UNIVERSIDADE FEDERAL DO PARANÁ

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RESOLUÇÃO PASSO A PASSO DE EXERCÍCIOS DA OBRA “VETORES E GEOMETRIA ANALÍTICA”, DE PAULO WINTERLE

CURITIBA

2018



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1. OBJETIVO E ESCOPO

A obra “Vetores e Geometria Analítica”, de Paulo Winterle, em sua segunda edição, é livro utilizado na disciplina de Geometria Analítica em vários cursos superiores do Brasil. Além dos exemplos resolvidos, o livro contém aproximadamente 800 exercícios e suas respostas, mas não a resolução passo a passo desses exercícios.

No segundo semestre de 2018, o autor, como parte do currículo do curso de Engenharia Industrial Madeireira na UPFR, cursou a disciplina de Geometria Analítica e utilizou, por escolha do docente da disciplina, essa obra como meio didático principal. Buscando aprofundar o estudo e constituir registro de estudo, o autor fez anotações completas dos exercícios da obra e de cada passo de sua resolução.

Não é necessário explicar que tais anotações podem ser de grande valia para outros alunos que utilizem a mesma obra, e portanto torna-se valiosa a disseminação do conhecimento produzido na resolução desses exercícios.

Sendo assim, este trabalho consiste da resolução completa dos exercícios da obra de Paulo Winterle. Os exercícios são apresentados em sua resolução passo a passo, e, quando possível e necessário, com gráficos simplificados.

Em “Vetores e Geometria Analítica”, os exercícios são apresentados de forma graduada, e por isso diversos exercícios, especialmente os iniciais, são de resolução imediata e simples. Tais exercícios foram omitidos desta resolução. Com isso quer-se dizer que o escopo desta resolução exclui aqueles exercícios resolvidos pela mera aplicação de conhecimento trivial sobre geometria analítica.

Da mesma maneira, alguns exercícios consitituem repetição explícita para prática, e tais exercícios também são excluídos do escopo deste trabalho.

Os exercícios aqui apresentados são cópia das anotações do autor, feitas a mão, o que normalmente auxiliará o leitor pela disposição espacial dos cálculos e também por outros artifícios gráficos rudimentares. O autor espera que tal seja de auxílio para o leitor.

Naturalmente, aprender Geometria Analítica significa praticar Geometria Analítica. A resolução de cada exercício apresentada aqui não deve ser vista como única forma ou sequer como resolução correta, mas somente como “troca de idéias” para quem já dedicou esforço à resolução de um exercício e está em busca de sugestões. O autor

lembra que o conteúdo apresentado aqui é o resultado do esforço de um aluno, e não de um professor da disciplina. Erros, portanto, são possíveis e até prováveis.

AGRADECIMENTOS

O autor agradece ao professor que o acompanhou durante a disciplina de Geometria Analítica, prof. Ademir Ribeiro, da Universidade Federal do Paraná, por suas aulas inspiradoras.

O autor agradece também à profa. Tanise Carnieri Pierin, também da Universidade Federal do Paraná, que, na disciplina de Cálculo I, demonstrou imensa paciência, determinação e organização, pavimentando a estrada de acesso do autor à Engenharia Industrial Madeireira.

p. 38, exercícios

1) $u = 2i - 3j$, $v = i - j$, $w = -2i + j$

a) $2(2,-3) - (1,-1) = (3,-5)$

b) $\frac{1}{2}(2,-3) - 2(1,-1) - (-2,1) = (1, -\frac{1}{2})$

c) $(1,-1) - (2,-3) + 2(-2,1) = (-5,4)$

2) $u = (3,-4)$, $v(-1,2)$

$\vec{x} = ?$

a) $4((3,-1) - (-1,2)) + \frac{1}{3}(x,y) = 2(3,-1) - (x,y)$

$\Rightarrow 4(4,-3) + \frac{1}{3}(x,y) = (6,-2) - (x,y)$

$\Rightarrow (16,-12) + \frac{1}{3}(x,y) = (6,-2) - (x,y)$

$\Rightarrow \frac{1}{3}(x,y) + (x+y) = (6,-2) - (16,-12)$

$\Rightarrow \frac{4}{3}(x,y) = (-10,10) \Rightarrow (x,y) = \frac{3}{4}(-10,10) = \frac{3}{2}(-5,5)$

$= (-\frac{15}{2}, \frac{15}{2})$

$-\frac{3}{2} + 2 - 1 = \frac{-3+4-2}{2}$

$= -\frac{1}{2}$

b

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3) $A(-1,3), B(2,5), C(3,-1), O(0,0)$

a) $OA - AB$

$= (A-O) - (B-A) = ((-1,3)-(0,0)) - ((2,5)-(-1,3)) = (-1,3) - (3,2) = (-4,1)$ ✓

c) $3BA - 4CB$

$= 3(A-B) - 4(B-C) = 3((-1,3)-(2,5)) - 4((2,5)-(3,-1))$

$= 3(-3,-2) - 4(-1,6) = (-9,-6) - (-4,24)$

$= (-5,-30)$ ✓

4) $u(2,-4), v(-5,1), w(-12,6)$

$w = a_1 u + a_2 v$

$a_1 = ?, \quad a_2 = ?$

$(-12,6) = a_1(2,-4) + a_2(-5,1) \Rightarrow (-12,6) = (2a_1, -4a_1) + (-5a_2, a_2)$

