

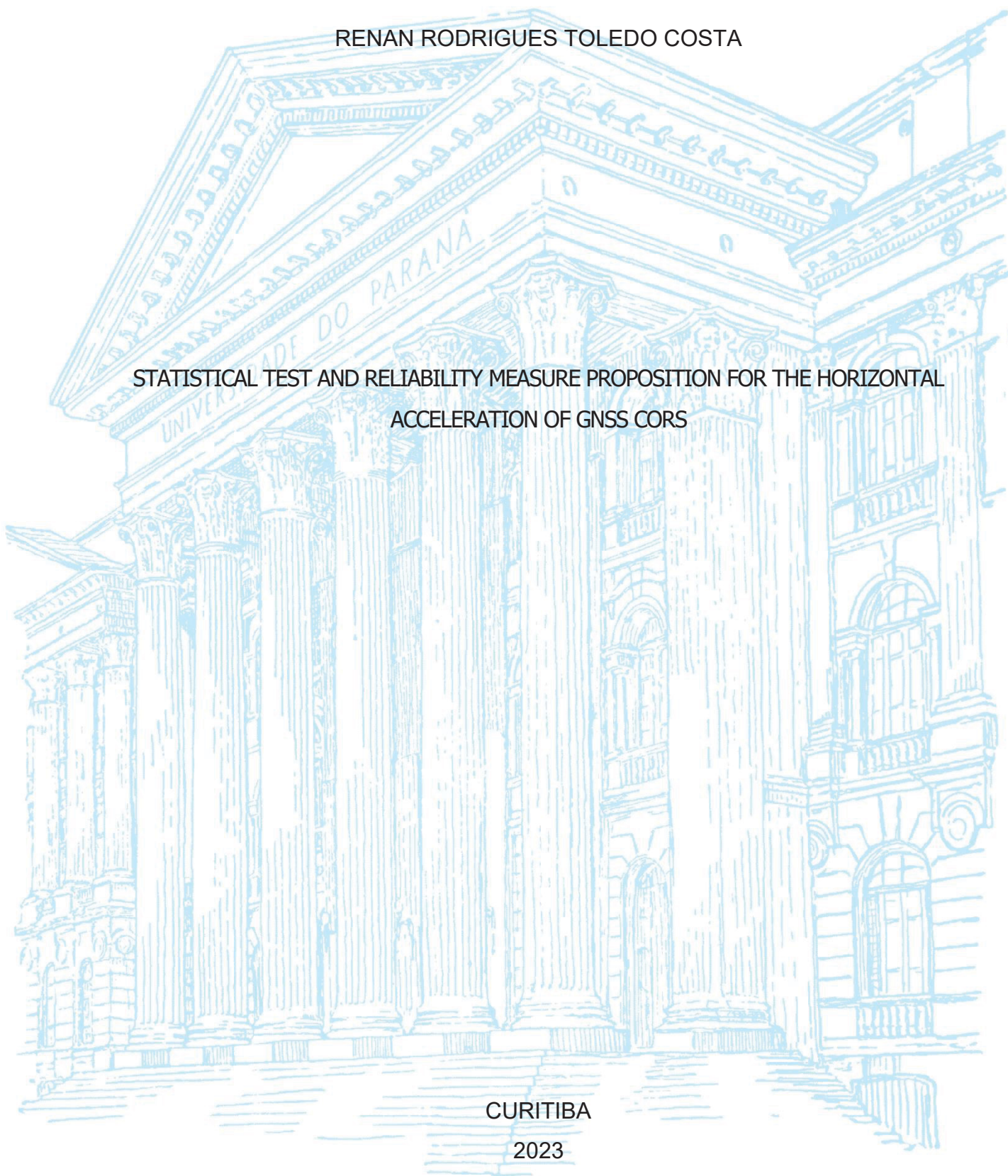
UNIVERSIDADE FEDERAL DO PARANÁ

RENAN RODRIGUES TOLEDO COSTA

STATISTICAL TEST AND RELIABILITY MEASURE PROPOSITION FOR THE HORIZONTAL
ACCELERATION OF GNSS CORS

CURITIBA

2023



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STATISTICAL TEST AND RELIABILITY MEASURE PROPOSITION FOR THE
HORIZONTAL ACCELERATION OF GNSS CORS

Dissertação apresentada ao curso de Pós-Graduação em Ciências Geodésicas, Setor de Ciências da Terra, Universidade Federal do Paraná, como requisito parcial à obtenção do título de Mestre em Ciências Geodésicas.

Orientador: Prof. Dr. Ivandro Klein

Coorientador: Prof. Dr. Paulo Sergio de Oliveira Junior

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"Science is more than a body of knowledge; it is a way of thinking, a way of interrogating the universe with a fine understanding of human fallibility."

(Carl Sagan)

RESUMO

As redes de estações ativas dos sistemas de posicionamento por satélites GNSS (Global Navigation Satellite System) têm sido utilizadas em diversos estudos relacionados à geodésia e geodinâmica, com o objetivo de modelar deformações e coletar dados que nos permitem entender o comportamento e o movimento das placas tectônicas. Devido ao deslocamento das placas litosféricas, as coordenadas sofrem mudanças contínuas ao longo do tempo, levando a uma dependência temporal em relação ao período de observação. As pesquisas atuais têm-se concentrado na análise de séries temporais de coordenada estimadas a partir das medidas registradas por redes ativas. No entanto, na maioria dos casos, apenas os efeitos de velocidade e cíclicos são incluídos no modelo funcional das séries temporais de coordenadas. O software Hector (HS) é comumente utilizado como ferramenta para extrair não somente os componentes periódicos, mas também os efeitos de tendência, tais como velocidade e aceleração. O HS emprega diferentes abordagens para a seleção do modelo, combinando Estimativa de Máxima Verossimilhança (MLE) com o Critério de Informação de Akaike (AIC) e o Critério de Informação Bayesiano (BIC). Nosso objetivo é realizar um teste estatístico derivado do teste da razão de verossimilhança generalizada para avaliar a significância da aceleração no modelo. Ao comparar os resultados dos métodos de seleção de modelo AIC e BIC utilizados no HS com o teste estatístico proposto, o teste pode servir como uma ferramenta adicional para a seleção do modelo, pois tem como uma vantagem fornecer uma análise mais objetiva quando comparado ao AIC e BIC, que aplicam penalidades aos modelos mais complexos. Além disso, esta pesquisa aborda a aplicação da teoria de confiabilidade de Baarda em um contexto diferente, com o objetivo de determinar a Menor Aceleração Horizontal Detectável (MDHA). Foram selecionados dados de estações do centro de análise do Nevada Geodetic Laboratory (NGL) para o estudo. Os experimentos mostram-se eficazes com 100% de concordância entre os resultados do teste e a MDHA.

Palavras-chave: deformação; séries temporais; Sistema Global de Navegação por Satélite; teste estatístico; critérios de informação.

ABSTRACT

Continuous Operating Reference Stations (CORS) have been widely used in various geodesy and geodynamic studies to model deformations and collect data that allow us to understand the behavior and movement of tectonic plates. Due to the displacement of the lithospheric plates, the coordinates undergo continuous changes over time, leading to a temporal dependence on the observation period. Current research has focused on analyzing time series from active networks of the Global Navigation Satellite System (GNSS). However, in most cases, only the effects of velocity and cyclic variations are included in the functional model of the GNSS coordinate time series. Hector software (HS) is commonly used as a tool to extract not only periodic components but also trending effects such as velocity and acceleration. HS employs different approaches for model selection, combining Maximum Likelihood Estimation (MLE) with the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). Here, we aim to perform a statistical test derived from the generalized likelihood ratio test to assess the significance of acceleration in the model. By comparing the results of model selection methods AIC and BIC used in HS with the proposed statistical test, the test can serve as an additional tool for model selection, as it has the advantage of providing a more objective analysis when compared to AIC and BIC, which apply penalties to more complex models. Additionally, this research addresses the application of Baarda's reliability theory in a different context, aiming to determine the Minimal Detectable Horizontal Acceleration (MDHA) of CORS. CORS data from the analysis center of the Nevada Geodetic Laboratory (NGL) were selected for the study. The experiments are shown to be effective with 100% agreement between the test results and the MDHA.

Keywords: deformation; time series; Global Navigation Satellite System; statistical test; information criteria.

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ABBREVIATION LIST

AIC	- Akaike Information Criterion
BIC	- Bayesian Information Criterion
CATS	- Create and Analyze Time Series
CORS	- Continuously Operating Reference Station
GGM	- Generalised Gauss Markov
GNSS	- <i>Global Navigation Satellite System</i>
GPS	- Global Positioning System
InSar	- Interferometric Synthetic Aperture Radar
ITRS	- International Terrestrial Reference System
ITRF	- International Terrestrial Reference Frame
JPL	- Jet Propulsion Laboratory
LiDAR	- Light Detection and Ranging
LLR	- Lunar Laser Ranging
MDB	- Minimal Detectable Bias
MDHA	- Minimal Detectable Horizontal Acceleration
MVC	- Covariance Matrix
NGL	- <i>Nevada Geodetic Laboratory</i>
SLR	- Satellite Laser Ranging
TEC	- Total Electron Content
VLBI	- Very Long Baseline Interferometry

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1 INTRODUCTION

The Earth's shape undergoes continuous changes due to mass redistribution at its surface and near-surface. Geodetic networks rely on GNSS (Global Navigation Satellite Systems), including GPS (Global Positioning System), to monitor these changes over time. Through GNSS position time series, researchers can capture small yet measurable displacements induced by loads on the Earth's surface. These loads result from various factors, such as fluctuations in sea level, atmospheric circulation, glacier and ice cap evolution, snowfall, and rainfall, as well as variations in water storage within rivers, lakes, soils, and groundwater aquifers (WHITE et al., 2022).

Coordinate time series play a vital role in the fields of high-precision geodesy and geodynamics. The precise and continuous measurements provided by coordinate time series are crucial for maintaining geodetic reference frames, which serve as the basis for accurate positioning and navigation systems. It is important for determining Earth's deformation processes, such as tectonic plate movements and vertical crustal deformation (KENYERES; BRUYNINX, 2004; ALINIA et al., 2017; REN et al., 2021; WHITE et al., 2022).

The use of GNSS has been used for different purposes, due to a series of innovations in recent years. As a result, GNSS has become important in different areas such as Geodesy, Geophysics, Hydrology, and Meteorology, among others. In parallel, GNSS has been used in modeling crustal movement based on time series data.

