^{-1}(X'Y) \quad (8)$$

Δ_{β_t} is calculated from equation 9

$$\Delta_{\beta_t} = \sqrt{(X'X)^{-1} \frac{\sum(y-\hat{y})^2}{N-k}} \quad (9)$$

Where $y - \hat{y}$ is the difference between the regression realized and the predicted value, N is the number of observations and k is the number of parameters. As it can be seen, if $\Delta_{\beta_{t-1}}$ is higher than Δ_{β_t}, the Kalman Filter will give a higher weight on β_t than on β_{t-1}. Then, as closer KG is to 1, the most

Kalman filter will rely on β_t. On the other side, as closer KG is to 0, the most Kalman filter will rely on β_{t-1}.

The next step is to calculate the resulting variance after the interaction of β_t and β_{t-1}. Based on Biezen (2015), the uncertainty of β_{t+1} can be calculated by equation 10:

$$\Delta_{\widehat{\beta_{t+1}}} = \frac{\Delta_{\beta_{t-1}} * \Delta_{\beta_t}}{\Delta_{\beta_{t-1}} + \Delta_{\beta_t}} \tag{10}$$

Also based on Biezen (2015), equation 10 can be expressed according to equation 11:

$$\Delta_{\widehat{\beta_{t+1}}} = (1 - KG) * \Delta_{\beta_{t-1}} \tag{11}$$

Once equation 11 is concluded, equations 5, 6, 7, 8, and 9 will be repeated for the following periods. In the following period, β_{t+1} will be β_{t-1}, which will be used to calculate $\Delta_{\beta_{t-1}}$, KG, and so on.

To assess and contrast the moving average approach with the Kalman Filter approach, a test was conducted using a multifactor cost-of-capital model. The betas for each factor were computed through two methods: one using a 5-year OLS rolling window and the other utilizing the Kalman Filter. It is important to highlight that a total of one thousand comparative tests were conducted between both methodologies, and the model employing the Kalman Filter exhibited a lower mean square error in 88.9% of these tests.

The tests consider 4 independent variables, with an initial matrix of 520 observations, each observation representing the weekly variation of each of the independent variables. The dependent variable of each of the 520 observations was defined as the sum of the product of each independent variable by its respective beta and a random error represented by the uncertainty of the beta measurement, plus an idiosyncratic error, according to equation 12:

$$\Delta\%Y = \sum \beta_{jt} * \Delta\%X_j * \varepsilon_j + \epsilon \tag{12}$$

Where

$\Delta\%Y$ = percentage variation of the dependent variable in each of the 520 observations

β_{jn} = the value of beta j for year t

$\Delta\%X_1$= percentage variation of the dependent variable j in each of the 520 observations

ε_j= the uncertainty of beta j measurement in each of the observations

ϵ = idiosyncratic error

Also, the betas of the tests were defined as random and independent of each other for the first year, and for subsequent years each of the 4 betas suffers random shocks with a mean of 0, according to equation 13.

$$\beta_{jt} = \beta_{jt-1} * (1 + random\ noise\ 0) \tag{13}$$

Bearing in mind that one of the methodologies considers a moving window of 5 years (260 weeks), the results of the methodologies were compared from the sixth year onwards.

Once the betas were calculated using both methodologies, the results were compared between the actual variations and the variations predicted by the methodologies – the moving average and the Kalman filter – for each of the 260 remaining observations. Next, the mean squared error was calculated for each observation and then the mean of the squared errors, according to equation 14.

$$MSE = \frac{\sum\left(\widehat{\Delta\%Y_i} - \Delta\%Y_i\right)^2}{n} \tag{14}$$

Where:

MSE is the mean square error

$\Delta\%Y$ is the predicted change of the dependent variable in each of the observations

$\Delta\%Y_i$is the realized variation of the dependent variable in each of the observations

n is the number of observations

The test presented in the previous paragraph was repeated another 999 times and in 88.9% of the cases, the mean square error of the methodology that uses the Kalman Filter generated a value lower than that calculated using a 5-year moving OLS rolling window.

3 THE METHODOLOGY

3.1 COMBINING PARAMETERS UNCERTAINTIES

The excess return on the single factor model well-diversified portfolio can be expressed as equation 14.

$$R_p = E(R_p) + \beta_p * M \tag{14}$$

Where R_p is the portfolio return, $E(R_p)$ is the expected portfolio return, β_p is the portfolio beta and M is the market premium. In addition, $E(R_p)$ can be expressed as equation 15.

$$E(R_p) = \sum w_i E(R_i) \tag{15}$$

And β_p can be expressed as equation 16.

$$\beta_p = \sum w_i \beta_i \tag{16}$$

The return of an asset can be measured by the variation of that asset's value in addition to the cash flow of that asset. When transaction costs are not considered, the expected return of security equals the expected price and dividends of that security, divided by its initial price minus 1 (Kwan, 1999).

Therefore, this paper considers the expected return of a security as the expected variation of that security's market cap, given by the ratio of the analyst's market cap expectation to the security's real market cap at time t, plus the expected dividends of that company set by analysts. Additionally, the parameter uncertainties will be measured by the variance of all estimates (market cap and dividends).

Hence, the expected excess return of an asset can be stated as shown in equation 17[4].

$$R_{i_{t+1}} = \frac{\hat{P}}{P_o} + \frac{\hat{D}}{P_o} + \sum_{j=1}^{J} \beta_i \, I_{jt} \tag{17}$$

Where $\hat{P}$ is the price target set by analysts, $\hat{D}$ is the target dividend set by analysts.

Fama and French (1997) state that the valuation of projects and firms is very imprecise due to uncertainty about either the factor risks or the expected cash flows. The parameter uncertainties in this dissertation will be measured by the variance of all estimates, where $\hat{P}$ and $\hat{D}$ will be set by the variance of the analysts' estimations and β_i will be the uncertainty calculated by the Kalman Filter.

Assuming that $\beta_i \, I_{jt}$ is a product of two Gaussian PDFs and independent from each other, this dissertation will follow Smith's (2011) method in order to combine those curves. Equations 18 and 19 demonstrate the Gaussian equations for β_i and I_{jt}.

$$\beta_i = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(t-\mu_1)^2}{\sigma_1^2}} \tag{18}$$

$$I_{jt} = \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{(t-\mu_2)^2}{\sigma_2^2}} \tag{19}$$

Then, following Smith's (2011) method, the mean of the product of $\beta_i \, I_{jt}$ is as shown in equation 20:

[4] Which is consonant to the APT model (Ross, 1976) – see equation 4

$$\mu = \frac{\frac{\mu_1}{2\sigma_1^2} + \frac{\mu_2}{2\sigma_2^2}}{\frac{1}{2\sigma_1^2} + \frac{1}{2\sigma_2^2}} = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_2^2 + \sigma_1^2} \tag{20}$$

Still, based on Smith (2011), the variance of the product of $\beta_i\, I_{jt}$ is as shown in equations 21:

$$\sigma^2 = \frac{1}{\frac{1}{1\sigma_1^2} + \frac{1}{\sigma_2^2}} = \frac{\sigma_2^2\sigma_1^2}{\sigma_2^2 + \sigma_1^2} \tag{21}$$

Considering equations 17, 20, and 21, the mean and variance of β_i are calculated from the Kalman Filter, while the mean of I_{jt} will be considered as zero[5] and the variance will be set by the historical variance. Once the mean and variance of $\beta_i\, I_{jt}$ are calculated, in order to discover the distributional curve of $R_{i_{t+1}}$it is still necessary to calculate the sum of distributional curves of $\frac{\hat{P}}{P_o}$, $\frac{\hat{D}}{P_{o,}}$ and $\sum_{j=1}^{J} \beta_i\, I_{jt}$.

To do that, and assuming that the parameters are independent of each other, by applying the normal sum theorem, the "mean and variance of a sum of statistically independent random variables is the sum of the individuals' means and variances". (Lemons, 2002). Therefore, $R_{i_{t+1}}$ point estimate is the sum of $\frac{\hat{P}}{P_o}$ and $\frac{\hat{D}}{P_o}$ point estimates, given that the point estimate of $\beta_i\, I_{jt}$ is 0, as claimed in the former paragraph. Moreover, the variance of $R_{i_{t+1}}$ is the sum of the uncertainties of $\frac{\hat{P}}{P_o}$; $\frac{\hat{D}}{P_o}$; and the variances of $\beta_i\, I_{jt}$, as explained in equation 20.

[5] As stated on section 2.4.2, the betas of the APT Model are the sensitity of a security/portfolio to unexecpeted news (surprise) of a specific macroeconomic factor. This study consider that those surprise have a expected variance but with zero mean

In addition, each portfolio uncertainty – either the long and the short one – is calculated as proposed by Markowitz (1952)[6]. Also, the uncertainty of the long and short portfolios combined is calculated by the product of the vector of the portfolios` weights[7] transposed by the portfolio variance-covariance matrix multiplied by the vector of the portfolios` weights. The optimum weights of each security and the long and short portfolios are calculated by applying the decision criteria indicated in the following sub-section.

Finally as indicated by Markowitz (1952), the expected return of a portfolio can be calculated by the weighted average of the expected return from each asset included in that portfolio. In the same direction, the portfolio's beta of each factor will be also the weighted average of the asset's respective beta.

3.2 THE DECISION CRITERIA

Considering the mean-variance approach, first proposed by Markowitz (1952), equation 22 demonstrates that the optimum "zero beta" portfolio in this study will be the one with the highest value of the expected return divided by its estimated variance, given that the individual betas of the long and short must respect equation 1. In addition, the portfolio expected return will be set by the difference between the expected return of the long positions and the expected return of the short position, also referred as the expected spread.

$$\max \frac{w_l * Er_l - (1 - w_l) * Er_s}{estimated\ variance} \tag{22}$$

Where:

w_l is the weight of portfolio long

Er_l is the expected return of portfolio long

Er_s is the expected return of portfolio short

Such as Markowitz (1952), the expected returns of the portfolios (either the long or the short ones) are set by the weighted average of the individual assets' expected returns. Also, to pursue Markowitz's (1952) maximum mean

[6] By the product of the vector of the securities weights transposed and the securities´ variance-covariance matrix multiplied by the vector of the securities weights.

[7] As a mentioned before, the weight of the long and the short portfolio does not necessarily need to be equal in order to result in a market neutral portfolio.

variance, the variance of each asset is equal to the sum of the combined uncertainties of its betas, market cap target, and dividends. Finally, this study will state a naive assumption for the covariance uncertainty, that the uncertainties of the parameters between the assets are independent.

The decision criteria of maximizing the ratio of the expected spread divided by the parameters uncertainties is supported by Kwan's (1999) study, which proposes a long-short optimization approach that maximizes the mean-variance ratio – similar to the tangency portfolio – for two portfolios with market neutrality. This strategy is also consonant with Göncü & Akyldirim's (2016) study, which considers not only the spread between two assets but also the uncertainties of the parameters.

Given that the investor is concerned not only to maximize his/her return but also to minimize the risk, maximizing the ratio obtained by the expected spread[8] divided by the parameters uncertainties is consonant with Markowitz (1952 and 1991). Also, Kemaloglu, Inan, & Apaydin (2018) apply two different approaches to apply robust optimization: the risk aversion formula based on the classical Markowitz formula, which maximizes the expected return for a given uncertainty, and the max-min, which minimizes the worst case scenario. It is worth mentioning that both approaches performed well according to Kemaloglu, Inan, and Apaydin's (2018) study.

3.3 THE DATA AND THE PORTFOLIOS' CONSTRUCTION

All the data, such as the market cap, the target market cap, the market cap estimation standard deviation, the distributed dividends, and the expected dividends and their estimation standard deviations were collected from the Refinitiv Eikon Database[9].

[8] The difference between the weighted expected return of the long and the short portfolios;

[9] Appendix 1 presents with better details how the data were collected from the Refinitiv Eikon Databse. The market cap target and the expected dividends are the statistical average of all broker estimates determined to be on the majority accounting basis, while the market cap estimation standard deviation and the dividends standard deviations are the statistical standard deviation of the analysts estimations;

To calculate the betas using the Kalman Filter, this study considered the weekly percentage change in the asset value. For Chen, Roll, and Ross's (1986) model, the following ETFs were used, respectively, as proxies for interest rate – long term minus short term –, inflation, industrial production, and the spread of high-grade bond minus low-grade bonds: iShares Short Treasury Bond ETF (SHV), iShares 20 Plus Year Treasury Bond ETF (TLT), Schwab US TIPS ETF[10] (SCHP), Vanguard Industrials Index Fund ETF (VIS), iShares iBoxx $ Inv Grade Corporate Bond ETF (LQD), iShares iBoxx $ High Yield Corporate Bond ETF (HYG).

The initial beta estimation was calculated by applying a linear regression from January 18th of 2013 until December 27 of 2013. The resulting coefficients for each asset were then applied as an estimator for 2014, as well as a new linear regression was calculated for each asset through the year 2014. Then, this study applied the Kalman Filter explained in section 2.4.2 to combine and estimate the beta for the following years, as well as their errors.

For the portfolio construction – even though there were calculated the yearly betas and their uncertainties considering the Chen, Roll, and Ross (1986) of more than two thousand securities – due to computational limitations to optimally weight the assets, the portfolio considers only ninety-nine security per year. In this study, the long and short portfolios were first built at the beginning of 2015 and then updated at the beginning of each of the following years until 2022.

As already set, for each year, the chosen portfolio is the one with the highest value of the expected spread[11] divided by the combined uncertainties. Also, at the end of each period, the portfolios will be measured and compared to a portfolio that uses the same parameters, but without considering their uncertainties.