$\Rightarrow -12 = 2a_1 - 5a_2$

$6 = -4a_1 + a_2 \quad | \cdot 2$

$-24 = 4a_1 - 10a_2$

$-18 = 0 - 9a_2 \Rightarrow \boxed{a_2 = 2}$

$6 = -4a_1 + 2 \Rightarrow 4 = -4a_1 \Rightarrow \boxed{a_1 = -1}$ ✓

5) $A(3,-4), B(-1,1)$

$v = (-2,3)$

a) $(B-A) + 2v$

$((-1,1)-(3,-4)) + 2(-2,3)$

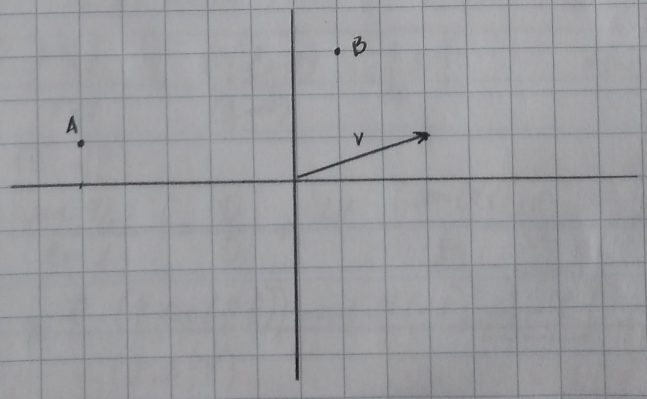
$(-4,5) + (-4,6) = (-8,11)$

6) $A(-5,1), B(1,3)$

$v = ?$

a) $(1,3) = (-5,1) + 2(x,y)$

$(6,2) = 2(x,y) \Rightarrow (x,y) = (3,1) \Rightarrow v = (3,1)$



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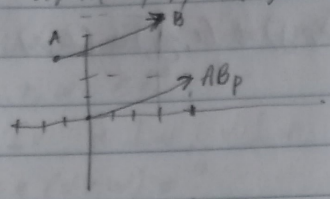
q 7. a) A(-1,3), B(3,5)



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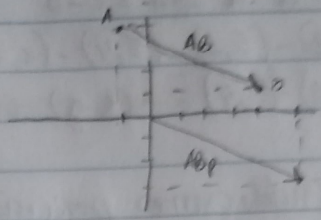

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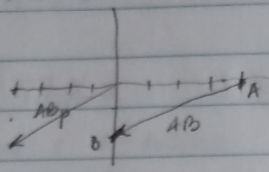


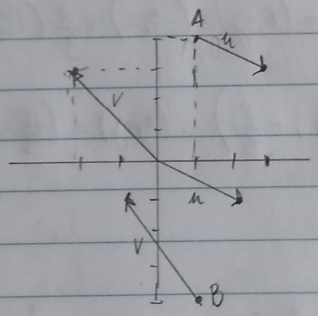
"Vetor posição" ou representante natural de $\overline{AB}$

q. 8 $V = (-1,3)$

$V - AB = B - A \Rightarrow (-1,3) = (0,1) - (x,y)$

$= (-4,2) = -(x,y) \Rightarrow (x,y) = (4,-2)$

q. 9)



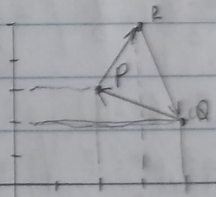
q. 10) P(2,3), Q(4,2), R(3,5)

a) $u = ?$ / $Q = P + u \Rightarrow (4,2) - (2,3) + (x,y)$

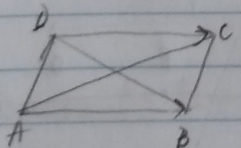
$v = ?$ / $R = Q + v$

$w = ?$ / $P = R + w$

b) $u + v + w = 0$



11)



a) $D = ?$ / $A(-3,-1), B(4,2), C(5,5)$

$AB = DC \Rightarrow B - A = C - D \Rightarrow (4,2) - (-3,-1) = (5,5) - (x,y)$

$\Rightarrow (7,3) - (5,5) = -(x,y) = (2,-2)$

$\Rightarrow (x,y) = (-2,2)$

q 4?

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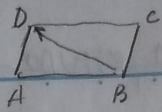
/ /

b) B = ? | A(5,1), B(7,3), C(3,4)



$\overrightarrow{AB} = \overrightarrow{DC} \Rightarrow B - A = C - D$

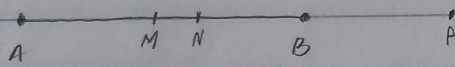
$\Rightarrow (7,3) - (5,1) = (3,4) - (x,y)$

$(2,2) - (3,4) = -(x,y) \Rightarrow (-1,-2) = -(x,y) \Rightarrow (x,y) = (1,2)$

12) 3 paralelogramos possíveis? ⚠

13) A(-3,2), B(5,-2)

M = ?, N = ? | $AM = \frac{1}{2} AB$, $AN = \frac{2}{3} AB$



a) $AM = MB \Rightarrow M - A = B - M \Rightarrow 2M = B + A$

$\Rightarrow M = \left(\frac{2}{2}, \frac{0}{2}\right) = (1,0)$

b) $AN = \frac{2}{3} AB \Rightarrow N - A = \frac{2}{3}(B - A) = (x,y) - (-3,2) = \frac{2}{3}((5,-2) - (-3,2))$

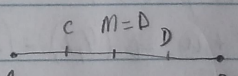
$\Rightarrow (x,y) - (-3,2) = \frac{2}{3}(8,-4) = \left(\frac{16}{3}, -\frac{8}{3}\right) = (x,y) = \left(\frac{16}{3}, -\frac{8}{3}\right) + (-3,2)$

$(x,y) = \left(\frac{7}{3}, -\frac{2}{3}\right) = N$

$\frac{16}{3} - 3 = \frac{16-9}{3} = \frac{7}{3}$ | $-\frac{8}{3} + 2 = \frac{-8+6}{3} = -\frac{2}{3}$

14) a) A(-2,3), B(6,-3)

C = ?, D = ?, E = ? | $= \frac{AB}{4}$

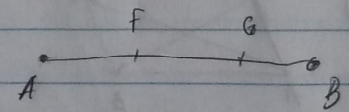


$\rightarrow \overrightarrow{AB} = B - A = (8,-6)$

$C = (x,y) = \frac{1}{4}(AB) \Rightarrow (x,y) = (2,-\frac{3}{2})$ X

$\rightarrow D = (x,y) = \frac{1}{2}(A+B) = \frac{1}{2}(4,0) = (2,0)$

$\rightarrow C = \frac{1}{2}(A+D) = \frac{1}{2}(0,0) = (0,\frac{3}{2})$



b) F = ?, G = ? | $= \frac{2}{3} AB \Rightarrow F = \frac{1}{3}(A+B) = \frac{1}{3}(4,0) \Rightarrow (\frac{4}{3}, 0)$

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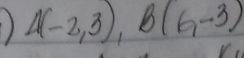
a) A(-2,3), B(6,-3)