Various positioning systems are employed in geodesy that enable the monitoring of small changes on the Earth's surface. Examples include satellite InSAR (Interferometric Synthetic Aperture Radar), LiDAR (Light Detection and Ranging), Very Long Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR), Lunar Laser Ranging (LLR), DORIS, and the Global Navigation Satellite System (GNSS) – a system that uses satellite signals to accurately determine location (PLAG; PEARLMAN, 2009).

The so-called lithospheric plates are constantly moving in different directions and magnitudes. As a result of this motion, the position of a point located on one of the plates also varies over time, becoming dependent on the epoch in which it was determined. The geodetic coordinates of a point on the Earth's surface are determined concerning a specific geodetic reference frame, which is defined and realized in

advance. The internationally recognized reference frame used is the International Terrestrial Reference Frame (ITRF) (PEREZ, 2002).

The establishment and upkeep of a terrestrial reference frame (TRF) are of utmost importance in comprehending and investigating the linear and nonlinear changes in the Earth's solid shape over time. These changes are induced by a myriad of geophysical processes, including plate tectonics, glacial isostatic adjustment, and significant variations in surface mass related to the atmosphere, snow, glaciers, soil moisture, groundwater storage, and sea level rise. The TRF forms the foundational framework for studying and analyzing these dynamic processes, providing valuable insights into the Earth's ever-evolving shape and its geophysical phenomena. (LIU et al., 2021).

The coordinates of the stations used in the determination of the reference system, which is situated on the Earth's surface, require periodic updates. Consequently, multiple versions of the ITRF have been developed. For instance, ITRF2020 incorporates precise modeling of the nonlinear motions of stations, capturing seasonal signals (such as annual and semi-annual variations) observed in the positional time series. These regular updates play a vital role in maintaining accurate and current reference systems (ALTAMIMI et al., 2016, 2023).

In the time series of GNSS coordinates, it is generally assumed that each component of the coordinate series is described by the sum of a linear rate (velocity) and several periodic terms. Furthermore, the parameters of interest in GNSS coordinate time series are often the velocities and their associated uncertainties, which need to be determined with the utmost reliability. However, it is known that not all GNSS position time series follow this simple linear behavior (BOGUSZ et al., 2016).

According to Bogusz et al. (2016), since time series do not always follow a linear behavior, a quadratic polynomial term of acceleration is added to the model to better describe the nonlinear motion in the position time series.

Time series analysis by researchers has been increasingly employed in various applications, such as crustal deformation monitoring, seasonal effects elimination in GNSS coordinates, noise monitoring in dam monitoring data, analysis of multipath effects in GNSS signals at continuous monitoring stations, trend estimation, and determination of station velocities (GEIRSSON, 2003; ROSA, 2008; ALVES et al.,

2013; DIDOVA et al., 2016; KLOS et al., 2018; ALINIA et al., 2017; GHASEMI KHALKHALI et al., 2021).

However, there are few studies in the literature regarding the inclusion of acceleration in the analysis of time series. Many of these studies are related to structural monitoring or noise analysis. (KLOS; BOGUSZ, 2015; BOGUSZ et al., 2016; DURDAG et al., 2018,2020).

In the context of time series analysis, our research utilizes the so-called Hector, which is an open-source software developed in C++ and serves as a powerful tool for analyzing time series data obtained from GNSS stations (BOS et al., 2013). This study aims to contribute to the modeling of temporal series, proposing the use of the likelihood ratio test instead of the AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) adopted in the Hector scientific software. Additionally, the reliability theory proposed by Baarda (1968) has been used in a novel context. Instead of error detection in observations, the reliability theory will be employed with the main purpose of determining the Minimal Detectable Horizontal Acceleration (MDHA) of some GNSS CORS (Continuously Operating Reference Stations).

Thus, an algorithm based on reliability theory was developed for determining the MDHA. To apply this algorithm, the SCILAB software was used, as Hector does not provide the required adjustment matrices for this particular application. Additionally, data were collected every 6 months over 10 years, as it is not feasible to perform Hector's complete processing strategy daily in SCILAB. Therefore, our contribution lies in providing a more realistic modeling approach for the GNSS time series.

Hence, a preliminary analysis of the available data on the Nevada Geodetic Laboratory (NGL) website was conducted. For the comparison of the statistical test and the AIC and BIC models from Hector, the following stations were used: CLL1 and QLAP, located in Chile, and MIZU, located in Japan. These stations were chosen because they exhibit a graph with nonlinear behavior and a clear presence of acceleration in one of their components and because they are situated in a region of the tectonic plate boundary, meaning they are in a geodynamical active region.

However, the MIZU station exhibits different behavior due to the high incidence of seismic events, such as the earthquake that struck the Fukushima region with a magnitude of 9.1 on the Richter scale. Therefore, data from 2012 is used for the MIZU

station since the graph shows a stabilization after the period of the Fukushima earthquake in 2011. If the complete series were to be used, it would require the application of post-earthquake modeling within the ITRF, which is beyond the scope of this project.

For the application of the MDHA algorithm in SCILAB, in addition to the previously mentioned CLL1 station, the stations BRAZ and BELE located in Brazil were chosen. These Brazilian stations are located in the center of a tectonic plate. Stations in the central regions of tectonic plates tend to exhibit lower rates of movement, meaning the stations are moving more uniformly. The displacements recorded by the GNSS stations in these areas are more predictable, as they are in a more stable region.

In this research, the vertical direction was disregarded as it generally exhibits more complex cyclic variations, such as a sinusoidal pattern, mainly due to loading effects (FERREIRA et al., 2019).

All of the above needs to define the purpose of this dissertation, which is:

- Propose a statistical test to analyze the significance of taking into account the acceleration when modeling positional time series of GNSS CORS. Additionally, the research aims to determine the Minimal Detectable Horizontal Acceleration (MDHA) of some GNSS CORS.

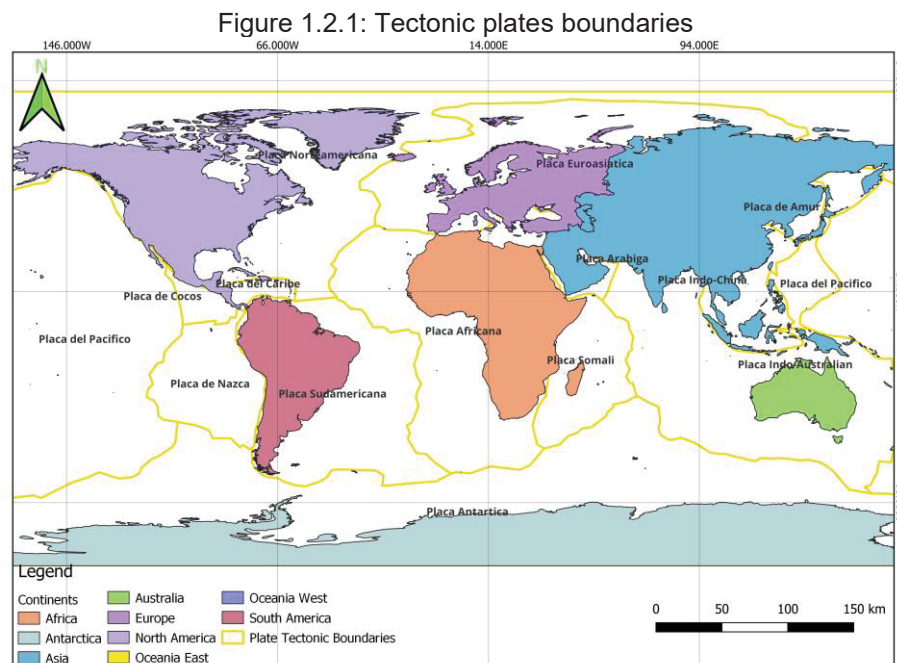
1.1 MAIN PURPOSES

- a) Use HS to obtain AIC and BIC values with and without acceleration models;
- b) Apply the statistical test to the models with and without acceleration in the GNSS CORS;
- c) Compare the results of AIC and BIC with the proposed statistical test;
- d) Propose and develop a mathematical model of MDHA using conventional reliability theory;
- e) Test the proposed new metric (MDHA) with data from the GNSS CORS;
- f) Results analysis by determining for which stations/locations the acceleration parameter is statistically significant or not.

1.2 JUSTIFICATION

The Earth's layer composed of the upper mantle and crust, during the process of lithospheric segmentation, has given rise to several irregularly shaped plates known as lithospheric plates. As a result, due to the release of heat from internal sources of the Earth, these plates continuously move over time in different directions and with varying magnitudes (ROSA, 2008).

The movements of the plates cause changes in the coordinates of stations over time, making them dependent on the epoch of observation. Therefore, the investigation and monitoring of plate motion often rely on the analysis of the temporal variation of position (positional time series) of points within geodetic networks. The temporal variation of position has been widely used to analyze and understand geophysical phenomena (DE FREITAS et al., 2022). Figure 1.2.1 shows the boundaries of the tectonic plates



Source: The author (2023).

50 years ago, before Alfred Wegener's theory of continental drift (or tectonic plates) was widely accepted and long before modern space geodetic techniques were developed, geodesists performed their geodetic surveys and described their geodetic networks by assigning three spatial coordinates to each geodetic station. Initially, these

coordinates were considered constant over time. Static coordinates are more appropriate for a static Earth, but they can also be used for a dynamic Earth as long as the existing positioning methods lack the precision to detect tectonic movements over long periods. However, with the advent of VLBI, SLR, and GPS, plate motions were easily resolved within one or two years and eventually within even shorter periods. (BELVIS; BROWN, 2014).

In recent years, the use of GNSS coordinate time series has become more frequent in several published works in the field, for example, De Freitas et al. (2022); Ren et al. (2021); Montillet; Bos (2020); Klos et al. (2018); Bogusz et al. (2016); Kreemer et al. (2014). Furthermore, the ITRF2020 was developed, which better models the nonlinear movements of the station, including seasonal signals (annual and semi-annual) of station positions and post-seismic deformation that has been subjected to major earthquakes. (ALTAMIMI et al., 2023).

Traditionally, the analysis of a time series can be accomplished by decomposing it into three types of components: trend, seasonality, and noise (random component). These effects need to be modeled to have a more accurate time series.

The inclusion of acceleration in the modeling of time series of coordinates is important to obtain a more precise representation of the trajectory over time, especially necessary for applications that require high precision, such as mapping and monitoring of tectonic plate movements. Neglecting acceleration in this modeling can lead to significant errors in position estimation.

As mentioned previously, one of the primary challenges of this study is to analyze the acceleration of GNSS CORS, thereby making a significant contribution to the research on the Earth's surface crustal dynamics. Moreover, we have introduced a novel statistical test that offers an objective criterion, distinct from the subjective classification criteria like AIC and BIC used in HS. This test can serve as a valuable supplementary tool in the process of model selection.

