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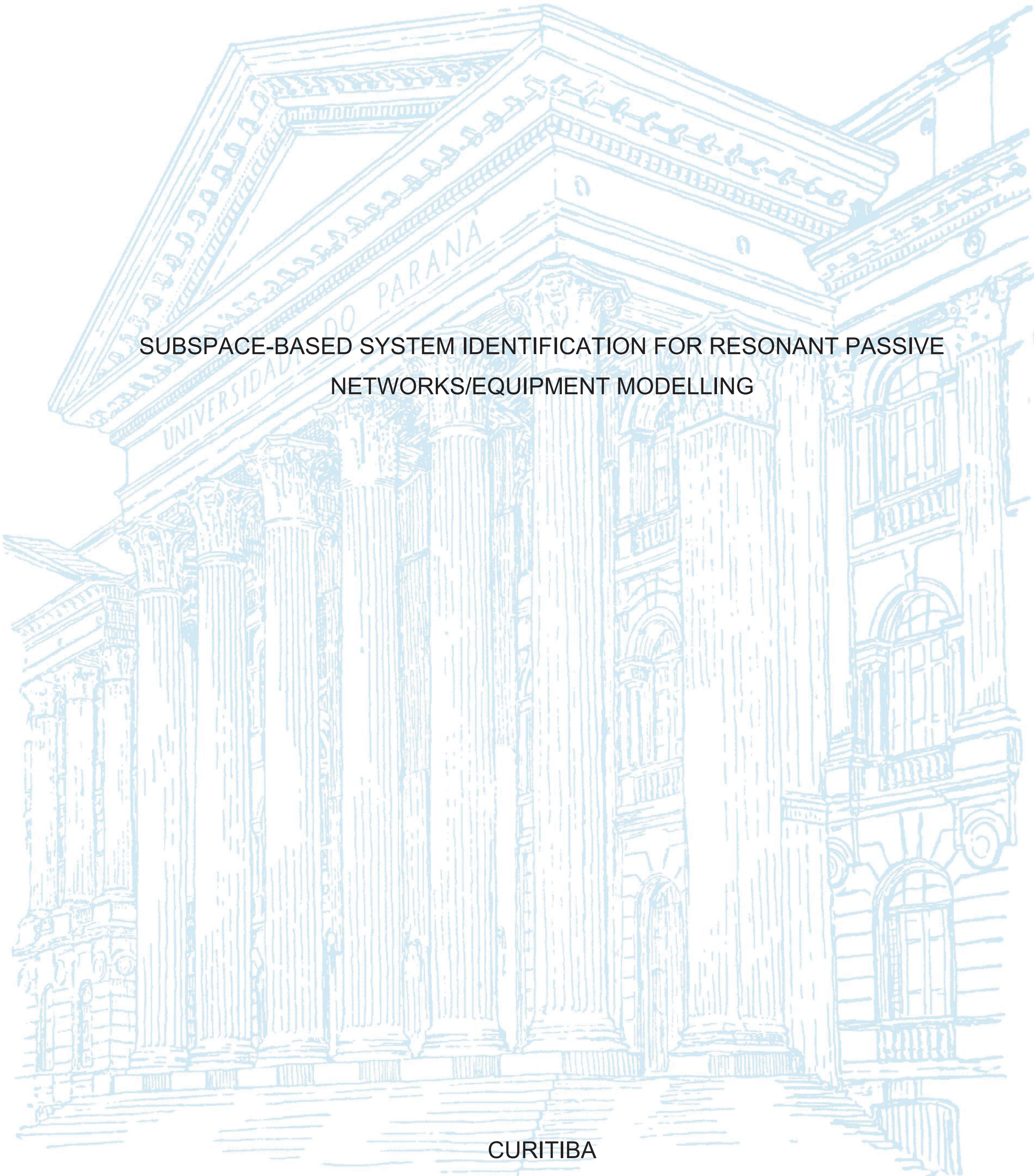

SUBSPACE-BASED SYSTEM IDENTIFICATION FOR RESONANT PASSIVE NETWORKS/EQUIPMENT MODELLING

CURITIBA

2020

LUCAS FARIAS MACIEL RODRIGUES

SUBSPACE-BASED SYSTEM IDENTIFICATION FOR RESONANT PASSIVE NETWORKS/EQUIPMENT MODELLING

Dissertação apresentada ao Programa de Pós-Graduação em Engenharia Elétrica, Área de Concentração Controle e Automação, Departamento de Engenharia Elétrica, Setor de Tecnologia, Universidade Federal do Paraná, como parte das exigências para obtenção do título de Mestre em Engenharia Elétrica.

Orientador: Prof. Dr. Gustavo Henrique da Costa Oliveira

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2020

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SETOR DE TECNOLOGIA
UNIVERSIDADE FEDERAL DO PARANÁ
PRÓ-REITORIA DE PESQUISA E PÓS-GRADUAÇÃO
PROGRAMA DE PÓS-GRADUAÇÃO ENGENHARIA ELÉTRICA - 40001016043P4

TERMO DE APROVAÇÃO

Os membros da Banca Examinadora designada pelo Colegiado do Programa de Pós-Graduação em ENGENHARIA ELÉTRICA da Universidade Federal do Paraná foram convocados para realizar a arguição da Dissertação de Mestrado de **LUCAS FARIAS MACIEL RODRIGUES** intitulada: **Subspace-Based System Identification For Resonant Passive Networks/Equipment Modelling**, sob orientação do Prof. Dr. GUSTAVO HENRIQUE DA COSTA OLIVEIRA, que após terem inquirido o aluno e realizada a avaliação do trabalho, são de parecer pela sua aprovação no rito de defesa.

A outorga do título de mestre está sujeita à homologação pelo colegiado, ao atendimento de todas as indicações e correções solicitadas pela banca e ao pleno atendimento das demandas regimentais do Programa de Pós-Graduação.

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Avaliador Interno Pós-Doc (UNIVERSIDADE FEDERAL DO PARANÁ)

DIEGO ECKHARD
Avaliador Externo (UNIVER. FEDERAL DO RIO GRANDE DO SUL)

ROMAN KUIAVA
Avaliador Interno (UNIVERSIDADE FEDERAL DO PARANÁ)

MINISTÉRIO DA EDUCAÇÃO
SETOR DE TECNOLOGIA
UNIVERSIDADE FEDERAL DO PARANÁ
PRÓ-REITORIA DE PESQUISA E PÓS-GRADUAÇÃO
PROGRAMA DE PÓS-GRADUAÇÃO ENGENHARIA ELÉTRICA - 40001016043P4

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ATA DE SESSÃO PÚBLICA DE DEFESA DE MESTRADO PARA A OBTENÇÃO DO GRAU DE MESTRE EM ENGENHARIA ELÉTRICA

No dia vinte e sete de fevereiro de dois mil e vinte às 14:00 horas, na sala PL07, Bloco PL, Centro Politécnico, foram instaladas as atividades pertinentes ao rito de defesa de dissertação do mestrando **LUCAS FARIAS MACIEL RODRIGUES**, intitulada: **Subspace-Based System Identification For Resonant Passive Networks/Equipment Modelling**, sob orientação do Prof. Dr. GUSTAVO HENRIQUE DA COSTA OLIVEIRA. A Banca Examinadora, designada pelo Colegiado do Programa de Pós-Graduação da Universidade Federal do Paraná em ENGENHARIA ELÉTRICA, foi constituída pelos seguintes Membros: GUSTAVO HENRIQUE DA COSTA OLIVEIRA (UNIVERSIDADE FEDERAL DO PARANÁ), LUCAS PIOLI REHBEIN KURTEN IHLENFELD Pós-Doc (UNIVERSIDADE FEDERAL DO PARANÁ), DIEGO ECKHARD (UNIVER. FEDERAL DO RIO GRANDE DO SUL), ROMAN KUIAVA (UNIVERSIDADE FEDERAL DO PARANÁ). A presidência iniciou os ritos definidos pelo Colegiado do Programa e, após exarados os pareceres dos membros do comitê examinador e da respectiva contra argumentação, ocorreu a leitura do parecer final da banca examinadora, que decidiu pela aprovação. Este resultado deverá ser homologado pelo Colegiado do programa, mediante o atendimento de todas as indicações e correções solicitadas pela banca dentro dos prazos regimentais definidos pelo programa. A outorga de título de mestre está condicionada ao atendimento de todos os requisitos e prazos determinados no regimento do Programa de Pós-Graduação. Nada mais havendo a tratar a presidência deu por encerrada a sessão, da qual eu, GUSTAVO HENRIQUE DA COSTA OLIVEIRA, lavrei a presente ata, que vai assinada por mim e pelos demais membros da Comissão Examinadora.

CURITIBA, 27 de Fevereiro de 2020.

GUSTAVO HENRIQUE DA COSTA OLIVEIRA
Presidente da Banca Examinadora (UNIVERSIDADE FEDERAL DO PARANÁ)

LUCAS PIOLI REHBEIN KURTEN IHLENFELD
Avaliador Interno Pós-Doc (UNIVERSIDADE FEDERAL DO PARANÁ)

DIEGO ECKHARD
Avaliador Externo (UNIVER. FEDERAL DO RIO GRANDE DO SUL)

ROMAN KUIAVA
Avaliador Interno (UNIVERSIDADE FEDERAL DO PARANÁ)

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RESUMO

O tema de identificação do sistema é recorrente em várias áreas da engenharia onde modelos matemáticos preciso são necessários para fins de simulação, previsão, controle, etc. Dentre as áreas de aplicação de identificação de sistemas, tem-se interesse neste trabalho os sistemas dinâmicos que possuem a propriedade de apresentar variados pontos de ressonância. No caso de sistemas pouco amortecidos, com altos picos de ressonância, a resposta em frequência apresenta muitas características relevantes em uma ampla faixa de frequência, o que requer rotinas de identificação de sistema eficientes. Sistemas altamente ressonantes são objeto de muitas áreas de estudo em engenharia, como em análise de vibrações, aeronáutica e redes de energia. Diversas métodos de identificação de sistemas já foram propostas na literatura. Nas últimas décadas, a classe de métodos baseados em subespaço vem chamando a atenção dos pesquisadores devido ao seu uso conveniente em plantas multivariáveis. Embora algoritmos baseados no subespaço sejam bastante simples de serem implementados, eles requerem ampla aplicação de conceitos matemáticos e ferramentas geométricas, além dos princípios de identificação do sistema e da teoria da álgebra linear. Os primeiros algoritmos foram formulados para a identificação no domínio do tempo e, poucos anos depois, um procedimento para identificação no domínio da frequência foi introduzido. Atualmente, existe uma quantidade extensa de algoritmos baseados em subespaço para estimar os parâmetros de sistema dinâmicos com dados no domínio do tempo. No que diz respeito a utilização de dados no domínio da frequência, poucos pesquisadores tratam do procedimento de identificação por subespaços na frequência. Dentre estes, métodos que não requerem dados de medição equidistantes entre si são de particular interesse prático pois esta classe de dados é a mais frequentemente encontrada. Dentre as aplicações para métodos de estimação de parâmetros com dados no domínio da frequência, tem-se as redes elétricas passivas. Como os sinais de potência que flui constantemente por tais redes são afetados pelas interconexões e pela dinâmica do sistema, eles devem ser caracterizados com precisão em uma ampla faixa de frequências, para que esses efeitos sejam avaliados em simulações no nível do sistema. No caso particular da modelagem de redes, garantir a passividade do modelo é fundamental, caso contrário, a simulação do sistema como um todo pode levar à instabilidade. Este documento apresenta um novo método de identificação de sistemas baseado em subespaços que possui a seguinte propriedades: a) não requer a utilização de dados de medição equidistantes na frequência; b) implementa mecanismos para superar problemas de mal condicionamento numérico; c) contém uma proposta de ponderação para melhorar a aproximação do modelo em sistemas com largas faixas de picos de ressonância. Por fim, em se tratando de redes elétricas passivas, o algoritmo garante que o modelo obtido será também passivo. Nesse trabalho, mostra-se que o algoritmo delineado apresentou melhor eficiência na identificação de frequências quando comparado à ferramenta de identificação existente baseada em subespaços, o N4SID. Além disso, verificou-se a efetividade da ferramenta de garantia de passividade do modelo estimado.

Palavras Chave: sistemas ressonantes. identificação de sistemas no domínio da frequência. macromodelos de redes elétricas. passividade em sistemas dinâmicos. técnicas baseadas no subespaço.

ABSTRACT

The area of system identification finds application in various areas of engineering where precise mathematical models are required for simulation, prediction, control, etc. Among the applications, this thesis is interested in analyzing dynamic systems with the property of presenting various resonance points. In the case of low-damped systems with high resonant peaks, the frequency response presents many relevant dynamics over a broad frequency band, which requires efficient system identification routines. Highly resonant systems are the object of many study areas in engineering, as in vibration analysis, aeronautics, and energy networks. Several methods of system identification have been proposed in the literature. In recent decades, subspace-based methods have been drawing researchers' attention due to their convenient use in multivariable plants. Although subspace-based algorithms are quite simple to implement, it requires a wide application of mathematical concepts and geometric tools, in addition to system identification principles and linear algebra theory. The first algorithms were formulated for time-domain identification, then a few years later a frequency-domain identification procedure was introduced. Currently, there is a large amount of subspace-based algorithms for estimating dynamic system parameters using time-domain data. However, as far as frequency domain data is concerned, few researchers deal with the frequency domain subspace identification procedure. Among these algorithms, the methods that do not require equidistant measurement data are of particular practical interest because this class of data is the most frequently available. Among the applications for parameter estimation based on frequency domain data, passive electric networks can be understood as a grid that comprises smaller interconnected subsystems. Power signals are constantly flowing through such networks, and since they are affected by interconnections and system dynamics, they must be accurately characterized over a wide frequency range so that these effects can be evaluated in system-level simulations. In the particular case of network modeling, ensuring the model passivity is critical, otherwise simulating the system as a whole can lead to instability. This document presents a new subspace-based system identification method which has the following properties: (a) it does not require the use of frequency equidistant measurement data; b) implementation of mechanisms to overcome problems of numerical ill-conditioning; c) contains a weighting methodology to improve model approximation in highly resonant systems. Finally, in the case of passive electric networks, the algorithm guarantees that the obtained model will also be passive. In this work, it is shown that the outlined algorithm presented better efficiency in frequency identification when compared to the existing subspace-based identification tool, N4SID. Also, the effectiveness of the passivity assurance tool of the estimated model is verified.

Keywords: resonant systems. frequency identification. electrical network macromodel. passivity in dynamic systems. subspace based techniques.

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NOMENCLATURE AND ABBREVIATIONS

$(\cdot)^T$ denotes the transpose matrix of a given matrix

$eig_{max}\,(\,\cdot\,)$ denotes the largest eigenvalue of a given matrix

$eig_{min}\,(\,\cdot\,)$ denotes the smallest eigenvalue of a given matrix

$\mathrm{eig}\,(\,\cdot\,)$ denotes the eigenvalues of a given matrix

$(\cdot)^\dagger$ denotes the Moore-Penrose pseudo-inverse, the generalized inverse matrix of a given matrix

$(\cdot)^*$ denotes the conjugate transpose matrix

$(\cdot)^\perp$ denotes the perpendicular space formed by a given matrix

$(\cdot)^c$ denotes a complex valued matrix or vector

$(\cdot)^H$ denotes the Hermitian operator

$(\cdot)_F$ denotes a matrix or vector defined using Forsythe recursions

$\dot{x}$ denotes the time-domain derivate of the state vector x

γ denotes the extended observability matrix

$\Im$ denotes the imaginary part of a given vector or matrix

$\mathcal{G}$ denotes the real part of an admittance matrix

$\mathcal{W}$ denotes the type of weight

$\otimes$ denotes the Kronecker product

Φ denotes a matrix relative to the assessment of passivity

ϕ denotes a vector containing the frequency response samples

$\prod$ denotes an operator for the orthogonal projection

$\Re$ denotes the real part of a given vector or matrix

Σ denotes the matrix related to singular values resulted from the singular value decomposition

Ξ denotes the weight matrix

A/B denotes the orthogonal projection of A onto B

A denotes a matrix

a denotes a column or line vector

H(s) or H(w) denote a system realization based on frequency-domain parameters

S scattering parameters

Y admittance parameters

Z impedance parameters

a denotes a scalar

BIBO **B**ounded-**I**nput **B**ounded-**O**utput

BMI **B**ilinear-**M**atrix-**I**nequality

BNC **B**ayonet **N**eill **C**oncelman

cond($\cdot$) the conditioning number of a given matrix

EMTP **E**lectro**M**agnetic **T**ransients **P**rogram

IFBW **I**ntermediate -**F**requency **B**and-**W**idth

IPT **I**nductive **P**ower **T**ransformer

LMI **L**inear-**M**atrix-**I**nequality

LS **L**east **S**quares

LTI **L**inear **T**ime **I**invariant

MIMO **M**ultiple-**I**nput **M**ultiple-**O**utput

N4SID **N**umerical Algorithms for **S**ubspace **S**tate-**S**pace **S**ystem **ID**entification

OSP **O**rthogonal **S**ubspace **P**rojection

PRL **P**ositive **R**eal **L**emma

SISO **S**ingle-**I**nput **S**ingle-**O**utput

VNA **V**ector **N**etwork **A**nalyzer

CONTENTS

1 Introduction

The problem of modeling dynamical systems is recursively confronted in many areas of study. The main question is how to determine a set of rules to describe the behavior of variables in a system. These variables, called state variables, could be anything susceptible to be summarized algebraically. One may think of that as an abstract concept and, indeed, it is. For example, it could be the position of a particle, the current that flows in a coil, or even the population of a nation. A dynamic model is a mathematical construction based on a set of rules that relate to the states of a system. Along with history, many researchers developed conceptual frameworks and methods to accurately describe the behavior of dynamic systems.

Resonant systems and vibration analysis are the object of many study areas in engineering, widely applied in structural analysis, civil construction, aeronautics, electrical engineering, and others. Several recent scientific research about resonant systems are addressed willing to obtain a detailed description of its models, to pattern its resonant poles and behavior (JEONG *et al.*, 2013; NOSHADI *et al.*, 2016; KUMON *et al.*, 2000). Due to the oscillatory nature of resonant systems, it is highly recommended to investigate their operational limits via reliable simulations in order to avoid disruptions and injuries to equipment. Therefore, procedures to obtain an accurate description of the system behavior is of great interest. Parametric and non-parametric identification attempt to reproduce characteristics of a real system based on exogenous data samples. A considerable number of studies have focused on computing resonant systems models in the field of structural analysis, such as in El-Gazzar (2017), Dou e Jensen (2015), Ruzziconi, Younis e Lenci (2013). An important challenge facing researchers is to investigate and monitor the effect of oscillations in the system's stability as addressed in Zhao, Hu e Yin (2018), Yao e Li (2015). Likewise in the area of energy supply, given the growing complexity in operating power grids, to supervise the stability of the system became increasingly important.

In an electrical power system, resonant equipment is interconnected in the network, where there is an ongoing power exchange among the components. In such cases, even small perturbations added to contingencies, can cause large power variations and lead to instability. In particular, power transformers are essential equipment to a reliable steady-state operation but also very sensitive to the occurrence of transient events. As a consequence of its high-cost equipment and its importance to connect different parts of the grid many authors are seeking ways to guarantee a safe life cycle of such equipment (CIGRè-JWG-A2/C4.52, 2020; CIGRè-JWG-A2/C4.39, 2014). Brazilian study shows that for the 20 step-up transformers inspected due to failure, 6 of the failures were associated with resonant transients generated on the system (BECHARA, 2010; SHIPP *et al.*, 2011a). Moreover, literature shows that the effect of switching in Gas Insulated Substations (GIS) can damage connected power and instrument transformers due to the occurrence of Very Fast Transient Over-voltages (VFTO) (GORAYAN; CHANDRAKAR, 2013; SHU *et al.*, 2013). This author would like to refer to Appendix A to discuss further on the importance of system identification in terms of dealing with practical issues of power systems.

Simply put, system identification deals with estimating models of a dynamical system based on input and output data records. The estimated models may be further used in transient simulations, in a software package such as Electromagnetic Transients Program (EMTP), on the condition of using a suitable model. A suitable model reflects every desired feature of the actual system to be emulated.