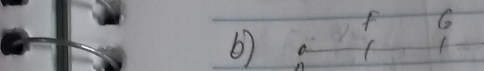


$M = \frac{A+B}{2} = \left(\frac{4,0}{2}\right) = (2,0)$

$C = \frac{A+M}{2} = \left(\frac{0,3}{2}\right) = (0, 3/2)$

$* \frac{8}{3} - 2 = \frac{8-6}{3} = \frac{2}{3}$

b)



$AF = \frac{1}{3} AB \Rightarrow F - A = \frac{1}{3}(B-A) \Rightarrow F - (-2,3) = \frac{1}{3}(8,-6)$

$\Rightarrow F - (-2,3) = \left(\frac{8}{3}, -2\right) \Rightarrow F = \left(\frac{8}{3}, -2\right) + (-2,3) = F = \left(\frac{2}{3}, 1\right)$

16) u(1,-1), v(-3,4), w(8,-6)

a) $|u| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

b) $|w| = \sqrt{8^2 + (-6)^2} = 10$

c) $|2u - w| = \sqrt{(-6)^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$

$2u = (2,-2) \Rightarrow (2,-2) - (8,-6) = (-6,4)$

52	2
26	2
13	

d) $\frac{v}{|v|} = \left(\frac{-3}{5}, \frac{4}{5}\right)$

e) $|v| = \sqrt{(-3)^2 + 4^2} = 5$

12	2
6	2
3	

17) $|u| = 4$

$u = (a,-2) \Rightarrow$

$|u|^2 = a^2 + (-2)^2$

$4^2 = a^2 + 4 \Rightarrow a^2 = 12$

$a = \pm\sqrt{12} = \pm 2\sqrt{3}$

18) a = ? / $|u| = 1$

$|u|^2 = a^2 + \frac{1}{4} \Rightarrow 1 = a^2 + \frac{1}{4} \Rightarrow a^2 = \frac{3}{4} \Rightarrow a = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

19) Para provar: 1) $|AB| = |BC| = |CD| = |DA|$

2) $|AC| = |BD|$

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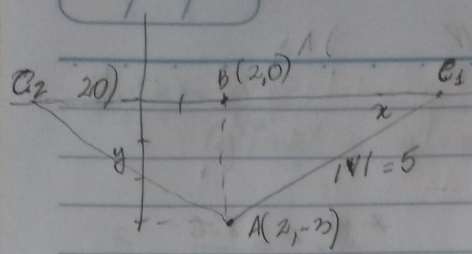


$|V|^2 = (-3)^2 + (x)^2$

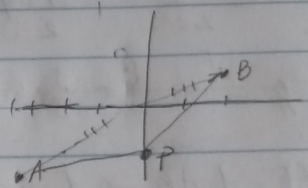
$25 = 9 + x^2 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

$\Rightarrow C_1 = (6,0)$

$C_2 = (-2,0)$

24) A(-4, 3), B(2, 1)

P = ?



a) $P(x,0)$, $AP = PB \Rightarrow P - A = B - P$

$\Rightarrow (x,0) - (-4,3) = (2,1) - (x,0) = (x+4, -3) = (2-x, 1)$

$x + 4 = 2 - x \Rightarrow 2x = -2 \Rightarrow x = -1$

$P(-1,0)$

a) $P(0,y)$, $|AP| = |PB| \Rightarrow |P - A| = |B - P|$

$\Rightarrow |(0,y) - (-4,3)| = |(2,1) - (0,y)|$ Módulo !

$= |(4, y-3)| = |(2, 1-y)|$

$\Rightarrow \sqrt{16 + y^2 - 6y + 9} = \sqrt{4 + 1 - 2y + y^2}$

$-6y + 2y = -25 + 5$

$-4y = -20 \Rightarrow \boxed{y = 5}$

(0,5) ?

b) $|AP| = |PB| \Rightarrow |P - A| = |B - P| \Rightarrow |(2x,x) - (-4,3)| = |(2,1) - (2x,x)|$

$P(2x, x)$

$\Rightarrow |(2x-4, x+3)| = |(2-2x, 1+x)|$

$|AP| = \sqrt{(2x-4)^2 + (x+3)^2}$

$|PB| = \sqrt{(2-2x)^2 + (1+x)^2}$

? $\Rightarrow 4x^2 - 8x + 16 + x^2 + 6x + 9 = 4 - 8x + 4x^2 + 1 + 2x + x^2$

$\Rightarrow 6x - 2x = -25 + 5 \Rightarrow 4x = -20 \Rightarrow \boxed{x = -5}$

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22) $\frac{V}{|V|} = ?$, $\frac{-V}{|V|} = ?$

a) $V = -i + j$

$(x_v, y_v) = (-x_i, -y_i) + (x_j, y_j) = (x_j - x_i, y_j - y_i)$

$\Rightarrow |V| = \sqrt{x_j^2 - 2x_j x_i + x_i^2 + y_j^2 - 2y_j y_i + y_i^2}$

c) $V = (1, \sqrt{3})$ b) ?

$\frac{V}{|V|} = \frac{(1, \sqrt{3})}{\sqrt{1+3}} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

d) $V = (0, 4)$

$\frac{V}{|V|} = \frac{(0,4)}{\sqrt{0+16}} = (0, 1)$

↳ a) $-i = (-1, -1)$, $j = (1, 1)$

$V = -i + j = (-1, -1) + (1, 1) = (2, 2)$

$\Rightarrow \frac{V}{|V|} = \frac{(2,2)}{\sqrt{4+4}} = \frac{(2,2)}{2\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