Besides the objective of optimizing the ratio of the long-short expected return (spread) by the uncertainty, it is worth mentioning the other criteria applied:

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[10] Index the Bloomberg US Treasury Inflation Protected Notes (TIPS)

[11] The expected spread can be defined as follows: it is equal to the expected return of the long positions within the portfolio multiplied by the weight assigned to those long positions, minus the expected return of the short positions within the portfolio multiplied by the weight allocated to those short positions.

1. For every risk factor[12], the weight of the portfolio long multiplied by the factor value minus the weight of the portfolio short multiplied by the factor has to be equal to zero – in consonance with equation 1.
2. The weight of the portfolio short equals 1 minus the weight of the portfolio long;
3. The sum of the weights of all securities – for each portfolio (long and short – equals to one;
4. The weight of all securities cannot be negative;
5. There are two variables that can be adjusted: the weights of the securities and the weight of the overall portfolio.

Therefore, the portfolios are constructed by applying the following steps:

1. **Step 1:** Collect the data from the Refinitiv Eikon Database[13];
2. **Step 2**: Calculate the betas of companies and their uncertainties, for the periods of 2015 to 2022, considering the macroeconomic factors[14] proposed by Chen, Roll, and Ross (1986) by applying the Kalman Filter approach[15];
3. **Step 3:** Calculate the securities' expected return and their uncertainties using analysts' estimations about price targets and dividends[16];
4. **Step 4:** Combine the parameters uncertainties of the securities and the uncertainties of the portfolios[17];
5. **Step 5:** Model optimum zero beta portfolios, yearly, from 2015 to 2022, that aim to maximize the ratio of the expected spread divided by the uncertainty and others for the same period that maximize the expected spread but neglects the uncertainty[18];
6. **Step 6:** Analyze the realized return of the portfolios

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[12] interest rate – long term minus short term –, inflation, industrial production, and the spread of high-grade bond minus low-grade bonds
[13] See Appendix 1
[14] Interest rate – long term minus short term –, inflation, industrial production, and the spread of high-grade bond minus low-grade bonds
[15] Appendix 2 shows with more details the betas and their uncertainty using the Kalman Filter.
[16] Appendix 3 presents with result for each security expected return
[17] Appendix 3 presents the securities and portfolio expected uncertainties
[18] Appendix 3 presents the modeled portfolios for each year

4 THE RESULTS

Appendix 3 shows with details the yearly optimized weight of the long-short portfolios (see the second line of each table) and the weights of each individual security.

The first and the third columns of Table 1 present the portfolio's expected spread[19] after running the optimization tool, considering the parameters uncertainties (first column) and not considering the parameters uncertainties (third column). Additionally, the second and fourth columns present the realized spread[20] of the respective portfolios. From now on, the “zero beta” portfolios that maximized the ratio between the expected spread by the parameters' uncertainties will be called long-short stochastic portfolios, while the “zero beta” portfolios that simply maximized the expected spread, neglecting the parameters uncertainties, will be called as long-short normal portfolio.

Table 1 - Portfolios expected and realized spread per year

Year	Long-Short stochastic portfolio expected spread	Long-Short stochastic portfolio realized spread	Long-Short normal portfolio expected spread	Long-Short normal portfolio realized spread
2015	13.24%	-0.17%	27.41%	-8.11%
2016	11.88%	-3.58%	79.57%	-18.10%
2017	9.57%	5.09%	43.65%	2.20%
2018	6.78%	2.24%	30.60%	-1.46%
2019	7.31%	-1.25%	70.57%	-3.39%
2020	9.35%	-2.03%	45.54%	-6.53%
2021	17.48%	3.56%	43.78%	-0.75%

[19] The expected spread can be defined as follows: it is equal to the expected return of the long positions within the portfolio multiplied by the weight assigned to those long positions, minus the expected return of the short positions within the portfolio multiplied by the weight allocated to those short positions.
[20] The realized spread can be defined as follows: it is equal to the realized return of the long positions within the portfolio multiplied by the weight assigned to those long positions, minus the realized return of the short positions within the portfolio multiplied by the weight allocated to those short positions.

2022	21.00%	-1.63%	44.62%	3.20%

Source: Self elaboration

As depicted in Table 1, the actual returns realized each year significantly diverged from the expected returns initially anticipated for the portfolio. In every instance, the realized returns fell considerably short of the expected values, and in certain cases, they even turned out to be negative. Several factors could contribute to these disparities, including inaccurate predictions of expected returns and potential omissions of other critical risk factors within the model. Regardless of the precise underlying causes, future research endeavors should delve deeper into the analysis of these variations to gain a more comprehensive understanding.

A significant result from this research pertains to the square root error of both portfolios, as computed. The stochastic portfolio yielded a considerably smaller root square error, as shown in Table 2. As a result, when we compared the stochastic portfolio to the normal portfolio, we found that the stochastic portfolio had a substantially lower root mean square error—11.34% compared to 52.47%. This indicates that the stochastic portfolio's predictions were, on average, much closer to the actual outcomes, suggesting it may be a more accurate and reliable choice.

Table 2 - Yearly square root error for the stochastic and normal portfolios

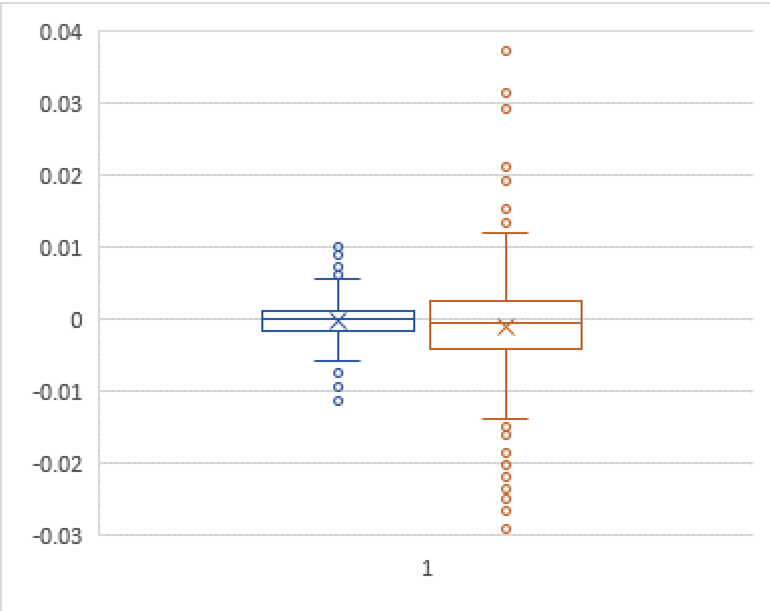
Year	Root Square Error: Long-Short stochastic	Root Square Error: Long-Short normal portfolio
2015	13.41%	35.51%
2016	15.46%	97.66%
2017	9.57%	43.65%
2018	6.78%	30.60%
2019	7.31%	70.57%
2020	9.35%	45.54%
2021	17.48%	43.78%
2022	21.00%	44.62%

Source: Self Elaboration

As anticipated, the long-short stochastic portfolios exhibited greater stability over time in comparison to the long-short normal portfolios. This stability is evident when we examine the weight distributions of the two types of portfolios, as detailed in Appendix 3. Specifically, the long-short normal portfolios tend to concentrate their investments in just a few securities, while the long-short stochastic portfolios are more diversified.

To visually illustrate this difference, Figure 3 displays a box plot that provides a snapshot of the weekly spreads of both portfolio types. The blue box plot represents the weekly spread of the long-short stochastic portfolio, while the orange box plot represents the weekly spread of the long-short normal portfolio. This graphical representation allows us to observe the dispersion and consistency of returns for each portfolio over time.

Figure 3 – box plot comparing the weekly returns of the robust and normal "zero beta" portfolios



Source: Self elaboration

Since the long-short stochastic portfolio had a higher accumulated realized spread from 2015 to 2022 and a lower standard deviation, compared to the long-short normal portfolio, consequently the stochastic portfolio resulted in a higher ratio of realized spread divided by the variance, as shown in table 2.

Table 3 – Portfolios return and standard deviation

Portfolio	Total realized spread	Standard deviation	Realized spread divided by the variance
Long-Short Stochastic	1.94%	0.256%	2,974.06
Long-Short Normal	-30%	0.744%	(5,395.01)

Source: Self elaboration

5 DISCUSSION AND FUTURE STUDIES

The findings of this dissertation indicate that taking into account the uncertainties associated with parameter estimation in the construction of investment portfolios appears to yield significant benefits for investors, fund managers, and industry practitioners. Portfolios constructed using zero beta models that incorporate parameter uncertainties consistently exhibited lower ex-post root mean square errors for each individual year analyzed, thereby resulting in a reduced overall root mean square error across the entire assessment period. These findings suggest that forecasting models for portfolios optimization that explicitly account for parameter uncertainties tend to produce more accurate ex-post predictions.

Moreover, portfolios that incorporate parameter uncertainties displayed lower ex-post standard deviations, indicating greater stability for investors. Furthermore, the inclusion of parameter uncertainty considerations in portfolio construction led to higher realized returns when compared to portfolios that did not account for such uncertainties. This unexpected outcome merits further comprehensive quantitative and qualitative investigation in future research endeavors.

The long-short stochastic portfolio, formulated as part of this research, demonstrated an overall positive cumulative return over the period spanning from 2015 to 2022. Nonetheless, it is worth noting that there were specific years within this time frame where the returns turned negative. These empirical findings align with the assertions of Do and Faff (2010), who contend that statistical arbitrage, previously a profitable strategy over an extended horizon, has encountered a diminishing trend in profitability. Consequently, it is advisable for investors, fund managers, and industry practitioners to exercise prudence and circumspection when contemplating the application of the statistical arbitrage strategy elucidated in this dissertation.

Subsequent research endeavors should contemplate the incorporation of transaction costs, a facet previously explored by Bowen (2016), to comprehensively gauge their influence on the strategy's performance. Additionally, the inclusion of supplementary risk factors, such as those advanced within the framework of the five-factor asset pricing model postulated by Fama and French (2015), could provide a more nuanced understanding of the strategy's risk-return profile. Furthermore, extending the temporal scope of analysis to encompass longer observation periods and diversifying the study across alternate financial markets, particularly those characterized as potentially less efficient than the American market, as recommended by Caneo and Kristjanpoller (2020), may yield valuable insights into the strategy's robustness and adaptability.

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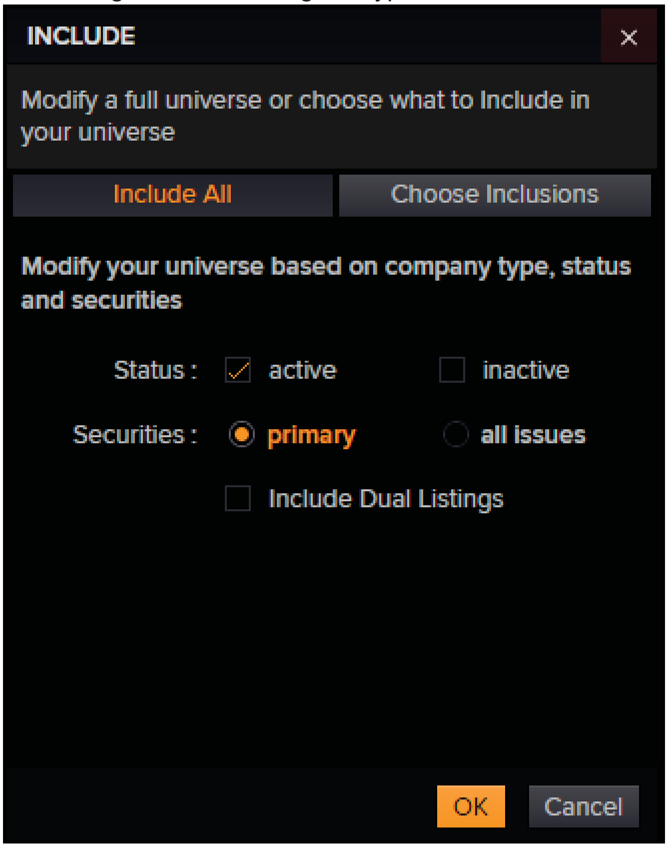
7 APPENDIX 1

The data of multiple companies were collected using the app Screener of the Refinitiv Eikon Database. The first step was to set the currency as the US dollar. Then, in the next step, I consider all active companies and the primary securities, as shown in Figure 4.

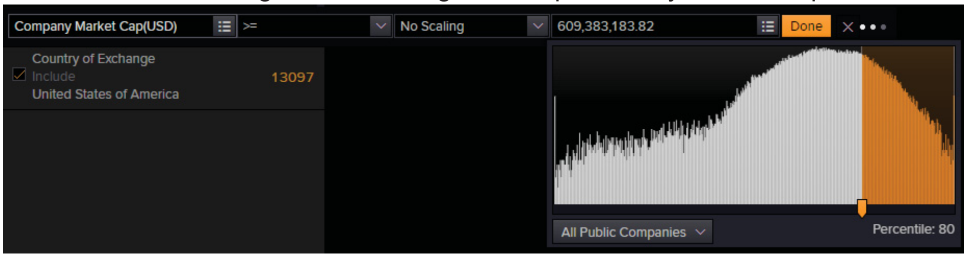
Figure 4 - Defining the types of securities



Source: Refinitiv Eikon Database

Next, the country of exchange was chosen as the United States of America. Given the vast universe of more than 13 thousand companies, a filter was applied to display the fifth quintile, categorized by market capitalization size, as depicted in Figure 5.