It is well known in the system identification literature, see for instance (AGUIRRE, 2007), that the model identification problem can be classified in three types: *white-box models* comprise those based on the physical laws about the system behavior, which require rich information on the properties of the actual system; *black-box models*, in its turn, requires little or no prior knowledge of the system and it is based on exogenous data; *grey-box models*, stands as a middle term in between *white* and *black-box models*, is based on any additional information beyond the input-output data. In this context, black-box models based on exogenous experimental data are highlighted as an efficient procedure for describing the dynamic behavior of the resonant system (GRIVET-TALOCIA; GUSTAVSEN, 2016).

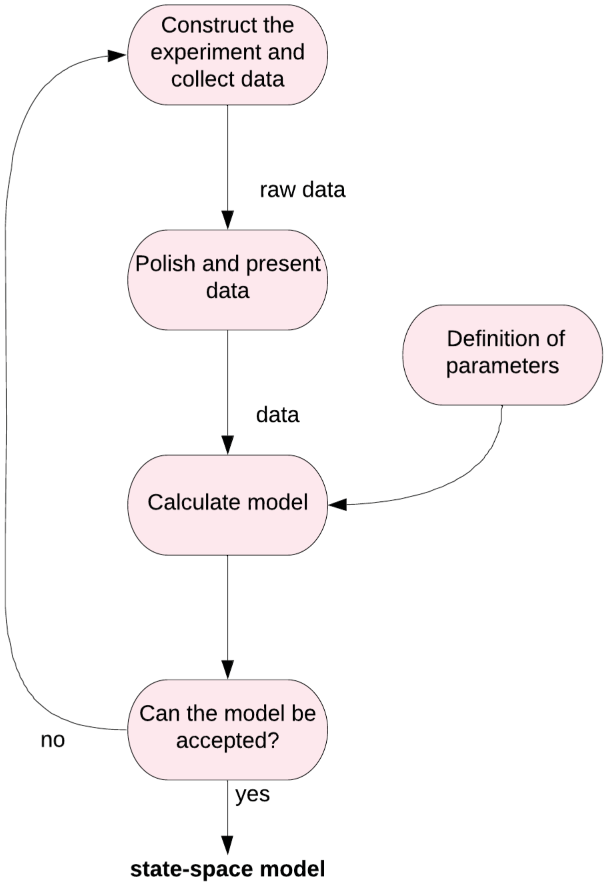

Figure 1 – System identification flowchart.

However, obtaining a black-box model can be a complex and error-prone task at various stages. In Figure 1, a generic system identification procedure is summarized. When frequency data is available from a time-domain transformation, two features are typical. One is the presence of noise and therefore, the necessity to polish data. Another characteristic is that the samples are equidistantly spaced, which might cause numerical issues in system matrices (CERONE; REGRUTO, 2015). The system identification algorithm to be proposed in this manuscript is fed with data descending from a measurement procedure, which avoids the presence of noise in the acquired dataset. Secondly, the computation of the model estimate shall be accomplished by many system identification procedures available in the literature, which usually requires a set of parameters to be previously defined. Lastly, the model is validated and further applied in a simulation environment.

In the particular case of estimating models for networks or electrical power systems equipment, the identification procedure requires a few other steps as illustrated in Figure 2. For instance, electrical network simulations require interconnected models to be passive. Therefore, it is quite common to have a passivity enforcement stage to ensure that the estimated model satisfies this property. Pre-processing approaches applied to raw data and post-processing approaches aim to achieve the final passive behavior, via different strategies.

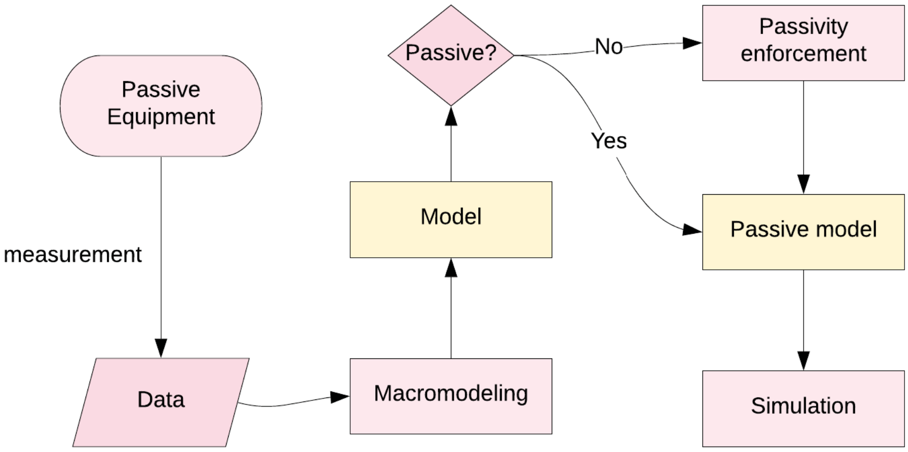

Figure 2 – Passive macromodeling flowchart.

For electrical power system requisitions, it is common to perform the data acquisition and system identification using frequency-domain data samples. Although equidistantly distributed data is widely applied in time-domain applications, it is less common in wide-band frequency-domain proposals. The use of equidistantly distributed data in the logarithmic scale is a common tool to overcome the issues concerned with working with big sets of samples. Moreover, literature shows a frequent use of data analyzers to obtain information in the frequency-domain by collecting samples non-uniformly spaced (MCKELVEY, 1995; LJUNG, 2007; KEYSIGHT, 2019). Frequency response samples for parameter estimation are commonly obtained from three main sources. One may be from a non-parametric identification when time-domain data is used to estimate the system's frequency response (LJUNG, 1999). Instead, it is possible to obtain an estimate for frequency-domain data based on the Fourier Transform of the dataset, originally in time-domain (PINTELON; SCHOUKENS, 2012a). Because these cases use time-domain data to provide the frequency-response samples, they are highly susceptible to noise influence. Therefore, to that purpose, not only filters should be applied but also a stochastic approach may be necessary.

Alternatively, it is possible to obtain direct measurement of the frequency response observations. Despite this approach is less common in traditional system identification, there are several applications reported in Electrical Power Systems (EPS) (OLIVEIRA; RODIER; IHLENFELD, 2016; SCHUMACHER; OLIVEIRA, 2017; RODRIGUES *et al.*, 2019; SCHUMACHER *et al.*, 2018) and in electronics (SARASWAT; ACHAR; NAKHLA, 2004; PORDANJANI *et al.*, 2011; WALKEY *et al.*, 2007). With regard to EPS applications, measurement data naturally characterized in terms of the system's frequency response is largely applied to estimate power and voltage transformers models in order to predict equipment failures (A2/C4.39, 2014).

Traditionally, the problem of fitting a model based on rational functions to a given measurement data has been addressed by many authors (TUTUNJII, 2016; BOTTEGAL; RISULEO; HJALMARSSON, 2016; RONGSHAN; YING; NAMBIAR, 2014; BAKIR, 2011; PARK; NOH, 2011; MCKELVEY; AKÇAY; LJUNG., 1996). The solution to this problem is achieved by a successive numerical search, and to find a feasible solution this procedure requires convergence. Given the difficulty of some system identification problems, different methodologies are applied with satisfactory efficiency. Tutunjii (2016), for example, relates the adjust of the synaptic weights of an artificial neural network to the coefficients of a transfer function. Bottegal, Risuleo e Hjalmarsson (2016) proposes an improvement to Kernel-based methods, which are an innovative approach to system identification that is as efficient as traditional parametric techniques. His study claims to build robustness in this identification procedure.

Among non-iterative methods to system identification, the algorithms which identify state-space models via a subspace approximation are commonly known as subspace methods and have received much attention in the late years. It is a widely explored group whose applications are addressed in a couple of papers as in Rongshan, Ying e Nambiar (2014), Bakir (2011), Park e Noh (2011). This approach does not involve parametric optimization and is based on concepts from system theory, algebra and statistics (MCKELVEY; AKÇAY; LJUNG., 1996). When compared to traditional identification methodologies, this approach is advantageous because it runs without initial parametrization, but also due to its effortlessness on computing the state sequence directly from input-output data. In classical prediction error minimization, initial parametrization is necessary for most model structures. Another highlight is that according to (MCKELVEY; AKÇAY; LJUNG., 1996) there is a minor difference between SISO and MIMO identification for subspace-based algorithms. Furthermore, the idea of estimating one single state-space model in a minimal realization makes the technique attractive.

The early subspace identification methods are based on time-domain data, firstly formalized by Overschee e Moor (1991) in which a novel algorithm to system identification by using stochastic models was proposed. Matter fact, this algorithm works well for systems of small order. Latter, Overschee e Moor (1996b) have described more robust algorithms in their book, eliciting different approaches for time-domain system identification. Arguably, one may say that whereas the estimated order increases, the user faces numerical problems in some matrices along with the algorithm, which may lead the procedure to undesired results. In (CHIUSO; PICCI, 2004), the issues related to ill-posed problems, caused by mathematical operations with ill-conditioned matrices, are discussed; not surprisingly, the authors show the poor performance of standard subspace methods for a few experimental conditions. Although subspace system identification methods based on time-domain data are widely used in the literature, much less works addressing subspace methods for non-uniformly spaced frequency domain wide-band data are found, especially for resonant system study. Here, it should be mentioned the works of (MCKELVEY; AKÇAY; LJUNG., 1996) and (OVERSCHEE; MOOR, 1996a).

With reference to Figure 2, a final stage is the validation of the estimated model based on the performances and the approximation to data. Given the complexity of some system's dynamics, progress has been made in subspace algorithms so that they could render accurate approximations to data. However, up to this point, little is known about modeling power/inductive transformers in wide-band frequency via this method. Additionally, it is an open question if subspace techniques can ensure to return a passive model, once there is no record in regard to passivity in subspace-based system identification. In order to quantify the growing relevance of subspace methods in the area, the keyword "subspace identification" was searched in *Web of Science*. Figure 3 illustrates the number of publications related to subspace from 1990 until October 2019.

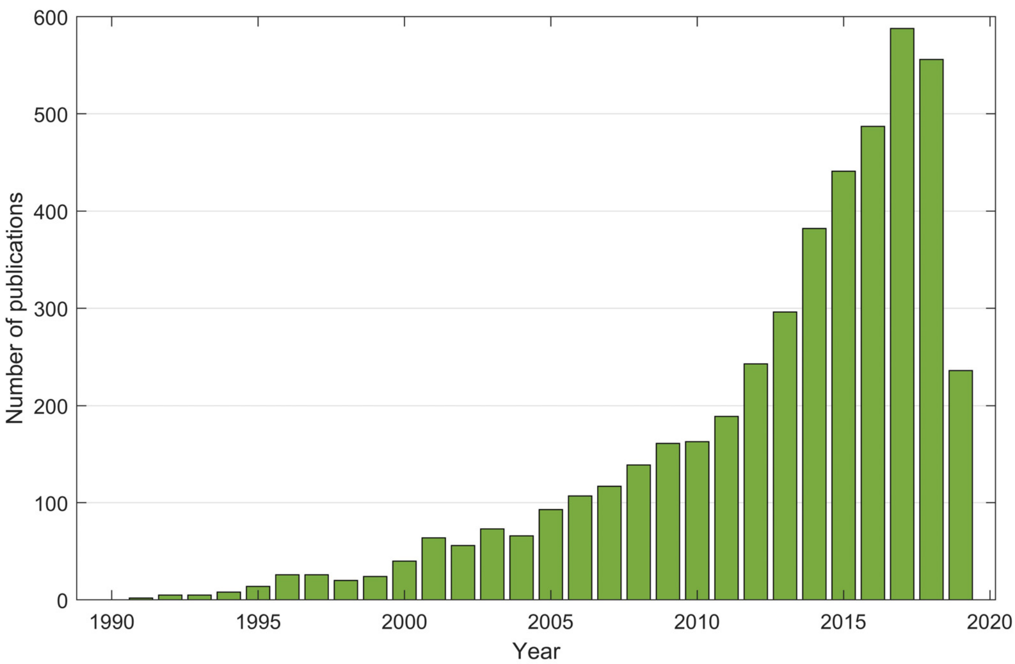

Figure 3 – Number of publications in *Web of Science* related to "subspace identification" from 1990 to 2019.

In the particular case of modeling equipment that operates in interconnected networks, the passivity property must be verified in order to obtain a consistent model. To summarize, passive equipment is not capable to generate energy by itself. Most linear elements encountered in networks such as resistors, capacitors, and inductors are passive, meaning that they always consume energy. Consequently, it is important that models of passive components themselves be passive, or inconsistent behavior may be expected in time-domain simulations. In fact, it is already proved that merely a sufficiently faithful approximation is not a strong condition to rely on when devising coherent models. According to (TRIVERIO *et al.*, 2007), accuracy is a major concern but it is neither the only nor the most important characteristic of any model. The passivity, stability, and causality are fundamental properties that models of passive equipment must bear (IHLENFELD, 2015a).

1.1 Objectives

The general objective of this work is the design of a numerically robust algorithm based on subspace theory to estimate models for resonant equipment or passive networks from non equidistantly spaced frequency-domain measurement data.

Therefore, the specific objectives of this work are:

1. To improve numerically robustness of frequency-domain subspace algorithm;
2. To apply subspace algorithms to power systems networks and equipment such as power transformers, potential transformers and other high voltage equipment;
3. To develop a new weighted subspace methodology for continuous-time models with high resonant behavior;
4. To develop a new framework for passive and stability enforcement of models estimated within subspace algorithm.

1.2 Research contributions

In terms of this manuscript, the specific objective number 1 is achieved in Chapter 5, where actual measurement data is used. Objective number 2 is discussed in Chapter 3 and number 3 is derived in Chapter 4.

As far as publications are concerned, preliminary results of this research have been published in Simpósio Brasileiro de Automática Inteligente (SBAI) 2019, with the paper entitled "On the use of Frequency-Domain Subspace-based System Identification for Estimating Resonant Systems". This paper applies a subspace-based algorithm for estimating high order systems, overcoming the occurrences of ill-posed problems.

The main contribution of this research is specific objectives number 3 and 4. A check in the literature shows that there is no record of facing the challenge of estimating passive models via subspace methods. Considering that, results involving passivity shall be presented in further publications.

1.3 Thesis organization

The remainder chapters of this thesis are organized as follows.

Chapter 2: Background This chapter presents fundamental concepts used all over the manuscript. Linear system theory, network descriptions and concepts from algebra are the basis for the adopted methodology.

Chapter 3: Subspace Methods in Frequency Domain In this chapter it is addressed the problem of formulating subspace algorithms for frequency domain applied to estimation of the dynamics (modes) of resonant systems. In addition, a weighted methodology is presented and improvements are made in the traditionally used algorithm.

Chapter 4: Subspace Methods for Passive Models Here, a novel subspace based algorithm is developed to obtain passive models with a stage of passivity enforcement, which is latter validated in some case studies.

Numerical examples presented in **Chapter 5** focus on synthetic and real measurement data. This chapter validate the methodologies proposed in the manuscript.

Chapter 6: Conclusions

This chapter addresses the conclusions of this work.

Appendix A: Application in electrical power systems

Here, a discussion about the effects of electromagnetic transients on a network, highlighting the particular case of gas insulated facilities.

2 Background

2.1 Introduction

This chapter concerns certain basic concepts of linear systems, algebra and some mathematical tools. It commences setting notations to be used in all this thesis, and then the discussion on linear system descriptions, representations and properties are revisited. Yet, some geometric concepts that are applied in subspace identification algorithms are discussed.

2.2 Dynamic linear time invariant systems

The problem of reproducing the dynamics of a Linear Time-Invariant (LTI) system by fitting frequency domain samples to an algebraic expression is a complex and error-prone task. A parametric procedure concerns in determining the variables of a model structure by successive numerical search, and in order to find a feasible solution, many times this procedure requires convergence. The idea of estimating one single model in a minimal realization makes state-space representations attractive (BOSCH, 1994; GRIVET-TALOCIA; GUSTAVSEN, 2016; CHEN, 1999)

In the context of continuous-time, LTI systems, a generic state-space model is represented by (2.1) where the matrices $\mathbf{A} \in \mathbb{R}^{nxn}$, $\mathbf{b} \in \mathbb{R}^{nxm}$, $\mathbf{c} \in \mathbb{R}^{pxn}$, $d \in \mathbb{R}^{pxm}$ and the state vector $\mathbf{x} \in \mathbb{R}^{n}$. Given frequency domain measurements samples of the input $\mathbf{u}(\mathbf{s})$ and the output $\mathbf{y}(\mathbf{s})$, where $s = jw$ is a complex variable.

$$\begin{aligned} s\mathbf{x}(s) &= \mathbf{A}\mathbf{x}(s) + \mathbf{b}\mathbf{u}(s) \\ \mathbf{y}(s) &= \mathbf{c}\mathbf{x}(s) + d\mathbf{u}(s) \end{aligned} \tag{2.1}$$

The assumption made here is that (2.1) denotes a state-space controllable and observable representation and the system initially relaxed $\mathbf{x}(0) = 0$. According to (CHEN, 1999), taking the Laplace transform of (2.1) and rearranging it, the following realization is achieved.

$$\mathbf{G}(\mathbf{s}) = \mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} + d \tag{2.2}$$

2.3 Behavioral descriptions of electrical components

Much electrical equipment can be represented by a complex frequency-dependent network with several nodes, branches and a set of ports that connects the component with the other systems (or network). Thinking in these terms, the way one equipment sees the other is via their port properties, which is similarly described as an equivalent circuit whose behavior is fully delivered via their connections. In this field, a port is defined as a pair of terminals, in which the current into one terminal is equivalent to the current in the other terminal (KUO, 1968). When using this notation, it is automatically implied that we are merely interested in the external characteristics of a network. Thus, a one-port is a black box and the only variables of interest are the voltage v and the port currents i (DESOER; KUH, 1969). See Alexander e Sadiku (2016) for a recent review on this subject. Figure 4(a) illustrates a 1-port, 2 terminals system, while Figure 4(b) illustrates a n-port generic system.

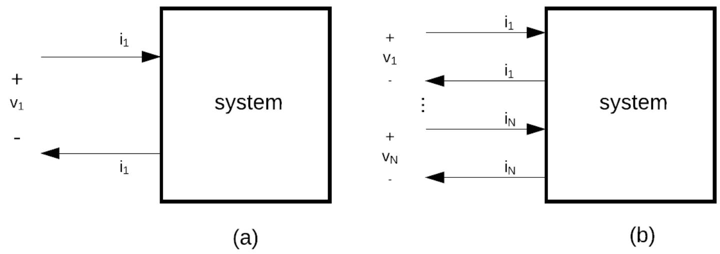

Figure 4 – (a) 1-port generic system; (b) n-port generic system.

Then, an n-port system may be characterized by an n-port matrix, which elements are network functions. In this sense, two kinds of descriptions are usual (GUSTAVSEN *et al.*, 2020): i) interacting descriptions and ii) non-interacting descriptions.

Interacting descriptions, such as *admittance and impedance parameters* are interesting when one concern is about analyzing the interaction between the model and other interconnected systems. Admittance representation is commonly used to describe interacting descriptions of elements of a electrical power system, such as transformers, reactors, etc. The impedance seen into a terminal is frequency-dependent and it also depends on the terminal conditions at the other system terminals. The admittance/impedance matrix contains open-circuit/short-circuit driving points and they are based on a relation between the port-voltage vector v_{ij} and the port-current vector i_{ij}. (DESOER; KUH, 1969).

In the frequency domain, a n-terminal admittance description of an electrical component can be expressed as follows:

$$\mathbf{i}(\omega) = \mathbf{Y}(\omega)\mathbf{v}(\omega) \tag{2.3}$$

where ω is the frequency and the matrix $\mathbf{Y}(w)$ is associated to each ω in the spectrum. $\mathbf{v}(\omega)$ and $\mathbf{i}(\omega)$ are n dimensional column vectors that contains, respectively, voltage and current at each terminal. In order to obtain a more complete and detailed characterization, a wide band frequency measurement is requested.