23) $V = (1, -3)$

a) $V_1 = ?$ $V_1 =$

$V_1 = (-1, +3)$

$|V| = \sqrt{1+9}$

$|V| = \sqrt{1+9} = \sqrt{10} \Rightarrow |V_1| = 2\sqrt{10}$

$|V_1| = 2|V| =$ ↳ $(-k)^2 + (-k, -3)^2$

$|V_1| = 2\sqrt{10}$

$(2\sqrt{10})^2 = (-k)^2 + (3k)^2 = k^2 + 9k^2 = 10k^2$

$\Rightarrow 40 = 10k^2 \Rightarrow k^2 = 4 \Rightarrow [k = -2]$

↳ $V_1 = (-2, 6)$

b) $V_1 = kV$ $|V_1| = 2 \Rightarrow |kV| = 2$

$\sqrt{k^2 + 9k^2} = 2 \rightarrow 10k^2 = 4 \Rightarrow k^2 = \frac{2}{5}$

$\Rightarrow k = \pm\sqrt{\frac{2}{5}}$

$V = \left(\sqrt{\frac{2}{5}}, -3\sqrt{\frac{2}{5}}\right)$

k tem que ser (+) (mesmo sentido)

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ex. 23 $V = (1, -3)$, , $V_1 = ?$

a) $V_1 = -kV$, $|V_1| = 2|V|$

$|V| = \sqrt{1+9} \Rightarrow \boxed{|V_1| = 2\sqrt{10}}$

$|V_1|^2 = (k.1)^2 + (k.-3)^2$

$(2\sqrt{10})^2 = k^2 + 9k^2 \Rightarrow 10k^2 = 40$

$\Rightarrow k = \pm 2$

$V_1 = \ominus kV \Rightarrow \boxed{k = -2}$

$\Rightarrow V = (-2.1, -2, -3) = (-2, 6)$

b) $V_1 \oplus kV$, $|V_1| = 2$

$2^2 = (k.1)^2 + (-3k)^2 \Rightarrow 10k^2 = 4 \Rightarrow k^2 = \frac{4}{10} \Rightarrow k = \frac{2}{\sqrt{10}}$ (k ⊕)

$\Rightarrow V_1 = \left(\frac{2}{\sqrt{10}}, \frac{-6}{\sqrt{10}}\right)$

c) $V_1 = -kV$, $|V_1| = 4$

$4^2 = (-k.1)^2 + (-k.-3)^2$

$\Rightarrow 16 = k^2 + 9k^2$

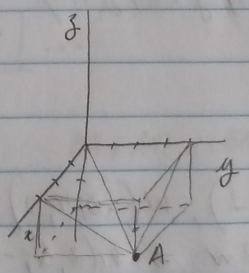
$\Rightarrow 10k^2 = 16 \Rightarrow \boxed{k = \pm \frac{4}{\sqrt{10}}}$

k ⊖

$\Rightarrow V_1 = \left(\frac{-4}{\sqrt{10}}.1\right), \left(\frac{-4}{\sqrt{10}}.-3\right)$

$V_1 = \left(\frac{-4}{\sqrt{10}}, \frac{12}{\sqrt{10}}\right)$

ex 28) $A(3, 4, -2)$



d) distância até O_x

$d^2 = -2^2 + 4^2 = 20 = \sqrt{20}$

$= 2\sqrt{5}$

20 | 2
10 | 2
5 | 5

e) distância a O_y

$d^2 = 9 + 4 = \sqrt{13}$

f) distância a O_z

$d^2 = 9 + 16 = \sqrt{25} = 5$

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29) $A=(2,-1,2)$
$B=(2,-3,2)$ ...
$C=(3,-3,2)$

31) $A(2,-2,3)$, $B(1,1,5)$, $v(1,3,-4)$
a) $A+3v$
$(2,-2,3)+(3,9,-12)=(5,7,-9)$ = ponto
b) $(A-B)-v$
$(1,-3,-2)-(1,3,-4)=(0,-6,+2)$ = vetor
c)

d) $2v-3(B-A)$
$(2,6,-8)-3(-1,3,2)=(2,6,-8)-(-3,9,6)=(5,-3,-14)$ vetor

32) $A(3,-4,-2)$, $B(-2,1,0)$, $N=?$ | $AN=\frac{2}{5}AB$

$N-A=\frac{2}{5}(B-A)=(x,y,z)-(3,-4,-2)=\frac{2}{5}((-2,1,0)-(3,-4,-2)$

$\Rightarrow (x-3,y+4,z+2)=\frac{2}{5}(-5,5,2)=(-2,2,\frac{4}{5})$

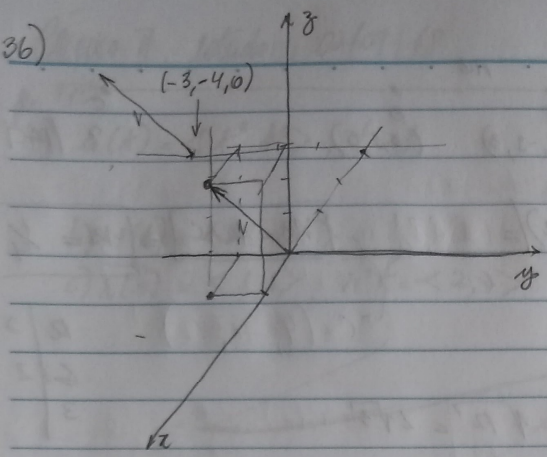
33) $(B-A)+(D-C)=0$

35) $u(2,3,-1)$, $v(1,-1,1)$, $w(-3,4,0)$
$3w-v+x=4x+2w \Rightarrow 3(2,3,-1)-(1,-1,1)-2(-3,4,0)=3\vec{x}$
$\vec{x}=?$

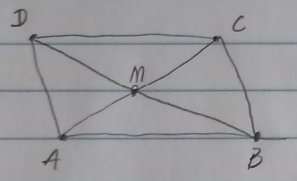
$\Rightarrow (6,9,-3)-(1,-1,1)-(-6,8,0)=3x$
$=(-\frac{11}{3},\frac{2}{3},-\frac{4}{3})=\vec{x}$

tilibra

36)



37)



$A(2,-5,3), B(7,2,-1), M(4,-3,3)$

$C = ?$, $D = ?$

$AM = MC$

$AB = DC$

$DM = MB$

$AM = \frac{1}{2} AC \Rightarrow M - A = \frac{1}{2}(C - A)$

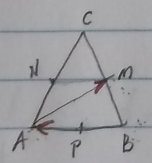
$\Rightarrow (4,-3,3) - (2,-5,3) = \frac{1}{2}((x_c, y_c, z_c) - (2,-5,3))$

$\Rightarrow ((2,+2,-1) \cdot 2) + (2,-5,3) = C$

$= (4,4,-2) + (2,-5,3) = C = (6,-1,1)$

$\Rightarrow AB = DC \Rightarrow \boxed{B - A = C - D}$ ...