While there have been numerous studies in the literature on estimating the horizontal movement of active GNSS stations, typically only velocity and cyclical effects are included in the functional model of GNSS coordinate time series, neglecting non-linearity. In this regard, we have proposed a metric based on reliability theory to estimate the MDHA, which cannot be derived from AIC and BIC. This metric provides a reliability measure for the model's MDHA.

The importance of estimating MDHA based on Baarda's theory is crucial for establishing the system's detection limit. This is particularly relevant in sensitive applications such as remote sensing, geodetic monitoring, and studies involving subtle movements or low-magnitude events. Furthermore, determining MDHA allows for the assessment of the system's measurement quality. Higher MDHA implies greater sensitivity and capability of the system to detect small variations in measured accelerations. This is essential for ensuring precise and reliable results in experiments and scientific studies.

Therefore, knowing MDHA is vital for validating the results obtained from the measurement system. It enables us to determine whether the observed variations are above or below the detection limit, which is crucial for ensuring result reliability and avoiding misleading conclusions.

The estimation of MDHA in this study involves a direct physical interpretation, incorporating velocity, acceleration, and cyclical effects into the model being fitted.

1.3 DISSERTATION STRUCTURE

This dissertation is structured into four chapters, which encompass the research through the development of an extended abstract and a paper submitted to a journal that revolves around the proposed approach.

Chapter 1 provides the introduction to the research, which includes the research's main purposes and justification.

Chapter 2 focuses on the modeling of acceleration in the time series of some GNSS CORS. It presents the extended abstract titled "Acceleration estimation of GNSS CORS ", presented at the XII Brazilian Colloquium of Geodetic Sciences and V Brazilian Symposium on Geomatics in 2022. In this abstract, the acceleration at a GNSS CORS was analyzed. The results showed that the acceleration is statistically significant in both cases, indicating that considering acceleration leads to more accurate modeling of displacements in both the east and north directions for the CLL1 station. These findings highlight the importance of estimating acceleration in the time series of GNSS CORS.

Chapter 3 presents the paper titled "Estimation of the minimal detectable horizontal acceleration of GNSS CORS which will be submitted to a scientific journal in

2023. This paper introduces a new statistical test that provides an objective criterion, unlike the subjective classification criteria of AIC and BIC information criteria. Additionally, a metric based on reliability theory is proposed to estimate the MDHA, which cannot be derived from AIC and BIC. This metric provides a reliability measure for the model's MDHA. The proposed algorithm for obtaining MDHA was applied in the SCILAB software, and the results showed 100% agreement between the test results and MDHA.

Overall, this dissertation contributes to the field by analyzing acceleration in GNSS CORS and introducing a new statistical test and reliability measure for the estimation of MDHA, providing a more accurate understanding of the temporal series of GNSS CORS.

Chapter 4 presents the final considerations of this research, encompassing the conclusions reached and recommendations for future work.

In this chapter, the extended abstract titled "Acceleration estimation of GNSS CORS" is presented, which was presented at the XII Brazilian Colloquium of Geodetic Sciences and V Brazilian Symposium on Geomatics in 2022. In this abstract, the acceleration at the CLL1 station was analyzed.

This abstract received the award in the Master's category - Geodesy and Surveys at the XII CBCG and V SBG, IEEE/GRSS Brazil.

The format presented below follows the extended abstract pattern.

2 ACCELERATION ESTIMATION OF GNSS CORS

Kinematics studies are developed to model deformations, thus obtaining data that allow us to know the behavior and/or movement of tectonic plates, as an example we have the NNR-NUVELL1-A (No Net Rotation - Northern University Velocity Model) (LEICK, 2004; MCCARTHY; PETIT, 2004).

These displacements are caused due to the position of the terrestrial crust on the mantle of fluid material and the interactions between the multiple plates that form the terrestrial crust, due to this dynamic the coordinates undergo continuous alteration over time, thus causing a temporal dependence on the time in which they were observed, and therefore the temporal variation of position has been used as one of the main tools to analyze and understand geophysical phenomena, through its vectorial decomposition (ROSA, 2008; DE FREITAS, 2022).

Thus, the time series of GNSS (CORS are an object of study, and for the most part, velocity estimation is the main focus (BAYRAK et al., 2015). There are a few kinds of research in the literature on the subject related to velocity and acceleration, in our knowledge all of these are related to the geodetic monitoring of structures (LIMA et al., 2013, 2014).

Hence, this research aims to analyze the acceleration of GNSS CORS, thus contributing to studies that consider the crustal dynamics of the Earth's surface. The methodology of this research made use of data from a GNSS CORS made available by the University of Nevada (Nevada Geodetic Laboratory - NGL) (NGL, 2022), these files with calculated coordinates and their respective precisions, using the GipsyX software (version 1.0) from JPL (Jet Propulsion Laboratory) (NGL,2022).

The processing method contains individual station data files of daily position time series in ITRF2014 concerning the position at the mid-time series epoch. These positions are expressed in meters in the local frame (North, East, and Up). The reference position and the 3D velocity (obtained using MIDAS) in the local coordinate system (East, North, Up) are provided in the header of each file (NGL, 2022).

After an analysis of the data available on the NGL website, the CLL1 station was selected for the behavior of the graph of its coordinates, with evident acceleration in the east component. The CLL1 station is part of a network located at the Universidad de Concepción, in the city of Chillan, in a volcanic region of the Ñuble Region in Chile.

A reference epoch was chosen, 06/01/2011, and the analyzed period comprises 10 years, that is, between 06/01 to 06/01/2021, considering coordinates displacement values in the “East” (E) directions and “North” (N) in annual periods. The vertical direction was disregarded, as it generally presents more complex cyclic variations, such as a sinusoidal pattern. After selecting the reference epoch for the initial count on June 1, 2011, data for each of the 365 days was utilized, as displayed in Table 2.1.

Table 2.1 – Data considered in the experiment

Date	Time: $\Delta t^{0 \rightarrow i}$ (anos)	Displacement East: $\Delta E^{0 \rightarrow i}$ (m)	Displacement North: $\Delta N^{0 \rightarrow i}$ (m)	Standard deviation: East (m)	Standard deviation: North (m)
06/01/2011	0	0	0	0.0008	0.0008
06/01/2012	1	-0.0647	0.0132	0.0007	0.0008
06/01/2013	2	-0.1075	0.0333	0.0007	0.0008
06/01/2014	3	-0.1297	0.0515	0.0007	0.0008
06/01/2015	4	-0.1459	0.0710	0.0007	0.0008
06/01/2016	5	-0.1623	0.0794	0.0006	0.0008
06/01/2017	6	-0.1686	0.0939	0.0006	0.0008
06/01/2018	7	-0.1750	0.1053	0.0007	0.0008
06/01/2019	8	-0.1756	0.1212	0.0007	0.0008
06/01/2020	9	-0.1795	0.1356	0.0007	0.0008
06/01/2021	10	-0.1760	0.1468	0.0007	0.0008

Source: The authors (2022).

The adjustment matrices for the parametric model to obtain velocity and acceleration were prepared considering the classic equations of uniformly accelerated rectilinear motion in Physics (Equation 1). In this case, the Jacobian matrix (matrix A) is obtained through the adjustment model, which expresses the observations (displacements) as a function of the parameters to be estimated (velocity and

acceleration), while the standard deviations (initially of the coordinates) allow obtaining the variance-covariance matrix (MVC) of the displacements and, therefore, the adjustment weight matrix (LIMA et al., 2014).

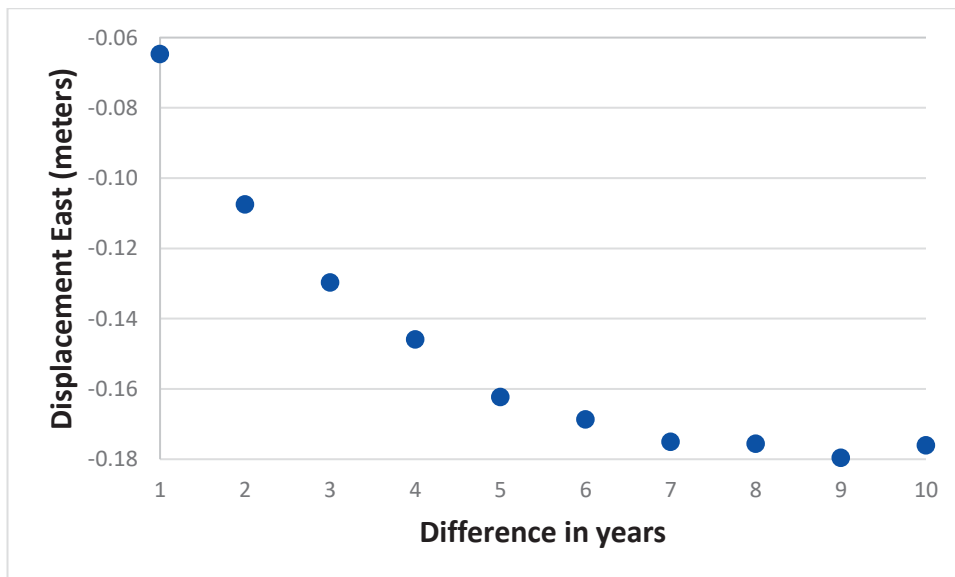
$$\Delta E^{0 \rightarrow i} = V_E \Delta t^{0 \rightarrow i} + \frac{1}{2} a_E (\Delta t^{0 \rightarrow i})^2$$

$$\Delta N^{0 \rightarrow i} = V_N \Delta t^{0 \rightarrow i} + \frac{1}{2} a_N (\Delta t^{0 \rightarrow i})^2$$
(1)

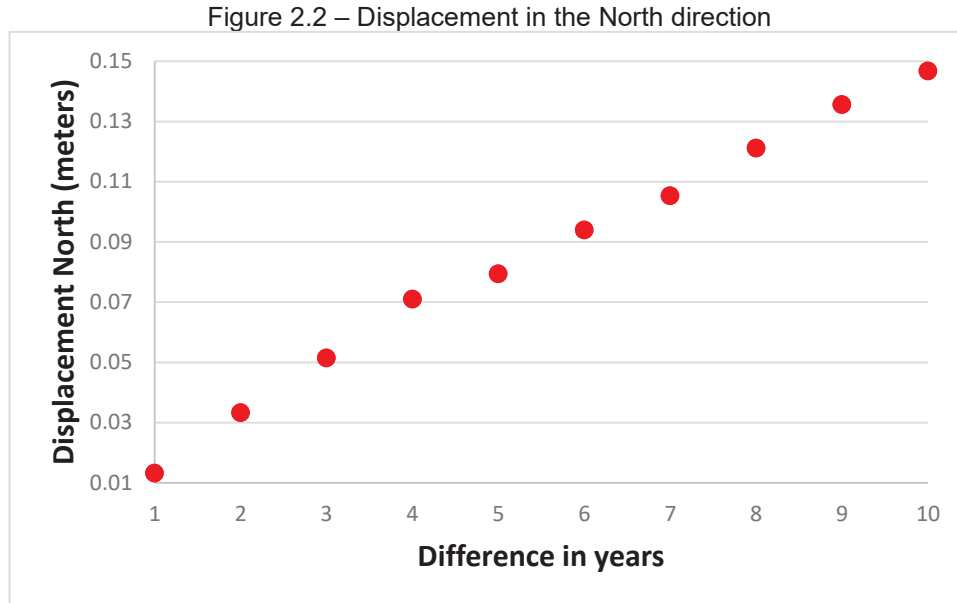
The vector of observations (displacements) is given by the elements of the third column, starting from the second row of Table 2.1 for the east component, and by the elements of the fourth column, starting from the second row of Table 2.1 for the north component. It is observed that the adjustment model is linear concerning the parameters (velocity and acceleration).