For a system with n terminals, the above mentioned description corresponds to:

$$\begin{bmatrix} \mathbf{i}_1(\omega) \\ .. \\ \mathbf{i}_n(\omega) \end{bmatrix} = \begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) & ... & Y_{1n}(\omega) \\ Y_{21}(\omega) & Y_{22}(\omega) & ... & Y_{2n}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1}(\omega) & Y_{n2}(\omega) & ... & Y_{nn}(\omega) \end{bmatrix} \times \begin{bmatrix} \mathbf{v}_1(\omega) \\ .. \\ \mathbf{v}_n(\omega) \end{bmatrix} \tag{2.4}$$

It follows that the impedance matrix is the inverse of the admittance matrix.

$$\mathbf{Z} = \mathbf{Y^{-1}} \tag{2.5}$$

One other interesting description related to interacting description are the *scattering parameters*. Unlike admittance/impedance representations, that are characterized in terms of voltages or currents, **S** parameters characterize the relation between injection (or incident) and reflection (or collected) signals at the equipment port when terminated by a reference impedance, illustrated in Fig. 5 as z_0. Typically, 50-Ω or 75-Ω reference

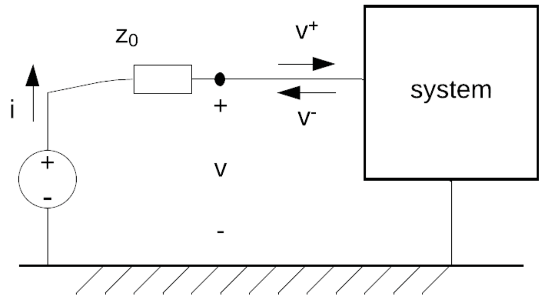

Figure 5 – Generic scattering representation of signal injection and reflection.

impedances are used, matching the characteristic impedance of common measurement cables (GUSTAVSEN; SILVA, 2013). Consider the system shown in Figure 5, where v^+ and v^- are the injection and reflection collected signals, for a n-port device.

$$\mathbf{v}^-(w) = \mathbf{S}(w)\mathbf{v}^+(w) \tag{2.6}$$

Equation (2.6) is the compact form of:

$$\begin{bmatrix} \mathbf{v_1^-}(\omega) \\ .. \\ \mathbf{v_n^-}(\omega) \end{bmatrix} = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & ... & S_{1n}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & ... & S_{2n}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1}(\omega) & S_{n2}(\omega) & ... & S_{nn}(\omega) \end{bmatrix} \times \begin{bmatrix} \mathbf{v_1^+}(\omega) \\ .. \\ \mathbf{v_n^+}(\omega) \end{bmatrix} \tag{2.7}$$

Note that in (2.7), a diagonal element S_{ii} defines the reflected wave at port i due to an incident wave at the same port. Also, an offdiagonal element S_{ij} defines the reflected wave at port i due to an impinging wave on port j.

In (KUO, 1968), the input column vector v^- and the output column vector v^+ are defined as linear combinations of the port variables. As referred to (2.8), $v^n(w)$ and $i^n(w)$ are the port voltages and current, for a n-port equipment. Eventually, a pair of scattering variables (input-output) are assigned to each i-th port.

$$\begin{aligned} \mathbf{v}^+(w) &= \frac{1}{2}\left(\mathbf{v^n}(w) + \mathbf{i^n}(w)\right) \\ \mathbf{v}^-(w) &= \frac{1}{2}\left(\mathbf{v^n}(w) - \mathbf{i^n}(w)\right) \end{aligned} \tag{2.8}$$

Non-interacting descriptions makes use of some signals from the electrical network to evaluate the equipment behavior. An example is to compute internal voltage nodes as a function of external terminal voltages. However, in situations where the calculation of internal nodes response, either in terms of voltage, current, etc, is required, a non-interacting description may be used. A typical example is the calculation of voltages on internal nodes, when one's interest is to monitor its behavior on time. In such situations, it is sufficient to establish a relation between external quantities (voltages or currents) and internal voltages. Then, it is established a relationship between external terminals $\mathbf{v}$ and some internal points, from now on referred to internal terminals, or $\mathbf{v}_{int}$. Description for that, in the frequency domain, can be given by (2.9):

$$\mathbf{v}_{int}(\omega) = \mathbf{H}(\omega)\mathbf{v}(\omega) \tag{2.9}$$

in which $\mathbf{H}(\omega)$ is the transfer function frequency response between terminals. More information on these descriptions can be found on (GUSTAVSEN *et al.*, 2020).

In (GUSTAVSEN *et al.*, 2017; GUSTAVSEN *et al.*, 2018; GUSTAVSEN *et al.*, 2020), the use of these representations for evaluating internal resonances in power transformers is presented. In (MITCHEL; OLIVEIRA, 2015), this concept is used for internal resonances analysis, using gray-box models and a non-interacting description.

Therefore, the problem is stated as to how to derive reliable wide-band models based on $\mathbf{Y}(w)$ or $\mathbf{S}(w)$ measurements to simulate the interaction between the model and the rest of the systems. In this context, in Rodrigues *et al.* (2019) and Schumacher *et al.* (2018) the authors discuss the evaluation of internal terminals resonance caused by electromagnetic transients in power systems for different scenarios.

2.3.1 Conversion of matrices parameters

In the foregoing section, we concerned ourselves with a few types of behavioral descriptions for models. Explicitly in (2.5), the relation between admittance and impedance representations are given, and here, we expand this investigation to how scattering and admittance/impedance parameters talk to themselves. In fact, there are many other representations, e.g., transmission matrices (or T *parameters*), particularly useful for cascade connections, relate to the input port variables and the output port variables (ANDERSON; VONGPANITLERD, 1973).

Although many representations are possible, it is possible to convert from one to another via a few formulations as typically discussed in network analysis books and papers (ANDERSON; VONGPANITLERD, 1973; DESOER; KUH, 1969; CARLIN, 1956), whereas it is fully examined in (FRICKEY, 1994). We choose the particular case of a tow-port model representation to typify the conversion between scattering and admittance parameters. The reason why we downplay impedance representation is twofold: firstly, because the literature shows a more frequent use of measurement equipment to collect admittance parameters from resonant equipment and also due to the fact that the case studies presented in Chapter 5 are destined to admittance and scattering.

Given $\mathbf{Z}_0$ a diagonal matrix whose the j-th entry holds the reference impedance at the j-th port, and $\mathbf{I}$ the identity matrix. Using (2.10) it is possible to convert admittance parameters (or impedance parameters) on scattering parameters.

$$\mathbf{S} = \left(\mathbf{I} + \sqrt{\mathbf{Z}_0}\mathbf{Y}\sqrt{\mathbf{Z}_0}\right)^{-1}\left(\mathbf{I} - \sqrt{\mathbf{Z}_0}\mathbf{Y}\sqrt{\mathbf{Z}_0}\right) \tag{2.10}$$

From (2.10), one can deduce a transformation from S-parameters to Y-parameters:

$$\mathbf{Y} = \sqrt{\mathbf{Z}_0}^{-1}\left(\mathbf{I} - \mathbf{S}\right)\left(\mathbf{I} + \mathbf{S}\right)^{-1}\sqrt{\mathbf{Z}_0}^{-1} \tag{2.11}$$

2.4 Energy balance and passivity

Given the necessity to study the behavior of the energy in time-varying networks, the term "passive element" comes as a clear formulation of elements that absorb energy. The passivity is a primal property of several physical systems, it qualifies the energy balance of a system in an input-output sense. A system is said to be passive when it is not capable of generating energy by itself. In other words, it can consume energy only from the sources that excite it (GRIVET-TALOCIA; GUSTAVSEN, 2016; MAHANTA; YAMIN; ZADEHGOL, 2017).

Generally, passive equipment only absorbs active power given any voltage excitation, in any frequency range. In order to be physically consistent, electrically interconnected models must satisfy causality, stability and passivity properties. Typically, the model is subjected to a passivity reinforcement stage with the intention to avoid further instability in simulations, as illustrated in Figure 2 (KUO, 1968; TRIVERIO *et al.*, 2007).

The passivity property either can be assessed in data or in a model, and literature shows a few formulations with different levels of details (BRUNE, 1931; WILLEMS, 1972; RAISBECK, 1954). If one is interested in the passivity assessment in raw data, then the techniques based on a model structure can not be employed. In Subsection 2.4.1, the sweep method checks passivity over measured frequencies, and it is appropriate to verify the property in raw data. Should a state-space realization be available, the evaluation of the passivity can be made using Linear Matrix Inequalities (LMI), as discussed in Subsection 2.4.2. In fact, many other techniques are accessible and a broad discussion on passivity assessment methods based on technical literature is delivered in (IHLENFELD, 2015b).

Given the importance of the passivity property in some complex systems, especially the ones composed by interconnected models, many authors direct their research to develop techniques to ensure passivity in models as further discussed in (GRIVET-TALOCIA; UBOLLI, 2008; GAO *et al.*, 2005; IHLENFELD, 2015a). There are available a few passivity enforcement schemes for LTI lumped systems. Some are based on direct enforcement of Positive Real Lemma (PRL) constraints via optimization, some others via discrete frequency samples and there is also a class based on Hamiltonian eigenvalue perturbation. All of them are intended to ensure the model's dynamic to be strictly passive. This thesis is focused on passivity enforcement via PRL using a Linear Matrix Inequality (LMI).

2.4.1 Frequency sweeping assessment

With regart to (TRIVERIO *et al.*, 2007), and recalling the LTI system described in (2.2), with $\mathbf{G}(s)$ assuming either admittance or scattering representations, it is possible to define

$$\mathbf{\Phi} = \begin{cases} \mathbf{I} - \mathbf{S}(\omega)^H \mathbf{S}(\omega) & (scattering\ \ representation) \\ \mathbf{Y}(\omega)^H + \mathbf{Y}(\omega) & (other\ \ representations) \end{cases} \tag{2.12}$$

where $(\cdot)^H$ denotes an Hermitian matrix.

The admittance/ impedance or scattering parameter matrix of a passive component is a positive real matrix rational function. A positive real matrix of the complex variable $j\omega$ (or simply ω) must satisfy analyticity, conjugacy, and positive definiteness. In other words, $\mathbf{G}(\omega)$ is positive real if the following conditions are satisfied.

1. $\mathbf{G}(\omega)$ is defined and analytic;
2. $\mathbf{G}(\omega^*) = \mathbf{G}^*(w)$;
3. $\mathbf{\Phi}(\omega) > \mathbf{0}$.

These three conditions are equivalent to suggest that the systems must have a "positive resistivity". As noted in (TRIVERIO *et al.*, 2007), condition 1 implies BIBO (Bounded-Input Bounded-Output) stability, while 2 ensures that the system impulse response to be real. Condition 3 is a generalization for the fact that any passive device has a positive real part. In other words, conditions 1,2 and 3 implies bounded-realness (WOHLERS, 1969).

As a result, a simple assessment for passivity, widely used in literature is:

$$\mathbf{\Phi}(\omega) > 0 \quad for\ \ all\ \ \omega, \tag{2.13}$$

Bringing the attention back to behavioral description of electric equipment in network analysis, admittance representations are quite common. In the particular case where $\mathbf{G}(\omega) = \mathbf{Y}(\omega)$, a simple frequency-sweeping assessment test is:

$$if \quad \mathbf{Y}^H(\omega) + \mathbf{Y}(\omega) > 0 \quad then \quad \mathcal{G}(\omega) > 0 \tag{2.14}$$

in such cases, to guarantee the passivity property, should the inequality in (2.14) hold. In other words, passivity requires the conductance matrix $\mathcal{G}(\omega) = Re[\mathbf{Y}(\omega)]$ to be positive real. If all eigenvalues of $\mathcal{G}$ are strictly positive, so is the smallest one. Consequently, the passivity criteria can take the even simpler form in (2.15).

$$eig_{min}(\mathcal{G}(\omega)) > 0. \tag{2.15}$$

Now, considering scattering representations, the positive-realness condition for $\mathbf{\Phi}(\omega)$ produces

$$\mathbf{I} - \mathbf{S^H}(\omega)\mathbf{S}(\omega) > 0 \tag{2.16}$$

That is, the largest singular value must be smaller than 1 to satisfy the passivity criteria.

$$eig_{max}(\mathbf{S}(\omega)) \leq 1. \tag{2.17}$$

Still, the violation of passivity can occur locally, and even if in one sampled frequency ω_k the passivity is violated, it is not true to imply that in ω_{k+k_1} the assessment will do so.

2.4.2 LMI based assessment

The idea of the passivity enforcement scheme based on LMI arose from control systems theory (KALMAN, 1964; KUO, 1966; KUO, 1968) where the authors state the relation between positive realness of a transfer matrix and feasibility of a state-space realization. In this thesis, we are particularly interested in this method. When imposing passivity, we fix system matrices $\mathbf{A}$ and $\mathbf{b}$ and compute $\mathbf{c}$ and d as passive realizations. Therefore, linear inequality is an attractive tool to that function.

If we take $\mathbf{Y}$ parameters represented in a state-space realization as in (2.1), given $\mathbf{P} \in \mathbb{R}^{n\times n}$ a positive definite symmetric matrix, a sufficient and necessary condition to call a system passive is to satisfy the following LMI.

$$\begin{bmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{PA} & \mathbf{Pb} - \mathbf{c}^T \\ \mathbf{b}^T\mathbf{P} - \mathbf{c} & -d^T - d \end{bmatrix} \preceq 0 \tag{2.18}$$

If we take $\mathbf{S}$ parameters represented in a state-space realization as in (2.1), given $\mathbf{P} \in \mathbb{R}^{n\times n}$ a positive definite symmetric matrix, a sufficient and necessary condition to call a system passive is to satisfy the following LMI.

$$\begin{bmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{b} & \mathbf{c}^T \\ \mathbf{b}^T\mathbf{P} & -1 & -d^T \\ \mathbf{c} & d & -1 \end{bmatrix} \preceq 0 \tag{2.19}$$

2.5 Mathematical tools

This section brings a brief on some geometric projections and mathematical tools that will be handled in the subspace-based identification.

2.5.1 Orthogonal Subspace Projection

The geometric tool examined in this subsection is capable to reveal some intrinsic characteristics of a system. This mechanism may be used when one's interest is to highlight the presence of a specific feature of a space formed by a linear combination of vectors and/or matrices. The ambient space of a matrix $\mathbf{A}$ is defined by the vector spaces whose coordinates are given by the elements of the matrix rows. For further details, the following references are clarifying (OVERSCHEE; MOOR, 1996b; CHANG, 2005; TAO, 2010).

Let us define the operator $\prod_B$

$$\prod_{\mathbf{B}} = \mathbf{B}^T(\mathbf{B}\mathbf{B}^T)^\dagger\mathbf{B}, \tag{2.20}$$

$\mathbf{A}/\mathbf{B}$ is the projection of the row space of $\mathbf{A}$ on the row space of $\mathbf{B}$:

$$\mathbf{A}/\mathbf{B} = \mathbf{A}\prod_{\mathbf{B}}. \tag{2.21}$$

This is identical to

$$\mathbf{A}/\mathbf{B} = \mathbf{A}\mathbf{B}^T\left(\mathbf{B}\mathbf{B}^T\right)^\dagger\mathbf{B}. \tag{2.22}$$

Given the operator that projects the row space of a given matrix onto the orthogonal complement of the row space of the matrix $\mathbf{B}$ is defined as (2.23), where $\mathbf{I}$ the identity matrix.

$$\prod_{\mathbf{B}^\perp} = \mathbf{I} - \prod_{\mathbf{B}} \tag{2.23}$$

The operator that projects the row space of $\mathbf{A}$ onto the orthogonal complement of the row space of the $\mathbf{B}$ is defined as follows.

$$\mathbf{A}/\mathbf{B}^{\perp} = \mathbf{A} \prod_{\mathbf{B}^{\perp}} . \tag{2.24}$$

The combination of the operators decomposes matrix $\mathbf{A}$ into two matrices of which the row spaces are orthogonal (OVERSCHEE; MOOR, 1996b).

$$\mathbf{A} = \mathbf{A} \prod_{\mathbf{B}} + \mathbf{A} \prod_{\mathbf{B}^{\perp}} . \tag{2.25}$$

Based on (2.24) and (2.25), the following property holds.

$$\mathbf{A}/\mathbf{A}^{\perp} = \mathbf{0} \tag{2.26}$$

A graphical interpretation of the projections for the two-dimensional case is shown in Figure 6. $\mathbf{A}/\mathbf{B}$ is formed by projecting the row space of $\mathbf{A}$ on the row space of $\mathbf{B}$. $\mathbf{A}/\mathbf{B}^{\perp}$ is performed by projecting the row space of $\mathbf{A}$ on the orthogonal complement of the row space of $\mathbf{B}$.

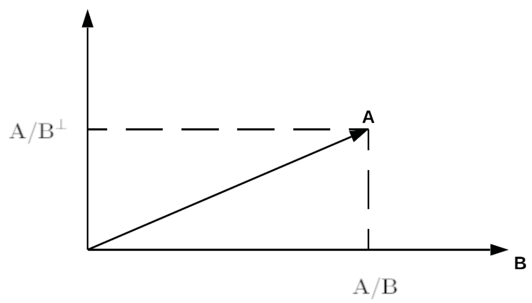

Figure 6 – Orthogonal projection in the two-dimensional row space.

2.5.2 Singular value decomposition

Consider a $k \times t$ matrix $\mathbf{A}$, real or complex, it can be decomposed as follows:

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \begin{bmatrix} \mathbf{U_1} & \mathbf{U_2} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V_1}^T \\ \mathbf{V_2}^T \end{bmatrix} \tag{2.27}$$

where $\mathbf{U}$ is a $k \times k$ dimensional matrix, $\mathbf{\Sigma}$ is a $k \times t$ dimensional matrix that contains the singular values of $\mathbf{A}$ and $\mathbf{V}$ $t \times t$ dimensional matrix.

Here, the idea is to avoid mathematical operations with big sized matrices such as $\mathbf{A}$. In such situations, the economy-sized decomposition saves both time and storage by producing smaller sized $\mathbf{U}, \mathbf{\Sigma}, \mathbf{V}$.

2.5.2.1 Using SVD in an optimization problem

Consider the following optimization problem where $\mathcal{A} \in \mathbb{R}^{k \times t}$, $\mathbf{b} \in \mathbb{R}^t$ and $k \geq t$.

$$\min_{\mathbf{x}} \qquad \|\mathcal{A}\mathbf{x} - \mathbf{b}\|_2 \tag{2.28}$$

A powerful tool for solving LS problems is the SVD, it is capable to considerably reduce the size of one matrix, what facilitates numerical operations. Lets consider

$$\mathcal{A} = \mathbf{U\Sigma V}^T \tag{2.29}$$

where $\mathbf{U} \in \mathbb{R}^{k \times k}$, $\mathbf{V} \in \mathbb{R}^{t \times t}$ and a diagonal matrix $\mathbf{S} \in \mathbb{R}^{t \times t}$.

If $\mathbf{y} = \mathbf{V}^T\mathbf{x}$ and $\mathbf{c} = \mathbf{U}^T\mathbf{b}$, it holds

$$\|\mathcal{A}\mathbf{x} - \mathbf{b}\|_2 = \left\|\mathbf{USV}^T\mathbf{x} - \mathbf{b}\right\|_2 = \|\mathbf{Sy} - \mathbf{c}\|_2 \tag{2.30}$$

For $r = rank(A)$ and $j = 1, ..., r$, it follows

$$y_j = \frac{c_j}{\sigma_j}, \tag{2.31}$$

Then, the pseudo norm solution is given by

$$\mathbf{x} = \mathbf{U_1}^T\mathbf{b}\mathbf{V_1}\mathbf{S_1}^{-1}. \tag{2.32}$$

2.5.3 QR decomposition

The QR factorization is commonly applied in the least squares problem solving and plays an important role in the QR algorithm (an effective tool to compute the eigenvalues of a given matrix). If we take $\mathbf{A}$, a real or complex matrix with $k \times t$ dimension, it can be decomposed as follows

$$
\begin{aligned}
\mathbf{A} &= \mathbf{QR} \\
\mathbf{A} &= \mathbf{Q} \begin{bmatrix} \mathbf{R_1} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{A} &= \begin{bmatrix} \mathbf{Q_1} & \mathbf{Q_2} \end{bmatrix} \begin{bmatrix} \mathbf{R_1} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{A} &= \mathbf{Q_1 R_1}
\end{aligned}
\tag{2.33}
$$

where $\mathbf{Q}$ is a $k \times k$ dimensional matrix and $\mathbf{R}$ is a $k \times t$ dimensional matrix. If the components of $\mathbf{A}$ are real numbers, then $\mathbf{Q}$ is an orthogonal matrix.