Figure 5 - Filtering the sample size by Market Cap



Source: Refinitiv Eikon Database

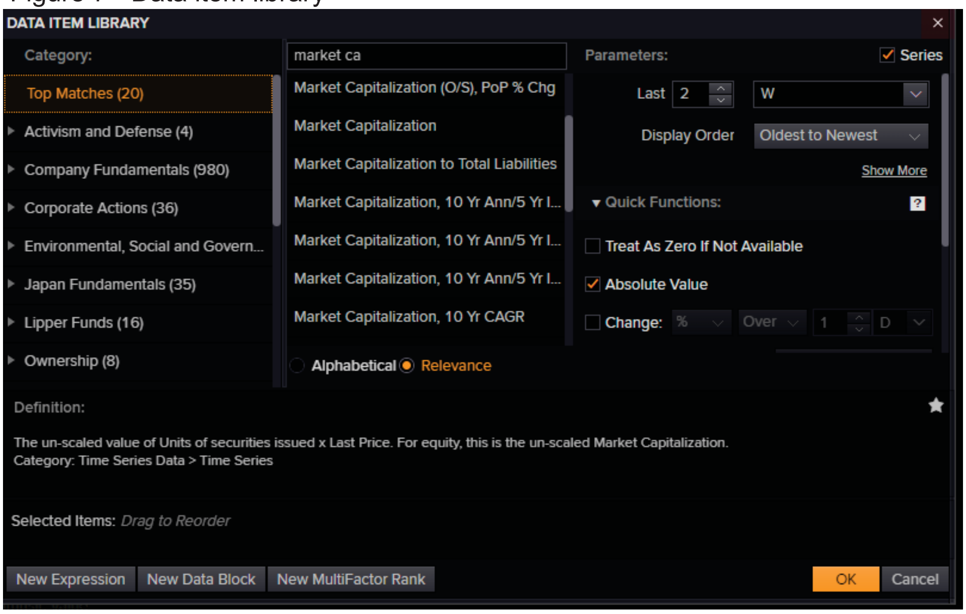
After displaying the sample companies, they were arranged in descending order based on their market capitalization values. Subsequently, the "add column" option was selected, as shown in Figure 6.

Figure 6 - Add column button

Source: Refinitiv Eikon Database

For the next step, I select “series”, last “700”, “weekly”. On the “Display order,” I changed to “oldest to newest”. As “Output” I selected “Value” and “Date”. On “Quick Functions” I check for “absolute value”. Then, I clicked “ok” – see figure 7. Finally, on the Excel symbol, I clicked to export all as values.

Figure 7 - Data item library



Source: Refinitiv Eikon Database

Similarly to those steps I used to extract the price, the price target – mean[21], price target standard deviation[22], the dividend per share – mean[23], dividend per share standard deviation

To retrieve the ETFs required for calculating the betas, I initiated a search by entering their names in the search field. Afterward, I selected "Price & Charts" and then proceeded to "Price History" (refer to Figure 8). Subsequently, I customized the data to match the same date range as the other datasets and downloaded it in Excel format.

Figure 8 - Extracting historical data of ETFs

Source: Refinitiv Eikon Database

[21] The statistical average of all broker estimates determined to be on the majority accounting basis

[22] The statistical standard deviation of the analysts price target estimations

[23] The statistical average of all broker estimates determined to be on the majority accounting basis

8 APPENDIX 2

As set, this study calculates the beta using more than 2 thousand companies. The aim of this appendix is simply to show the main results of the betas and the expected variance, calculated using the Kalman Filter approach for 2015 to 2023.

Table 4 – The expected betas considering the macroeconomic factors proposed by Chen, Roll, and Ross (1986) calculated using the Kalman Filter Approach

Year	Expected Beta	AAPL.O Q	MSFT.OK	GOOGL. OQ	AMZN.O Q	BRKa.N	JNJ.N	UNH.N	V.N	XOM.N	JPM.N
2015	Long-term minus short term	- 0.113	- 0.180	0.017	0.313	0.117	- 0.006	0.070	0.023	0.132	0.134
2015	Inflation	- 0.384	- 0.144	- 0.106	0.365	- 0.023	0.154	- 0.326	- 0.220	0.424	- 0.792
2015	Industrial Production	0.561	0.502	0.995	1.189	0.559	0.551	0.537	1.000	0.319	0.909
2015	High-grade bond minus low grade	0.046	0.039	0.463	0.735	- 0.212	- 0.149	0.515	0.309	- 1.127	0.495
2016	Long-term minus short term	0.058	0.159	- 0.052	0.029	0.082	0.145	0.054	0.126	- 0.133	0.096
2016	Inflation	- 0.110	- 0.883	0.003	0.583	- 0.031	0.507	0.568	- 0.355	0.569	- 0.853
2016	Industrial Production	0.775	1.147	1.231	0.740	0.519	0.483	0.550	0.638	0.474	0.759
2016	High-grade bond minus low grade	- 0.307	0.430	0.620	- 0.083	- 0.299	- 0.013	0.039	- 0.059	- 0.925	- 0.056
2017	Long-term minus short term	0.073	- 0.023	0.004	- 0.023	0.004	- 0.104	0.107	- 0.033	- 0.104	0.071
2017	Inflation	- 0.066	0.030	0.520	1.331	- 0.253	0.330	0.139	0.150	- 0.060	- 1.012
2017	Industrial Production	0.597	0.656	0.637	0.377	0.641	0.482	0.582	0.530	0.471	1.001
2017	High-grade bond minus low grade	- 0.637	- 0.348	- 0.429	- 1.240	- 0.124	0.164	- 0.133	- 0.292	- 0.180	- 0.242
2018	Long-term minus short term	0.319	0.043	0.084	- 0.031	- 0.010	0.081	0.062	0.063	0.088	- 0.022
2018	Inflation	0.320	0.438	0.764	1.387	- 0.737	0.104	- 0.109	0.329	- 0.509	- 1.184

Year	Expected Beta	AAPL.OQ	MSFT.OK	GOOGL.OQ	AMZN.OQ	BRKa.N	JNJ.N	UNH.N	V.N	XOM.N	JPM.N
2018	Industrial Production	0.191	0.680	0.856	0.475	0.605	0.515	0.434	0.437	0.365	0.596
2018	High-grade bond minus low grade	- 0.994	- 0.202	- 0.234	- 0.939	- 0.188	0.628	- 0.435	- 0.100	- 0.132	- 0.983
2019	Long-term minus short term	0.462	0.061	0.226	0.025	- 0.040	0.068	- 0.051	0.021	0.067	- 0.043
2019	Inflation	1.372	1.276	1.773	2.211	- 0.415	0.654	0.735	1.682	0.375	- 0.993
2019	Industrial Production	0.366	0.715	0.878	0.611	0.897	0.640	0.521	0.558	0.562	0.904
2019	High-grade bond minus low grade	- 1.093	- 0.399	- 0.386	- 1.769	0.142	0.493	- 1.086	- 0.887	- 0.563	- 0.154
2020	Long-term minus short term	0.200	0.059	0.119	- 0.097	- 0.008	0.059	- 0.107	0.055	0.126	0.033
2020	Inflation	- 0.298	0.930	0.964	1.674	- 0.835	0.604	1.060	1.573	1.410	- 0.928
2020	Industrial Production	0.814	0.628	0.465	0.503	0.822	0.538	0.504	0.551	0.530	0.762
2020	High-grade bond minus low grade	0.427	- 0.332	- 0.718	- 1.462	0.369	0.236	- 0.741	- 0.643	- 1.254	- 0.416
2021	Long-term minus short term	0.016	0.008	0.055	- 0.104	- 0.107	- 0.156	- 0.279	0.073	- 0.070	- 0.102
2021	Inflation	- 0.399	- 0.087	0.263	1.199	- 0.601	0.067	- 0.967	0.367	1.241	- 0.975
2021	Industrial Production	0.724	0.579	0.486	0.277	0.755	0.512	0.978	0.672	0.729	0.932
2021	High-grade bond minus low grade	0.317	0.183	- 0.462	- 0.829	0.231	0.662	1.040	- 0.364	- 0.757	- 0.516
2022	Long-term minus short term	- 0.051	- 0.008	- 0.022	- 0.007	- 0.089	- 0.164	- 0.102	- 0.171	- 0.136	- 0.105
2022	Inflation	- 0.500	0.019	0.041	0.111	- 0.317	0.004	- 0.597	0.087	0.376	- 0.936
2022	Industrial Production	0.701	0.566	0.681	0.456	0.661	0.437	0.652	0.687	0.837	0.877
2022	High-grade bond minus low grade	0.210	0.270	- 0.397	0.156	- 0.099	0.380	0.276	- 0.450	- 1.424	- 0.725

Year	Expected Beta	AAPL.OQ	MSFT.OK	GOOGL.OQ	AMZN.OQ	BRKa.N	JNJ.N	UNH.N	V.N	XOM.N	JPM.N
2023	Long-term minus short term	- 0.076	- 0.008	- 0.022	- 0.007	- 0.089	- 0.164	- 0.102	- 0.171	- 0.136	- 0.105
2023	Inflation	- 0.500	0.019	0.041	0.111	- 0.317	0.004	- 0.597	0.087	0.376	- 0.936
2023	Industrial Production	0.701	0.566	0.681	0.456	0.661	0.437	0.652	0.687	0.837	0.877
2023	High-grade bond minus low grade	0.210	0.270	- 0.397	0.156	- 0.099	0.380	0.276	- 0.450	- 1.424	- 0.725

Source: Self Elaboration

Table 6 demonstrates the expected variance considering the macroeconomic factors proposed by Chen, Roll, and Ross (1986) calculated using the Kalman Filter approach.

Table 5 - The expected variance considering the macroeconomic factors proposed by Chen, Roll, and Ross (1986) calculated using the Kalman Filter Approach

Year	Expected Variance	AAPL.OQ	MSFT.OQ	GOOGL.OQ	AMZN.OQ	BRKa.N	JNJ.N	UNH.N	V.N	XOM.N	JPM.N
2015	Long-term minus short term	0.052	0.030	0.023	0.029	0.004	0.006	0.025	0.019	0.005	0.012
2015	Inflation	0.257	0.146	0.112	0.142	0.020	0.029	0.123	0.091	0.027	0.061
2015	Industrial Production	0.111	0.063	0.049	0.062	0.009	0.013	0.053	0.040	0.012	0.026
2015	High-grade bond minus low grade	0.703	0.401	0.307	0.388	0.054	0.081	0.337	0.250	0.073	0.166
2016	Long-term minus short term	0.012	0.014	0.023	0.027	0.003	0.002	0.007	0.005	0.005	0.005
2016	Inflation	0.120	0.145	0.240	0.282	0.027	0.019	0.068	0.050	0.056	0.050
2016	Industrial Production	0.020	0.024	0.039	0.046	0.004	0.003	0.011	0.008	0.009	0.008
2016	High-grade bond minus low grade	0.080	0.097	0.160	0.188	0.018	0.012	0.045	0.033	0.037	0.033

Year	Expected Variance	AAPL.OQ	MSFT.OQ	GOOGL.OQ	AMZN.OQ	BRKa.N	JNJ.N	UNH.N	V.N	XOM.N	JPM.N
2017	Long-term minus short term	0.036	0.021	0.021	0.037	0.005	0.011	0.018	0.011	0.012	0.011
2017	Inflation	0.291	0.175	0.170	0.299	0.038	0.086	0.149	0.093	0.097	0.091
2017	Industrial Production	0.032	0.019	0.019	0.033	0.004	0.009	0.017	0.010	0.011	0.010
2017	High-grade bond minus low grade	0.192	0.115	0.112	0.197	0.025	0.056	0.098	0.061	0.064	0.060
2018	Long-term minus short term	0.052	0.025	0.034	0.062	0.011	0.021	0.029	0.018	0.019	0.026
2018	Inflation	0.427	0.207	0.279	0.507	0.087	0.169	0.239	0.151	0.152	0.214
2018	Industrial Production	0.084	0.041	0.055	0.100	0.017	0.033	0.047	0.030	0.030	0.042
2018	High-grade bond minus low grade	0.418	0.202	0.273	0.497	0.085	0.166	0.234	0.148	0.149	0.209
2019	Long-term minus short term	0.089	0.025	0.037	0.069	0.020	0.028	0.038	0.018	0.034	0.017
2019	Inflation	0.829	0.235	0.343	0.648	0.184	0.265	0.357	0.171	0.321	0.156
2019	Industrial Production	0.028	0.008	0.011	0.022	0.006	0.009	0.012	0.006	0.011	0.005
2019	High-grade bond minus low grade	0.424	0.120	0.175	0.331	0.094	0.136	0.182	0.087	0.164	0.080
2020	Long-term minus short term	0.022	0.011	0.022	0.016	0.008	0.011	0.041	0.007	0.008	0.010
2020	Inflation	0.584	0.298	0.580	0.416	0.203	0.298	1.087	0.182	0.201	0.264
2020	Industrial Production	0.045	0.023	0.045	0.032	0.016	0.023	0.084	0.014	0.015	0.020
2020	High-grade bond minus low grade	0.379	0.193	0.376	0.270	0.132	0.193	0.705	0.118	0.130	0.171

Year	Expected Variance	AAPL.OQ	MSFT.OQ	GOOGL.OQ	AMZN.OQ	BRKa.N	JNJ.N	UNH.N	V.N	XOM.N	JPM.N
2021	Long-term minus short term	0.046	0.035	0.022	0.059	0.010	0.024	0.037	0.026	0.034	0.017
2021	Inflation	0.215	0.164	0.104	0.277	0.045	0.111	0.173	0.123	0.161	0.082
2021	Industrial Production	0.010	0.008	0.005	0.013	0.002	0.005	0.008	0.006	0.008	0.004
2021	High-grade bond minus low grade	0.182	0.139	0.088	0.234	0.038	0.094	0.146	0.104	0.136	0.069
2022	Long-term minus short term	0.044	0.031	0.041	0.049	0.010	0.012	0.034	0.041	0.054	0.017
2022	Inflation	0.488	0.339	0.455	0.541	0.113	0.133	0.375	0.457	0.600	0.192
2022	Industrial Production	0.045	0.031	0.042	0.050	0.010	0.012	0.035	0.042	0.056	0.018
2022	High-grade bond minus low grade	0.405	0.282	0.378	0.449	0.093	0.111	0.311	0.379	0.498	0.159
2023	Long-term minus short term	0.021	0.045	0.067	0.133	0.020	0.025	0.038	0.045	0.113	0.036
2023	Inflation	0.255	0.238	0.352	0.698	0.103	0.129	0.200	0.236	0.593	0.189
2023	Industrial Production	0.049	0.045	0.067	0.133	0.020	0.025	0.038	0.045	0.113	0.036
2023	High-grade bond minus low grade	0.297	0.277	0.410	0.813	0.121	0.151	0.233	0.275	0.691	0.220

Source: Self Elaboration

9 APPENDIX 3

The tables below display the annual optimized weights for the long-short portfolios (refer to the second line in each table) and the individual security weights. In these tables, the fourth and fifth columns represent the portfolio weights that take into account parameter uncertainties, while the sixth and seventh columns depict the weights in which parameter uncertainties are disregarded during optimization.