Figures 2.1 and 2.2, relative to the time series in the analyzed period, clearly indicate a non-linear displacement (with acceleration) in the east component Figure 2.1, and a relatively linear displacement, with or without low acceleration, in the north component Figure 2.2.

Figure 2.1 – Displacement in the East direction



Source: The authors (2022).



Source: The authors (2022).

Table 2.2 shows the results obtained considering the estimation of velocity and acceleration in both cases in the adjustment; while Table 2.3 presents the results obtained considering the estimation of velocity only in the adjustment (considering zero acceleration) in both cases.

Table 2.2: Estimated velocity and acceleration (East and North components)

Elements	East	North
v(mm/year)	-49.9 ± 0.21	17.6 ± 0.23
a(mm/year ²)	6.7 ± 0.05	-0.6 ± 0.06

Source: The authors (2022).

Table 2.3: Estimated Velocity considering zero acceleration (East and North components)

Component	East	North
v (mm/year)	-23.6 ± 0.05	15.3 ± 0.06

Source: The authors (2022).

The velocities estimated by the NGL (NGL, 2022) were -0.97 mm/year and 14.34 mm/year in the east and north components, respectively. The values obtained for the north component are in agreement with those estimated by the NGL, but the velocity in the east component is very discrepant, which requires a deeper analysis of the strategy adopted by the NGL.

Finally, a statistical test was carried out at a significance level of 5% to verify whether the estimated acceleration is statistically significant in both cases, using the likelihood ratio, with the results presented in Table 2.4. In this case, the statistics of the

test are given by the difference between the weighted sum of squared residuals of the model without acceleration (null hypothesis) and the model with acceleration (alternative hypothesis). If this test statistic is greater than the critical value according to the stipulated significance level, the alternative hypothesis is not rejected (TEUNISSEN, 2006).

Table 2.4: Statistical results testing the significance of the estimated acceleration (5% significance level)

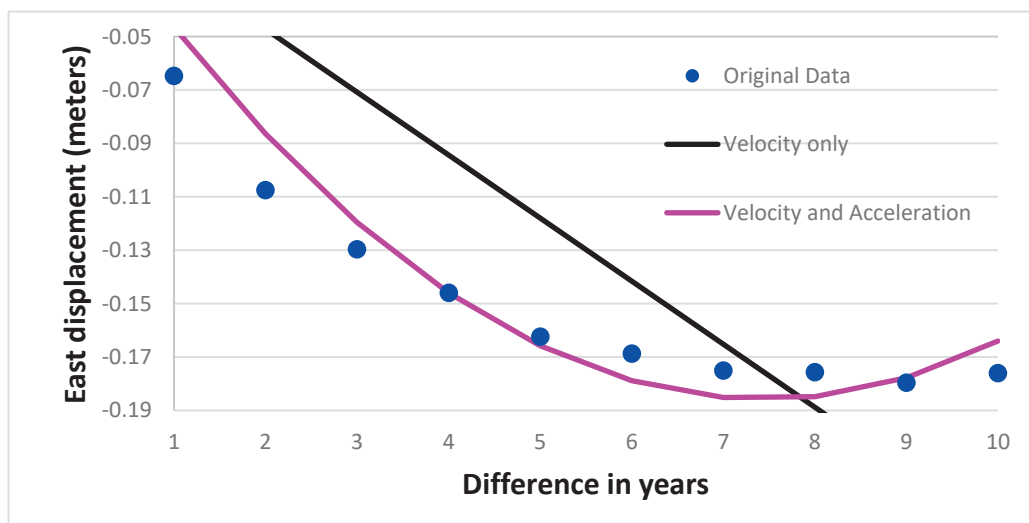
Critical Test Value	Calculated Test Statistic	
	East	North
3.84	17047.52	102.95

Source: The authors (2022).

Analyzing Table 2.4, as the calculated statistics exceed the critical test value, it is noted that the acceleration was statistically significant in both cases, that is, the consideration of acceleration produces a more adequate modeling of the displacements both in the east and in the east direction. North direction for the GNSS CORS considered in the analyzed period.

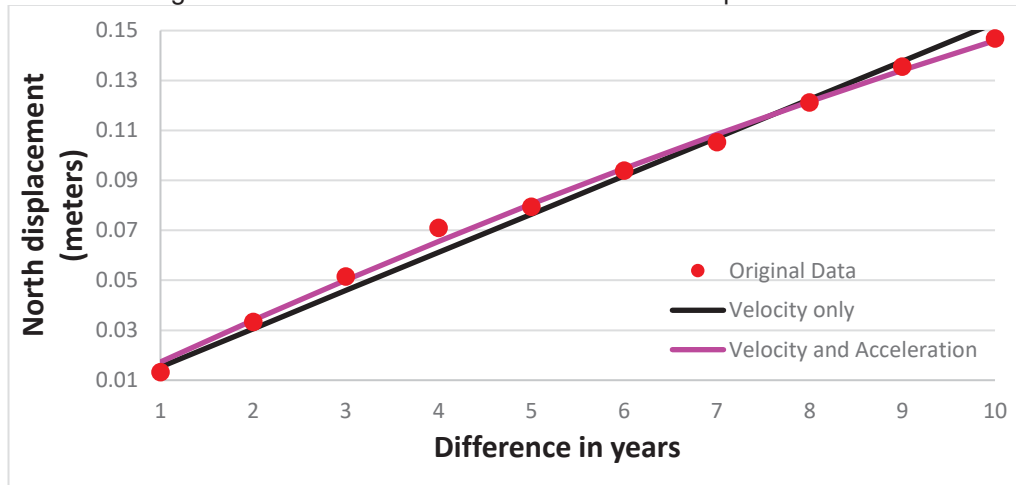
To visualize these differences, Figures 2.3 and 2.4 show the straight line (in black) of the model with only velocity and the curve (in magenta) of the model with velocity and acceleration for the east and north components, respectively. Although visually the difference is greater in the east component, it is noteworthy that the estimated acceleration was statistically significant in both cases.

Figure 2.3 – Estimated models for the East component series



Source: The authors (2022).

Figure 2.4 – Estimated models for the North component series



Source: The authors (2022).

These preliminary results demonstrate the importance of estimating acceleration in the time series of GNSS CORS, as proposed in this research. For its continuity, it is intended to analyze more GNSS CORS in a larger area, being able to investigate joint velocity and acceleration estimations (same acceleration and “block” velocity for all CORS considered), as well as individual ones (such as carried out in this first experiment), aiming to better understand the pattern of geodynamic deformations that occur in these regions. In the next stages of this research, other sources of data for the time series of GNSS CORS besides the University of Nevada should also be used, such as data from the SIRGAS-CON network (SIRGAS, 2022). In addition, new alternatives will be proposed for measuring the degree of reliability, to obtain values that express the “minimal detectable acceleration of the model”, depending on the adjustment matrices and the conventional reliability theory proposed in Baarda (1968).

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In this chapter, we present the paper titled "Estimation of the minimal horizontal detectable acceleration of GNSS CORS," which will be submitted to a scientific journal in 2023.

3 ESTIMATION OF THE MINIMAL DETECTABLE HORIZONTAL ACCELERATION OF GNSS CORS

ABSTRACT

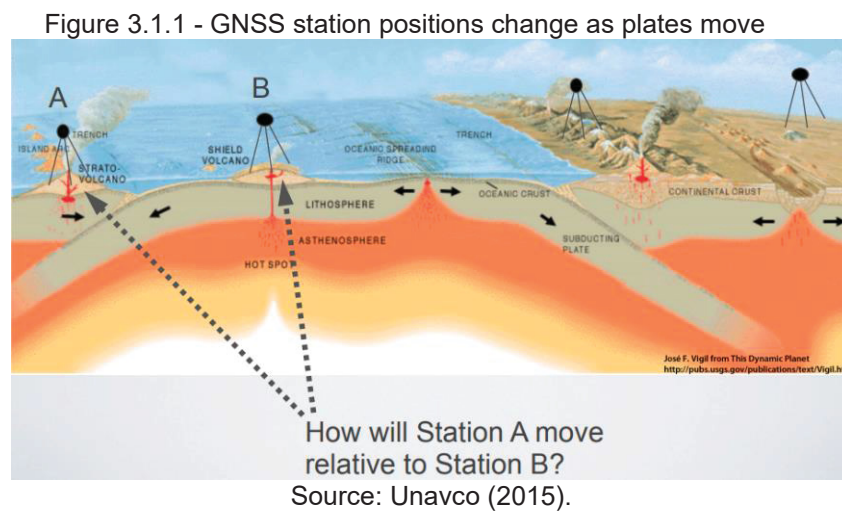
CORS (Continuously Operating Reference Stations) network of GNSS (Global Navigation Satellite System) has been extensively used in geodynamic studies to model deformations and understand the movement of tectonic plates. These station coordinates undergo continuous changes due to the ongoing movement, creating a temporal dependence on the observation period. The Hector software (HS) is commonly employed to analyze these temporal series and extract periodic and trend components from GNSS time series data such as velocity and acceleration. The HS utilizes different approaches for model selection. These approaches combine Maximum Likelihood Estimation (MLE) with the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). Our objective is to contribute to the trajectory model selection by proposing a statistical test derived from the generalized likelihood ratio test to analyze the significance of the acceleration in the model. By comparing the results of the AIC and BIC models used in HS with the proposed statistical test, the statistical test can serve as an additional source for the important task of model selection. Another aspect addressed in this paper is the application of Baarda's reliability theory in a different context, aiming to determine the Minimal Detectable Horizontal Acceleration (MDHA) of CORS. CORS data from the NGL (Nevada Geodetic Laboratory) analysis center were selected for the study. The experiments demonstrated a good agreement between the test results and the MDHA, validating the effectiveness of the proposed statistical test in this context.