2.5.3.1 Using QR decomposition in an optimization problem

Consider the optimization problem defined in (2.28), where $\mathcal{A} \in \mathbb{R}^{k \times t}$, $\mathbf{b} \in \mathbb{R}^t$ and $k \geq t$.

$$
\min_{\mathbf{x}} \qquad \|\mathcal{A}\mathbf{x} - \mathbf{b}\|_2 \tag{2.34}
$$

If $\mathcal{A} = \mathbf{QR}$, with $\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix}$, $\mathbf{R}_1 \in \mathbb{R}^{t \times t}$, an upper triangular matrix, and because $Q \in \mathbb{R}^{k \times k}$ is orthogonal, then

$$
\|\mathcal{A}\mathbf{x} - \mathbf{b}\|_2 = \left\|\mathbf{Q}(\mathbf{Rx} - \mathbf{Q}^T\mathbf{b})\right\|_2 = \left\| \begin{bmatrix} \mathbf{R}_1\mathbf{x} - \beta_\mathbf{1} \\ \beta_\mathbf{2} \end{bmatrix} \right\|_2 \tag{2.35}
$$

where $\mathbf{Q}^T\mathbf{b} = \begin{bmatrix} \beta_\mathbf{1} \\ \beta_\mathbf{2} \end{bmatrix}$, and the unique solution is

$$
\mathbf{x} = \mathbf{R}_1^{-1}\beta_\mathbf{1}. \tag{2.36}
$$

2.5.4 A similarity transform

Recalling the generic state-space realization of a LTI system in (2.1), where $\mathbf{A}$ maps the $\mathbb{R}^n$ space into itself and $\mathbf{x}$ is the $\mathbb{R}^n$ state vector.

$$s\mathbf{x}(s) = \mathbf{A}\mathbf{x}(s) + \mathbf{b}\mathbf{u}(s)$$
$$\mathbf{y}(s) = \mathbf{c}\mathbf{x}(s) + d\mathbf{u}(s)$$

Intrinsically associated to this state-space, there is an orthonormal basis. If one selects a different set of basis, then matrix $\mathbf{A}$ has a different representation (CHEN, 1999). Let $\mathcal{P}$ be an $n \times n$ real nonsingular matrix and let $\bar{\mathbf{x}} = \mathcal{P}\mathbf{x}$. Then, the state-space equation

$$\begin{aligned} s\bar{\mathbf{x}}(s) &= \bar{\mathbf{A}}\bar{\mathbf{x}}(s) + \bar{\mathbf{b}}\mathbf{u}(s) \\ \mathbf{y}(s) &= \bar{\mathbf{c}}\bar{\mathbf{x}}(s) + \bar{d}\mathbf{u}(s) \end{aligned} \tag{2.37}$$

where $\bar{A} = \mathcal{P}\mathbf{A}\mathcal{P}^{-1}$, $\bar{B} = \mathcal{P}\mathbf{B}$, $\bar{C} = \mathbf{C}\mathcal{P}^{-1}$ and $\bar{d} = d$. is said to be algebraically equivalent to (2.1) and $\bar{\mathbf{x}} = \mathcal{P}\mathbf{x}$ is an equivalence transformation (CHEN, 1999). Equivalent systems have same eigenvalues and same frequency responses.

One possible transformation is the one which states that if we consider $(\mathbf{A},\ \mathbf{b},\ \mathbf{c}, d)$ a state-space representation, then the quadruple $(\mathbf{A}^T, \mathbf{c}^T, \mathbf{b}^T, d)$ is said equivalent, or simply

$$(\mathbf{A}, \mathbf{b}, \mathbf{c}, d) \qquad \textit{is equivalent to} \qquad (\mathbf{A}^T, \mathbf{c}^T, \mathbf{b}^T, d)$$

2.5.5 Ill posed problems

In Hadamard (1932) the concept of well posed and ill posed problems has been introduced. According to the author, a problem is well posed (or correctly-set) if

1. it is solvable;
2. its solution is unique;
3. its solution depends continuously on system parameters[1].

[1] If the solution depends continuously on data and parameters, there is no need to concern about small errors in measurement producing large errors in predictions.

Otherwise, the problem is *ill posed*, in a sense that the underlying model is wrong. Hadamard believed that mathematical models should satisfy the aforementioned properties. A trigger that commonly causes ill-posed problems is the bad conditioning number of matrices. An ill-conditioned matrix is almost singular, and the computation of its inverse, or solution of a linear system of equations is prone to large numerical errors.

The complication related to such cases, is to not satisfy property number 3, meaning that arbitrary small perturbation of the data can cause arbitrary large perturbation of the solution and therefore lead to untrustworthy results.

2.5.6 Model evaluation statistics

In this topic, two measurements of error are presented. As stated in Chai e Draxler (2014), every statistical measure condenses a large number of data into a single value, and this way provides one projection of the model errors.

Lets denote $(H(jw_1), \ \ldots, \ (H(jw_N))$ as the set of data, and $(\hat{H}(jw_1), \ \ldots, \ (\hat{H}(jw_N))$ the model response samples as previously referred in many papers (KUMARESAN; BURRUS, 1991; RAN *et al.*, 1989; MCKELVEY; AKÇAY; LJUNG., 1996; MCKELVEY; AKCAY, 1994; LIU; JACQUES; MILLER, 1996). The aforementioned notation will be used to define two measurements of error to be applied along this manuscript.

2.5.6.1 Root Mean Square Error

RMSE stands for the Root Mean Square Error, defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{H}(jw_k) - H(jw_k))^2} \tag{2.38}$$

2.5.6.2 Relative Root Mean Square Error point-by-point

The relative Root Mean Square Error point-by-point (or simply, WSE) is computed as follows.

$$WSE = \sum_{k=1}^{N}\left(\frac{\hat{H}(jw_k) - H(jw_k)}{|H(jw_k)|}\right)^2 \tag{2.39}$$

3 Subspace methods in frequency domain

3.1 Introduction

This chapter is dedicated to present the problem of frequency-domain system identification based on subspace techniques. It is first shown a simple algorithm in order to demonstrate its ability to compute the system matrices in a few steps. Subsequently, we overcome the numerical problem of matrix conditioning using some Forsythe recursions. Finally, improvements in the algorithm approximation to the better estimate frequency response of resonant systems by weighting parameters are formulated.

3.2 A review on subspace methods

Subspace algorithms have been firstly formalized by Overschee e Moor (1991) in which a novel algorithm to system identification by using stochastic models was proposed. Matter fact, the algorithm works well for systems of small order. Arguably, one may say that whereas the estimate order increases, the user faces some problems related to the conditioning number of some matrices, which may lead the procedure to undesired results. Latter, Overschee e Moor (1996b) have described more robust algorithms in their book, eliciting different approaches for time-domain system identification. After that, one can find many approaches of subspace methods in the literature (YI, 2016; YUEPING; HAITAO, 2007; JIE; SONGHAO; CAILING, 2013; ZHANG *et al.*, 2018; SUGIE; INOUE; MARUTA, 2017).

Literature shows a frequent use of subspace methods for time-domain identification, while fewer researches take frequency-domain algorithms into consideration. For frequency identification, the algorithm shall transform complex-valued data into real values to avoid working with complex numbers. In such cases, it is also pertinent the manner data is sampled.

Just a minor number of authors have addressed their research to the problem of frequency-domain subspace-based identification using not equidistantly spaced data. Although equidistantly distributed data is widely applied in time-domain system identification, it is less common in practical applications for wide-band frequency-domain data. The use of equidistantly distributed data in the logarithmic scale is a common tool to overcome the issues concerned with working with big sets of samples.

In this context, two works stand out: one, described in Overschee e Moor (1996a), whose objective is twofold: *i*) to present a simple frequency-domain system identification algorithm; *ii*) to overcome the problem of estimating models of high order systems with a better-conditioned algorithm. The second work was later described in McKelvey, AkÇay e Ljung. (1996) where the authors presented two algorithms for frequency domain identification, one based on equidistantly spaced data and another one which accepts non equidistantly spaced data. Nonetheless, in Rodrigues, Oliveira e Santo (2019), the authors show that despite the different approaches, both methods can compute models with the same eigenvalues. Here, the difference between the methods is highlighted in Subsection 3.4.1.

3.3 Problem statement

Consider a LTI continuous-time Single-Input Single-Output (SISO) system described by the following state-space equations:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{b}u(t), \tag{3.1a}$$

$$y(t) = \mathbf{c}x(t) + du(t), \tag{3.1b}$$

with $u(t)$ and $y(t)$ input and output signals. Also assume that the system is of finite order n and is considered initially relaxed.

Using Laplace transform, the time-domain description in (3.1) turns to a frequency-domain representation as in (3.2).

$$sx(s) = \mathbf{A}x(s) + \mathbf{b}u(s), \tag{3.2a}$$

$$y(s) = \mathbf{c}x(s) + du(s), \tag{3.2b}$$

The transfer function associated to (3.2) is

$$\frac{y(s)}{u(s)} = d + \mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}. \tag{3.3}$$

Applying $u(s) = 1$, the impulse signal, the frequency response vector can be indicated as

$$G(s) = d + \mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}. \tag{3.4}$$

Consider $G(j\omega_k)$ the measured frequency response of an unknown device under test or equipment, where ω_k, $k = 1, ..., N$, are the measured frequencies, which may not necessarily be equidistantly spaced and N is the number of samples. Then $\{G(j\omega_1), ..., G(j\omega_N)\}$ is the set of samples.

The problem addressed in this paper is the one to use subspace techniques to fit a state-space model in terms of the quadruple $(\mathbf{A}, \mathbf{b}, \mathbf{c}, d)$, given a set of non-equidistantly spaced, wide-band, frequency domain samples $\{G(j\omega_1), ..., G(j\omega_N)\}$.

3.4 Subspace-based identification algorithms

In general, subspace-based algorithms follow a few steps to obtain the system matrices $\mathbf{A}, \mathbf{b}, \mathbf{c}$ and d. The overall procedure illustrated in Figure 7 is separated into two parts: transforming data and computing system's matrices. Overall, five steps are necessary to compute the quadruple, which are deeper discussed in 3.4.1. Initially, input/output basis matrices are determined using the system input/response vectors. Then, 3.4.1.2 indicates an orthogonal projection to be applied in data matrices, which provides specific information about the system's characteristics. The SVD generates three matrices $(\mathbf{U}, \mathbf{S}, \mathbf{V}^T)$, from which it is possible to estimate the order of the model and compute the extended observability matrix. Note that, up to this point, the algorithm was able to obtain information on the system just based on the transformation of data. From now on, we use this information to compute the system matrices. In 3.4.1.4, the extended observability matrix is used to obtain an estimate for matrices **A** and **c**. Lastly, in 3.4.1.5, **b** and d is determined through a least squares solution of a linear set of equations.

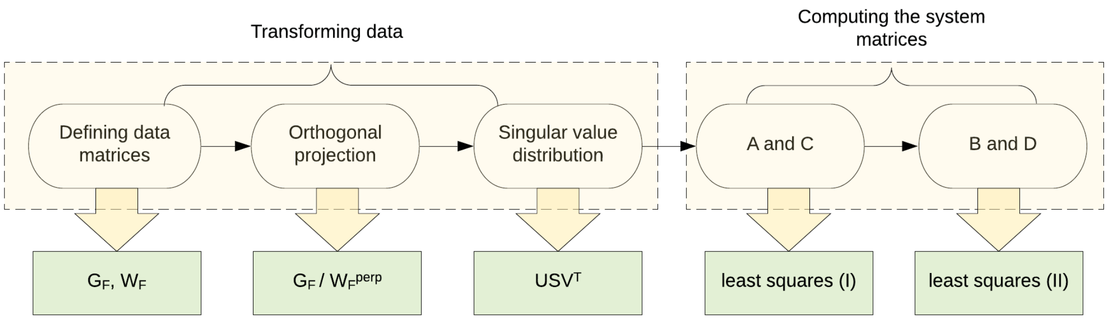

Figure 7 – Subspace based identification flowchart.

3.4.1 A simple frequency identification algorithm

Inspired by the frameworks in McKelvey, AkÇay e Ljung. (1996), Overschee e Moor (1996a), and changing McKelvey, AkÇay e Ljung. (1996) to a continuous-time model estimation approach, a subspace algorithm is described as follows.

This author would like to state that despite the fact the formulation in this subsection is made for SISO systems, it shall be easily comprised of Multiple-Input Multiple-Output (MIMO) systems. Also important to highlight that the content in this chapter is valid for any system represented by a frequency response matrix that relates system input-output observations.

In order to start the algorithm, an initial guess about the model order is necessary, which is defined as q. This value will be related to the dimension of the first matrices applied in the algorithm. Eventually, it may be changed along with the algorithm.

3.4.1.1 Step 1- Computing the data matrices

The procedure starts defining the data matrices $\mathbf{G}^c$ and $\mathbf{W}^c$ as in (3.5) and (3.6), where the superscript c stands for complex.

$$\mathbf{G}^c = \begin{bmatrix} G(jw_1) & \cdots & G(jw_N) \\ (jw_1)G(jw_1) & \cdots & (jw_N)G(jw_N) \\ \vdots & \ddots & \vdots \\ (jw_1)^{(q-1)}G(jw_1) & \cdots & (jw_N)^{(q-1)}G(jw_N) \end{bmatrix}, \tag{3.5}$$

$$\mathbf{W}^c = \begin{bmatrix} 1 & \cdots & 1 \\ (jw_1)1 & \cdots & (jw_N)1 \\ \vdots & \ddots & \vdots \\ (jw_1)^{(q-1)}1 & \cdots & (jw_N)^{(q-1)}1 \end{bmatrix}. \tag{3.6}$$

Because the input/output basis are complex matrices, it is necessary to make them real matrices, and the following definition shall be used throughouth the manuscript.

$$\begin{aligned} \mathbf{G} &= [\Re(\mathbf{G}^c) \quad \Im(\mathbf{G}^c)] \\ \mathbf{W} &= [\Re(\mathbf{W}^c) \quad \Im(\mathbf{W}^c)] \end{aligned} \tag{3.7}$$

3.4.1.2 Step 2 - Orthogonal projection

Recalling Chapter 2 where the orthogonal projection is deeper detailed, in this step (3.10) is applied to project $\mathbf{G}$ onto the perpendicular space of $\mathbf{W}$.

$$\mathbf{G}/\mathbf{W}^{\perp} = \mathbf{G} - (\mathbf{G}\mathbf{W}^T(\mathbf{W}\ \ \mathbf{W}^T)\mathbf{W}) \tag{3.8}$$

Because

$$\mathbf{W}\ \ \mathbf{W}^T = \mathbf{I}_q \tag{3.9}$$

where $\mathbf{I}_q$ is the q-dimensional identity matrix. Equation (3.8) is shortened to

$$\mathbf{G}/\mathbf{W}^{\perp} = \mathbf{G} - \mathbf{G}\ \ \mathbf{W}^T\mathbf{W} \tag{3.10}$$

3.4.1.3 Step 3- The SVD and its partitions

$$\mathbf{G}/\mathbf{W}^{\perp} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{T}. \tag{3.11}$$

In accordance with Chapter 2, the decomposition of the matrix $\mathbf{G}/\mathbf{W}^{\perp}$ provides its singular values, what permits an estimate of the order of the model based on the largest singular values. Ideally, the number of singular values different from zero determines the order of the model. In practical applications, we look for a gap between a set of dominant singular values that correspond to the dynamics of the system and a set of small singular values due to the noise. One possibility is to use the rank of matrix Σ as a possible estimate of the order.

3.4.1.4 Step 4- Computing the system matrices **A** and **c**

In this step, the system matrices $\mathbf{A}$ and $\mathbf{c}$ are computed. Traditionally, there are two different approaches to compute them, both are based on block shift property of the observability matrix, defined using information from the SVD.

In Overschee e Moor (1996a), $\gamma \in \Re^{q\times n}$ is defined as

$$\gamma = \mathbf{U_1}\mathbf{\Sigma_1}^{1/2}. \tag{3.12}$$

where in (3.12) an element-by-element root square operation is performed. Shall we denote $\overline{\overline{\gamma}}$ for γ without the first 2 rows; $\underline{\overline{\gamma}}$ be γ without the first row and without the last row; $\underline{\underline{\gamma}}$ be γ without the last 2 rows. Also γ_p as the first row of γ. Consider the partitions of γ, $\mathbf{A}$ is given by the following optimization problem based on least squares

$$\begin{aligned} \min_{A} \qquad & \left(\underline{\gamma}\mathbf{A} - \overline{\gamma}\right)^{T}\left(\underline{\gamma}\mathbf{A} - \overline{\gamma}\right), \\ & \mathbf{A} = \left[\underline{\gamma}\right]^{\dagger}\overline{\gamma}. \end{aligned} \tag{3.13}$$

Vector $\mathbf{c}$ is computed directly from the extended observability matrix.

$$\mathbf{c} = \gamma_p. \tag{3.14}$$

In McKelvey, AkÇay e Ljung. (1996), the authors define matrices J_1, J_2 and J_3 to use specific information from matrix $\mathbf{U}$, from the SVD.

$$\begin{aligned} \min_{A} \qquad & \left(\mathbf{J_1U_1A} - \mathbf{J_2U_1}\right)^T \left(\mathbf{J_1U_1A} - \mathbf{J_2U_1}\right), \\ & \mathbf{A} = (\mathbf{J_1U_1})^{\dagger}\mathbf{J_2U_1}. \end{aligned} \tag{3.15}$$

$$\mathbf{c} = \mathbf{J_3U_1}. \tag{3.16}$$

where

$$\mathbf{J_1} = [\mathbf{I_{(q-1)}} \;\; \mathbf{0_{(q-1)}}], \tag{3.17a}$$

$$\mathbf{J_2} = [\mathbf{0_{(q-1)}} \;\; \mathbf{I_{(q-1)}}], \tag{3.17b}$$

$$\mathbf{J_3} = [1 \;\; \mathbf{0_{(q-1)}}]. \tag{3.17c}$$

where $\mathbf{I_i}$ denotes the i-th order identity matrix and $\mathbf{0_{i\times j}}$ denotes the $i \times j$ order zero matrix.

3.4.1.5 Step 5- Computing the system matrices **b** and d

Lastly, $\mathbf{b}$ and d is determined through a least squares solution of (3.18).

$$\min_{\mathbf{b},d} \left(\phi - \mathbf{M}\begin{bmatrix}\mathbf{b}\\ d\end{bmatrix}\right)^T \left(\phi - \mathbf{M}\begin{bmatrix}\mathbf{b}\\ d\end{bmatrix}\right) \tag{3.18}$$

for ϕ and $\mathbf{M}$ defined as follows

$$\phi = \begin{pmatrix} G(jw_1) \\ G(jw_2) \\ \vdots \\ G(jw_N) \end{pmatrix} \tag{3.19}$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{c}(jw_1\mathbf{I}_n - \mathbf{A})^{-1} & 1 \\ \mathbf{c}(jw_2\mathbf{I}_n - \mathbf{A})^{-1} & 1 \\ \vdots & \vdots \\ \mathbf{c}(jw_N\mathbf{I}_n - \mathbf{A})^{-1} & 1 \end{pmatrix} \tag{3.20}$$

where $\mathbf{I}_n$ is a n-dimensional identity matrix.

3.4.2 Towards a better conditioning

Ill-posed problems arise quite naturally if one's interest is to determine the internal structure of a physical system from the system's measured behavior, or in determining the unknown input that gives rise to a measured output signal. In this subsection, the application of some mathematical tools is derived to overcome matrices numerical issues related to the conditioning number of them.

3.4.2.1 Step 1 - the use of Forsythe recursions

As proposed in Overschee e Moor (1996a), by using Forsythe recursions in Step 1, it is possible to construct well-conditioned basis for the spaces of $\mathbf{W}$ and $\mathbf{G}$. Here the subscript F is used to refer to Forsythe recursions. Let us start defining $\mathbf{W_F}$ as the input basis, and $\mathbf{G_F}$ as the output basis formed from the frequency responses of a system. Next, it is presented how Forsythe recursions are used to obtain a well-conditioned basis $\mathbf{G_F}$ and $\mathbf{W_F}$.