Table 6 - Portfolio 2015

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
			51,68%	**48,32%**	**45,43%**	**54,57%**
Apple Inc	0,14	0,13	0,12%	0,04%	0,10%	0,06%
Microsoft Corp	0,16	0,10	0,16%	0,07%	0,12%	0,16%
Alphabet Inc	0,08	0,21	0,75%	0,09%	0,34%	0,04%
Amazon.com Inc	0,11	0,15	0,70%	1,96%	0,28%	0,62%
UnitedHealth Group Inc	0,23	0,09	0,92%	0,00%	0,04%	0,00%
Johnson & Johnson	0,11	0,07	0,58%	0,91%	0,22%	0,92%
Exxon Mobil Corp	0,10	0,09	0,37%	2,71%	0,03%	0,01%
Tesla Inc	0,27	0,23	0,25%	0,03%	0,22%	0,04%
NVIDIA Corp	0,17	0,08	0,81%	1,19%	0,43%	0,52%
Walmart Inc	0,09	-0,02	0,03%	0,06%	0,02%	0,01%
Visa Inc	0,20	0,05	0,21%	1,05%	0,22%	0,90%
JPMorgan Chase & Co	0,10	0,11	1,97%	2,10%	0,09%	0,03%
Procter & Gamble Co	0,13	0,04	0,90%	0,84%	0,16%	0,06%
Eli Lilly and Co	0,09	0,05	0,22%	0,51%	0,06%	0,07%
Chevron Corp	0,10	0,13	3,18%	2,46%	0,46%	0,02%
Mastercard Inc	0,25	0,11	0,76%	1,40%	0,18%	0,06%
Home Depot Inc	0,09	0,02	0,19%	1,43%	0,17%	2,67%
Meta Platforms Inc	0,10	0,13	0,47%	0,09%	0,06%	0,32%
Pfizer Inc	0,14	0,20	0,18%	0,02%	0,13%	0,01%
Abbvie Inc	0,15	0,10	0,74%	1,02%	0,33%	0,04%
Merck & Co Inc	0,07	0,21	4,30%	0,00%	0,46%	0,68%
Coca-Cola Co	0,15	0,07	0,39%	0,46%	0,12%	0,03%
Bank of America Corp	0,22	0,03	0,40%	1,92%	0,09%	1,14%
PepsiCo Inc	0,09	0,10	1,71%	0,00%	0,29%	0,00%
Broadcom Inc	0,17	0,15	1,25%	0,00%	0,35%	0,00%
Oracle Corp	0,15	0,00	0,03%	1,39%	0,10%	0,89%
Thermo Fisher Scientific Inc	0,18	0,13	1,07%	0,01%	0,06%	0,01%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Costco Wholesale Corp	0,16	0,15	1,38%	0,41%	0,35%	0,66%
McDonald's Corp	0,07	0,06	2,13%	1,94%	0,26%	1,08%
Cisco Systems Inc	0,26	0,01	0,04%	0,38%	0,09%	0,36%
Danaher Corp	0,35	0,43	1,41%	0,00%	0,80%	0,00%
Abbott Laboratories	0,12	0,04	0,20%	0,98%	0,12%	0,95%
Nike Inc	0,12	0,07	0,64%	0,13%	0,10%	0,01%
Accenture PLC	0,10	0,05	0,06%	0,91%	0,04%	0,77%
T-Mobile US Inc	0,14	0,32	0,14%	0,00%	0,15%	0,00%
Nextera Energy Inc	0,07	0,05	0,97%	3,75%	0,29%	0,88%
Linde PLC	0,07	0,09	3,18%	2,24%	0,37%	0,02%
Adobe Inc	0,07	0,15	0,95%	0,92%	0,31%	0,14%
Verizon Communications Inc	0,24	0,17	0,30%	0,36%	0,22%	0,50%
Walt Disney Co	0,16	0,04	0,99%	1,69%	0,39%	0,77%
Wells Fargo & Co	0,06	0,03	0,05%	0,25%	0,05%	0,33%
Bristol-Myers Squibb Co	0,16	-0,01	0,05%	0,71%	0,07%	0,93%
Philip Morris International Inc	0,08	0,09	0,83%	1,01%	0,29%	0,88%
Texas Instruments Inc	0,15	0,00	0,42%	3,16%	0,09%	2,21%
United Parcel Service Inc	0,08	0,05	0,54%	2,41%	0,27%	2,34%
Charles Schwab Corp	0,13	0,04	1,05%	3,94%	0,09%	2,16%
Morgan Stanley	0,21	-0,02	0,01%	2,35%	0,05%	1,25%
Raytheon Technologies Corp	0,07	0,84	14,79%	0,00%	77,90%	0,01%
Conocophillips	0,13	0,20	2,46%	0,26%	0,56%	0,01%
Honeywell International Inc	0,19	0,14	1,18%	0,03%	0,41%	0,01%
Amgen Inc	0,14	0,08	0,46%	0,56%	0,15%	0,01%
Netflix Inc	0,19	0,22	0,17%	0,05%	0,16%	0,12%
Deere & Co	0,20	-0,01	0,04%	2,05%	0,12%	0,94%
Salesforce Inc	0,12	0,19	0,69%	0,64%	0,39%	0,44%
AT&T Inc	0,10	0,45	6,83%	0,00%	0,32%	0,12%
International Business Machines Corp	0,17	0,13	0,41%	0,01%	0,07%	0,00%
Qualcomm Inc	0,28	0,11	1,25%	0,00%	0,34%	0,00%
Union Pacific Corp	0,16	0,09	0,91%	1,35%	0,38%	0,62%
Lockheed Martin Corp	0,13	0,02	0,12%	0,62%	0,07%	0,55%
Caterpillar Inc	0,24	0,19	1,64%	0,65%	0,58%	0,07%
CVS Health Corp	0,20	0,08	0,39%	0,38%	0,15%	0,60%
Lowe's Companies Inc	0,14	0,02	0,08%	0,78%	0,12%	0,86%
Elevance Health Inc	0,20	0,08	0,14%	0,10%	0,05%	0,24%
Goldman Sachs Group Inc	0,13	-0,02	0,02%	3,84%	0,06%	3,31%
Boeing Co	0,13	0,17	0,28%	0,00%	0,23%	0,00%
Starbucks Corp	0,20	0,13	1,99%	0,06%	0,44%	0,02%
Intel Corp	0,21	0,01	0,74%	1,94%	0,11%	1,88%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Intuit Inc	0,38	0,06	0,05%	0,13%	0,05%	0,19%
American Express Co	0,10	0,07	1,75%	2,53%	0,37%	0,02%
Advanced Micro Devices Inc	0,24	0,16	0,41%	0,57%	1,89%	0,06%
BlackRock Inc	0,09	0,08	0,02%	3,70%	0,32%	1,23%
Gilead Sciences Inc	0,14	0,30	0,80%	0,01%	0,32%	0,08%
Prologis Inc	0,04	0,09	0,63%	1,46%	0,35%	0,76%
Medtronic PLC	0,09	0,13	0,28%	0,18%	0,18%	0,02%
Automatic Data Processing Inc	0,16	-0,02	0,02%	2,86%	0,09%	9,90%
American Tower Corp	0,30	0,13	0,33%	0,21%	0,21%	0,02%
Intuitive Surgical Inc	0,14	0,03	0,05%	0,00%	0,04%	0,00%
Blackstone Inc	0,14	0,27	0,57%	0,24%	0,33%	0,23%
Stryker Corp	0,13	0,01	0,03%	0,34%	0,04%	0,36%
Mondelez International Inc	0,16	0,15	0,30%	0,14%	0,21%	0,02%
TJX Companies Inc	0,16	-0,01	0,61%	2,89%	0,10%	37,60%
Chubb Ltd	0,10	0,03	0,19%	0,74%	0,12%	0,79%
General Electric Co	0,06	0,57	11,18%	0,00%	0,00%	0,01%
Applied Materials Inc	0,19	0,09	0,31%	0,58%	0,13%	0,34%
Estee Lauder Companies Inc	0,09	0,09	0,42%	0,74%	0,23%	0,78%
Citigroup Inc	0,09	0,11	0,10%	0,21%	0,13%	0,21%
Analog Devices Inc	0,25	0,06	0,97%	1,95%	0,22%	2,18%
Marsh & McLennan Companies Inc	0,10	0,04	0,16%	1,09%	0,13%	1,25%
Northrop Grumman Corp	0,16	0,00	0,25%	2,27%	0,15%	1,53%
Altria Group Inc	0,09	0,03	0,55%	1,87%	0,10%	1,55%
Regeneron Pharmaceuticals Inc	0,13	-0,01	0,03%	0,85%	0,13%	0,41%
ServiceNow Inc	0,08	0,09	0,01%	1,40%	0,02%	0,63%
Duke Energy Corp	0,07	0,03	1,96%	2,44%	0,28%	0,03%
Southern Co	0,06	-0,01	0,21%	4,78%	0,11%	1,10%
Booking Holdings Inc	0,07	0,17	0,18%	0,14%	0,21%	0,06%
EOG Resources Inc	0,20	0,25	1,91%	1,00%	0,55%	0,17%
Schlumberger NV	0,14	0,22	2,79%	0,96%	0,69%	0,13%
Progressive Corp	0,55	0,03	0,30%	1,10%	0,15%	2,87%
Vertex Pharmaceuticals Inc	0,16	0,05	0,04%	0,02%	0,03%	0,06%

Source: Self Elaboration

Table 7 - Portfolio 2016

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
			51,57%	**48,43%**	**41,75%**	**58,25%**