Keywords: Global Navigation Satellite System; time series; statistical test; information criteria.

3.1 INTRODUCTION

Geodesy has provided important contributions to geodynamic studies, mainly with the development of positioning and navigation by GNSS (Global Navigation Satellite System). The rise of GNSS systems and long-term accumulated GNSS observations from global reference stations have provided valuable information data for Geodesy and Geodynamics studies since the 1990s. The GNSS CORS, serve not only as a static geodetic reference but their long-term observation series provide also

(geo) kinematical information (KENYERES; BRUYNINX, 2004), being important to determine Earth's deformation processes such as the tectonic plate movements as well as the vertical deformation of the crust (REN et al., 2021). The so-called lithospheric plates are constantly moving in different directions and displacement sizes. Because of this movement, the position of a point located in a lithospheric plate also varies with time Figure 3.1.1.



The geodetic coordinates on the Earth's surface are determined by a specific geodetic reference system, previously defined and realized. The reference used internationally is the International Terrestrial Reference System (ITRS) and its realization is the International Terrestrial Reference Frame (ITRF). In addition, the ITRF materialization consists of a set of positions associated with a certain reference epoch and respective velocities. The stations coordinates used in the determination of the reference system, located on the Earth's surface, need to be periodically updated. As a result, several versions of the ITRF are available. For example, ITRF2020 accurately models the nonlinear station motions for seasonal signals (annual and semi-annual) present in the station's positional time series. These updates are crucial for maintaining accurate and up-to-date reference systems (ALTAMIMI et al., 2012, 2023). As cited by Plag and Pearlman (2009), the main purpose of the ITRF is to combine positions and velocities of the stations under the responsibility of different analysis centers, using observations from different space geodetic techniques (eg. VLBI, SLR/LLR, DORIS, and GNSS) and make available

lists of coordinates and velocities of this group of stations with the respective accuracies.

In recent years, the use of GNSS coordinates time series has become more frequent in several kinds of research, such as Kreemer et al. (2014); Bogusz et al. (2016); Klos et al. (2018); Montillet; Bos (2020); Ren et al. (2021). Obtaining daily solutions of time series coordinates of GNSS CORS, considering data from more than 10 years, represent a considerable sampling for analysis of station components and allows the assessment of the spatial and temporal evolution.

When it comes to GNSS coordinate time series analysis, usually, it is assumed that each of the coordinate component series is described by the sum of the linear rate and several periodic terms. The annual and semi-annual seasonal signals affect GNSS time series coordinates (VAN DAM et al., 2001; DONG et al., 2002). Furthermore, frequently, the parameters of interest in the time series of GNSS coordinates are the velocities and their associated uncertainties, which must be determined with higher reliability. However, it is well-known that not all GNSS position time series follow this simple linear behavior. According to Bogusz et al. (2016), since the time series do not always follow a linear behavior, an acceleration term in the form of a quadratic polynomial function is added to the model to better describe the non-linear movement. This non-linear motion may be a response to purely geophysical processes.

In this paper, we used Hector software (HS), an open-source academic software package that can be used to estimate a trajectory model, for example, a linear trend with an annual and semiannual signal, in time series with temporally correlated noise. The HS is used to obtain the periodic and trend terms in the horizontal directions of the GNSS time series (BOS et al., 2013). The trend can be linear or a higher degree polynomial; one can also estimate periodic signals, offsets, and post-seismic deformation. Together they represent the model that is fitted to the observations.

In the past decades, there are several studies in the literature on estimating the horizontal movement of CORS, however, without taking into account the non-linearity trend (KREEMER et al., 2014). Recently the improved modeling of non-linear station movements had been addressed, e.g., in recent ITRF2014 and ITRF2020 realizations (ALTAMIMI et al., 2016, 2023). It should be noted that acceleration

estimation has already been investigated in GNSS networks for monitoring artificial structures (DURDAG et al., 2018).

This study aims to contribute to the modeling of CORS time series by introducing the application of the generalized likelihood statistical test proposed by Teunissen (2006), as an alternative to the information criteria or Bayes criteria employed in the HS method. Additionally, the reliability theory proposed by Baarda (1968) will be used in a novel context, aiming at the determination of the Minimal Detectable Horizontal Acceleration (MDHA).

In this study, CORS data were selected from the NGL (Nevada Geodetic Laboratory) analysis center. Thus, the QLAP and CLL1 stations located in Chile, and the MIZU station located in Japan were selected for processing in HS. The choice of these stations was based on their evident nonlinear behavior in at least one of their horizontal components. For MDHA analysis, the station CLL1 in addition to the stations BRAZ and BELE located in Brazil (in a central region of the tectonic plate) was chosen.

The paper is structured as follows:

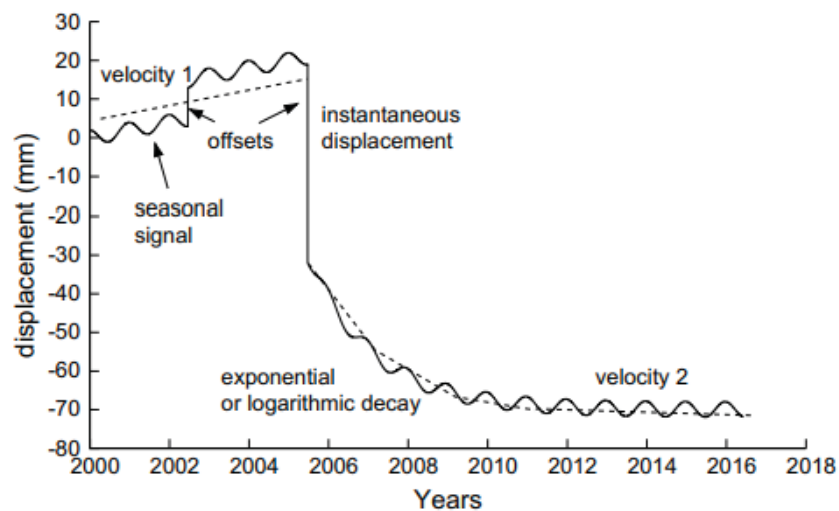
- The "Time Series" section provides a comprehensive definition and the mathematical model used for the analysis of time series. The "Hector (Analysis of Time Series)" section describes the processing strategies employed to obtain the AIC and BIC selection models.
- The "Proposed Statistical Approach to Testing the Acceleration" section outlines the steps for applying the proposed statistical test.
- In the "Hector Criteria versus Statistical Test" section, statistical tests are conducted and compared with the corresponding values of the AIC and BIC models calculated by the HS.
- The "Minimal Detectable Horizontal Acceleration (MDHA)" section introduces a new metric proposed for detecting the smallest detectable acceleration based on Baarda's reliability theory.
- "Results and Discussions" section presents the assessments and outcomes of this study.
- "Conclusions" section provides final considerations and recommendations for future research.

3.2 TIME SERIES

As shown by Bos et al. (2020), a geodetic time series consists of a set of observations at various epochs. When it comes to the time series of GNSS coordinates, it can be stated as a sequence of position data from a station, collected by GNSS receivers over time.

As demonstrated by Montillet; Bos (2020), it can be said that, by definition, coordinate time series are trajectories, therefore, the kinematic models that geodesists and geophysicists use to describe these time series are trajectory models. Figure 3.2.1 illustrates an exemplary integration of all these models into a unified trajectory model.

Figure 3.2.1 - Sketch of a trajectory model containing common phenomena



Source: Montillet and Bos (2020).

The Earth undergoes various geodynamic processes that can have a direct impact on the computation of coordinates, affecting the deformation of the Earth's surface. Therefore, seismic events in the vicinity of positioning points can introduce errors in the horizontal component (LANGBEIN et al., 2006; BEVIS; BROWN, 2014). The analysis of the GNSS position time series involves a preliminary assessment of the linear trend v_i , seasonal elements (annual and semiannual), the coefficients a_p and b_p represent the amplitudes of the harmonic terms of cosine and sine, respectively. Additionally, angular velocities ω_p are modeled as a combination of sine and cosine, defined by time difference Δt (BOGUSZ et al., 2016). In our research, we consider the inclusion of the acceleration in the form of a quadratic term allowing us

to describe non-linear motion in the functional model. This approach aims to provide a more realistic representation of the time series.

According to Bevis; Brown (2014), a suitable way to describe the GNSS station's movement is through functional (or trajectory) models. We assume that the observations are the sum of a deterministic model and stochastic noise, where Δx_{0-k} is the displacement from the reference epoch t_0 to epoch t_k (in mm), Δt_{0-k} is the time difference from the reference epoch t_0 to considered epoch t_k (in years), v_i is the velocity of the horizontal component (in mm per year), ∇_i is the acceleration of the horizontal component (in mm per year²), a_p e b_p are the amplitude coefficients for the sine and cosine terms of the periodic signal, and ω_p is the angular velocity. The observed motion of each site in each direction can be written as shown below (MONTILLET; BOS, 2020):

$$\Delta x_{0-k} = \Delta t_{0-k} v_i + \frac{\Delta t_{0-k}^2}{2} \nabla_i + \sum_{j=1}^{n_j} c_j H(t_k - t_j) + \sum_{p=1}^2 a_p \sin(\omega_p t_k) + b_p \cos(\omega_p t_k) \quad (01)$$

where H is the Heaviside or unit step function, c_j describes the direction and magnitude of the jump which occurs at the time t_j , n_j is the number of jumps.

3.2.1 ANALYSIS OF TIME SERIES BY HECTOR SOFTWARE (HS)

HS is an open-source software developed in C++ and serves as a powerful tool for analyzing time series data obtained from GNSS stations (BOS et al., 2013).

Based on the Maximum Likelihood Estimation (MLE), HS utilizes a robust framework that jointly estimates deterministic and stochastic models. This approach enhances the accuracy and reliability of the obtained results. Moreover, the methodology employed by HS has been adopted by other software packages such as CATS (Create and Analyze Time Series) and East_noise, further attesting to its effectiveness. The obtained parameters that describe the position, seasonal signals, velocity, acceleration, and the combination of GGM (Generalised Gauss Markov) and white noise model were estimated using MLE (BOS et al., 2013).

The HS employs two analysis models: 1) MLE supplemented with model selection using AIC (Akaike Information Criterion) and 2) BIC (Bayesian Information Criterion), and spectral analysis. In this article, our focus will be on the first approach,

which utilizes MLE in conjunction with the AIC and the BIC. The AIC and BIC are employed to select the most suitable model from a set of competing models. The AIC was initially proposed by Akaike (AKAIKE, 1974) as an alternative to compare different models based on a given outcome. On the other hand, the BIC, introduced by Schwarz (SCHWARZ, 1978), serves as a criterion for model selection among a finite set of models. The methodology begins with MLE and subsequently incorporates penalties to the model to prevent overfitting.