Let $\mathbf{D_w}$ be a complex matrix with the measured frequencies, as follows

$$\mathbf{D_w} = diag\left((jw_1)(jw_2)...(jw_N)\right) \tag{3.21}$$

To find $\mathbf{G}_F$ and $\mathbf{W}_F$, first initialize defining r_0, r_1 and z_0, z_1 as in (3.22) to (3.25).

$$\mathbf{r_0} = \left(G(jw_1), G(jw_2), ..., G(jw_N)\right), \tag{3.22}$$

$$z_0 = diag\left(diag(\mathbf{r_0}\mathbf{r_0^*})\right), \tag{3.23}$$

$$\mathbf{r_1} = \mathbf{r_0}\mathbf{D_w}, \tag{3.24}$$

$$z_1 = diag\left(diag(\mathbf{r_1}\mathbf{r_1^*}\right). \tag{3.25}$$

Then, compute the recursion for $k = 2$ to $q - 1$, where q is the estimate order.

$$\mathbf{r_k} = \mathbf{r_{k-1}}\mathbf{D_w} + \frac{z_{k-1}}{z_{k-2}}\mathbf{r_{k-2}} \tag{3.26}$$

$$z_k = diag\left(diag(\mathbf{r_k}\mathbf{r_k^*}\right). \tag{3.27}$$

Note that $r_k \in \mathbb{C}^{q\times N}$ and $z_k \in \mathbb{C}^{q\times p}$. Now, defining $\mathbf{D_1}$ and $\mathbf{D_2}$, diagonal matrices of order $q-2$ as in (3.28) and (3.29), respectively.

$$\mathbf{D_1} = \begin{bmatrix} \frac{z_1^{1/2}}{z_2^{1/2}} & 0 & \cdots & 0 \\ 0 & \frac{z_2^{1/2}}{z_3^{1/2}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \frac{z_{q-2}^{1/2}}{z_{q-1}^{1/2}} \end{bmatrix}, \tag{3.28}$$

$$\mathbf{D_2} = \begin{bmatrix} \frac{z_1}{(z_0 z_2)^{1/2}} & 0 & \cdots & 0 \\ 0 & \frac{z_2}{(z_1 z_3)^{1/2}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \frac{z_{q-2}}{(z_{q-3} z_{q-1})^{1/2}} \end{bmatrix}. \tag{3.29}$$

Finally, the output basis, with the superscript c standing for complex.

$$\mathbf{G_F^c} = \begin{bmatrix} \frac{\mathbf{r_0}}{z_0^{1/2}} \\ \frac{\mathbf{r_1}}{z_1^{1/2}} \\ \cdots \\ \frac{\mathbf{r_{q-1}}}{z_{q-1}^{1/2}} \end{bmatrix}. \tag{3.30}$$

Hence, $\mathbf{G_F} = (\Re\left[\mathbf{G_F^c}\right] \;\; \Im\left[\mathbf{G_F^c}\right])$, with $\Re(\cdot)$ and $\Im(\cdot)$ meaning the real and imaginary part of $(\cdot)$. Then,

$$\mathbf{G_F}\mathbf{G_F}^T = \mathbf{I}_q, \tag{3.31}$$

$$cond(\mathbf{G_F}) = 1. \tag{3.32}$$

To compute the input basis, make $R_0 = (1 \;\; 1...1)$ and the following steps are equivalent. Then, $\mathbf{W_F} = (\Re\left[\mathbf{W_F^c}\right] \;\; \Im\left[\mathbf{W_F^c}\right])$[1].

[1] Note that (3.31) and (3.32) holds for the input basis as well.

3.4.2.2 Step 4 and 5- solving ill posed LS problems

Referring to Chapter 2 where it is provided the necessary background on ill posed problems, and a few tools to overcome issues related to them. In brief, a very large conditioning number of one matrix implies ill conditioning issues. Even then, such matrix is almost singular and the computation of its inverse is prone to large numerical errors. Here, problems related to ill conditioned matrices that appear in LS problems, in step 4 and 5, are solved.

Step 4- using SVD

It is well known that the computation of matrix $\mathbf{A}$ is based on the solution of

$$\begin{aligned} \mathbf{A} &= \left[\underline{\gamma}\right]^{\dagger}\overline{\gamma}, \\ \min_{A} \quad & \left(\underline{\gamma}\mathbf{A} - \overline{\gamma}\right). \end{aligned} \tag{3.33}$$

If $\underline{\gamma}$ is a ill conditioned matrix, then we shall decompose it via SVD in order to avoid ill posed problems. The approach is made as follows

$$\underline{\gamma} = \mathbf{U\Sigma V}^T \tag{3.34}$$

where $\mathbf{U} \in \mathbb{R}^{k\times k}$, $\mathbf{V} \in \mathbb{R}^{t\times t}$ and a diagonal matrix $\Sigma \in \mathbb{R}^{t\times t}$.

If $\mathbf{y} = \mathbf{V}^T\mathbf{A}$ and $\mathbf{c} = \mathbf{U}^T\overline{\gamma}$, it holds

$$\left\|\underline{\gamma}\mathbf{A} - \overline{\gamma}\right\|_2 = \left\|\mathbf{U\Sigma V}^T\mathbf{A} - \overline{\gamma}\right\|_2 = \|\Sigma\mathbf{y} - \mathbf{c}\|_2 \tag{3.35}$$

For $r = rank(\underline{\gamma})$ and $j = 1, ..., r$, it follows

$$y_j = \frac{c_j}{\sigma_j}, \tag{3.36}$$

Then, the pseudo norm solution is given by

$$\mathbf{A} = \mathbf{U_1}^T\overline{\gamma}\mathbf{V_1}\Sigma_1^{-1}. \tag{3.37}$$

Step 5- using QR factorization

Consider $\mathbf{x} = \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix}$. We know that the solution of $\mathbf{x}$ is given by

$$\begin{aligned} \mathbf{x} &= [\mathbf{M}]^{\dagger}\,\phi, \\ \min_{\mathbf{x}} \quad & (\phi - \mathbf{M}\mathbf{x})\,. \end{aligned} \tag{3.38}$$

If $\mathbf{M}$ is a ill conditioned matrix, then we shall decompose it via QR in order to avoid ill posed problems. Initially make $\mathbf{M} = \mathbf{QR}$, with $\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix}$, $\mathbf{R}_1 \in \mathbb{R}^{t\times t}$, an upper triangular matrix, and because $Q \in \mathbb{R}^{k\times k}$ is orthogonal, then

$$\|\phi - \mathbf{M}\mathbf{x}\|_2 = \left\|\mathbf{Q}(\mathbf{R}\mathbf{x} - \mathbf{Q}^T\phi)\right\|_2 = \left\|\begin{bmatrix} \mathbf{R}_1\mathbf{x} - \beta_{\mathbf{1}} \\ \beta_{\mathbf{2}} \end{bmatrix}\right\|_2 \tag{3.39}$$

where $\mathbf{Q}^T\phi = \begin{bmatrix} \beta_{\mathbf{1}} \\ \beta_{\mathbf{2}} \end{bmatrix}$, and the unique solution is

$$\mathbf{x} = \mathbf{R}_1^{-1}\beta_{\mathbf{1}}. \tag{3.40}$$

3.5 A weighted frequency identification algorithm

In (3.18) all measurements are equally weighted. When one's interest is to put more emphasis on one measurement with respect to the other, a weighing approach is recommended. Pintelon e Schoukens (2012b) remark that in some practical applications, it is not always clear what weighting should be used. If, for example, the model errors are known, the user may choose to apply weighting in order to keep the model error small in some specific regions. This can be done to make the difference between measurement and model smaller. Also, when it comes to stochastic arguments, if the covariance of the noise is known, it is interesting to downplay measurements with high uncertainty and emphasize those with low uncertainty. In such cases, the weighting is dictated by the covariance matrix.

Here, the motivation to weight measurements arose while attempting to estimate more complex high-order resonant systems, especially based on actual measurement data.

Subspace based algorithms showed difficulties in capturing some peaks of resonance with low magnitude distribution. This is because the algorithm understands that the error is lower in small magnitude zones when related to larger magnitudes. In cases like this, literature shows that a weighted approach can improve the performance of the algorithm. The framework discussed in this manuscript has been presented in other papers (SCHUMACHER; OLIVEIRA; MITCHELL, 2015; PORDANJANI *et al.*, 2011; UBOLLI; GRIVET-TALOCIA, 2007) where the strategy of relative error preservation is applied.

Now, the question the reader may ask is how to define the weight matrix Ξ. Since we want to normalize the impact of the error throughout the frequency spectrum, then it is pertinent to define Ξ as a diagonal matrix of the inverse of the frequency response samples. Consider $\mathcal{W} = diag(\mathcal{W}(jw_k))$, for $\mathcal{W}(jw_k)$ defined as follows. Here, three types of weight are presented. Quadratic weighting is applied when one's objective is to have a higher weighting when the system's frequency response values are too small to improve fitness. Moreover, a squared weighting is a weak methodology, and linear weighting is able to satisfy in many cases.

Linear weighting For a linear weighing, $\mathcal{W}(jw_k) = (\frac{1}{|G(jw_k)|})$.

Quadratic weighting For a quadratic weighing, $\mathcal{W}(jw_k) = (\frac{1}{|G(jw_k)|^2})$.

Squared weighting For a squared weighing, $\mathcal{W}(jw_k) = (\frac{1}{\sqrt{|G(jw_k)|}})$.

for $k = 1, 2, \cdots, N$.

In fact, normally, using any weighting type is possible to obtain an accurate model. However, in some cases, when the response samples have too deep valleys, due to high resonant behavior, the model approximation to data may be miss-fitted.

Finally, from (3.18), it is possible to add the weighting methodology to the lease squares optimization problem.

$$\min_{\mathbf{b},d} \left(\phi - \mathbf{M}\begin{bmatrix}\mathbf{b}\\ d\end{bmatrix}\right)^T \boldsymbol{\Xi} \left(\phi - \mathbf{M}\begin{bmatrix}\mathbf{b}\\ d\end{bmatrix}\right) \tag{3.41}$$

where $\Xi = (\mathcal{W}^T \ \ \mathcal{W}) \in \mathbb{R}^{N \times N}$, a diagonal matrix.

4 Subspace methods for passive models

4.1 Introduction

In the previous chapter, not only it was discussed how to develop a system identification routine based on subspace methods to compute the system matrices ($\mathbf{A}$, $\mathbf{b}$, $\mathbf{c}$, d), but also how to improve the approximation and to guarantee numerical robustness to the algorithm. The theory presented was stated to be applicable to system identification as a whole, either when using simulated or real data, for mechanical systems, electrical systems, and others. In this chapter, the interest in passivity lays more specifically in devices that are interconnected, such as networks. Of course, it is also extendable to any other system request, whatever is the formulation of the problem.

An issue usually associated with the macromodeling procedure is the necessity to impose passivity, especially in the context of systems that are interconnected, either because of consistency purpose or in order to uphold stable results on general-purpose time-domain simulation. Returning to Figure 2, after the estimation of the model, a passivity assessment scheme decides on the necessity to impose passivity. While passive models are ready to be applied in simulations, in the cases where passivity has somehow violated the step related to enforcing passivity is essential.

This chapter presents a novel approach for black-box macromodeling based on subspace algorithms with passivity enforcement, what, as stated, plays a fundamental role in the system identification procedure in order to obtain a computationally feasible solution for practical applications. Here, it is shown how to use admittance representations and scattering representations to compute passive state-space models from devices that operate in interconnected networks. In this sense, shall this author emphasize the application of $\mathbf{Y}/\mathbf{S}$ parameters to estimate models of equipment related to electric power systems and the concerns on passivity, necessary to stable analysis.

4.2 Passivity enforcement based on PRL

In Chapter 2, the discussion on passivity was raised and then stated the importance to estimate passive models in the identification routine. However, it is valuable to reinforce that passivity is a property of systems (just like stability) and therefore it does not depend on any system identification technique. In Ihlenfeld (2015b), the author state that passivity has been broadly explored in the literature as a golden model property and has been fully incorporated as a modeling task as well. Post-processing approaches are intended to ensure that the model behavior is strictly passive under any circumstances; among them, convex optimization methods assume a prior knowledge on the system matrix $\mathbf{A}$, and via a few constraints on the optimization problem, the other parameters are changed.

The passivity enforcement scheme based on PRL has been addressed by a few authors, and the majority of them apply the Vector Fitting (VF)- or any modification to its original formulation- to provide prior knowledge of the system poles (OLIVEIRA; RODIER; IHLENFELD, 2016; GUSTAVSEN; SEMLYEN, 2009; IHLENFELD; OLIVEIRA; SANS, 2016; OLIVEIRA; RODIER; IHLENFELD, 2014; COELHO; PHILLIPS; SILVEIRA, 2004). Still in the VF case, the enforcement is formulated by fixing system matrices $\mathbf{A}$ and $\mathbf{b}$ and adapting $\mathbf{c}$ and d to attain a passive realization. The idea of computing the quadruple in two steps is to avoid non-linearity along with the method.

4.3 Passivity enforcement for subspace based algorithms

As previously stated, system identification algorithms widely used in literature, firstly compute $\mathbf{A}$ and $\mathbf{b}$ and then estimate $\mathbf{c}$ and d via an optimization scheme. On the other way around, in subspace methods matrices $\mathbf{A}$ and $\mathbf{c}$ are firstly computed and then $\mathbf{b}$ and d are estimated. Therefore, in order to integrate the passivity enforcement using the LMI (previously defined in (2.18) and (2.19)) in the algorithm, a similarity transform which allows the use of $\mathbf{c'}$ instead of $\mathbf{b}$ is applied (see Section 2.5).

In summary, recalling the algorithm discussed in Subsection 3.4.1, we want to change the description of the optimization problem found in the subspace algorithm (more specifically in step 5) in such a way to write LMI as constraints concerning to impose the passivity property to the estimate state-space realization. The idea is to transform this problem into a convex programming approach for generating guaranteed passive approximations. One topology is presented in Oliveira, Rodier e Ihlenfeld (2014). Although it is functional, the processing time to obtain a feasible solution seemed too long, as the data set increases. In this sense, a computationally practical methodology for changing the optimization cost function to further incorporating passivity constraints is presented in Coelho, Phillips e Silveira (2004), to which we refer in the following description.

In 3.4.2.2 the QR decomposition was stated as a powerful tool to improve the conditioning number of a matrix. Usually $\mathbf{M}$, from (3.18), is susceptible to suffer from ill-conditioning issues. The next steps transform (3.18) in a well-conditioned LMI, which has been proved to be efficiently solved (COELHO; PHILLIPS; SILVEIRA, 2004). If we make

$$\mathbf{M} = \mathcal{Q}\mathcal{R}, \tag{4.1}$$

then, (3.18) can be writen as

$$\min_{\mathbf{b},d} \left(\mathcal{Q} \left(\mathcal{Q}^T \phi - \mathcal{R} \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix} \right) \right)^T \left(\mathcal{Q} \left(\mathcal{Q}^T \phi - \mathcal{R} \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix} \right) \right), \tag{4.2}$$

or, it can be further expanded as

$$\min_{\mathbf{b},d} \left(\mathcal{R}\mathcal{R}^T \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix}^T - \mathcal{R}^T \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix}^T \phi \mathcal{Q}^T - \mathcal{R} \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix} \mathcal{Q}\phi^T + \phi\phi^T \right). \tag{4.3}$$

Equation (4.3) can be shortened to:

$$\min_{\mathbf{b},d} \mathbf{E}^T\mathbf{E} + \delta^2 \tag{4.4}$$

where

$$\begin{aligned} \delta^2 &= \phi^T \left(\mathbf{I} - \mathcal{Q}^T \mathcal{Q} \right) \phi \\ \mathbf{E} &= \left(\mathcal{R} \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix} - \mathcal{Q}^T \phi \right) \end{aligned} \tag{4.5}$$

It is possible to rewrite the problem in a way incorporate an upper limit to the cost function in order to obtain the following optimization over contraints.

$$\begin{aligned} \min_{\mathbf{b},d} \quad & \mu \\ \text{subject to} \quad & \\ & \mathbf{E}^T\mathbf{E} + \delta^2 \prec \mu \\ & \mu \succeq 0 \end{aligned} \tag{4.6}$$

Literature shows that a computationally fast way to solve these equations is to use the Schur complement to transform each quadratic inequality in (4.6) to an LMI (BOYD *et al.*, 1994; VANDENBERGHE; BOYD, 1996; COELHO; PHILLIPS; SILVEIRA, 2001). However, one may understand that it is more efficient to use an algorithm that treats (4.6) directly, rather than translating to the LMI form due to its need for semidefinite programming (COELHO; PHILLIPS; SILVEIRA, 2004).

To attain a passive realization of (3.2), we add the LMIs defined in (2.18) (for admittance representations) and in (2.19) (for scattering representations) in the optimization problem as constraints.

Then, (4.7) and (4.8) are for admittance and scattering representation, respectively.

$$\begin{aligned} \min_{\mathbf{b},d} \quad & \mu \\ \text{subject to} \quad & \\ & \mathbf{E}^T\mathbf{E} + \delta^2 \prec \mu \\ & \mu \succeq 0 \\ & \mathbf{P} \succ 0 \\ & \begin{bmatrix} \mathbf{AP} + \mathbf{PA}^T & \mathbf{Pc}^T - \mathbf{b} \\ \mathbf{cP} - \mathbf{b}^T & -d - d^T \end{bmatrix} \preceq 0 \end{aligned} \tag{4.7}$$

$$
\begin{aligned}
\min_{\mathbf{b},d} \quad & \mu \\
\text{subject to} \quad & \\
& \mathbf{E}^T\mathbf{E} + \delta^2 \prec \mu \\
& \mu \succeq 0 \\
& \mathbf{P} \succ 0 \\
& \begin{bmatrix} \mathbf{AP} + \mathbf{PA}^T & \mathbf{Pc}^T & \mathbf{b} \\ \mathbf{cP} & -1 & -d \\ \mathbf{b}^T & d^T & -1 \end{bmatrix} \preceq 0
\end{aligned}
\tag{4.8}
$$

Therefore, with $\mathbf{A}$ and $\mathbf{c}$ previously estimated and computing the optimal pair $[\mathbf{b}, d]^T$ as (4.7) or (4.8), according to the representation adopted, a passive state-space realization is obtained.

4.4 A weighted approach

In Section 3.5, a weighting strategy to subspace algorithms has been introduced. At that time, the weighting matrix Ξ was defined and showed how to be introduced in an optimization problem based on least squares. In this section, it is provided how to incorporate the weighting matrix in an optimization problem based on LMIs.

In order to add the weight matrix, we change the QR decomposition to

$$
\Xi \;\mathbf{M} = \mathcal{Q}_\Xi \mathcal{R}_\Xi \tag{4.9}
$$

Defining $\mathbf{E}_\Xi$ and δ^2_Ξ as:

$$
\begin{aligned}
\delta^2_\Xi &= \Xi \;\phi^T \left(\mathbf{I} - \mathcal{Q}_\Xi^T \mathcal{Q}_\Xi\right) \Xi \;\phi \\
\mathbf{E}_\Xi &= \left(\mathcal{R}_\Xi \begin{bmatrix} \mathbf{b} \\ d \end{bmatrix} - \mathcal{Q}_\Xi^T \;\Xi \;\phi\right)
\end{aligned}
\tag{4.10}
$$

Then, the optimization problem with the weighting methodology incorporated is

$$
\begin{aligned}
\min_{\mathbf{b},d} \quad & \mu \\
\text{subject to} \quad & \\
& \mathbf{E}_{\Xi}^{T}\mathbf{E}_{\Xi} + \delta_{\Xi}^{2} \prec \mu \\
& \mu \succeq 0
\end{aligned}
\tag{4.11}
$$

5 Case studies

5.1 Introduction

In this section, some case studies based on fundamentals from the previous chapters are presented. Starting in Subsection 5.2 by confronting the theoretical affirmation, widely applied in subspace algorithms, that the order of the model is extracted from the SVD. In 5.3, actual measurement data from a flexible beam is used to evaluate the efficiency of the proposed method to frequency domain system identification. Still in 5.3, the estimation result is compared with the one obtained using the toolbox available on Matlab: Numerical algorithms for Subspace State Space System Identification $N4SID$. Using equivalent parameters, it is shown the proposed algorithm presents more attractive approximations.

Network dynamic systems are analysed in the following subsections where Y and S parameters are used to estimate models for coaxial cables (5.4), a power transformer (5.5) and voltage transformers (5.6). In addition, a passive macromodeling procedure is delivered for these cases. For these three last case studies, besides the passivity enforcement, it was necessary to impose stability as well. So, all eigenvalues computed are positioned in the left half-plane of s.