38) $A_1B_1C_1 \Delta = ?$



$BA + AM = BM$ $\quad BM = \frac{1}{2} BC$

$PA + AM = PM$

$(A - P) + (M - A) = M - P$

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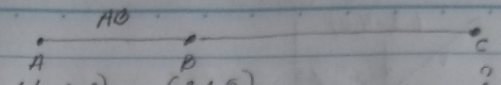
39) $A(1,-1,3)$, $B(3,1,5)$



$$\underset{A\,(1,-1,3)}{\bullet}\overset{AB}{\text{———}}\underset{B\,(3,1,5)}{\bullet}\text{———}\underset{C}{\bullet}$$

? $|AC| = 4|AB|$

$|C-A| = 4|B-A|$

$|(x-1),(y+1),(z-3)| = 4|(2,2,2)| =$ $\boxed{AB = k\,AC} = \boxed{k = \frac{1}{4}}$

$\to C = (4, -4, 12)$

$$\begin{array}{r|l} 12 & 2 \\ 6 & 2 \\ 3 & \end{array}$$

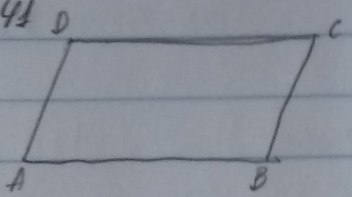
$|AB| = |B-A| = \sqrt{2^2+2^2+2^2} = \sqrt{12} = 2\sqrt{3}$

$\to \boxed{|AC| = 4|AB| = 8\sqrt{3}}$

EA, ESTUDO

Whitrik p. 43

ex. 41



$AA' = ?$

ex. 42) a) $(x, 0, 0)$

b) $(0, 0, z)$

c) $(x, y, 0)$

d) $(0, y, z)$

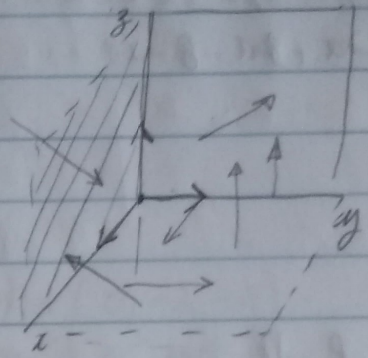
e) $(x, 0, z)$

f) $(x, y, 0)$

g) $(0, 0, z)$ ← ortogonal a xy

h) $(0, y, 0)$ ← " a xz

> sempre representado na origem!



ex. 43) $u\ (4, -6, 2)$

$v\ (-6, 9, -3)$

$-\frac{3}{2} \quad -\frac{3}{2} \quad -\frac{3}{2}$

$u \parallel v \parallel t$ ✓

$\frac{u}{w}\ (4, -6, 2)$ ✗ $\frac{u}{t}\ \frac{(4, -6, 2)}{(10, -15, 5)}$ ✓

MANUELLE, P. 44

44) $w(3,2,5)$

$a=?, b=?$ | $u(3,2,1) \parallel v = (a,6,b) + 2w$

$(3,2,1) = k\left((a,6,b) + (6,4,10)\right)$

$(3,2,1) = k(a+6, 10, b+10)$

$\boxed{k_y = \frac{1}{5}}$

$\Rightarrow k(a+6) = 3 \Rightarrow \frac{1}{5}a + \frac{6}{5} = 3 = 15 \Rightarrow \boxed{a=9}$

45) $\overline{AB} \parallel \overline{CD}$

$A(-2,5,1), B(1,3,0), C(3,-1,-1), D(0,m,n)$

$m=?, n=?$

$AB \parallel CD$

$B-A = k(D-C)$

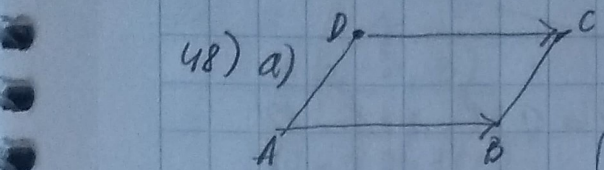
$(3,-2,-1) = k(-3, m+1, n+1)$

$\Rightarrow \boxed{k=-1}$

$\Rightarrow -m-1 = -2 \Rightarrow \boxed{m=1}, -n-1 = -1 \Rightarrow \boxed{n=0}$

WINTERLE P. 43

48) a)



$AB = DC$

$B - A = C - D$

$(2,1,-1) - (3,-2,5) = D = (-1,3,-6) = (1,3,6)$

b) igual até aqui

$(1,1,2) - (3,2,5) = D = (-2,-1,-3) = (3,1,3)$

49) $|u| = \sqrt{3}$, portanto $x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\vec{u} \neq \frac{u}{|u|}$

$|v| = \sqrt{\frac{1}{6} + \frac{4}{6} + \frac{1}{6}} = 1 \Rightarrow$ unitário

50) $1^2 = n^2 + \frac{1}{4} + \frac{9}{16} = n^2 + \frac{4+9}{16} \Rightarrow n^2 + \frac{13}{16} = 1 \Rightarrow n = \pm\frac{\sqrt{3}}{4}$

WINTERLE, P. 43

51) $a = ? \;/\; \vec{u} = \frac{u}{|u|}$

$u = (a, -2a, 2a)$

$|u| = \sqrt{a^2 + 4a^2 + 4a^2} = \sqrt{9a^2} = \pm 3a$

$\Rightarrow \frac{u}{|u|} = \left(\frac{a}{3a}, -\frac{2a}{3a}, \frac{2a}{3a}\right)$

$\Rightarrow \boxed{a = \frac{1}{\pm 3}}$

$\pm$

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52) $m = ?$

$\vec{V} = m \cdot \vec{AC} + \vec{BC}$ $\Rightarrow V = m(0,2,1) + (-3,0,1) = (-3, 2m, m+1)$

$|V| = 7$ $\Rightarrow |V| = 7 = \sqrt{9 + 4m^2 + m^2 + 2m + 1}$

$AC = C - A = (0,2,1)$
$BC = C - B = (-3,0,-1)$

$\Rightarrow 49 = 5m^2 + 2m + 10$

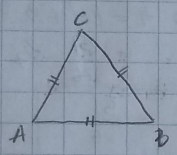
$\Rightarrow 5m^2 + 2m - 39 = 0$

$\Delta = b^2 - 4ac$
$\Delta = 4 - 4 \cdot 5 \cdot (-39)$?
$= 796$

Alternativa:
$V = m(-3,2,0) \Rightarrow (-3m, 2m, 0)$

$\Rightarrow 7 = \sqrt{9m^2 + 4m^2}$
$\Rightarrow 49 = 13m^2 \Rightarrow m^2 = \frac{13}{49} \Rightarrow m = \pm \frac{\sqrt{13}}{7}$?