The rule for selecting a model based on the AIC is that a smaller AIC value indicates a better-fitting model. Similarly, for the BIC, a smaller BIC value indicates a better model fit. In general, the HS incorporates robust noise analysis methods to treat the characteristics of the noise signals. The total variance of the noise is set by $\hat{\sigma}^2$, which is usually called the 'driving' noise (BOS et al., 2013; HE et al., 2018; MONTILLET; BOS, 2020).

The definition of the log-likelihood is expressed as follows:

$$\ln(L) = -\frac{1}{2}[N \ln(2\pi) + \ln \det(C) + \hat{v}^T C^{-1} \hat{v}] \quad (02)$$

where N is the actual number of observations (gaps do not count), \hat{v} is the residual vector and C is the covariance matrix $C = \sigma^2 \bar{C}$, which is a function of the sum of various noise models and the estimated standard deviation of the residuals.

The number of parameters is denoted by K, which is the sum of the parameters present in the design matrix and the noise models and also the variance of the driving white noise process $\hat{\sigma}^2$. The formulas for analyzing different noise models using AIC and BIC standards are as follows (BOS et al., 2013):

$$AIC = 2K + 2\ln(L) \quad (03)$$

$$BIC = K \ln(N) + 2\ln(L) \quad (04)$$

$$BIC_{tp} = K \ln\left(\frac{N}{2\pi}\right) + 2\ln(L) \quad (05)$$

The model selection process in HS involves choosing the model with the minimum value of either the AIC or the BIC. It is important to understand that these criteria serve as relative measures for comparing different model choices rather than absolute criteria. In addition to AIC and BIC, HS offers BIC_{tp} and BIC_c as options for model selection. These additional criteria provide a more comprehensive assessment

of the models by considering extra penalties and factors. BIC_c in particular, allows for a more nuanced evaluation compared to the standard BIC. For details, (see HE et al. 2019).

3.3 PROPOSED STATISTICAL APPROACH TO TESTING THE ACCELERATION

The sample space, which is the set of all possible outcomes for the experiment, is divided into two parts: 1) the rejection region, also known as the critical region and 2) the acceptance region, also known as the non-rejection region, of the null test hypothesis (TEUNISSEN, 2006). In this case, the null hypothesis H_0 is expressed assuming null acceleration ($\nabla_i = 0$). Here a statistical test is proposed to determine if there is significant acceleration in the East and North component, as represented below:

$$H_0: y = Ax: \Delta x_{0-k} = \Delta t_{0-k} v_i + \sum_{j=1}^{n_j} c_j H(t_k - t_j) + \sum_{p=1}^2 a_p \sin(\omega_p t_k) + b_p \cos(\omega_p t_k) \quad (06)$$

In such a case, the alternative hypothesis H_A the acceleration is significant ($\nabla_i \neq 0$). In other words, here it is tested whether the $q = 1$ additional parameter in the model (∇_i) is statistically significant or not according to the generalized likelihood ratio test (see TEUNISSEN, 2006):

$$H_A: y = Ax + C_{q=1} \nabla_{q=1}: \Delta x_{0-k} = \Delta t_{0-k} v_i + \frac{\Delta t_{0-k}^2}{2} \nabla_i + \sum_{j=1}^{n_j} c_j H(t_k - t_j) + \sum_{p=1}^2 a_p \sin(\omega_p t_k) + b_p \cos(\omega_p t_k) \quad (07)$$

Regarding the decision about the hypotheses, two types of errors can be made. Type I error is about rejecting H_0 when it is true, that is, the occurrence of a "false positive", with probability of occurrence designated by α (the significance level of the test). On the other hand, Type II error is about accepting H_0 when H_0 is false, that is, when H_A is true, whose probability of occurrence is designated by β . In other words, β corresponds to the probability of a "false negative".

The test statistic (T_q) is given by the difference between the weighted sum of squared residuals (w.s.s.r) under the null hypothesis ($\hat{v}^T W \hat{v}$) and the w.s.s.r under the alternative hypothesis ($\hat{v}_A^T W \hat{v}_A$) as demonstrated in Teunissen (2006), W is the weight matrix being the inverse of the covariance matrix of observations. With an unknown variance factor as considered in HS, the test statistic T_q follows the F -distribution with $q = 1$ degrees of freedom in the numerator and K degrees of freedom in the

denominator in H_0 . Otherwise, it follows the χ^2 -distribution with $q = 1$ degrees of freedom in H_0 . Thus, the test decision is given by:

$$T_q = \hat{v}^T W \hat{v} - \hat{v}_A^T W \hat{v}_A, \text{ do not reject } H_0 \text{ if } T_q \leq F_{q=1, K, \alpha} \quad (08)$$

However, HS does not provide the w.s.s.r in each model H_0 and H_A . Therefore, considering the relation between the estimated (a posteriori) variance factor $\hat{\sigma}^2$ and w.s.s.r (see, e.g., LEHMANN; LÖSLER, 2016), we have equation (09) for calculating $(\hat{v}^T W \hat{v})_{H_i}$ for cases with acceleration ($H_i = H_A$) and without acceleration ($H_i = H_0$), based on the respective STD_{H_i} (standard deviation of the driving noise) obtained in the HS output report:

$$(\hat{v}^T W \hat{v})_{H_i} = STD_{H_i}^2 (N - K_{H_i}) \quad (09)$$

Where N is the number of observations and K_{H_i} is the number of model parameters in H_i for H_0 or H_A : $K_{H_A} = K_{H_0} + 1$.

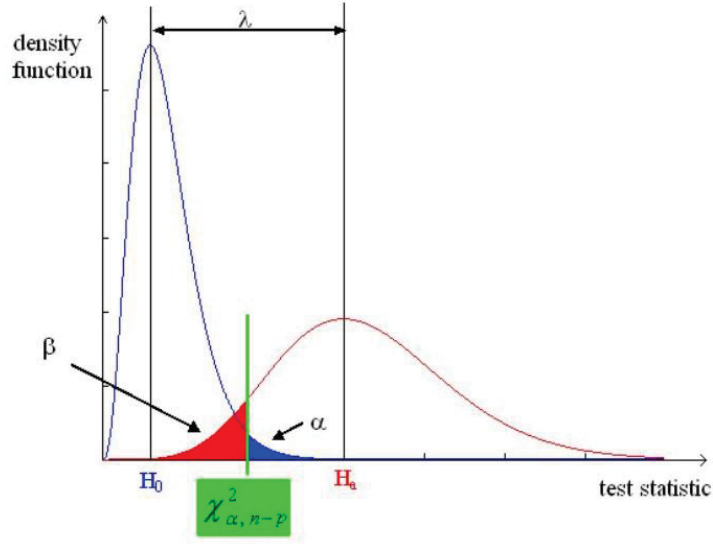
3.4 MINIMAL DETECTABLE HORIZONTAL ACCELERATION (MDHA)

One can also start from the conventional reliability theory proposed by Baarda (1968), where a study related to the detection and identification of non-random errors was proposed, using statistical tests for the detection and identification of these errors. Reliability theory employs appropriate measures to quantify the MDB ("Minimal Detectable Bias") of each observation. Baarda's reliability theory seeks to test the significance of the additional parameter of H_A , in that case, the possible bias in observation, and determine the threshold value at which this additional parameter becomes significant or detected by the statistical test.

Here, a novel reliability measure is presented considering the reliability theory proposed by Baarda (1968). In this sense, our additional parameter in the alternative hypothesis is not a bias in observation, but the acceleration of a horizontal component of a GNSS CORS. Thus, our objective will be to determine the MDHA (Minimal Detectable Horizontal Acceleration).

The separation between the mean vector of H_0 and H_A is given by the so-called non-centrality parameter of the model Figure 3.4.1, which in a general case with known variance factor is given by (TEUNISSEN, 2006; ROFATTO et al., 2018):

Figure 3.4.1 - The central and non-central density functions.



Source: Kuusniemi and Lachapelle, (2004).

$$\lambda = \nabla_q^T C_q^T W \Sigma_{\hat{\rho}} W C_q \nabla_q \quad (10)$$

where $C_q \nabla_q$ is the model error with q additional parameters ∇_q in H_A and C_q is a known (pre-stipulated) coefficient matrix. If the variance factor is unknown, then the non-centrality parameter is denoted as:

$$\lambda = \frac{\nabla_q^T C_q^T W \Sigma_{\hat{\rho}} W C_q \nabla_q}{\hat{\sigma}^2} \quad (11)$$

In our case, we have $q = 1$ additional parameter (acceleration term) in H_A and thus the non-centrality parameter is given by:

$$\lambda = \nabla_i c_i^T W \Sigma_{\hat{\rho}} W c_i \nabla_i \quad (12)$$

where $c_i = \begin{bmatrix} \frac{\Delta t_{0-1}^2}{2} \\ \frac{\Delta t_{0-2}^2}{2} \\ \vdots \\ \frac{\Delta t_{0-k}^2}{2} \end{bmatrix}$ (see Equation 07) and $\Sigma_{\hat{\rho}}$ is the covariance matrix of residuals in H_0 .

We suggest Teunissen (2006) for details. Isolating the term ∇_i results in:

$$\nabla_i^2 = \lambda(c_i^T W \Sigma_{\hat{\varphi}} W c_i)^{-1} \Rightarrow |\nabla_i| = \sqrt{\frac{\lambda}{c_i^T W \Sigma_{\hat{\varphi}} W c_i}} \quad (13)$$

However, the value of λ is unknown. So, considering a pre-stipulated value for $\lambda = \lambda_0$ (see BAARDA, 1968; TEUNISSEN, 2006), we have the MDHA absolute value as follows:

$$\text{MDHA} = \sqrt{\frac{\lambda_0}{c_i^T W \Sigma_{\hat{\varphi}} W c_i}} \quad (14)$$

The model's non-centrality parameter λ_0 is obtained as a function of the stipulated probability levels in F -distribution: $\lambda_0 = f(q = 1, N - K, \alpha_0, \beta_0)$ or χ^2 -distribution: $\lambda_0 = f(q = 1, \alpha_0, \beta_0)$, (see PRÓSZYŃSKI; ŁAPIŃSKI, 2021) for details. For example, if $q = 1$ (meaning only one parameter is added to the model), fixing the significance level of the test at $\alpha_0 = 0.05$ (5%), $\beta_0 = 0.2$ (20%), $N - K = 18$, the non-centrality parameter of the model in F -distribution is given by $\lambda_0 = 8.7774$.