5.2 Illustrative example

To start the validation of the technique discussed in the aforementioned chapters, a very simple synthetic system is chosen.

$$G(s) = \frac{s^3 + 4.2s^2 + 710.3s - 1445}{s^3 - 9.378s^2 + 1.008s - 72.28} \tag{5.1}$$

Alternatively, given $G(s)$ in terms of its poles and zeros.

$$G(s) = \frac{(s-2)(s+3.10-26.704i)(s+3.10+26.704i)}{(s+10)(s+0.3110-2.6704i)(s+0.3110+2.6704i)}$$

The frequency response of this third order system is illustrated in Figure 8. 87 samples logarithmic distributed from 10^{-1} rad/s to 10^3 rad/s are collected, and the magnitude is indicated in decibels.

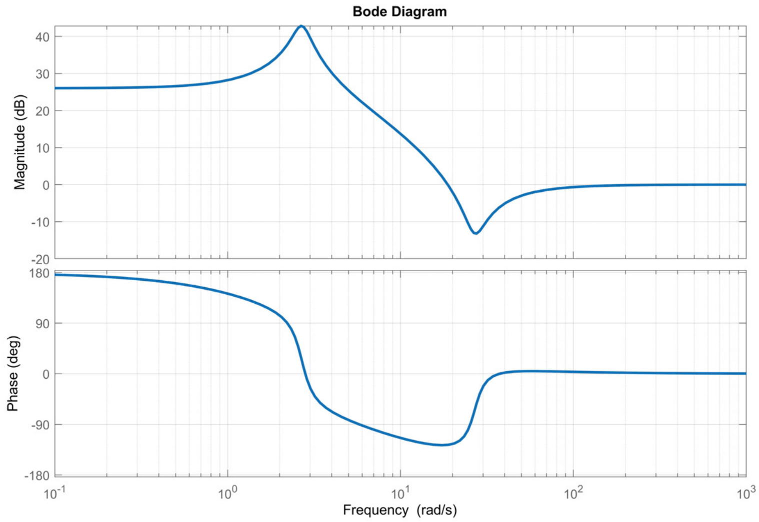

Figure 8 – Bode plot of synthetic illustrative example

Along Chapter 3, it has been stated that the information about the order of the estimate model is given by matrix Σ from SVD. Because it is known that the order of the system to be estimated is 3, this example is used to check that statement. Setting $q = 10$, a initial guess about the order of the system, and using $n = rank(\Sigma)$, then subspace algorithm would compute $n = 3$ and the following eigenvalues are estimated.

$$-9.9978 + 0.0000i$$

$$-0.3110 + 2.6707i$$

$$-0.3110 - 2.6707i$$

In Figures 9 and 10 it is shown the identification case for magnitude and phase, respectively. The dashed line is used for the model response samples, and a solid line denotes the data.

It has been shown the effectiveness of the rank information from matrix Σ to determine the model order. As the complexity of data and system dynamics increases, especially for the cases using data corrupted by noise, it has been checked that purely the

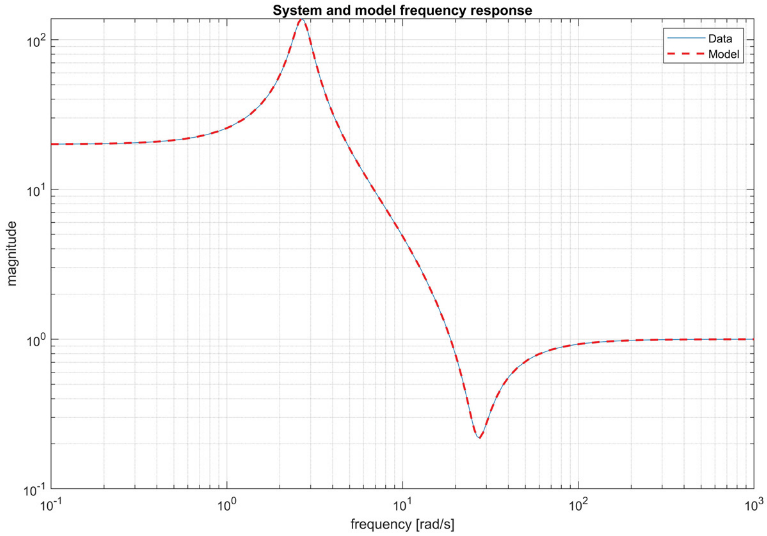

Figure 9 – Magnitude of frequency response for the identification case

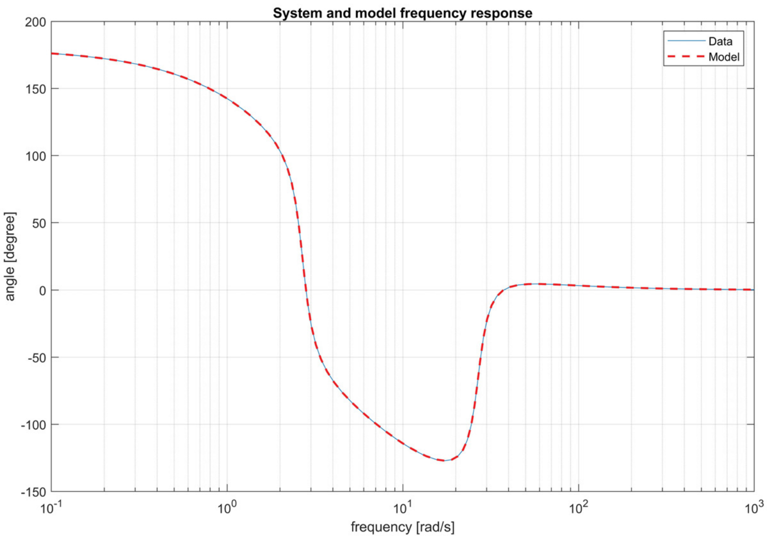

Figure 10 – Phase of frequency response for the identification case

rank information is a weak tool. Therefore, for such cases, the user must seek dominant singular values in a set with the size determined with the aid of the rank information.

5.3 Flexible beam

In this section, the efficiency of the proposed algorithm for subspace frequency identification is proved for a highly resonant system using real measurement data to obtain a state-space model. Here, many items discussed in previous chapters are employed, such

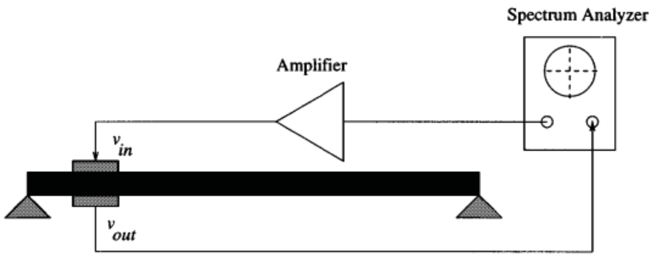

Figure 11 – Resonant beam experimental setup (GILSON; WELSH; GARNIER, 2018).

as the mathematical tools applied to improve the conditioning number of matrices (read as meaning the algorithm is able to estimate high order models), or the weighting methodology applied to improve the model approximation to data. In the end, the proposed algorithm returns a better approximation to response curves when compared to the N4SID, based on the RMSE.

In turn, this author would like to acknowledge professor James Welsh for sharing the dataset. The experimental setup and measurement samples have been presented in Welsh e Goodwin (2003), Gilson, Welsh e Garnier (2018). The frequency axis is defined from 10Hz to 600Hz and 1600 points were used to estimate the model.

Figure 11 illustrates the experimental setup described in Gilson, Welsh e Garnier (2018). The bar consists of a $60cm$ long uniform aluminum beam and a pair of piezoelectric elements are attached symmetrically to either side of the beam. In this experiment, a vector analyzer is used to determine the frequency response of the piezoelectric laminate and the experimental data span approximately two decades as illustrated in Figure 12.

The N4SID toolbox is capable of estimating state-space models based on time-domain and frequency-domain data. Adjusting its parameters to estimate a deterministic realization, with the weighting scheme for singular value decomposition set as automatically chosen, no weighting prefilter used, and with no regard to stability enforcement. All other options are set as default so that the comparison is fair. Then, we estimate a 19th order model with N4SID, and the computed RMSE is 0.8651, and the WSE is $2.4797{\times}10^3$.

Using the algorithm described in Chapter 3 with the same parameters used in

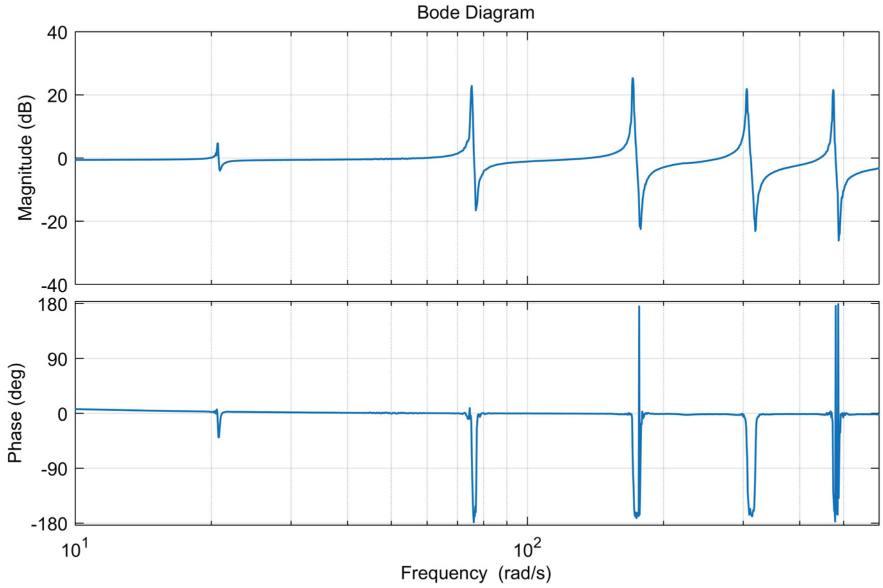

Figure 12 – Bodeplot of flexible beam.

N4SID, and weight matrix equal to the identity, that is, no weight applied, we estimate a model, called here Model 1, with RMSE equal to 0.4205 and WSE equal to 83.6328. Fig. 13 shows the models approximation to data. Clearly, the approximation of the proposed algorithm was more accurate when compared to N4SID.

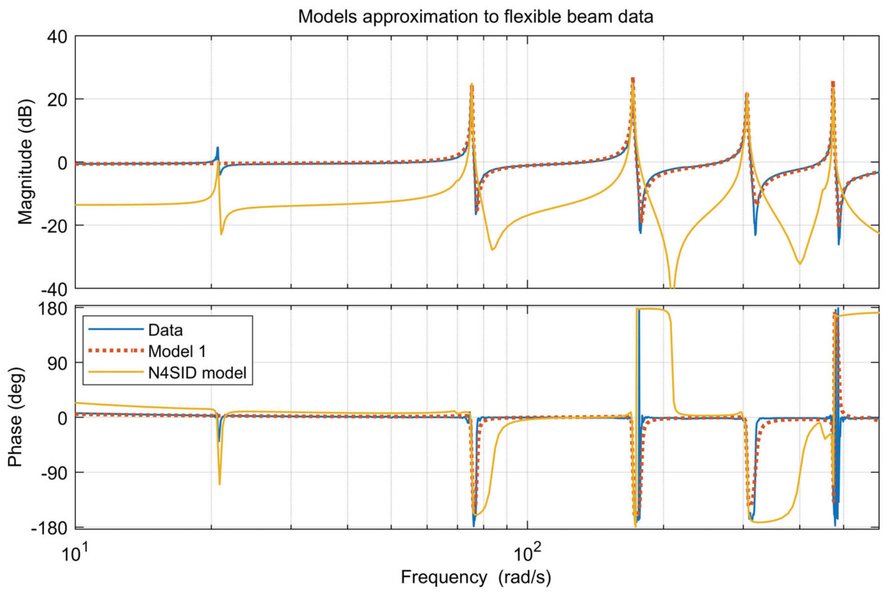

Figure 13 – Models and data approximation.

Now we show the efficiency of the linear weight in the approximation. Again, we estimate a 19th order model with linear weighting called here Model 2. Note that the weighted algorithm is capable to estimate lower magnitudes resonances because these response samples have turned to be equally relevant in the computation of the error measurements. The computed RMSE and WSE are, respectively, 0.4339 and 7.5918. The approximation is illustrated in Fig. 14.

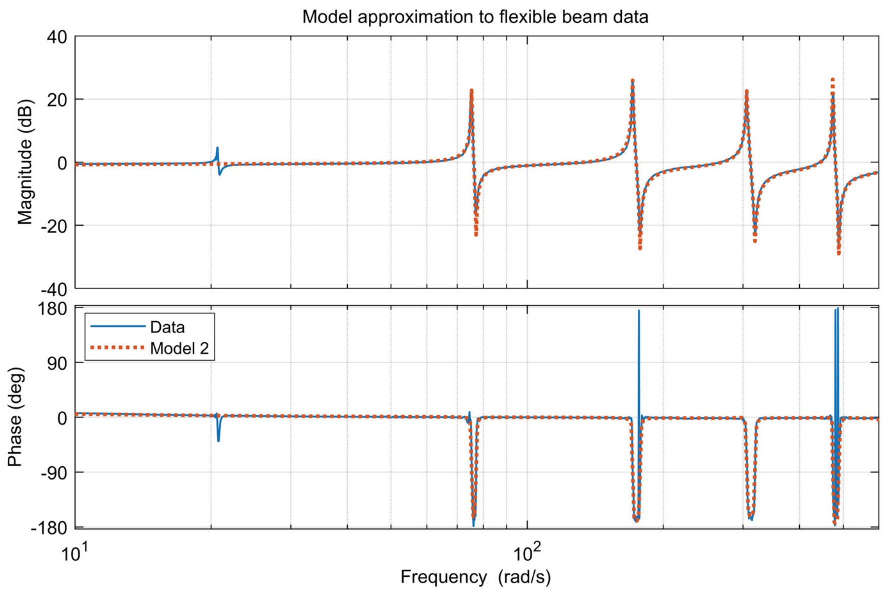

Figure 14 – Data and Model 2 frequency response.

A linear weighting approach has diminished the WSE between model and data considerably. Eventually, it is a powerful tool to obtain more accurate approximations in case of systems with high resonant behavior.

5.4 Coaxial cable

Coaxial cables are commonly used to connect frequency response analyzers to the equipment in the test. When the terminals of the devices are distant, one may use long cables to connect them. Literature shows that in such cases, the presence of cables impact on the collected observations, especially in high-frequency points, where the presence of cables manifests more evidently. As a consequence, when using measurement data to estimate a mathematical representation for given equipment, the user recognizes that the effects of cables are present in the dataset.

In the particular case of modeling transformers, the presence of high-frequency resonances may induce excessive overvoltage in the windings, causing dielectric stresses (GUSTAVSEN, 2010; GUSTAVSEN *et al.*, 2018). The reason why we highlight the transformers is that the next two subsections deal with transformer modeling, and especially in Subsection 5.6, a high-frequency modeling analysis is given.

While working with coaxial cables models, we noticed that in some cases, the standard algorithm could not compute a passive realization. Thus, this case study aims to collect frequency observations and estimate a passive model to cables, which validates the passivity enforcement methodology.

In this case study, we are interested in a 1-port system for coaxial cables. To that purpose, one terminal is connected to the frequency analyzer and in the other terminal, an open-circuit termination is plugged, as illustrated in Figure 15. Non equidistantly spaced data is acquired from a 4.26m coaxial cable, in a frequency sweep that spanned a few decades. Type N plugs were used in the connections, which avoid interference and noise pollution. The frequency response is determined from 1.3370 Mrad/s to 69.1840 Mrad/s within 1371 points collected, as illustrated in Figure 16.

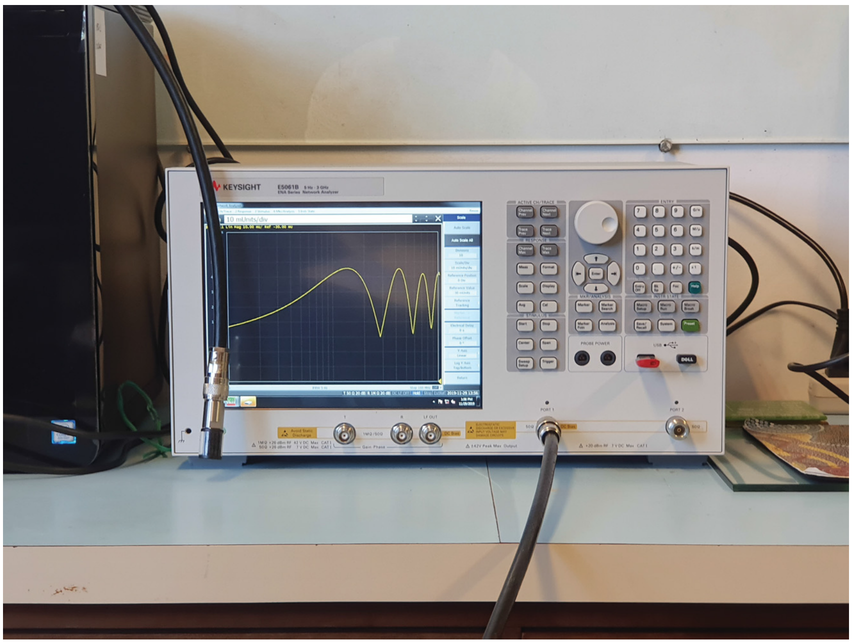

Figure 15 – Experimental setup photo.

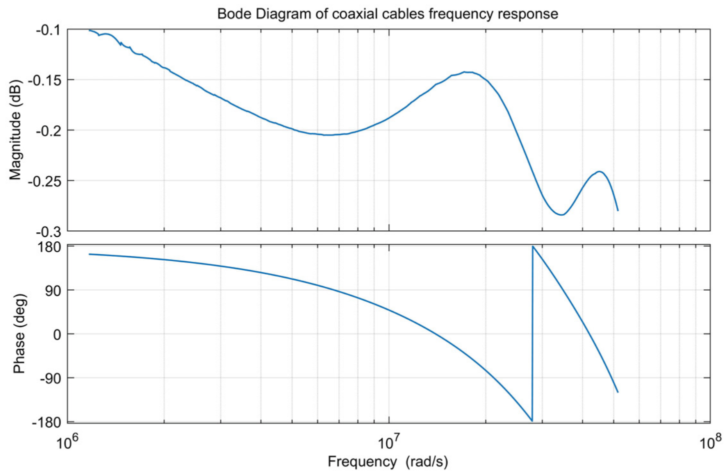

Figure 16 – Bode diagram of coaxial cable.

The objective of this section is to apply the method proposed in Chapter 4 using either *S parameters* and *Y parameters*. In an initial analysis, the dataset, described in a scattering sense, is used to obtain a passive state-space realization to the 4.26m long cable. Secondly, we convert data to admittance representation and then repeat the passive macromodeling procedure. Simply put, for data based on both Subsection 5.4.1 and Y parameters, two algorithms were used to estimate the cable's dynamics:

- **Model 1:** a simple subspace scheme with no respect to passivity;
- **Model 2:** a subspace based algorithm with a LMI constraint guaranteeing passivity.

In this sense, Subsection 5.4.1 uses scattering parameters, and Subsection 5.4.2 uses admittance parameters data. In the two subsections, we estimate a 10^{th} order model with stability enforcement applied and aided by all mathematical tools to improve the conditioning number of matrices. Also, to improve the model approximation, a quadratic weighting methodology is applied. SeDuMi optimization solver was used to solve the LMIs.

In accordance with Section 2.3, the sweep evaluation of the passivity property in the dataset indicated that it is passive. Meanwhile, there is often a misconception that passive data yields a passive model by simply estimating parameters to fit the response curve. This author intentionally chose this example that will illustrate that a passive dataset yields a not passive model.

5.4.1 S parameter data

Figures 17 and 18 illustrates the Model 1 magnitude and phase approximations to data. The computed RMSE = 8.9019×10^{-4} and WSE = 0.0011.