53) $y = ?$



$|BC| = |BA| = |CA|$

$BC = C - B = (-8, y, -2)$

$BA = A - B = (-6, 0, 6)$

$\Rightarrow |BA| = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$

$\Rightarrow 6\sqrt{2} = |BC| = |(-8, y, -2)| = \sqrt{64 + y^2 + 4}$

$\Rightarrow 72 = 68 + y^2 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

72 | 2
36 | 2 } 2
18 | 2
9 | 3 } 3
3 | 3

54) $P = ?$

$P = (x, 0, 0)$

$|AP| = |BP| \Rightarrow |AP|^2 = |BP|^2$

$AP = P - A = (x-3, 1, -4)$ $\Rightarrow |AP|^2 = x^2 - 6x + 9 + 1 + 16$
$BP = P - B = (x-1, 2, 3)$ $\Rightarrow |BP|^2 = x^2 - 2x + 1 + 4 + 9$

$|AP|^2 = |BP|^2 \Rightarrow x^2 - 6x + 26 = x^2 - 2x + 14$
$\Rightarrow -4x = -12 \Rightarrow x = 3$

$\Rightarrow P(3,0,0)$

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Wiederh. 43

55) $P(0,0,z) = ?$
$A(-1,2,-2)$

$|AP| = 3$
$AP = P - A = (1, -2, z+2)$
$\Rightarrow 9 = 1 + 4 + (z+2)^2 \Rightarrow 4 = z^2 + 4z + 4 \Rightarrow z(z+4) = 0$ $z_1 = 0$ $z_2 = -4$

56) $\vec{v}(2,-1,-3)$
a) $v'(-6,3,9)$

b) $|v| = \sqrt{4+1+9} = \sqrt{14}$ $\sqrt{14} \cdot k = 4 \Rightarrow \frac{4}{\sqrt{14}} = \frac{4\sqrt{14}}{14} = \frac{2}{7}\sqrt{14} =$

$\Rightarrow v'' = \left(\frac{4\sqrt{14}}{7}, -\frac{2}{7}\sqrt{14}, -\frac{6}{7}\sqrt{14}\right)$

c) $\sqrt{14} \cdot k = -5 \Rightarrow k = -\frac{5}{\sqrt{14}} \Rightarrow v''' \left(\frac{-10}{\sqrt{14}}, +\frac{5}{\sqrt{14}}, +\frac{15}{\sqrt{14}}\right)$

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Pag. 66

1) $u(2,-3,-1)$, $v(1,-1,4)$

a) $2u \cdot (-v) = (4,-6,-2) \cdot (-1,1,-4) = -4-6+8 = 2$

b) $(u+3v) \cdot (v-2u) = ((2,-3,-1)+(3,-3,12)) \cdot ((1,-1,4)-(4,-6,-2))$

$= (5,-6,11) \cdot (-3,5,6) = -15-30+66 = 21$ *

c) $(u+v) \cdot (u-v) = ((2,-3,-1)+(1,-1,4)) \cdot ((2,-3,-1)-(1,-1,4))$

$|u|^2 - |v|^2 = 14 - 18 = -4$

$|u| = \sqrt{4+9+1} = \sqrt{14}$; $|v| = \sqrt{1+1+16} = \sqrt{18}$

d) $|v|^2 - |u|^2 = 4$

* $u \cdot v - 2|u|^2 + 3|v|^2 - 6u \cdot v$

$\Rightarrow (-2 \cdot 14) + (3 \cdot 18) - 5(2+3-4)$

$= -28 + 54 - 5 = 21$

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2) ~~$u \cdot v = u \cdot v + u \cdot w + |v|^2 + v \cdot w$~~

$u \cdot w + vw + |v|^2 = 0$

$w(u+v) + |v|^2 = 0$

$w((2,a,-1)+(3,1,-2)) + (9+1+4) = 0$

$(2a-1,-2,4)(5, a+1, -3) = -14$

$10a - 5 - 2a - 12 - 2 = -14 \Rightarrow 8a = 5 \Rightarrow \boxed{a = 5/8}$

3) b) $\bar{AB} = B - A = (-2,-2,2)$

$2x + 2v = -4v$

$2x = -6v \Rightarrow x = -3v$

$\Rightarrow x = (3,6,-9)$

$AB \cdot u = -4 - 2 + 2 = -4$

b) $(BC \cdot v)x = (u \cdot v)v - 3x$

$BC = C - B = (-1,5,1)$

$BC \cdot v = 1 - 10 + 3 = -6$

$u \cdot v = -2 - 2 + 3 = -1$

$\Rightarrow -6x = -v - 3x$

$\Rightarrow 3x = v \Rightarrow x = \frac{v}{3} \Rightarrow x\left(-\frac{1}{3}, -\frac{2}{3}, 1\right)$

4) $v = ku \quad | \quad v \cdot u = -42$

~~$u(2,-1,3)$~~

~~$v \cdot (2,-1,3) = -42$~~

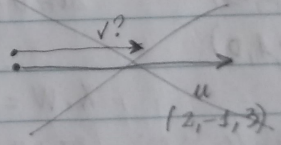
~~$\boxed{(2x, -y, 3z) = -42}$~~

~~$v \cdot u = |v||u| \cdot \cos\theta$~~

~~$v \cdot (2,-1,3) = |v| \cdot \sqrt{14}$~~

~~$|u| = \sqrt{4+1+9} = \sqrt{14} =$~~

~~Alternativa: $\text{proj}_u v = \left(\frac{v \cdot u}{|u|^2}\right) v$~~



$v \cdot u = -42 \Rightarrow k \cdot u \cdot u = -42$

$v = ku \qquad k|u|^2 = -42$

$k \cdot 14 = -42 \Rightarrow k = \frac{-42}{14} = \frac{-21}{7} = \boxed{-3}$

$v = -3(2,-1,3) = (-6,3,-9)$

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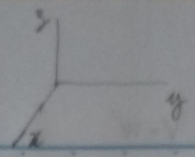