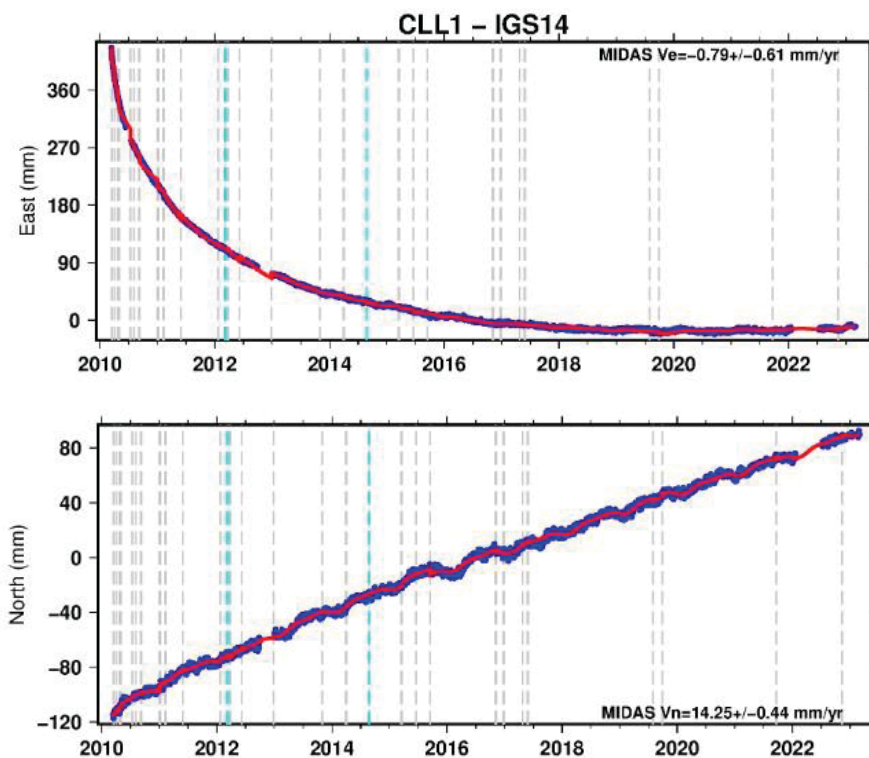
3.5. DATASET AND EXPERIMENTS DESCRIPTION

As already stated, in this research data from the GNSS CORS was provided by the NGL at the University of Nevada (BLEWITT et al., 2018). Such data consists of files containing calculated coordinates and their respective uncertainties, NGL collects and processes geodetic-quality GPS observations at more than 10 000 stations worldwide from many regional and commercial networks in addition to the commonly used International GNSS Service (IGS) network. NGL routinely processes the observations by using GipsyX version 1.0 software (BERTIGER et al., 2020). The processing method used by NGL is Precise Point Positioning (PPP) and the data is tied to the IGS14 (ITRF2014). This results in the provision of daily GNSS solutions called NGL14.

In this way, statistical tests are conducted and compared with the corresponding values of the AIC and BIC models calculated by the HS. The East component of the CLL1 station is shown in Figure. 3.5.1a, while the North component is represented in Figure 3.5.1b. The East and North components of the QLAP station are presented in Figure 3.5.2a and Figure 3.5.2b, respectively. However, the MIZU station, Figure 3.5.3a, and Figure 3.5.3b exhibit different behavior due to the high incidence of seismic events. Therefore, data from 2012 are used for the MIZU station

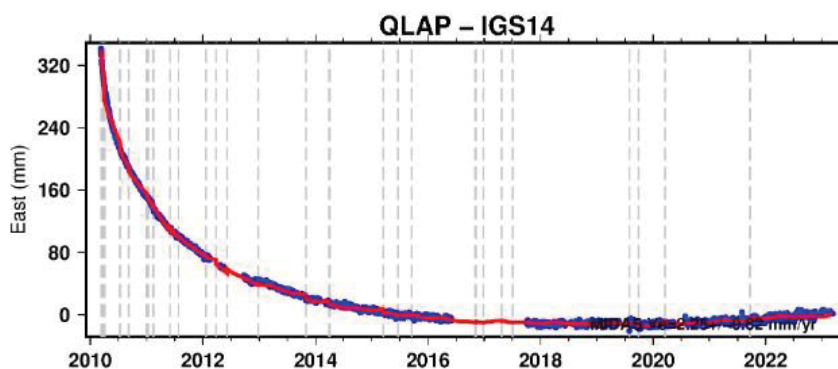
since the graph shows a stabilization after the period of the Fukushima earthquake in 2011, with a magnitude of 9.1. If the complete series were to be employed, it would necessitate the application of post-earthquake ITRF modeling, which is outside the scope of this study.

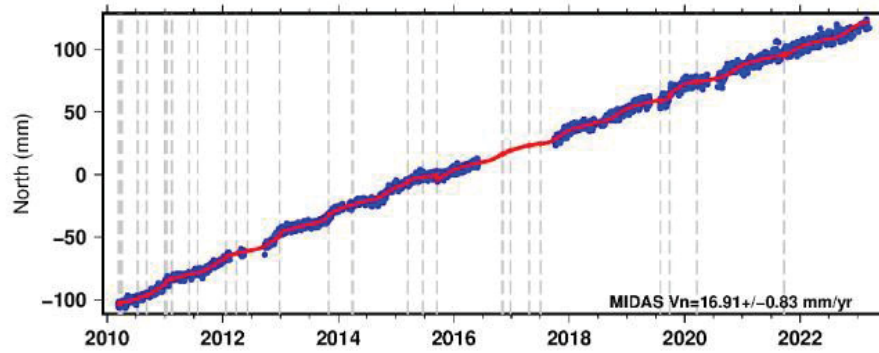
Figure 3.5.1 - a) East component (in mm/yr) from CLL1 and b) North component (in mm/yr)



Source: NGL (2023).

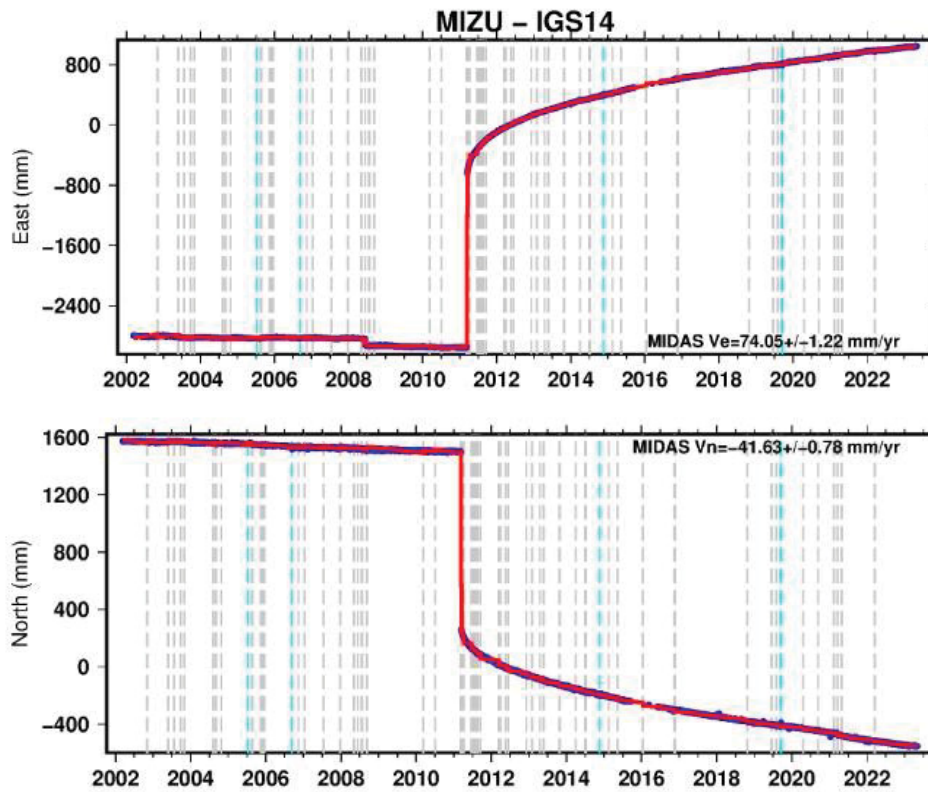
Figure 3.5.2 - a) East component (in mm/yr) from QLAP and b) North component (in mm/yr)





Source: NGL (2023).

Figure 3.5.3 - a) East component (in mm/yr) from MIZU and b) North component (in mm/yr)



Source: NGL (2023).

3.6 RESULTS AND DISCUSSIONS

The results obtained by HS and by the proposed approach are presented and discussed in this section.

3.6.1 HECTOR CRITERIA AND STATISTICAL TEST

Table 3.6.1.1 presents the results of the proposed statistical test (with a significance value of $\alpha = 5\%$) and the HS model selection criteria for the coordinates of the stations.

Table 3.6.1.1 Statistical test and HS criteria for the coordinates of the analyzed stations

Station	Coordinate	Test Result	HS model selection criteria			
			AIC	BIC	BIC_tp	BIC_c
CLL1	East	H_A (With Acceleration)	With Acceleration	With Acceleration	With Acceleration	With Acceleration
	North	H_0 (Without Acceleration)	With Acceleration	With Acceleration	With Acceleration	Without Acceleration
QLAP	East	H_A (With Acceleration)	With Acceleration	With Acceleration	With Acceleration	With Acceleration
	North	H_A (With Acceleration)	With Acceleration	With Acceleration	With Acceleration	With Acceleration
MIZU	East	H_A (With Acceleration)	With Acceleration	With Acceleration	With Acceleration	With Acceleration
	North	H_A (With Acceleration)	With Acceleration	With Acceleration	With Acceleration	With Acceleration

Source: The author (2023).

It can be observed that the model with acceleration is selected by AIC, BIC, and BIC_tp in all cases, but not for the North component of the CLL1 station in the BIC_c criterion. Our proposed statistical test also reveals an insignificant acceleration for the North component of the CLL1 station. Thus, these experiments showed that different criteria for model selection can lead to different results with the same dataset. In this sense, the proposed statistical test can be another criterion to be added in the HS to help the user in the important task of model selection choice. However, which criterion is the most suitable deserves future research and thus is outside the scope of this paper. Our goal here is just to propose a new way of model selection using a statistical test rather than AIC or BIC.