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Apple Inc	14,813%	0,43	3,33%	0,02%	0,37%	0,07%
Microsoft Corp	14,136%	0,05	0,03%	2,39%	0,02%	2,20%
Alphabet Inc	6,696%	0,10	0,01%	0,02%	0,01%	0,02%
Amazon.com Inc	9,359%	0,10	0,09%	0,52%	0,08%	0,34%
UnitedHealth Group Inc	31,603%	0,22	0,95%	0,00%	0,04%	0,00%
Johnson & Johnson	8,301%	0,08	0,04%	1,63%	0,02%	1,24%
Exxon Mobil Corp	13,086%	0,11	0,01%	2,31%	0,01%	0,02%
NVIDIA Corp	17,785%	-0,04	0,02%	2,67%	0,02%	24,21%
Visa Inc	15,770%	0,12	0,00%	0,96%	0,00%	1,18%
JPMorgan Chase & Co	4,853%	0,13	0,03%	0,02%	0,01%	0,01%
Eli Lilly and Co	16,258%	0,21	0,75%	0,17%	0,02%	0,53%
Chevron Corp	11,932%	0,15	0,38%	2,51%	0,11%	0,16%
Mastercard Inc	17,033%	0,15	1,01%	0,82%	0,25%	0,53%
Home Depot Inc	12,319%	0,08	0,02%	2,02%	0,01%	1,84%
Meta Platforms Inc	10,011%	0,19	3,15%	0,05%	0,22%	0,04%
Pfizer Inc	20,465%	0,36	2,46%	0,03%	0,31%	0,23%
Abbvie Inc	18,071%	0,28	1,45%	0,12%	0,30%	0,26%
Merck & Co Inc	9,075%	0,27	3,95%	0,27%	0,08%	0,70%
Coca-Cola Co	14,795%	0,09	0,18%	1,35%	0,04%	0,03%
Bank of America Corp	10,724%	0,13	0,18%	0,09%	0,10%	0,15%
PepsiCo Inc	5,323%	0,09	0,31%	4,14%	0,06%	2,50%
Broadcom Inc	14,175%	0,21	0,96%	0,84%	0,00%	0,08%
Oracle Corp	24,269%	0,22	0,15%	0,25%	0,02%	0,20%
Thermo Fisher Scientific Inc	6,732%	0,09	2,07%	0,00%	0,21%	0,00%
Costco Wholesale Corp	16,358%	0,15	0,08%	0,00%	0,02%	0,00%
McDonald's Corp	9,839%	0,03	0,04%	2,61%	0,03%	3,33%
Cisco Systems Inc	23,750%	0,16	0,00%	0,01%	0,00%	0,01%
Danaher Corp	16,694%	0,48	3,45%	0,01%	0,43%	0,12%
Abbott Laboratories	8,488%	0,16	2,05%	0,87%	0,22%	0,65%
Nike Inc	28,110%	0,17	1,08%	0,16%	0,23%	0,35%
Accenture PLC	9,563%	0,10	1,18%	0,00%	0,20%	0,00%
T-Mobile US Inc	18,294%	0,19	0,76%	0,06%	0,16%	0,36%
Nextera Energy Inc	19,606%	0,17	1,30%	0,09%	0,07%	0,02%
Linde PLC	9,680%	0,22	0,59%	0,18%	0,03%	0,28%
Adobe Inc	10,223%	0,10	2,01%	0,01%	0,08%	0,01%
Verizon Communications Inc	8,430%	0,13	2,06%	2,61%	0,09%	0,03%
Walt Disney Co	28,516%	0,14	0,56%	0,75%	0,04%	0,07%
Wells Fargo & Co	7,474%	0,11	0,06%	1,58%	0,02%	1,67%
Bristol-Myers Squibb Co	15,836%	0,10	0,03%	0,04%	0,02%	0,07%
Philip Morris International Inc	6,469%	0,05	0,05%	5,17%	0,08%	2,19%
Texas Instruments Inc	10,324%	0,10	0,11%	1,40%	0,04%	0,47%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
United Parcel Service Inc	8,681%	0,17	0,01%	0,65%	0,00%	0,73%
Charles Schwab Corp	12,048%	0,06	0,02%	2,34%	0,01%	6,69%
Comcast Corp	6,830%	0,27	0,23%	0,04%	0,11%	0,11%
Morgan Stanley	11,883%	0,22	2,21%	1,01%	0,06%	1,02%
Raytheon Technologies Corp	11,234%	0,96	10,47%	0,00%	90,77%	0,03%
Conocophillips	17,944%	0,36	1,77%	0,65%	0,16%	0,44%
Honeywell International Inc	19,554%	0,15	0,33%	0,02%	0,03%	0,01%
Amgen Inc	15,029%	0,18	0,26%	0,11%	0,03%	0,02%
Netflix Inc	20,694%	0,08	0,01%	0,63%	0,13%	0,01%
Salesforce Inc	9,933%	0,15	0,55%	2,83%	0,06%	0,07%
AT&T Inc	11,887%	0,51	5,43%	0,01%	0,40%	0,10%
International Business Machines Corp	12,373%	0,17	1,77%	1,66%	0,04%	1,18%
Qualcomm Inc	28,272%	0,29	1,08%	0,43%	0,17%	0,44%
Union Pacific Corp	19,793%	0,29	0,85%	0,05%	0,32%	0,09%
Lockheed Martin Corp	9,195%	0,08	1,89%	0,01%	0,00%	0,01%
Caterpillar Inc	17,822%	0,02	0,02%	0,00%	0,10%	0,00%
CVS Health Corp	14,328%	0,20	1,83%	0,53%	0,07%	0,84%
Elevance Health Inc	20,164%	0,29	2,37%	0,03%	0,10%	0,40%
Boeing Co	16,244%	0,16	0,00%	1,12%	0,00%	0,39%
Starbucks Corp	10,746%	0,15	0,04%	0,83%	0,01%	0,62%
Intel Corp	12,189%	0,08	0,07%	1,39%	0,05%	0,69%
Intuit Inc	14,775%	0,07	0,00%	1,89%	0,00%	2,28%
S&P Global Inc	8,306%	0,16	1,41%	1,72%	0,05%	1,13%
American Express Co	11,369%	0,18	0,01%	0,00%	0,00%	0,00%
Advanced Micro Devices Inc	34,649%	-0,21	0,00%	2,99%	0,07%	0,07%
BlackRock Inc	9,210%	0,13	0,16%	3,69%	0,10%	1,26%
Gilead Sciences Inc	8,613%	0,24	2,53%	1,83%	0,08%	0,31%
Prologis Inc	8,932%	0,13	2,26%	2,01%	0,05%	0,08%
Medtronic PLC	7,570%	0,16	0,46%	1,57%	0,17%	0,83%
Automatic Data Processing Inc	12,814%	0,07	0,08%	2,42%	0,03%	4,43%
American Tower Corp	13,718%	0,22	1,98%	0,94%	0,14%	0,60%
Blackstone Inc	15,918%	0,48	2,77%	0,03%	0,43%	0,15%
Stryker Corp	11,341%	0,16	1,27%	0,46%	0,05%	0,03%
Mondelez International Inc	16,476%	0,13	1,16%	1,51%	0,12%	1,23%
TJX Companies Inc	13,306%	0,15	1,21%	0,59%	0,05%	0,06%
Chubb Ltd	9,286%	0,08	0,15%	0,00%	0,03%	0,00%
General Electric Co	13,582%	0,38	2,91%	0,02%	0,37%	0,03%
Applied Materials Inc	21,363%	0,16	0,19%	1,18%	0,02%	0,63%
Analog Devices Inc	10,508%	0,24	0,11%	0,05%	0,06%	0,03%
Marsh & McLennan Companies Inc	9,859%	0,11	0,02%	1,99%	0,01%	1,71%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Northrop Grumman Corp	10,337%	0,08	0,13%	1,74%	0,05%	0,87%
Regeneron Pharmaceuticals Inc	6,895%	0,15	0,00%	0,04%	0,00%	0,02%
ServiceNow Inc	10,363%	0,05	0,02%	3,96%	0,02%	1,14%
Duke Energy Corp	5,730%	0,11	3,84%	2,66%	0,07%	1,32%
Southern Co	5,969%	0,03	0,09%	3,89%	0,07%	11,22%
Booking Holdings Inc	252,617%	0,19	0,14%	0,12%	0,09%	0,22%
EOG Resources Inc	29,007%	0,41	1,25%	0,60%	0,43%	0,45%
Schlumberger NV	15,383%	0,32	2,23%	0,24%	0,33%	0,48%
Progressive Corp	18,123%	0,01	0,03%	1,96%	0,03%	3,29%
Vertex Pharmaceuticals Inc	11,835%	0,16	0,11%	1,03%	0,08%	0,18%
Becton Dickinson and Co	7,497%	0,12	0,03%	1,79%	0,01%	1,08%
HCA Healthcare Inc	255,499%	0,30	0,00%	0,03%	0,00%	0,07%
General Dynamics Corp	6,924%	0,23	2,17%	0,40%	0,04%	0,49%
Illinois Tool Works Inc	12,577%	0,09	0,05%	0,22%	0,04%	0,14%
3M Co	16,047%	0,08	0,00%	0,00%	0,00%	0,00%
Colgate-Palmolive Co	10,872%	0,07	0,46%	2,14%	0,05%	2,81%
CSX Corp	9,374%	0,22	0,23%	0,69%	0,06%	0,17%
Target Corp	11,130%	0,17	2,41%	0,50%	0,07%	1,11%

Source: Self Elaboration

Table 8 - Portfolio 2017

Source: Self Elaboration

Table 9 - Portfolio 2018

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
			41,45%	**58,55%**	**38,50%**	**61,50%**
Apple Inc	13,496%	0,12	0,56%	0,22%	0,42%	0,19%
Microsoft Corp	14,032%	0,10	0,04%	0,94%	0,04%	0,76%
Alphabet Inc	6,599%	0,12	0,03%	0,02%	0,03%	0,02%
Amazon.com Inc	11,687%	0,09	1,49%	0,48%	0,72%	0,40%
Berkshire Hathaway Inc	5,688%	0,03	0,28%	0,00%	0,22%	0,00%
UnitedHealth Group Inc	32,384%	0,12	0,05%	1,63%	0,05%	1,15%
Johnson & Johnson	11,624%	0,08	0,01%	3,36%	0,01%	2,14%
Exxon Mobil Corp	9,135%	0,07	0,08%	4,91%	0,07%	2,15%
Tesla Inc	30,943%	0,00	0,00%	0,44%	0,00%	0,33%
NVIDIA Corp	30,161%	0,10	0,03%	0,01%	0,03%	0,01%
Walmart Inc	12,715%	0,05	0,10%	0,04%	0,09%	0,04%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Visa Inc	16,519%	0,10	0,74%	2,93%	0,46%	1,91%
JPMorgan Chase & Co	14,059%	-0,01	0,08%	0,30%	0,07%	0,29%
Procter & Gamble Co	10,368%	0,05	0,05%	2,40%	0,05%	1,78%
Eli Lilly and Co	11,764%	0,12	0,09%	1,49%	0,08%	1,44%
Chevron Corp	8,018%	0,06	0,43%	1,57%	0,31%	1,25%
Mastercard Inc	17,792%	0,09	1,14%	2,06%	0,58%	1,41%
Home Depot Inc	10,372%	0,03	2,36%	0,00%	0,58%	0,00%
Meta Platforms Inc	10,458%	0,18	4,85%	0,00%	0,77%	0,00%
Pfizer Inc	14,638%	0,15	1,14%	0,15%	0,61%	0,15%
Abbvie Inc	17,985%	0,06	0,21%	0,01%	0,17%	0,01%
Merck & Co Inc	11,316%	0,25	11,66%	0,96%	1,32%	0,62%
Coca-Cola Co	15,308%	0,10	0,10%	5,48%	0,09%	1,57%
Bank of America Corp	11,807%	0,04	0,54%	0,00%	0,38%	0,00%
PepsiCo Inc	8,339%	0,06	0,06%	0,00%	0,05%	0,00%
Broadcom Inc	20,581%	0,26	1,43%	0,18%	0,83%	0,17%
Oracle Corp	20,099%	0,20	0,00%	0,01%	0,00%	0,01%
Thermo Fisher Scientific Inc	4,137%	0,15	2,20%	0,42%	0,81%	0,38%
Costco Wholesale Corp	11,433%	0,11	0,26%	0,49%	0,22%	0,40%
McDonald's Corp	5,251%	0,07	0,33%	0,13%	0,25%	0,13%
Cisco Systems Inc	13,813%	0,05	0,39%	0,00%	0,28%	0,00%
Danaher Corp	10,966%	0,08	0,74%	0,00%	0,44%	0,00%
Abbott Laboratories	5,097%	0,10	0,58%	0,04%	0,40%	0,04%
Nike Inc	19,664%	0,05	0,01%	1,09%	0,01%	0,78%
Accenture PLC	11,166%	0,07	0,04%	0,01%	0,04%	0,01%
T-Mobile US Inc	10,968%	0,13	0,05%	0,08%	0,05%	0,08%
Nextera Energy Inc	6,860%	0,06	0,36%	4,19%	0,27%	3,10%
Linde PLC	10,151%	0,06	0,00%	0,81%	0,00%	0,78%
Adobe Inc	7,446%	0,12	0,04%	1,29%	0,04%	0,73%
Verizon Communications Inc	7,196%	0,02	0,19%	3,16%	0,16%	1,58%
Walt Disney Co	15,958%	0,06	0,36%	3,76%	0,29%	0,72%
Wells Fargo & Co	13,091%	0,01	0,01%	4,60%	0,01%	9,34%
Bristol-Myers Squibb Co	17,274%	0,07	0,02%	1,62%	0,02%	1,29%
Philip Morris International Inc	7,508%	0,20	0,10%	1,23%	0,09%	0,81%
Texas Instruments Inc	12,757%	-0,01	0,01%	0,02%	0,01%	0,02%
United Parcel Service Inc	12,845%	0,06	0,26%	0,14%	0,21%	0,14%
Charles Schwab Corp	10,492%	0,02	0,17%	2,49%	0,15%	1,46%
Comcast Corp	8,075%	0,14	0,42%	0,00%	0,31%	0,00%
Morgan Stanley	14,497%	0,06	0,11%	0,20%	0,10%	0,19%
Raytheon Technologies Corp	13,619%	0,74	1,45%	0,02%	37,49%	0,02%
Conocophillips	11,444%	0,07	0,31%	0,06%	0,26%	0,06%
Honeywell International Inc	10,444%	0,15	0,17%	1,10%	0,15%	0,87%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Amgen Inc	10,796%	0,12	0,03%	0,43%	0,03%	0,39%
Netflix Inc	16,288%	0,12	2,19%	0,01%	0,59%	0,01%
Deere & Co	17,691%	0,02	0,09%	0,00%	0,09%	0,00%
AT&T Inc	13,343%	0,41	7,65%	0,01%	1,70%	0,01%
Qualcomm Inc	16,529%	0,06	0,20%	0,00%	0,17%	0,00%
Union Pacific Corp	13,026%	-0,05	1,21%	2,59%	0,50%	4,63%
Lockheed Martin Corp	8,300%	0,05	0,05%	9,96%	0,05%	35,52%
Caterpillar Inc	17,798%	-0,05	0,00%	1,86%	0,00%	0,99%
CVS Health Corp	14,971%	0,21	0,03%	0,93%	0,03%	0,59%
Lowe's Companies Inc	19,054%	-0,02	0,15%	0,16%	0,13%	0,16%
Elevance Health Inc	11,498%	0,06	0,00%	1,58%	0,00%	1,33%
Goldman Sachs Group Inc	13,898%	0,03	0,00%	0,94%	0,00%	0,82%
Boeing Co	13,437%	0,00	0,01%	0,00%	0,01%	0,00%
Starbucks Corp	14,094%	0,10	0,71%	0,21%	0,46%	0,20%
Intel Corp	16,182%	0,04	1,88%	1,11%	0,68%	0,97%
Intuit Inc	14,910%	0,01	0,08%	0,55%	0,08%	0,55%
S&P Global Inc	6,152%	0,07	0,45%	0,00%	0,31%	0,00%
American Express Co	14,289%	0,01	0,05%	1,11%	0,05%	1,00%
Advanced Micro Devices Inc	29,600%	0,39	0,10%	0,68%	0,10%	0,34%
BlackRock Inc	8,143%	0,06	2,52%	1,58%	0,73%	1,40%
Gilead Sciences Inc	17,744%	0,22	3,23%	0,00%	0,00%	0,00%
Prologis Inc	6,904%	0,08	0,16%	7,26%	0,15%	1,03%
Medtronic PLC	6,280%	0,14	0,00%	0,12%	0,00%	0,12%
Automatic Data Processing Inc	10,579%	0,00	0,57%	0,97%	0,35%	0,96%
Cigna Corp	123,698%	0,09	0,79%	0,00%	0,47%	0,00%
American Tower Corp	6,730%	0,16	2,04%	0,28%	0,92%	0,24%
Intuitive Surgical Inc	13,320%	0,03	0,21%	0,01%	0,17%	0,01%
Blackstone Inc	11,302%	0,32	0,02%	0,15%	0,02%	0,13%
Stryker Corp	14,699%	0,05	0,02%	3,80%	0,02%	1,41%
Mondelez International Inc	10,955%	0,15	0,12%	0,14%	0,11%	0,14%
TJX Companies Inc	17,860%	0,08	0,00%	0,16%	0,00%	0,15%
Chubb Ltd	8,661%	0,13	1,58%	0,00%	0,63%	0,00%
General Electric Co	21,587%	0,62	17,19%	0,00%	36,87%	0,00%
Applied Materials Inc	10,498%	0,33	4,78%	0,00%	0,66%	0,00%
Estee Lauder Companies Inc	9,520%	0,06	0,32%	2,67%	0,25%	2,68%
Citigroup Inc	11,093%	0,08	1,10%	1,01%	0,58%	0,77%
Analog Devices Inc	11,489%	0,12	0,35%	0,10%	0,31%	0,09%
Marsh & McLennan Companies Inc	11,191%	0,11	0,28%	1,83%	0,22%	1,59%
Northrop Grumman Corp	11,482%	0,08	0,20%	0,05%	0,16%	0,05%
Altria Group Inc	10,027%	0,06	0,10%	0,00%	0,10%	0,00%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Regeneron Pharmaceuticals Inc	12,674%	0,21	0,00%	0,22%	0,00%	0,20%
ServiceNow Inc	8,770%	0,07	0,01%	0,03%	0,01%	0,03%
Duke Energy Corp	6,531%	0,09	0,68%	0,08%	0,42%	0,08%
Southern Co	6,100%	0,12	0,01%	0,00%	0,01%	0,00%
Booking Holdings Inc	6,574%	0,15	6,29%	0,15%	0,71%	0,15%
EOG Resources Inc	75,387%	0,13	0,00%	0,63%	0,00%	0,42%
Schlumberger NV	7,153%	0,14	6,43%	0,08%	1,06%	0,08%