5) $V = ?$

$|V| = 5$

$V = (0, y, z)$

$V \cdot W = 6 \Rightarrow 0 + 2y + 0 = 6 \Rightarrow \boxed{y = 3}$

$W = i + 2j = (1, 2, 0)$

$z = 0$

$y = 3$

$|V| = 5 \Rightarrow 25 = x^2 + 9 + 0 \Rightarrow x^2 = 16 \Rightarrow x = \pm\sqrt{16}$

$\Rightarrow \boxed{x = \pm 4}$

$|V|^2 = x^2 + y^2 + z^2$

6) $V = ?$

$V = (x, 0, z)$

$V \cdot V_1 = 8$

$V \cdot V_2 = -3$

$V_1 = (3, 1, -2)$

$V_2 = (-1, 1, 1)$

7) $u(1, 2, -3)$

$v(2, 0, -1)$

$w(3, 1, 0)$

$\vec{x} = ?$

$x \cdot u = -16$

$x \cdot v = 0$

$x \cdot w = 3$

$X \cdot V = 0$

$2x + 0 - z = 0 \Rightarrow \boxed{x = z/2} \Rightarrow \boxed{2x = z}$

$X \cdot W = 3$

$3x + y + 0 = 3 \Rightarrow \boxed{x = -y/3} \quad \boxed{y = -3x}$

$x \cdot u = -16$

$x + 2y - 3z = 16$

$\Rightarrow x + 2(-3x) - 3(2x) = -16$

$x - 6x - 6x = -16$

$-11x = -16 \Rightarrow x = 16/11$?

?

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8) $|u|=2$, $|v|=3$, $u \cdot v = -1$

a) $(u-3v) \cdot u$

$|u|^2 - 3u \cdot v = 4 - 3(-1) = 7$

b) $(2v-u) \cdot (2v) = 4|v|^2 - 2uv = 36 - 2(-1) = 38$

c) $(u+v) \cdot (v-4u) = uv - 4|u|^2 + |v|^2 - 4uv = -16 + 9 + 3 = -4$

d) $(3u+4v) \cdot (-2u-5v)$

9) $uv + uw + vw$

$u+v+w=0$

$|u|=2$, $|v|=3$, $|w|=5$

$(\cdot u)\ |u|^2 + uv + uw = 0 \Rightarrow uv + uw = -4$

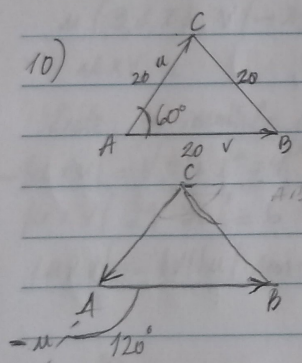
$(\cdot v)\ uv + 9 + vw = 0 \Rightarrow uv + vw = -9$

$(\cdot w)\ uw + wv + 25 = 0 \Rightarrow uw + vw = -25$

$2uv + 2uw + 2vw = -38$

$\dots = -19$

10)



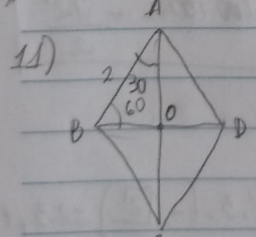
$u \cdot v = |u| \cdot |v| \cdot \cos\theta$

$= 20 \cdot 20 \cdot \frac{1}{2} = 200$

$-u \cdot v = |u| \cdot |v| \cdot \cos 120$

$20 \cdot 20 \cdot \left(-\frac{1}{2}\right) = -200$

11)



a) O

b) $|AB| \cdot |AD| \cdot \cos 60$

$2 \cdot 2 \cdot \frac{1}{2} = 2$

$AO \cdot AB = |AO| \cdot |AB| \cdot \cos 30 =$

$AC = AB + AD$

$|AB|^2 =$

$\cos\theta = \frac{AO}{2}$

$\frac{\sqrt{3}}{2} = \frac{AO}{2} \Rightarrow |AO| = \sqrt{3}$

$\cos 60 = \frac{BO}{2}$

$\frac{1}{2} = \frac{BO}{2} \Rightarrow |BO| = 1$

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c) $|BA||BC| \cdot \cos 120$

$2 \cdot 2 \cdot -\frac{1}{2} = -2$

d)

12) $|u+v| =$

$|u-v| =$

$(u+v)(u-v) = 7$

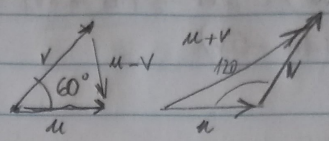
$|u| = 4$

$|v| = 3$

$|u|^2 - |v|^2 = 16 - 9 = 7$

$u \cdot v = |u||v| \cdot \cos 60 = 4 \cdot 3 \cdot \frac{1}{2} = 6$

$\boxed{u \cdot v = 6}$



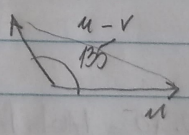
$\boxed{\text{Lei dos cossenos: } |u-v|^2 = |u|^2 + |v|^2 - 2|u||v| \cdot \cos\theta}$

$= 16 + 9 - 2 \cdot 4 \cdot 3 \cdot \frac{1}{2}$

$\boxed{|u-v|^2 = 13 \Rightarrow |u-v| = \pm\sqrt{13}}$

$|u+v| = ?$

13) $|u| = \sqrt{2}$, $|u| = 3$,



$\boxed{\cos 135 = -\frac{\sqrt{2}}{2}}$

$|u-v| = ?$

$|u+v| = ?$

$|u-v|^2 = |u|^2 + |v|^2 + 2|u||v| \cdot \cos\theta = 2 + 9 + 2\sqrt{2} \cdot 3 \cdot \left(-\frac{\sqrt{2}}{2}\right) = 2 \cdot 3 + 11 = 5$