3.6.2 MDHA

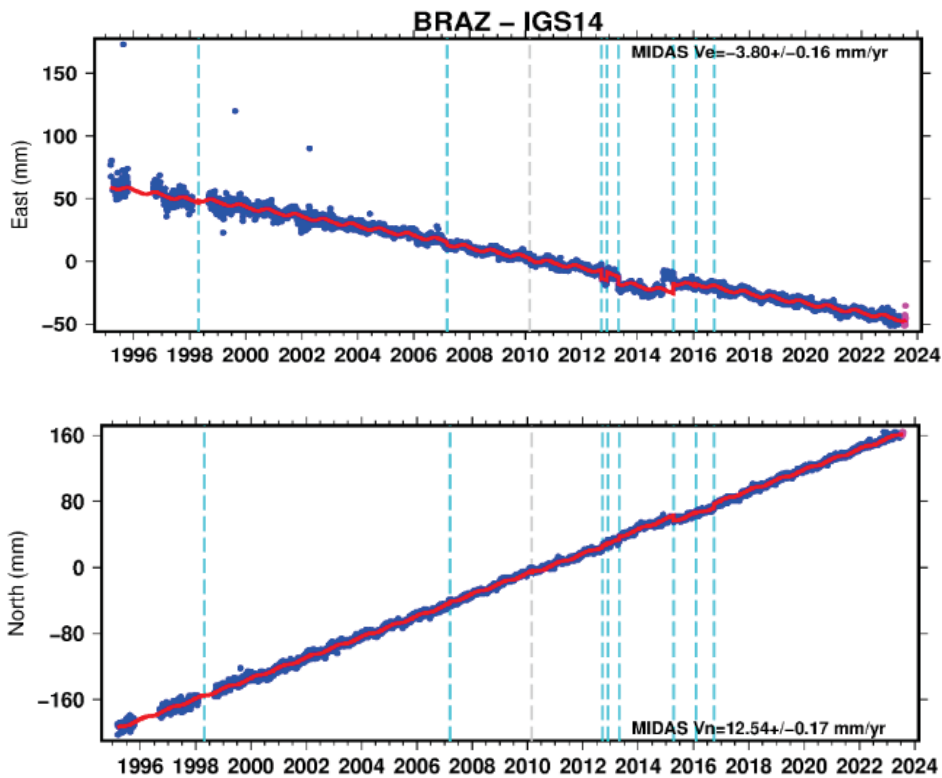
For the implementation of the algorithm developed to obtain the MDHA, the *Scilab software* was used, as HS does not provide the required matrices. In addition, data were considered every 6 months for 10 years, since it is not feasible to carry out Hector's entire processing in *Scilab*, due to computational limitations. Therefore, the BRAZ and BELE stations located in Brazil and the CLL1 station located in Chile were selected. The choice of these stations is due to their locations at two extremes, with a

station located on the edge of a tectonic plate, subject to constant seismic events, and other stations located in the middle of the plate. The data from CLL1, BRAZ, and BELE were obtained from the database of the Nevada Geodetic Laboratory (NGL) network at the University of Nevada (BLEWITT et al., 2018).

The East component of the BRAZ station is shown in Figure 3.6.2.1a, while the North component is represented in Figure 3.6.2.1b. The East and North components of the BELE station are presented in Figure 3.6.2.2a and Figure 3.6.2.2b, respectively.

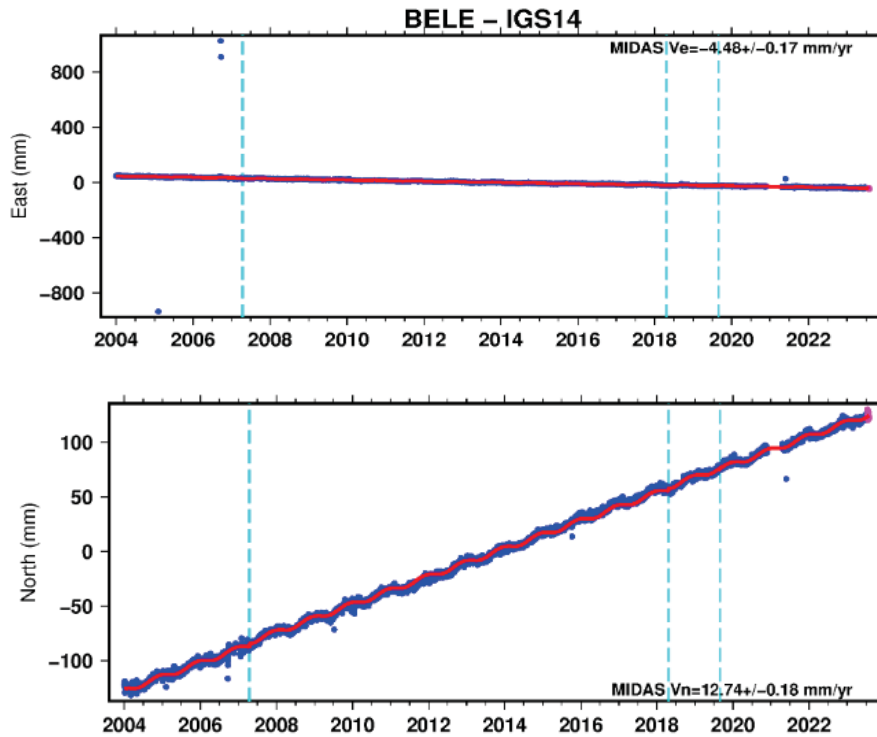
Each blue dot represents an individual estimate of site position, and the plot axes automatically scale to accommodate the data's period and range of positions (which have been demeaned). The red curve on the plots represents a model that fits the data. Gray and cyan vertical dashed lines indicate times of nearby earthquakes and known equipment change events, respectively (NGL, 2023).

Figure 3.6.2.1 - a) East component (in mm/yr) from BRAZ and b) North component (in mm/yr)



Source: NGL (2023).

Figure 3.6.2.2 - a) East component (in mm/yr) from BELE and b) North component (in mm/yr)



Source: NGL (2023).

Table 3.6.2.1, show the calculated acceleration values for each station coordinate and the results obtained for the MDHA with $\alpha_0 = 5\%$ and $\beta_0 = 20\%$. considering a known variance factor in the software Scilab.

Table 3.6.2.1 - Acceleration and the MDHA of the CLL1, BRAZ, and BELE stations (East and North)

Station	Coordinate	Acceleration(mm/year ²)	MDHA (mm/year ²)
CLL1	East	8.46	0.10
	North	-1.02	0.11
BRAZ	East	0.37	0.09
	North	-0.08	0.40
BELE	East	-0.04	0.10
	North	-0.25	0.09

Source: The author (2023).

When observing the results of the estimated acceleration and MDHA, it is noted that in the BRAZ and BELE stations, only one of the coordinates has an estimated acceleration higher than the MDHA. This is expected since these stations are located in a central region of the tectonic plate and thus have lower accelerations. As

for the CLL1 station, the estimated acceleration for both coordinates is significantly higher than the respective MDHA. The results also show that within the plate, acceleration is not absent.

To validate the proposed metric, a statistical test was also performed in Scilab software at the same significance level of MDHA values ($\alpha_0 = 5\%$) to verify if the estimated acceleration is statistically significant or not. The results are presented in Table 3.6.2.2.

Table 3.6.2.2 - Statistical test to analyze the significance of acceleration in the CLL1, BRAZ, and BELE stations

Stations	East	North
CLL1	Acceleration significant	Acceleration significant
BRAZ	Acceleration significant	Acceleration insignificant
BELE	Acceleration insignificant	Acceleration significant

Source: The author (2023).

When analyzing the MDHA and statistical test tables, we can observe that in coordinates where the estimated acceleration is greater than the MDHA, the statistical test indicated that the acceleration is statistically significant. This was the case for the east and north components at CLL1 station, the east coordinate of BRAZ station, and the north coordinate of BELE station. On the other hand, when the estimated acceleration was smaller than the MDHA, the statistical test indicated that the acceleration was not significant. Thus, the proposed metric showed to work in practice and this is an important advantage of the proposed statistical test over model selection criteria: we can derive reliability measures such as MDHA for this approach. Besides that, note that the value of MDHA is independent of the observations such as the MDB values of the original Baarda's reliability theory. In other words, the MDHA can be an important tool in the a priori analysis of GNSS CORS time series such as Baarda's reliability theory can be applied in the design of geodetic networks (see, e.g., ROFATTO et al., 2018).

3.7 CONCLUSIONS

This study follows current analyses involving the inclusion of nonlinear trends in the CORS time series. The proposed statistical test is important because the

nonlinear trend may not always be significant in modeling, as demonstrated by the results. We introduced a new statistical test that offers an objective criterion, unlike the relative classification criteria of AIC and BIC. This proposed test can serve as an additional classifier in the model selection process.

Furthermore, we proposed a metric based on reliability theory to estimate the MDHA (Minimal Detectable Horizontal Acceleration), which cannot be derived from AIC and BIC. This metric provides a reliability measure for the model's minimum detectable horizontal acceleration. The Scilab experiments demonstrated a good agreement between the test results and the MDHA.

One of the advantages of the proposed statistical test is its practical applicability, as it allows for the derivation of reliability measures such as MDHA. This sets it apart from model selection criteria. Additionally, the MDHA value remains independent of observations, such as the MDB values of Baarda's original reliability theory.

For future work, we suggest implementing the statistical test and MDHA in software packages like HS, enabling their analysis in daily solutions and the a priori analysis of time series data from GNSS CORS using the concept of MDHA.

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4 GENERAL CONCLUSIONS

The extended abstract presented illustrates the significance of estimating acceleration within the GNSS CORS time series, aligning with the research's proposed objectives. To ensure a seamless progression, the intention is to expand the analysis to encompass a broader array of GNSS CORS across a more expansive geographical expanse. This expanded approach will enable a thorough exploration of both collective velocity and acceleration estimations (applying uniform acceleration and "block" velocity across all considered CORS) and individualized estimations (akin to those undertaken in this initial experiment). This comprehensive investigation aims to provide deeper insights into the intricate patterns of geodynamic deformations that manifest in these regions.

The article presented in Chapter 3 highlights the preliminary results of a study on estimating acceleration parameters for positional time series of GNSS CORS. A new statistical test has been introduced, providing an objective criterion for model selection, unlike subjective classification criteria such as AIC and BIC. Additionally, a metric based on reliability theory has been proposed to estimate the Minimal Detectable Horizontal Acceleration (MDHA) of the model. The experiments demonstrated a good agreement between the test results and the MDHA.

One advantage of this proposed statistical test is its practical applicability, as it enables the calculation of reliability measures like MDHA, setting it apart from model selection criteria. For future work, implementing the statistical test and MDHA in software packages like HS is suggested, enabling daily solution analysis and prior analysis of time series data from GNSS CORS using the MDHA concept. Moreover, testing more stations and other regions is recommended, eventually considering a model with different nonlinear acceleration parameters.

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