Source: Self Elaboration

Table 10 - Portfolio 2019

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
			45,60%	**54,40%**	**42,36%**	**57,64%**
Apple Inc	15,509%	0,41	1,98%	0,21%	1,11%	0,20%
Microsoft Corp	13,326%	0,26	0,04%	1,50%	0,04%	1,00%
Alphabet Inc	7,432%	0,29	0,03%	0,02%	0,03%	0,02%
Berkshire Hathaway Inc	6,110%	0,23	2,75%	0,91%	1,09%	0,83%
UnitedHealth Group Inc	31,142%	0,26	0,30%	0,00%	0,27%	0,00%
Johnson & Johnson	9,298%	0,18	0,06%	2,70%	0,06%	1,92%
Exxon Mobil Corp	15,364%	0,31	0,01%	2,43%	0,01%	1,58%
Tesla Inc	25,806%	0,03	0,07%	5,41%	0,07%	1,39%
NVIDIA Corp	28,447%	0,71	0,00%	0,02%	0,00%	0,02%
Visa Inc	15,043%	0,25	0,03%	0,02%	0,03%	0,02%
JPMorgan Chase & Co	10,682%	0,26	0,10%	0,04%	0,10%	0,04%
Procter & Gamble Co	11,856%	0,02	0,25%	6,18%	0,22%	28,28%
Eli Lilly and Co	21,358%	0,04	0,08%	0,54%	0,07%	0,52%
Chevron Corp	5,915%	0,32	0,05%	3,43%	0,05%	1,52%
Mastercard Inc	13,538%	0,24	0,10%	5,34%	0,09%	1,70%
Home Depot Inc	14,123%	0,22	0,53%	1,58%	0,49%	0,94%
Meta Platforms Inc	14,944%	0,42	7,05%	0,41%	1,85%	0,34%
Pfizer Inc	13,040%	0,13	0,10%	0,00%	0,10%	0,00%
Abbvie Inc	25,939%	0,10	1,66%	0,00%	0,81%	0,00%
Merck & Co Inc	11,473%	0,16	0,69%	0,23%	0,53%	0,21%
Coca-Cola Co	16,469%	0,13	0,21%	0,01%	0,19%	0,01%
Bank of America Corp	8,376%	0,40	7,55%	3,85%	1,60%	0,59%
PepsiCo Inc	10,317%	0,09	0,10%	2,42%	0,09%	1,49%
Broadcom Inc	24,228%	0,19	0,30%	0,00%	0,25%	0,00%
Oracle Corp	17,916%	0,19	0,06%	0,00%	0,06%	0,00%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Costco Wholesale Corp	12,411%	0,22	1,98%	0,28%	1,39%	0,24%
McDonald's Corp	5,562%	0,12	0,00%	0,01%	0,00%	0,01%
Cisco Systems Inc	9,782%	0,25	3,23%	0,70%	1,47%	0,56%
Danaher Corp	12,622%	0,12	0,23%	1,22%	0,21%	0,94%
Abbott Laboratories	5,317%	0,12	0,26%	0,18%	0,23%	0,18%
Nike Inc	16,313%	0,18	0,44%	0,00%	0,37%	0,00%
Accenture PLC	28,308%	0,26	2,06%	0,00%	1,11%	0,00%
T-Mobile US Inc	12,101%	0,22	0,68%	0,04%	0,55%	0,04%
Nextera Energy Inc	7,155%	0,10	0,01%	3,51%	0,01%	3,73%
Linde PLC	53,214%	0,13	0,03%	0,01%	0,03%	0,01%
Adobe Inc	6,952%	0,28	0,06%	0,09%	0,05%	0,09%
Verizon Communications Inc	6,713%	0,09	0,29%	1,98%	0,26%	1,42%
Walt Disney Co	14,638%	0,15	0,00%	7,51%	0,00%	3,72%
Wells Fargo & Co	10,013%	0,37	0,04%	0,18%	0,04%	0,18%
Bristol-Myers Squibb Co	10,783%	0,16	0,26%	1,26%	0,22%	1,28%
Philip Morris International Inc	11,373%	0,46	1,24%	0,80%	0,90%	0,62%
Texas Instruments Inc	14,011%	0,20	0,01%	4,12%	0,01%	2,28%
United Parcel Service Inc	15,328%	0,30	0,02%	0,78%	0,02%	0,59%
Charles Schwab Corp	11,862%	0,32	0,10%	6,06%	0,09%	11,20%
Comcast Corp	11,259%	0,32	0,01%	0,02%	0,01%	0,02%
Morgan Stanley	13,523%	0,45	0,46%	0,15%	0,37%	0,14%
Raytheon Technologies Corp	10,485%	1,37	3,97%	0,02%	58,39%	0,02%
Conocophillips	9,999%	0,27	0,51%	0,00%	0,45%	0,00%
Honeywell International Inc	8,682%	0,30	0,14%	0,21%	0,13%	0,20%
Amgen Inc	9,523%	0,08	0,13%	0,02%	0,12%	0,02%
Netflix Inc	20,092%	0,46	1,11%	0,05%	1,09%	0,05%
Deere & Co	16,561%	0,20	0,21%	2,52%	0,18%	7,85%
AT&T Inc	13,506%	0,69	0,03%	0,06%	0,03%	0,06%
International Business Machines Corp	20,798%	0,44	9,66%	0,01%	2,83%	0,01%
Qualcomm Inc	13,899%	0,25	0,09%	0,00%	0,09%	0,00%
Union Pacific Corp	11,219%	0,24	1,50%	0,01%	0,88%	0,01%
Lockheed Martin Corp	9,393%	0,41	0,37%	0,00%	0,31%	0,00%
Caterpillar Inc	21,353%	0,26	1,07%	0,66%	0,71%	0,58%
CVS Health Corp	11,751%	0,44	0,06%	1,84%	0,06%	0,61%
Lowe's Companies Inc	12,488%	0,24	0,00%	0,09%	0,00%	0,09%
Elevance Health Inc	11,233%	0,24	0,05%	3,38%	0,04%	1,10%
Goldman Sachs Group Inc	15,423%	0,52	0,21%	0,16%	0,19%	0,15%
Boeing Co	13,223%	0,32	0,00%	1,33%	0,00%	0,96%
Starbucks Corp	11,521%	0,09	0,00%	2,80%	0,00%	2,59%
Intel Corp	17,800%	0,18	0,01%	0,00%	0,01%	0,00%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Intuit Inc	22,752%	0,17	0,86%	0,29%	0,70%	0,25%
S&P Global Inc	7,703%	0,23	0,40%	0,93%	0,34%	0,78%
American Express Co	8,744%	0,22	0,10%	1,29%	0,09%	1,07%
Advanced Micro Devices Inc	29,675%	0,30	0,25%	0,00%	0,24%	0,00%
BlackRock Inc	8,089%	0,24	0,06%	1,23%	0,06%	0,90%
Gilead Sciences Inc	14,335%	0,43	0,12%	0,41%	0,12%	0,33%
Prologis Inc	5,862%	0,25	5,22%	0,63%	2,29%	0,48%
Medtronic PLC	8,615%	0,18	3,01%	0,00%	1,36%	0,00%
Cigna Corp	8,530%	0,30	0,23%	0,44%	0,21%	0,41%
American Tower Corp	5,609%	0,09	0,00%	0,15%	0,00%	0,15%
Intuitive Surgical Inc	13,558%	0,22	0,75%	3,42%	0,60%	1,44%
Blackstone Inc	14,448%	0,54	8,44%	0,01%	2,09%	0,01%
Stryker Corp	7,702%	0,21	4,49%	0,44%	1,63%	0,38%
Mondelez International Inc	12,181%	0,24	0,26%	0,01%	0,23%	0,01%
TJX Companies Inc	22,809%	0,24	0,02%	0,22%	0,02%	0,20%
Chubb Ltd	9,783%	0,21	0,02%	1,86%	0,02%	1,72%
General Electric Co	29,021%	0,92	0,32%	0,02%	0,29%	0,02%
Applied Materials Inc	40,144%	0,50	0,00%	0,09%	0,00%	0,09%
Estee Lauder Companies Inc	8,912%	0,20	0,13%	0,00%	0,12%	0,00%
Citigroup Inc	13,198%	0,60	9,02%	0,00%	2,72%	0,00%
Northrop Grumman Corp	12,134%	0,35	7,03%	0,00%	1,32%	0,00%
Altria Group Inc	16,118%	0,31	0,37%	0,02%	0,32%	0,02%
Regeneron Pharmaceuticals Inc	10,755%	0,11	0,33%	3,79%	0,28%	3,38%
Duke Energy Corp	7,489%	0,06	0,41%	0,12%	0,32%	0,12%
Southern Co	6,928%	0,12	0,40%	4,18%	0,32%	2,97%
EOG Resources Inc	25,661%	0,67	0,44%	0,05%	0,43%	0,04%
Schlumberger NV	17,797%	0,71	0,18%	0,00%	0,17%	0,00%
Progressive Corp	43,038%	0,28	0,00%	0,21%	0,00%	0,20%
Vertex Pharmaceuticals Inc	8,878%	0,20	0,02%	0,03%	0,02%	0,03%
Becton Dickinson and Co	15,040%	0,24	1,17%	0,08%	0,79%	0,08%
Air Products and Chemicals Inc	11,160%	0,21	0,01%	0,00%	0,01%	0,00%
HCA Healthcare Inc	6,693%	0,22	0,16%	0,21%	0,15%	0,21%
General Dynamics Corp	9,813%	0,37	0,00%	0,41%	0,00%	0,39%
Illinois Tool Works Inc	11,168%	0,11	1,15%	0,13%	0,64%	0,13%