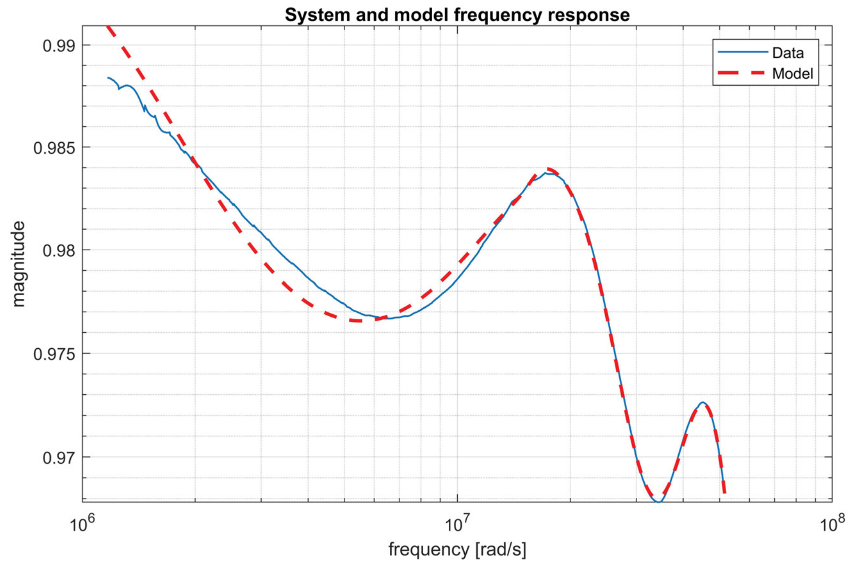

Figure 17 – Approximation between not passive model and data: magnitude plot.

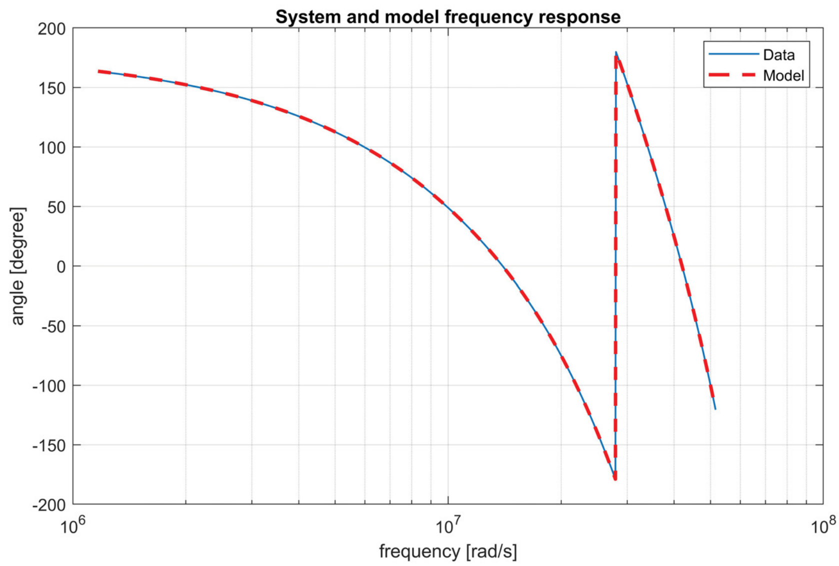

Figure 18 – Approximation between not passive model and data: angle plot.

After subjecting this model to the frequency sweep check, it has been stated that it violates the passivity in a percentage of 15.9767. This percentage represents the number of frequency points that violated the passivity criteria in relation to the total number of frequency samples evaluated.

The passivity enforcement scheme based on PRL is now added to the procedure. We highlight that the system identification parameters are the same as previously detailed. Figures 19 and 20 illustrate Model 2 approximation to data, with RMSE = 0.0011 and WSE = 0.0017. Comparing this metric with the one obtained for Model 1, it is possible to note that Model 2 shows a greater error. It is important to highlight that in spite of that fact, the properties of the system are preserved. In summary, Model 2 is a consistent representation of the equipment in terms of passivity, and therefore could be applied to simulation.

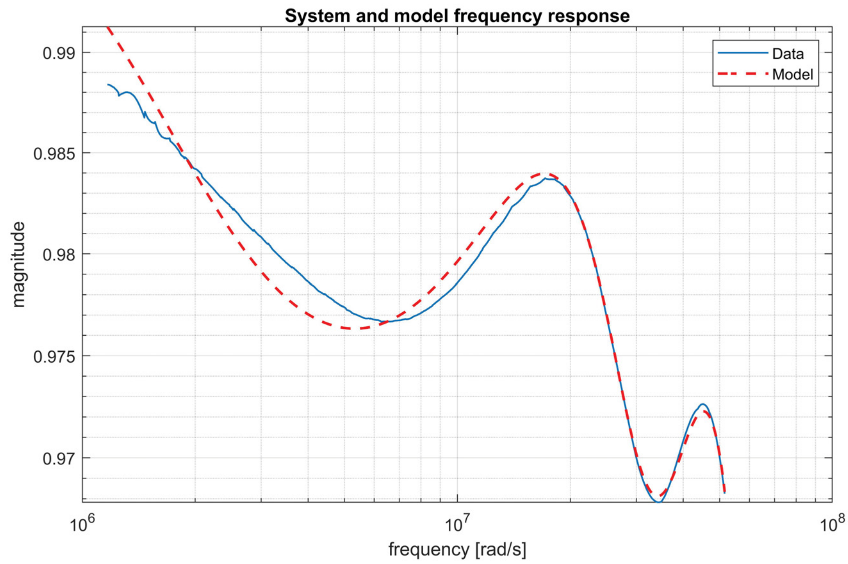

Figure 19 – Approximation between passive model and data: magnitude plot.

5.4.2 Y parameter data

Here, data is converted to admittance via a formulation described in Section 2.3. As stated, the data in admittance representation is passive. Again, two models are estimated: a first one with no concern about passivity property and a second one with passivity enforcement. Consequently, Model 1 is not passive (RMSE = 0.0028 and WSE = 0.0094), whereas Model 2 is passive (RMSE = 0.0134 and WSE = 0.1903).

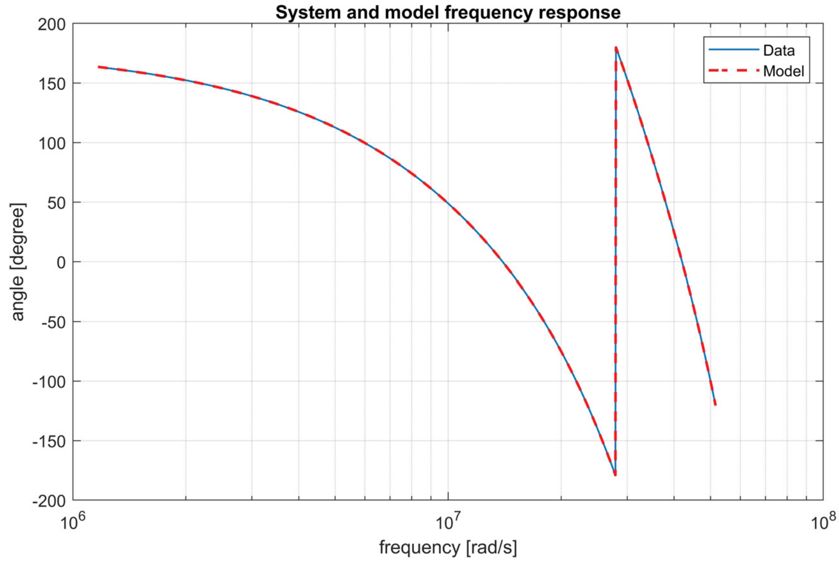

Figure 20 – Approximation between passive model and data: angle plot.

5.5 Power transformer from Itaipu hydropower plant

Because of the presence of many inductances and capacitances, understanding a transformer frequency response is such a complex problem. Therefore, obtaining a model is not straightforward even for a one winding in air transformer. In a real transformer, the winding becomes coupled by inductive and capacitive effects to the other windings, the core and the tank, which contribute to the transformer frequency response (PICHER *et al.*, 2017). These effects are observed along the spectrum and to simplify the analysis, it is common to divide the frequency response into frequency ranges.

In the first stage, at low frequency (below 12k rad/s), the magnitude decreases amplitude in a response characterized by the magnetizing inductance of the core. Then, there is the presence of a low-amplitude frequency resonance, due to the interaction between windings, usually observed below 65k rad/s. From 125k rad/s to 6M rad/s the winding structure itself is assumed to be the main contributor to the frequency response (GUSTAVSEN *et al.*, 2018; HUSSEIN; MATAR; IRAVANI, 2016; VAESSEN, 1988; LIANG *et al.*, 2006).

In this case study, actual measurement data from a power transformer in operation in the hydroelectric plant of Itaipu, in Foz do Iguaçu/PR, is used. The system response

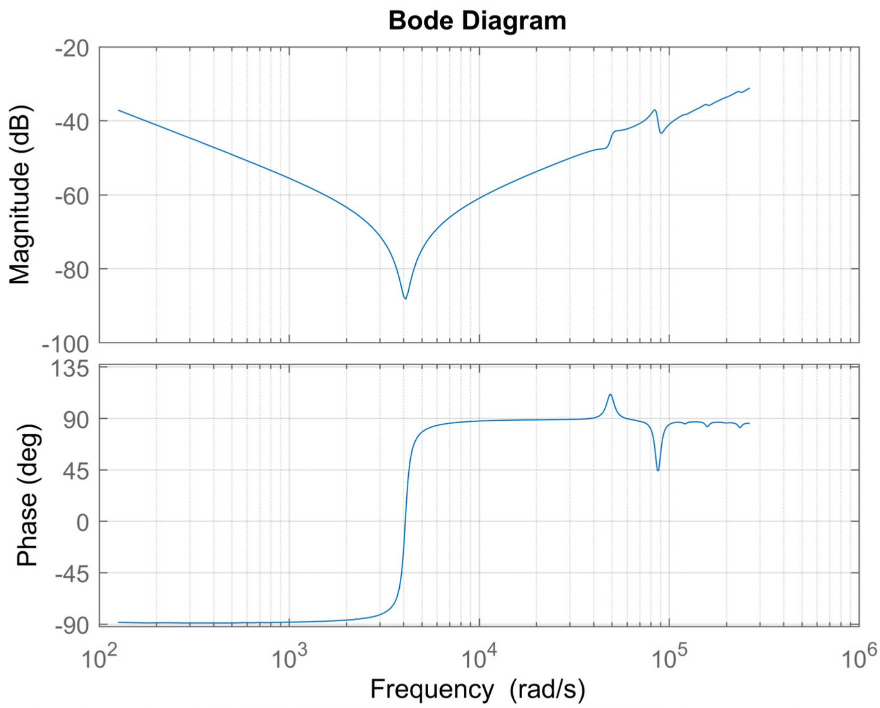

Figure 21 – Bode diagram of a inductive power transformer from Itaipu.

is illustrated in Figure 21 in which the frequency axis is defined from 125 rad/s to 265k rad/s. In this case study, we concern a SISO system identification based on observations for the Y_{11} case from (2.4).

The objective of this case study is twofold: i) to obtain the transformer model based on the measurement data; and ii) to check and ensure passivity to the model. Therefore, initially defining the input parameters applied to the code and then checking the passivity of the estimated model. If it shows to be not passive, then the parameter estimation is reset to compute a passive model for the transformer. SeDuMi optimization solver was used to solve the LMIs.

In subspace methods, the order estimate is aided by the information from matrix Σ from the SVD in step 3, although an initial guess is necessary. Setting this guess to 20, the algorithm suggests the model be of 15th order. In this sense, the computed error is evaluated as the order varied in the range of 10 to 20, and it is displayed in Table 1. Aided by that information, the order is set as 14.

Once the order is defined, the approximation can be improved with a weighted

order	RMSE ($\times 10^{-3}$)
10	1.8010
11	0.9388
12	1.1000
13	0.2819
14	0.1459
15	0.3784
16	0.5417
17	1.7000
18	1.9000
19	1.8000
20	2.1000

Table 1 – Values of RMSE according to order variation

algorithm. Three types of weighting methodology are tested and the results are presented in Table 2. Note that the WSE (see Equation (2.39)) was added, what provides an error measurement point-by-point.

weight	RMSE($\times 10^{-4}$)	WSE
no	1.4594	1.4137
linear	1.3976	0.1115
quadratic	12	0.9752
squared	1.4021	0.1428

Table 2 – Values of RMSE according to variation of weight type

From the aforementioned evaluation, it has been noted that the combination that returns the minor error measurement (RMSE and WSE) is a 14th order model with linear weighting. Figures 22 and 23 illustrate the model approximation to the data response considering the parameters selected.

Now, using the sweep assessment to check passivity in the estimate model, it is was noted that it violates in a percentage of 5.4945 in the frequency range of 34k rad/s to 77k rad/s. Also, the LMI test showed that the model does not satisfy the passivity criteria. One condition added to the algorithm is the necessity to compute only stable eigenvalues, so that it would be consistent to expect a passive model. Therefore, the algorithm is set to compute $\mathbf{b}$ and d with regard to passivity, as discussed in Chapter 4. The parameter are the same as previously defined. The frequency response of the passive model is illustrated in Figures 24 and 25. The approximation metrics for this model are: RMSE = 1.100e-3

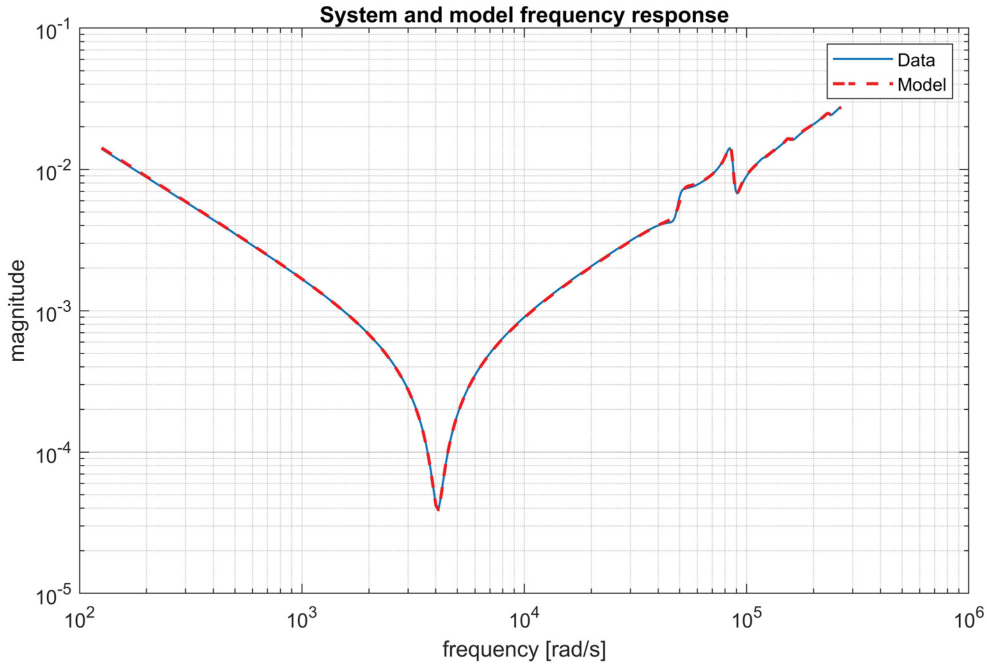

Figure 22 – Magnitude model approximation to data frequency response of a transformer.

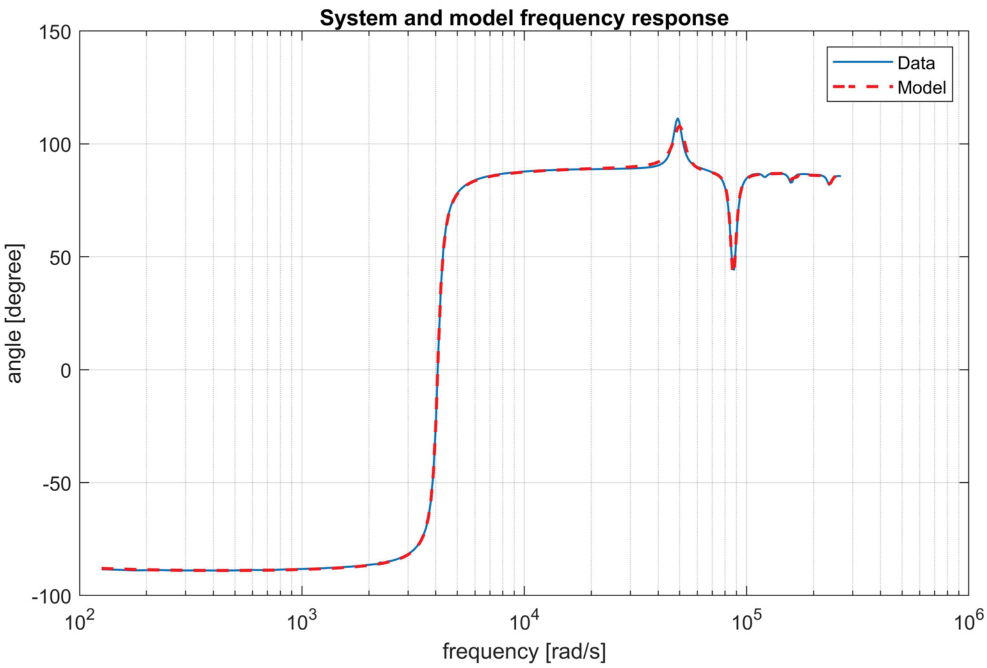

Figure 23 – Phase model approximation to data frequency response of a transformer.

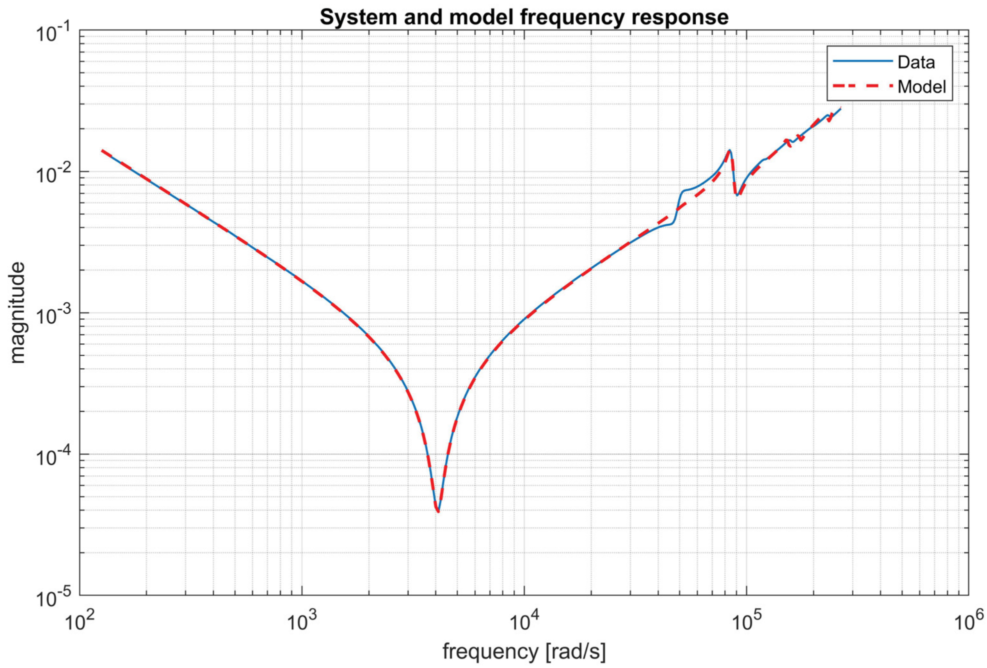

Figure 24 – Magnitude of passive model of a transformer.

and WSE = 2.0250.