Source: Self Elaboration

Table 11 - Portfolio 2020

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
			52,83%	**47,17%**	**46,60%**	**53,40%**
Apple Inc	19,242%	-0,08	0,12%	2,33%	0,10%	11,66%
Microsoft Corp	9,898%	0,05	0,06%	3,82%	0,05%	1,18%
Alphabet Inc	7,654%	0,10	0,04%	0,03%	0,03%	0,02%
Berkshire Hathaway Inc	4,240%	0,12	3,08%	0,75%	0,24%	1,16%
UnitedHealth Group Inc	11,824%	0,07	0,62%	0,00%	0,22%	0,00%
Johnson & Johnson	9,409%	0,08	0,12%	1,48%	0,06%	1,59%
Exxon Mobil Corp	13,453%	0,17	0,01%	0,00%	0,01%	0,00%
Tesla Inc	37,912%	-0,29	0,01%	2,63%	0,01%	15,58%
NVIDIA Corp	14,851%	0,01	0,00%	0,04%	0,00%	0,03%
Walmart Inc	7,472%	0,12	0,05%	0,02%	0,03%	0,02%
Visa Inc	13,308%	0,09	0,26%	0,07%	0,11%	0,05%
JPMorgan Chase & Co	11,405%	-0,05	0,08%	5,47%	0,07%	1,22%
Procter & Gamble Co	10,493%	0,05	0,21%	2,28%	0,08%	4,32%
Eli Lilly and Co	16,141%	0,01	0,07%	2,34%	0,06%	1,13%
Chevron Corp	7,890%	0,17	0,95%	0,00%	0,16%	0,00%
Mastercard Inc	8,595%	0,06	1,29%	6,82%	0,26%	1,07%
Home Depot Inc	9,979%	0,09	1,29%	2,51%	0,32%	0,53%
Meta Platforms Inc	12,523%	0,17	0,98%	0,00%	0,14%	0,00%
Pfizer Inc	13,141%	0,17	1,41%	0,00%	0,26%	0,00%
Abbvie Inc	16,039%	0,13	0,44%	0,13%	0,21%	0,14%
Merck & Co Inc	4,870%	0,15	1,57%	0,01%	0,21%	0,01%
Coca-Cola Co	5,955%	0,09	2,70%	0,76%	0,24%	1,61%
Bank of America Corp	12,063%	0,02	0,11%	2,95%	0,07%	0,10%
PepsiCo Inc	8,450%	0,05	2,72%	0,00%	0,16%	0,00%
Broadcom Inc	13,066%	0,15	0,16%	0,00%	0,08%	0,00%
Oracle Corp	21,572%	0,08	1,52%	1,75%	0,23%	0,57%
Thermo Fisher Scientific Inc	8,866%	0,00	0,00%	0,01%	0,00%	0,01%
Costco Wholesale Corp	12,456%	0,09	2,79%	7,61%	0,37%	0,46%
McDonald's Corp	7,826%	0,15	2,81%	0,40%	0,25%	0,53%
Cisco Systems Inc	12,967%	0,12	2,97%	0,25%	0,35%	0,10%
Danaher Corp	11,982%	0,02	0,19%	0,00%	0,10%	0,00%
Abbott Laboratories	5,369%	0,11	1,51%	0,00%	0,24%	0,00%
Nike Inc	13,610%	0,09	1,23%	0,06%	0,28%	0,05%
Accenture PLC	5,933%	0,06	0,01%	2,06%	0,01%	1,20%
T-Mobile US Inc	12,153%	0,15	0,05%	0,01%	0,04%	0,01%
Nextera Energy Inc	4,320%	0,04	0,11%	0,20%	0,06%	0,14%
Linde PLC	15,886%	0,05	0,68%	1,66%	0,20%	0,96%
Adobe Inc	6,679%	0,03	0,00%	9,19%	0,00%	0,96%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Verizon Communications Inc	13,996%	0,05	0,06%	2,39%	0,04%	0,41%
Wells Fargo & Co	11,382%	0,00	0,29%	0,90%	0,15%	1,69%
Bristol-Myers Squibb Co	10,159%	0,06	0,29%	0,05%	0,14%	0,05%
Philip Morris International Inc	10,584%	0,13	0,01%	0,00%	0,01%	0,00%
Texas Instruments Inc	13,900%	0,01	0,03%	0,98%	0,02%	0,94%
United Parcel Service Inc	12,610%	0,10	0,25%	1,55%	0,09%	2,61%
Charles Schwab Corp	10,036%	0,13	0,01%	0,03%	0,01%	0,03%
Comcast Corp	9,304%	0,16	3,55%	0,33%	0,38%	0,17%
Morgan Stanley	14,021%	0,11	2,82%	0,07%	0,32%	0,05%
Raytheon Technologies Corp	8,754%	0,85	11,57%	0,00%	71,37%	0,00%
Conocophillips	13,203%	0,15	2,07%	0,26%	0,26%	0,10%
Honeywell International Inc	7,158%	0,08	0,45%	0,02%	0,12%	0,02%
Amgen Inc	15,225%	-0,01	0,20%	0,19%	0,13%	0,09%
Netflix Inc	19,741%	0,12	0,23%	0,01%	0,36%	0,01%
Deere & Co	25,874%	0,05	0,03%	0,11%	0,03%	0,07%
Salesforce Inc	6,092%	0,17	3,76%	0,01%	0,39%	0,01%
Qualcomm Inc	18,788%	0,13	0,31%	0,00%	0,12%	0,00%
Union Pacific Corp	10,275%	0,03	0,39%	0,02%	0,23%	0,02%
Lockheed Martin Corp	8,770%	0,06	1,16%	0,00%	0,20%	0,00%
Caterpillar Inc	18,326%	0,01	0,37%	3,53%	0,21%	0,07%
CVS Health Corp	10,628%	0,12	0,10%	2,34%	0,07%	0,38%
Elevance Health Inc	9,197%	0,13	0,00%	0,13%	0,00%	0,10%
Goldman Sachs Group Inc	16,730%	0,09	0,04%	0,77%	0,03%	0,62%
Boeing Co	17,215%	0,15	0,26%	0,16%	0,17%	0,15%
Starbucks Corp	11,340%	0,10	0,00%	0,17%	0,00%	0,25%
Intel Corp	19,795%	-0,03	0,00%	2,65%	0,00%	34,66%
S&P Global Inc	8,585%	0,06	0,01%	0,00%	0,01%	0,00%
American Express Co	9,055%	0,07	3,43%	6,31%	0,28%	0,30%
Advanced Micro Devices Inc	23,443%	-0,18	0,01%	2,62%	0,01%	0,81%
BlackRock Inc	7,504%	0,08	0,19%	1,35%	0,10%	0,81%
Gilead Sciences Inc	11,278%	0,23	5,25%	0,00%	0,50%	0,00%
Prologis Inc	6,055%	0,08	0,10%	1,07%	0,06%	0,94%
Medtronic PLC	7,278%	0,10	1,07%	1,11%	0,12%	0,93%
Cigna Corp	234,023%	0,12	0,40%	0,23%	0,18%	0,33%
Intuitive Surgical Inc	5,188%	0,05	0,39%	0,00%	0,26%	0,00%
Blackstone Inc	10,157%	0,04	0,22%	0,88%	0,18%	0,49%
Stryker Corp	6,685%	0,13	0,00%	0,30%	0,00%	0,20%
Mondelez International Inc	6,936%	0,13	1,49%	0,64%	0,28%	0,58%
TJX Companies Inc	8,243%	0,09	3,24%	0,01%	0,52%	0,01%
Chubb Ltd	10,042%	0,07	2,42%	1,28%	0,23%	1,40%
General Electric Co	42,397%	0,24	1,71%	0,01%	0,00%	0,01%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Applied Materials Inc	25,778%	0,12	0,02%	0,10%	0,02%	0,09%
Estee Lauder Companies Inc	11,403%	0,02	0,02%	0,39%	0,02%	0,58%
Citigroup Inc	14,527%	0,11	3,14%	0,02%	0,29%	0,02%
Analog Devices Inc	24,642%	0,07	0,00%	0,40%	0,00%	0,15%
Marsh & McLennan Companies Inc	6,961%	-0,02	0,13%	0,00%	0,09%	0,00%
Northrop Grumman Corp	8,163%	0,17	1,93%	0,00%	0,24%	0,00%
Altria Group Inc	14,193%	0,14	1,29%	0,00%	0,34%	0,00%
Regeneron Pharmaceuticals Inc	11,311%	0,04	0,34%	0,03%	0,24%	0,02%
ServiceNow Inc	7,172%	0,07	0,27%	3,06%	0,29%	0,49%
Duke Energy Corp	5,515%	0,09	5,63%	0,19%	0,23%	0,18%
Southern Co	7,537%	0,01	0,94%	1,12%	0,18%	1,69%
Booking Holdings Inc	7,170%	0,03	0,25%	0,16%	0,18%	0,07%
EOG Resources Inc	18,247%	0,32	4,05%	0,00%	14,29%	0,00%
Schlumberger NV	12,185%	0,11	0,00%	2,34%	0,00%	0,15%
Progressive Corp	13,400%	0,19	0,02%	0,03%	0,02%	0,03%
Vertex Pharmaceuticals Inc	8,734%	0,07	0,31%	0,77%	0,31%	0,11%
Becton Dickinson and Co	10,135%	0,06	0,01%	0,01%	0,01%	0,00%
Air Products and Chemicals Inc	8,756%	0,08	0,12%	0,12%	0,11%	0,17%
HCA Healthcare Inc	5,531%	0,10	0,00%	0,99%	0,00%	0,37%
General Dynamics Corp	10,105%	0,19	1,11%	0,14%	0,25%	0,15%

Source: Self Elaboration

Table 12 - Portfolio 2021

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
			53,38%	**46,62%**	**50,33%**	**49,67%**
Apple Inc	19,034%	-0,02	0,03%	0,83%	0,03%	0,95%
Microsoft Corp	12,895%	0,09	0,27%	0,19%	0,16%	0,22%
Alphabet Inc	8,346%	0,10	2,09%	0,84%	0,33%	0,54%
Amazon.com Inc	7,136%	0,17	2,22%	0,46%	0,48%	0,35%
UnitedHealth Group Inc	13,228%	0,11	1,73%	0,00%	0,12%	0,00%
Johnson & Johnson	8,151%	0,09	0,50%	1,10%	0,18%	0,94%
Exxon Mobil Corp	18,378%	0,25	3,05%	0,00%	1,20%	0,00%
Tesla Inc	53,376%	-0,39	0,00%	1,91%	0,00%	45,97%
NVIDIA Corp	15,625%	0,14	0,00%	0,03%	0,00%	0,03%
Walmart Inc	9,585%	0,14	0,41%	0,16%	0,16%	0,19%
Visa Inc	9,812%	0,06	1,09%	1,94%	0,33%	0,61%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
JPMorgan Chase & Co	12,564%	0,02	1,99%	4,20%	0,10%	11,17%
Procter & Gamble Co	13,470%	0,11	1,04%	0,48%	0,30%	0,52%
Eli Lilly and Co	14,682%	0,06	0,08%	0,48%	0,05%	0,54%
Chevron Corp	7,047%	0,27	1,01%	0,00%	0,41%	0,00%
Mastercard Inc	16,127%	0,02	0,25%	1,22%	0,22%	0,57%
Home Depot Inc	9,847%	0,17	1,56%	0,16%	0,37%	0,12%
Meta Platforms Inc	10,672%	0,18	3,87%	0,00%	1,30%	0,00%
Pfizer Inc	16,237%	0,18	0,91%	0,00%	0,30%	0,00%
Abbvie Inc	12,394%	0,13	0,55%	0,14%	0,25%	0,19%
Merck & Co Inc	11,833%	0,26	5,09%	0,01%	0,26%	0,01%
Coca-Cola Co	7,150%	0,07	0,45%	0,99%	0,23%	0,63%
Bank of America Corp	13,977%	0,04	2,59%	3,06%	0,00%	2,21%
PepsiCo Inc	6,371%	0,07	0,14%	0,00%	0,09%	0,00%
Broadcom Inc	23,466%	0,09	0,24%	0,00%	0,21%	0,00%
Oracle Corp	10,977%	0,04	0,61%	0,65%	0,21%	0,61%
Thermo Fisher Scientific Inc	32,959%	0,09	0,00%	0,06%	0,00%	0,03%
Costco Wholesale Corp	67,805%	0,08	0,07%	1,50%	0,05%	1,51%
McDonald's Corp	6,535%	0,15	1,09%	0,36%	0,38%	0,30%
Cisco Systems Inc	13,410%	0,11	1,30%	1,17%	0,34%	0,55%
Danaher Corp	13,251%	0,18	1,62%	0,00%	0,36%	0,00%
Abbott Laboratories	8,254%	0,11	1,32%	0,00%	0,26%	0,00%
Nike Inc	13,702%	0,15	0,98%	0,46%	0,47%	0,24%
Accenture PLC	8,043%	0,08	0,01%	1,82%	0,01%	0,60%
T-Mobile US Inc	12,807%	0,12	0,02%	0,82%	0,02%	0,98%
Nextera Energy Inc	9,608%	0,04	0,02%	5,35%	0,03%	0,85%
Linde PLC	9,319%	0,07	2,44%	2,95%	0,11%	1,26%
Adobe Inc	7,610%	0,11	0,00%	0,13%	0,00%	0,22%
Verizon Communications Inc	7,684%	0,10	1,36%	1,74%	0,18%	1,29%
Walt Disney Co	76,733%	-0,02	0,04%	1,52%	0,04%	1,04%
Wells Fargo & Co	21,850%	0,16	0,69%	0,09%	0,42%	0,06%
Bristol-Myers Squibb Co	8,407%	0,23	5,66%	0,00%	0,53%	0,00%
Philip Morris International Inc	10,286%	0,15	1,17%	0,14%	0,39%	0,17%
Texas Instruments Inc	13,127%	0,00	0,01%	0,05%	0,01%	0,05%
United Parcel Service Inc	19,491%	0,05	0,02%	0,76%	0,01%	0,68%
Charles Schwab Corp	18,224%	-0,03	0,53%	0,61%	0,19%	0,78%
Comcast Corp	12,951%	0,04	1,41%	2,77%	0,22%	1,94%
Morgan Stanley	15,306%	0,00	0,12%	0,00%	0,14%	0,00%
Raytheon Technologies Corp	25,099%	0,16	0,47%	0,36%	0,38%	0,11%
Conocophillips	16,317%	0,30	2,45%	0,01%	21,43%	0,01%
Honeywell International Inc	17,771%	0,03	1,50%	4,57%	0,22%	0,20%
Amgen Inc	14,780%	0,14	0,41%	0,02%	0,23%	0,02%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Netflix Inc	19,039%	0,02	0,02%	1,20%	0,02%	0,59%
Deere & Co	24,884%	0,08	0,17%	0,01%	0,21%	0,01%
Salesforce Inc	10,923%	0,24	5,43%	0,00%	0,92%	0,00%
AT&T Inc	16,516%	0,53	7,27%	0,00%	29,49%	0,00%
Qualcomm Inc	24,541%	0,10	1,24%	0,00%	0,37%	0,00%
Union Pacific Corp	8,926%	0,05	2,03%	4,25%	0,19%	0,72%
Lockheed Martin Corp	8,321%	0,26	0,50%	0,02%	0,39%	0,02%
Caterpillar Inc	19,523%	-0,02	0,00%	4,73%	0,00%	0,55%
CVS Health Corp	11,059%	0,23	2,64%	0,37%	0,25%	0,20%
Lowe's Companies Inc	9,697%	0,20	2,52%	0,02%	0,74%	0,02%
Elevance Health Inc	36,894%	0,14	0,00%	1,25%	0,00%	0,94%
Goldman Sachs Group Inc	15,387%	0,06	0,00%	2,67%	0,00%	1,80%
Boeing Co	29,501%	0,09	0,01%	0,00%	0,01%	0,00%
Starbucks Corp	11,563%	0,00	0,28%	1,76%	0,15%	1,16%
Intel Corp	19,796%	0,07	0,01%	1,28%	0,01%	0,93%
Intuit Inc	23,360%	0,07	0,09%	0,28%	0,09%	0,23%
S&P Global Inc	4,078%	0,21	1,54%	0,00%	0,40%	0,00%
American Express Co	17,408%	0,00	0,09%	0,81%	0,12%	0,53%
Advanced Micro Devices Inc	22,727%	0,00	0,01%	1,91%	0,02%	0,66%
BlackRock Inc	7,631%	0,08	1,11%	1,80%	0,30%	0,66%
Gilead Sciences Inc	16,735%	0,29	1,67%	0,00%	0,46%	0,00%
Prologis Inc	9,676%	0,18	1,02%	1,45%	0,39%	0,41%
Medtronic PLC	9,625%	0,11	0,01%	0,84%	0,01%	0,42%
Automatic Data Processing Inc	17,466%	-0,02	0,03%	3,15%	0,03%	0,72%
Cigna Corp	207,785%	0,22	0,88%	0,01%	0,25%	0,01%
American Tower Corp	11,066%	0,27	1,86%	0,02%	0,44%	0,02%
Intuitive Surgical Inc	15,986%	-0,10	0,01%	5,69%	0,01%	1,32%
Blackstone Inc	14,354%	0,02	0,06%	0,40%	0,05%	0,28%
Stryker Corp	10,300%	-0,03	0,01%	5,35%	0,01%	1,70%
Mondelez International Inc	6,134%	0,12	1,85%	1,79%	0,32%	0,57%
TJX Companies Inc	13,421%	0,08	0,00%	0,30%	0,00%	0,20%
Chubb Ltd	8,877%	0,05	0,33%	0,01%	0,24%	0,01%
General Electric Co	19,016%	0,35	2,64%	0,00%	27,72%	0,00%
Applied Materials Inc	33,254%	0,06	0,12%	0,00%	0,18%	0,00%
Estee Lauder Companies Inc	18,326%	-0,06	0,02%	2,85%	0,02%	1,76%
Citigroup Inc	21,259%	0,17	0,76%	0,03%	0,44%	0,03%
Analog Devices Inc	24,056%	0,07	0,39%	0,70%	0,25%	0,46%
Marsh & McLennan Companies Inc	8,660%	0,04	0,10%	2,42%	0,09%	1,71%
Northrop Grumman Corp	10,872%	0,28	4,62%	0,01%	0,76%	0,01%
Altria Group Inc	18,626%	0,26	0,35%	0,00%	0,26%	0,00%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Regeneron Pharmaceuticals Inc	9,642%	0,38	0,00%	0,01%	0,00%	0,01%
ServiceNow Inc	16,085%	0,02	0,03%	3,30%	0,03%	0,13%
Duke Energy Corp	8,293%	0,11	0,59%	1,34%	0,26%	0,74%
Southern Co	11,069%	0,08	0,01%	0,01%	0,01%	0,01%
Booking Holdings Inc	12,291%	-0,11	0,01%	1,15%	0,01%	1,03%
EOG Resources Inc	16,671%	0,42	0,01%	0,01%	0,01%	0,01%
Schlumberger NV	55,101%	0,19	1,17%	0,64%	0,32%	0,11%