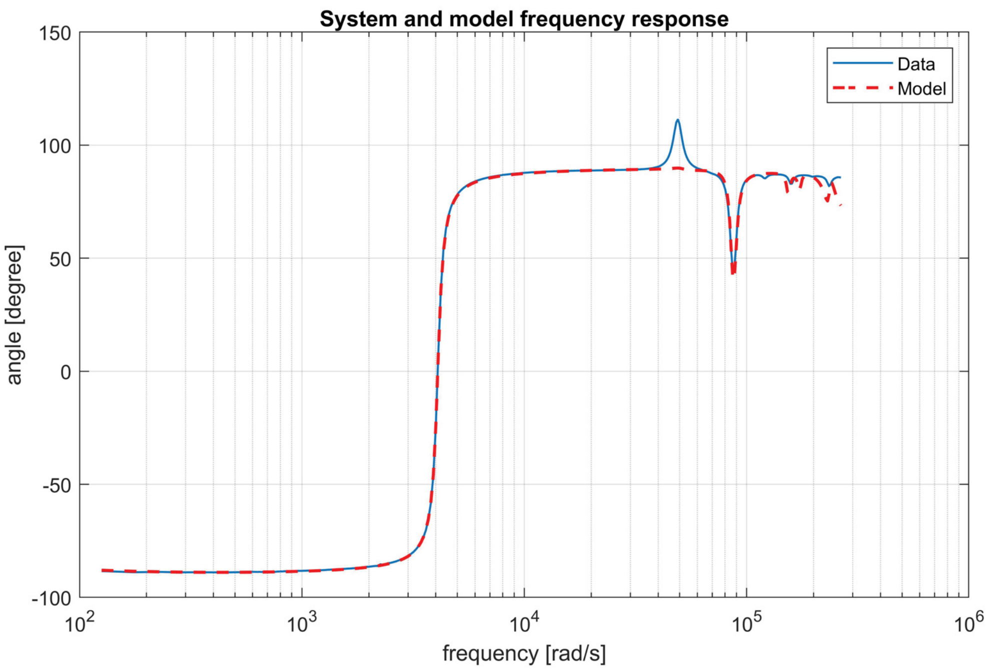

Figure 25 – Phase of passive model of a transformer.

5.6 Jirau's Potential Transformer

This case study aims to estimate a passive model of an operational potential transformer in the Jirau hydroelectric plant based on actual measurement data. The equipment operates in a GIS facility, which is isolated with SF6 gas. It is about a single-phase earthed potential transformer with a primary voltage $525/\sqrt{3}$ kV. The secondary is a two winding with voltage options of $121.2 - 67.4$ V each winding, regulated by a tap in accordance with Figure 26. Figure 27 illustrates the transformer rated values.

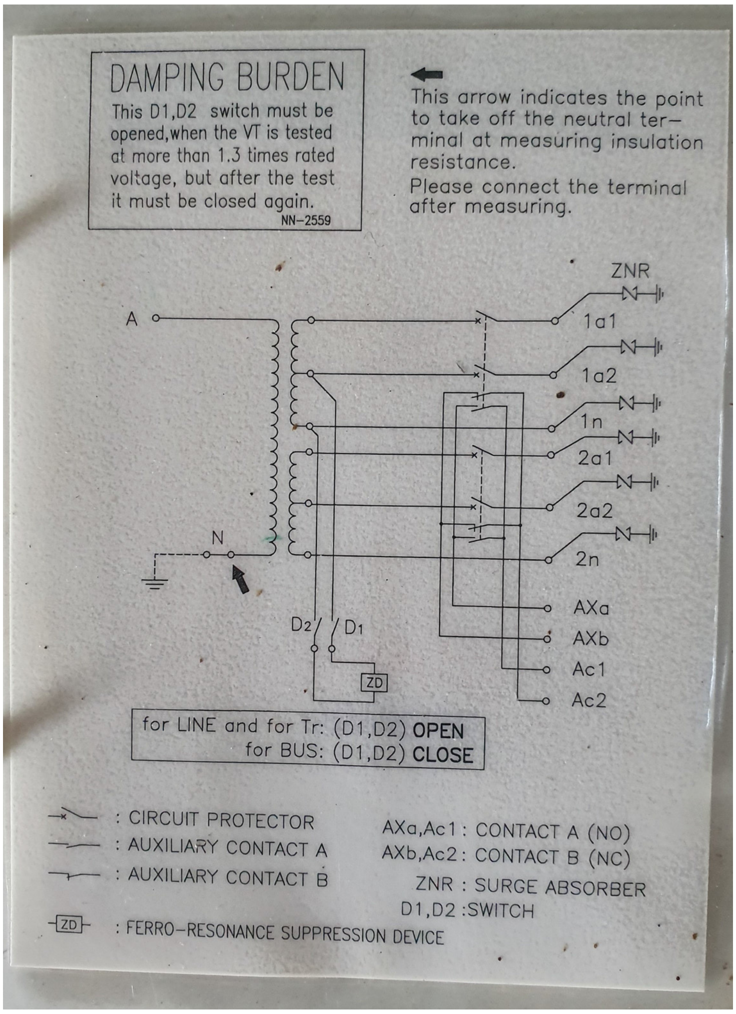

Figure 26 – Connections of the single-phase transformer.

The potential transformer has been placed in an area with satisfactory grounding, which avoid the flow of parasitic currents and reduces the presence of noise in measurement data. More importantly, an effective grounding is fundamental to assure a common point to either the transformer and other connected equipment. The measurements were performed

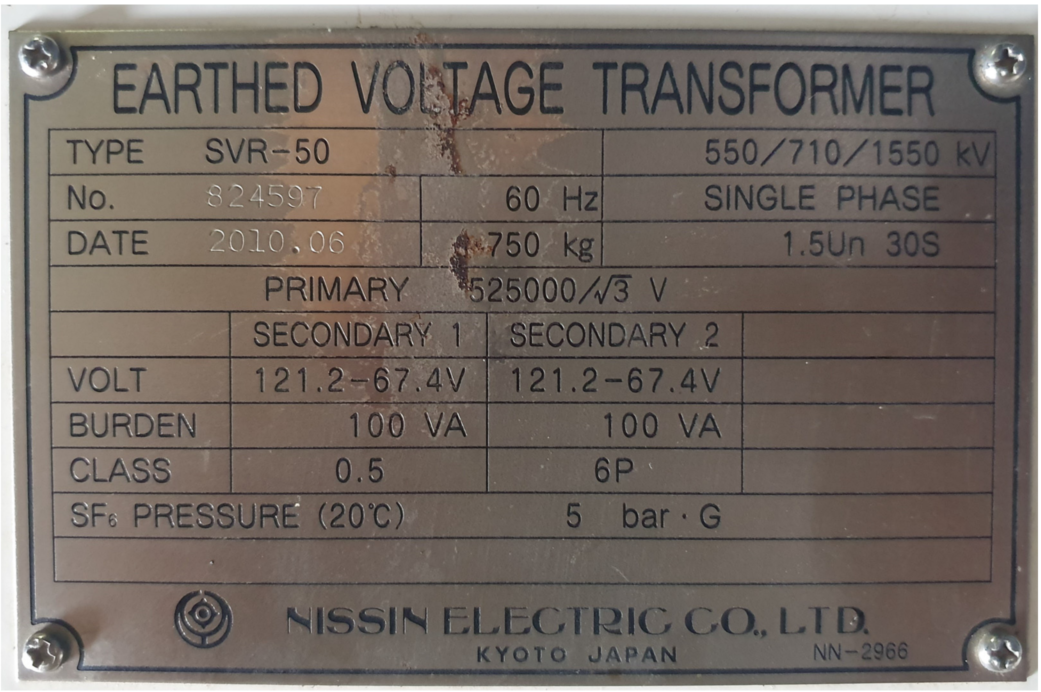

Figure 27 – Ratings of the single-phase transformer.

in a bunker where the transformer and the data acquisition equipment were put close enough to avoid using long coaxial cables. Although for practical applications, the presence of the cables must be compensated in the final transformer model. A picture of the experimental setup is given in Figure 28.

The frequency response in terms of admittance parameters is indicated in Figure 29. In this frequency experiment, we collected 1161 points along 4.35k rad/s to 314M rad/s. A more detailed description of the measurements performed in this power plant can be found in OLIVEIRA *et al.* (2019).

In Table 3, the performance of the algorithm with no weight applied is presented in terms of the RMSE, what yields to an order 7. So, with fixed model order, different weighting types are tested as demonstrated in Table 4. Aided by these results, it is defined the parameters: $7th$ order model, with linear weighting. SeDuMi optimization solver was used to solve the LMIs.

Once the identification parameters are set, a realization is obtained and its magnitude and phase are indicated in Figures 30 and 31, respectively. Surprisingly, the parameters estimate satisfied both the frequency sweep test and the LMI based passivity assessment.

Figure 28 – Experimental setup.

order	RMSE
5	0.0850
6	0.0059
7	0.0020
8	0.0036
9	0.0060
10	0.0988
11	0.2811
12	0.2336
13	0.2637
14	0.2133
15	0.1831

Table 3 – Values of RMSE according to order variation

weight	RMSE ($\times 10^{-3}$)	WSE
no	0.0020	113.8374
linear	0.0019	2.0965
quadratic	0.0026	4.2130
sqrt	0.0021	4.2900

Table 4 – Values of RMSE according to variation of weight type

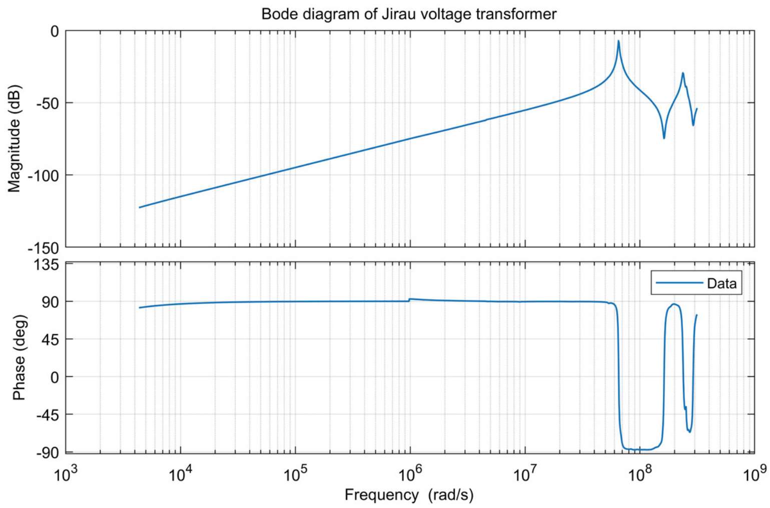

Figure 29 – Frequency response of Jirau's potential transformer.

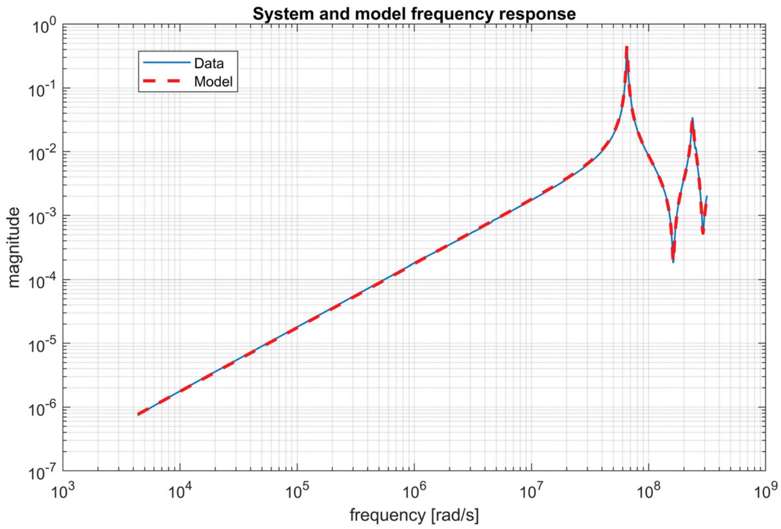

Figure 30 – Magnitude of frequency response of estimated model.

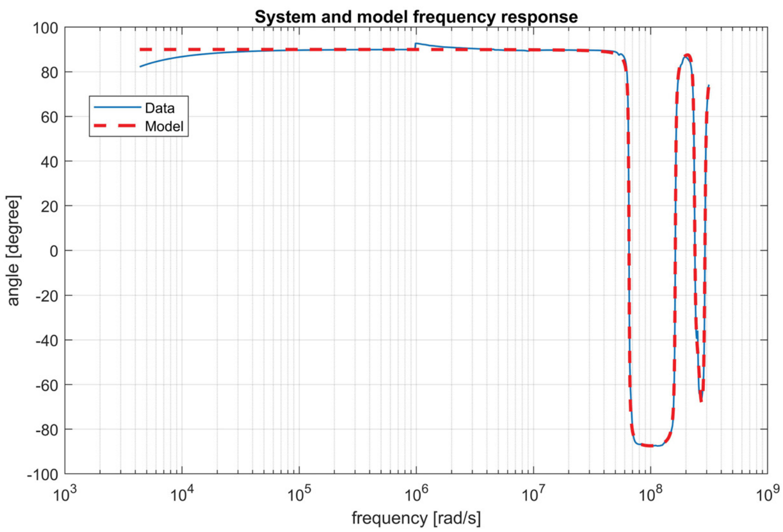

Figure 31 – Phase of frequency response of estimated model.

6 Conclusions

6.1 Introduction

This chapter exhibit a summary on the main results and principal contributions of this thesis. It is also shows how the research can move forward; open questions are then raised.

6.2 Overall considerations

This thesis can be interpreted as a contribution to subspace-based methods applied to frequency domain system identification. While many existing algorithms conceives frequency domain approximations to an arbitrarily close degree of accuracy, devising a passive model is mandatory to avoid anomalous behavior in simulations. This research has culminated in an original formulation for estimate parameters of a state-space system representation to passive resonant equipment via subspace algorithms.

Subspace technique easily computes the system matrices using the information provided by an orthogonal projection applied to the data matrices. From that understanding, many identification frameworks are derived and whereas the model complexity has risen, adjustments in the algorithm are made necessary. It should be stated, for instance, numerical problems in the algorithm that commonly occur as the number of eigenvalues increases. Along with a few steps of the subspace framework, the user will face equations related to matrix inversion, what are one reason to cause ill-posed problems. Therefore, it came the demand for developing a procedure capable of managing high order matrices with accuracy. In this sense, the QR decomposition played an important role throughout this research.

Synthetic data showed to go easy with subspace algorithms. In many cases, high order models could be estimated without computational stress. If one's interest is to obtain state-space parameters based on synthetic data, then perhaps only working with a better-conditioned algorithm is adequate for the job. Nevertheless, real problems require operational equipment to be tested, via a non destructively procedure that aims to apply input signals and collect output samples. However, the estimation based on actual measurement data is easily affected by noise. A non-linear model can be seen as a solution to such cases.

Although deterministic algorithms are not expected to be robust, depending on the dataset and the distribution of the orthogonal projection, the proposed algorithm showed interesting efficiency for estimating resonant equipment parameters based on measurement data. Indeed, the presence of noise in the case studies presented is not relevant. In many examples, the dataset was acquired from a measurement procedure using a VNA, and the equipment itself is capable to maximize the reduction of noise in the samples. Considering that fact, the authors understood that a deterministic realization should be enough for the proposed application.

Because of the magnitude characteristics of the frequency response behavior of resonant poles along the spectrum, sometimes subspace algorithms can give more importance to estimate resonances present in higher magnitudes and downplay the presence of resonance behavior in lower magnitudes. To overcome these related issues, a weighted procedure has been developed and proved to approximate better the curve of response data. For most cases studied, a linear weight matrix returned satisfactory accuracy.

When compared to the existing subspace-based system identification toolbox N4SID, the proposed algorithm has shown to be an exciting tool, being capable of approximate frequency domain data more accurately under the same circumstances.

Doubtless, the major achievement of this research is the frequency domain estimation of passive models. The incorporation of a positive real lemma that imposes passivity in the model construction was validated and showed effectiveness. Towards the thesis, the misconception that using data from passive equipment to estimate its model would return a passive model has been proved to be wrong. For instance, measurement data from coaxial cables were used to obtain its model and it showed a certain level of passivity violation. It also happens to more complex devices, such as an inductive power transformer, operational in the hydroelectric plant of Itaipu. Such examples demonstrate the necessity to enforce passivity in models used in interconnected networks.

6.3 Future directions

Throughout this research, the proposed algorithm had difficulties when trying to estimate models using frequency response with high-frequency samples. In very high-frequencies, non-linear behavior is predominant along the spectrum, therefore it is a challenge for linear models to accurately approximate to data. One further research direction in subspace-based algorithms is to obtain non linear realizations for systems whose dynamics are characterized above hundreds of megahertz. Another suggestion is to develop multiple models to represent specific partitions of the frequency spectrum, either modulated in magnitude or in frequency. In other words, the idea is to work not only with one model for all data but to obtain various realizations, each one dedicated to a partition of data with its own characteristics.

Moreover, the research also can lead to developing a stochastic algorithm with regard to passivity. The reason why estimate a passive model has been endorsed along with the manuscript. Now, the idea of implementing a stochastic algorithm raised when working with data not descending from measurements itself, but from other sources, like a non-parametric identification. In such cases, the presence of noise is significant and therefore, a stochastic methodology is necessary.

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Appendix

APPENDIX A – Application in electrical power systems

In this appendix, the authors refer to a common application related to wide band modelling in the context of electrical power systems. Examples of some common issues related to transient signals that frequently occur in those systems are given.

A.1 Introduction

Procedures for representing power system dynamic functionality have been addressed by many research. A detailed description of the system parameters and elements is necessary in order to understand its behavior beyond the usual steady-state analysis. During contingencies, wide-band electromagnetic transients are noticed in the interaction among the electrical system components. In order to mitigate the issues caused by transient over-voltages, reliable models that cope well with this frequency range are essential. Thus, trustworthy decisions about the system shall be taken.

The occurrence of transient over-voltage is a common cause in high voltage equipment dielectric failures (IEEE, 2011; CIGRE-A2/C4.39, 2014). When a high electric field is applied, the dielectric layer loses the characteristic of non-conducting, and begins to ionize and conduct electric current. Such a problem can damage high voltage equipment and cause system injuries (SHIPP *et al.*, 2011b; HORI *et al.*, 2007; OLIVEIRA; RODIER; IHLENFELD, 2014).

A.2 Very Fast Transient Overvoltage (VFTO)

The operation of electric power systems demands an ongoing distribution of energy among the systems components. Under normal operation conditions, the system behavior can be described by a couple of voltage and current phasors (IEEE, 2011). However, in disturbance or switching conditions, the power exchange among the systems elements may lead to a large voltage and current variation. The study about electromagnetic transients lays its basis on the importance to investigate the phenomena on power systems, which play fundamental role in the analysis of energy grid. Reliable decisions about the system shall be taken. The aforementioned study involves a frequency range from DC to 50MHz (CIGRÉ, 2000). Tabela 5 illustrate the origin of transients and its correspondent frequency spectrum (most common values).

Table 5 – Origin of electrical transients and associated frequency ranges

Origin	**Frequency range**
Transformer energization ferroresonance	0.1Hz - 1kHz
Load rejection	0.1Hz - 3kHz
Fault clearing	50/60 Hz - 3kHz
Fault initiation, line energization	50/60 Hz - 20kHz
Transient recovery voltage	50/60 Hz - 20/100 kHz
Multiple restrikes of circuit breaker	10kHz - 1MHz
Lightning surges, faults in substations	10kHz - 3Mhz
Disconnector switching (single restrike) and faults in GIS	100kHz - 50MHz

Source: Working Group 02 (Internal overvoltages) of Study Committee 33 (Overvoltages and Insulation Coordination). Guidelines for representation of network elements when calculating transients 02, Published by Cigrè, 2000.

Among the occurrences described in Tabela 5, transients may be classified in four groups, as shown in Tabela 6 categorized in accordance to the frequency of the electromagnetic wave. Also, a designation for each group is presented and examples of each occurrence is provided.

Table 6 – Classification of frequency ranges

Group	**Frequency range**	**Designation**	**Representation maninly for**
I	0.1 Hz - 3kHz	Low frequency oscillations	Temporary overvoltage
II	50/60 H - 20kHz	Slow front surges	Switching overvoltages
III	10kHz - 3Mhz	Fast front surges	Lightning overvoltages
IV	100kHz - 50Mhz	Very fast front surges	Restrike overvoltages

Source: Working Group 02 (Internal overvoltages) of Study Committee 33 (Overvoltages and Insulation Coordination). Guidelines for representation of network elements when calculating transients 02, Published by Cigrè, 2000.

In particular, literature shows that the effect of switching in Gas Insulated Substations (GIS) can cause system injury due to the occurrence of Very Fast Transient Overvoltages (VFTO) (YOSHIZUMI; MATSUDA; NITTA, 1982; V.HIMASAILA;

M.NAGAJYOTHI; T.NIREEKSHANA, 2017; PATHAK; BHATTI; IBRAHEEM, 2015). Because of the characteristics of the substation, these overvoltages increase considerably after some reflections in the junctions of the GIS. Problems have been reported in those facilities claiming that VFTO potentially damage GIS internal insulation and other components.