Source: Self Elaboration

Table 13 - Portfolio 2022

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
			46,23%	**53,77%**	**54,92%**	**45,08%**
Apple Inc	14,279%	0,00	0,01%	2,69%	0,01%	2,04%
Microsoft Corp	15,202%	0,10	0,11%	0,22%	0,10%	0,21%
Alphabet Inc	7,560%	0,15	0,22%	0,36%	0,19%	0,32%
Amazon.com Inc	6,142%	0,23	0,58%	0,06%	0,45%	0,06%
UnitedHealth Group Inc	13,896%	0,00	0,01%	0,01%	0,01%	0,01%
Johnson & Johnson	9,649%	0,11	0,11%	0,10%	0,10%	0,10%
Exxon Mobil Corp	16,475%	0,24	1,73%	0,00%	0,71%	0,00%
Tesla Inc	41,294%	-0,19	0,00%	1,15%	0,00%	1,09%
NVIDIA Corp	20,676%	0,16	0,00%	0,01%	0,00%	0,01%
Walmart Inc	9,171%	0,19	0,13%	0,04%	0,12%	0,04%
Visa Inc	19,254%	0,27	1,47%	0,04%	0,77%	0,04%
JPMorgan Chase & Co	10,314%	0,16	2,01%	5,65%	0,79%	0,00%
Procter & Gamble Co	10,455%	-0,03	0,01%	5,19%	0,01%	4,46%
Eli Lilly and Co	14,557%	0,04	0,48%	0,79%	0,37%	0,64%
Chevron Corp	10,743%	0,15	0,37%	0,00%	0,30%	0,00%
Mastercard Inc	16,806%	0,21	1,68%	1,67%	0,76%	1,25%
Home Depot Inc	10,705%	0,00	0,04%	1,54%	0,04%	0,96%
Meta Platforms Inc	9,795%	0,19	0,88%	0,00%	0,59%	0,00%
Pfizer Inc	18,058%	-0,03	0,03%	0,01%	0,03%	0,01%
Abbvie Inc	12,267%	0,03	0,05%	3,63%	0,05%	2,34%
Merck & Co Inc	14,405%	0,24	2,29%	0,01%	0,91%	0,01%
Coca-Cola Co	5,329%	0,09	0,55%	2,27%	0,42%	1,46%
Bank of America Corp	12,620%	0,13	0,12%	4,99%	0,12%	17,78%
PepsiCo Inc	5,952%	0,00	0,02%	0,01%	0,02%	0,01%
Broadcom Inc	26,448%	0,03	0,02%	0,00%	0,02%	0,00%
Oracle Corp	25,120%	0,19	0,41%	0,28%	0,33%	0,25%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Thermo Fisher Scientific Inc	7,934%	0,03	0,00%	2,78%	0,00%	3,33%
Costco Wholesale Corp	20,145%	-0,03	0,02%	1,65%	0,02%	1,48%
McDonald's Corp	7,594%	0,05	0,02%	0,25%	0,02%	0,23%
Cisco Systems Inc	12,238%	0,01	0,02%	2,53%	0,02%	2,12%
Danaher Corp	33,464%	0,07	0,16%	0,00%	0,14%	0,00%
Abbott Laboratories	13,371%	0,03	0,02%	0,03%	0,02%	0,02%
Nike Inc	10,770%	0,12	0,12%	0,08%	0,11%	0,08%
Accenture PLC	9,631%	0,06	0,01%	1,09%	0,01%	0,88%
T-Mobile US Inc	13,912%	0,45	9,28%	0,00%	13,41%	0,00%
Nextera Energy Inc	9,289%	0,02	0,05%	2,79%	0,05%	2,32%
Linde PLC	12,507%	0,04	0,06%	2,86%	0,06%	2,09%
Adobe Inc	6,428%	0,18	0,01%	0,69%	0,01%	0,49%
Verizon Communications Inc	8,037%	0,21	2,74%	0,79%	1,03%	0,62%
Walt Disney Co	10,449%	0,26	1,04%	0,05%	0,68%	0,05%
Wells Fargo & Co	21,146%	0,14	0,05%	0,34%	0,05%	0,32%
Bristol-Myers Squibb Co	12,084%	0,19	4,86%	0,00%	2,41%	0,00%
Philip Morris International Inc	9,385%	0,19	3,40%	0,97%	1,20%	0,71%
Texas Instruments Inc	12,642%	0,11	0,02%	0,53%	0,02%	0,44%
United Parcel Service Inc	23,702%	0,10	0,03%	1,33%	0,03%	1,14%
Charles Schwab Corp	11,065%	0,13	0,02%	0,55%	0,02%	0,52%
Comcast Corp	11,718%	0,29	1,82%	0,02%	0,95%	0,02%
Morgan Stanley	12,963%	0,17	2,74%	0,00%	0,93%	0,00%
Raytheon Technologies Corp	6,160%	0,23	3,61%	0,92%	1,33%	0,62%
Conocophillips	13,971%	0,28	0,38%	0,02%	0,30%	0,02%
Honeywell International Inc	12,688%	0,16	3,48%	1,54%	1,36%	0,94%
Amgen Inc	14,391%	0,10	0,34%	0,92%	0,29%	0,74%
Netflix Inc	16,239%	0,12	0,02%	2,96%	0,02%	2,00%
Deere & Co	27,982%	0,20	0,64%	0,01%	0,50%	0,01%
Salesforce Inc	11,718%	0,29	2,58%	0,00%	1,14%	0,00%
AT&T Inc	14,783%	0,75	8,09%	0,00%	48,23%	0,00%
International Business Machines Corp	10,463%	0,14	0,73%	0,00%	0,47%	0,00%
Qualcomm Inc	34,346%	0,16	1,56%	0,13%	1,05%	0,12%
Union Pacific Corp	7,486%	0,04	0,04%	4,53%	0,04%	2,39%
Lockheed Martin Corp	8,329%	0,11	0,00%	2,28%	0,00%	1,16%
Caterpillar Inc	14,223%	0,13	0,15%	0,28%	0,14%	0,26%
CVS Health Corp	8,929%	0,09	0,55%	1,18%	0,37%	1,16%
Lowe's Companies Inc	22,456%	0,08	0,00%	0,39%	0,00%	0,34%
Elevance Health Inc	20,074%	0,04	0,00%	3,51%	0,00%	5,64%
Goldman Sachs Group Inc	13,868%	0,21	2,33%	0,00%	0,97%	0,00%
Boeing Co	12,783%	0,29	3,60%	0,02%	2,42%	0,02%

Company Name	Combined Uncertainty	Expected Return	Weight Long (Stochastic)	Weight Short (Stochastic)	Weight Long (Normal)	Weight Short (Normal)
Starbucks Corp	15,250%	0,07	0,01%	1,65%	0,01%	0,98%
Intel Corp	16,431%	0,10	1,01%	2,02%	0,61%	1,45%
American Express Co	12,395%	0,18	0,71%	0,00%	0,50%	0,00%
Advanced Micro Devices Inc	15,316%	0,00	0,65%	1,48%	0,51%	1,10%
BlackRock Inc	10,853%	0,13	1,38%	1,02%	0,85%	0,71%
Gilead Sciences Inc	18,114%	0,10	0,46%	2,77%	0,35%	12,02%
Prologis Inc	7,787%	-0,01	0,03%	0,00%	0,03%	0,00%
Medtronic PLC	10,938%	0,32	4,02%	0,01%	1,38%	0,01%
Automatic Data Processing Inc	18,668%	-0,05	0,00%	2,63%	0,00%	2,49%
Cigna Corp	10,177%	0,15	0,68%	2,87%	0,38%	2,60%
American Tower Corp	8,328%	0,06	0,34%	0,05%	0,29%	0,04%
Intuitive Surgical Inc	9,589%	0,00	0,02%	0,85%	0,02%	0,76%
Blackstone Inc	18,471%	0,19	3,06%	0,22%	1,03%	0,17%
Stryker Corp	11,311%	0,08	0,23%	1,40%	0,20%	1,04%
Mondelez International Inc	13,085%	0,09	0,02%	1,91%	0,02%	1,40%
TJX Companies Inc	7,889%	0,15	6,00%	1,32%	1,96%	0,85%
General Electric Co	17,398%	0,64	0,04%	0,00%	0,02%	0,00%
Applied Materials Inc	19,773%	0,08	0,02%	0,01%	0,02%	0,01%
Estee Lauder Companies Inc	10,833%	0,00	0,01%	0,00%	0,01%	0,00%
Citigroup Inc	15,139%	0,40	3,77%	0,00%	1,19%	0,00%
Analog Devices Inc	18,235%	0,20	1,26%	0,27%	0,72%	0,26%
Northrop Grumman Corp	11,117%	0,05	0,02%	2,74%	0,02%	1,60%
Altria Group Inc	13,455%	0,21	0,63%	0,34%	0,46%	0,29%
Regeneron Pharmaceuticals Inc	10,993%	0,13	0,76%	0,41%	0,51%	0,36%
ServiceNow Inc	10,071%	0,13	0,23%	0,10%	0,21%	0,08%
Duke Energy Corp	5,969%	0,06	0,05%	0,01%	0,05%	0,01%
Southern Co	10,461%	0,02	0,00%	1,16%	0,00%	1,00%
Booking Holdings Inc	11,093%	0,13	0,05%	2,01%	0,05%	1,50%
EOG Resources Inc	53,749%	0,38	1,32%	0,23%	0,58%	0,19%
Schlumberger NV	11,917%	0,34	4,55%	0,01%	0,45%	0,01%
Progressive Corp	42,576%	-0,02	0,01%	1,91%	0,01%	2,02%
Vertex Pharmaceuticals Inc	15,686%	0,17	0,03%	0,02%	0,03%	0,02%
Becton Dickinson and Co	12,081%	0,11	0,71%	1,85%	0,47%	1,64%

Source: Self Elaboration