X $\boxed{|u-v| = \sqrt{5}}$

a) $|(2u-v) \cdot (u-2v)| = |2|u|^2 - 4uv - uv + 2|v|^2| = |4 + 18 - 5uv|$

$u \cdot v = |u||v| \cdot \cos\theta \Rightarrow u \cdot v = \sqrt{2} \cdot 3 \cdot \left(-\frac{\sqrt{2}}{2}\right) = -3$

$\boxed{37}$

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WINTERLE P. 67

ex 15) $\bar{a} \cdot \bar{b} = 0 \Leftrightarrow ?$

$a(\alpha, 2, -4)$, $b(2, (1-2\alpha), 3)$

$a \cdot b = 2\alpha + 2 - 4\alpha - 12$

$0 = -2\alpha + 10$

$2\alpha = 10 \Rightarrow \boxed{\alpha = 5}$

11

16) $(a+b)\cdot(c-a)=0 \Leftrightarrow \alpha = ?$

$a(2,1,2),\ b(\alpha+2,-5,2),\ c(2\alpha,8,\alpha)$

$u = a+b = (\alpha+4,-4,\alpha+2)$

$v = c-a = (2\alpha-2,7,0)$

$u\cdot v = (\alpha+4)\cdot(2\alpha-2)-28+0$

$0 = 2\alpha^2-2\alpha+8\alpha-8-28 = \boxed{2\alpha^2+6\alpha-36=0}$

$\Delta = b^2-4ac = 36-4\cdot 2\cdot(-36) = 324$

$\alpha = \frac{-6 \pm \sqrt{324}}{2\cdot 2} = \begin{matrix}\alpha_1 = 3 \\ \alpha_2 = -6\end{matrix}$

WINTERLE, 67

17) $A(-1,0,5)$, $B(2,-1,4)$, $C(1,1,1)$, $P(x,0,x-3)$

$x = ?$ / $AC \cdot BP = 0$

$AC = C - A = (2,1,-4)$

$BP = P - B = (x-3, +1, x-7)$

$\Rightarrow \cancel{2x} - 4 + 1 \cancel{-4x} + 28 = 0 \Rightarrow -2x = -25 \Rightarrow x = 25/2$

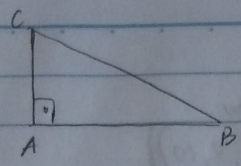
18) $AB \cdot AC = 0$ ou $AB \cdot BC = 0$

$AB = (-2,4,-3)$, $AC = (-3,5,-1)$, $BC(-1,1,2)$

$AB \cdot AC = 6 + 20 + 3 \Rightarrow AB \not\perp AC < \pi/2$

$AB \cdot BC = 2 + 4 - 6 = 0 \Rightarrow AB \perp BC$

11

19) 

$\Rightarrow AB \cdot AC = 0$

$A(m,1,0)$, $B(m-1,2m,2)$, $C(1,3,-1)$

$m = ?$

$\Rightarrow AB(-1, 2m-1, 2)$, $AC(1-m, 2, -1)$

$S_\Delta = ?$

$AB \cdot AC = m - 1 + 4m - 2 - 2 = 5m - 1$

$\hookrightarrow |AB \times AC|/2$

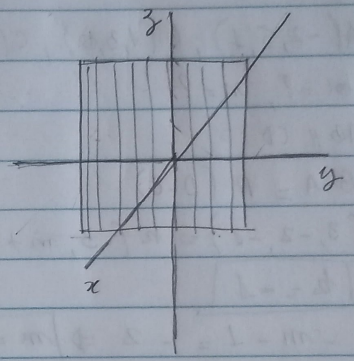
ou $|AB| \cdot |AC|/2$

$0 = 5m - 1 \Rightarrow \boxed{m = +\frac{1}{5}}$?

20) $u, w = ?$ / $u, w \perp v(4,1,-2)$

$u, w \parallel yOz \Rightarrow (0, y, z)$

ou $(x, 0, 0)$



$u \cdot v = 0$

$(0, y, z) \cdot (4, 1, -2) = 0$

$4 + y - 2z = 0 \Rightarrow \boxed{y = 2z - 4}$

ou

$(x, 0, 0) \cdot (4, 1, -2) = 0$

$\Rightarrow 4x = 0 \Rightarrow \boxed{x = 0}$

21) $u = ?$ $|u| = 2$

$u \cdot v = |u||v| \cdot \frac{\sqrt{2}}{2}$

$u \sphericalangle v = \pi/4$

$v(1,-1,0)$, $w(1,1,0)$

$\boxed{|v| = \sqrt{2}}$

$u \cdot w = 0$

$\boxed{u \cdot v = 2 \cdot \sqrt{2} = 2}$

$u \cdot v = x - y + 0z = 2$

$u \cdot w = x + y + 0z = 0$

$2x = 2$

$\Rightarrow \boxed{x = 1} \Rightarrow \boxed{y = -1}$

$|u| = 2 \Rightarrow 2 = \sqrt{1 + 1 + z^2}$

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21) $u = ?$

$|u| = 2$ $\qquad \theta = 45°$

$v(1,-1,0) \Rightarrow |v| = \sqrt{1+1} = \sqrt{2}$

$u \cdot v = |u| \cdot |v| \cdot \cos\theta$

$u \cdot v = 2 \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{2} \Rightarrow \boxed{u \cdot v = 2} \Rightarrow x - y = 2 \Rightarrow x = y + 2$

$w(1,1,0) \longrightarrow \Rightarrow x + y = 0 \Rightarrow y + 2 + y = 0 \Rightarrow 2y = -2$

$\boxed{y = -1}$

$\boxed{x = 1}$

$u \cdot w = 0$

$|u| = 2 \Rightarrow 4 = 1^2 + 1^2 + z^2 = z = \pm\sqrt{2} \Rightarrow u(1,-1,\sqrt{2})$ ou $(1,-1,-\sqrt{2})$

22) $u = ? \mid u \perp v$

a) $v(2,-1,1)$

$u \cdot v = 0 \Rightarrow (x,y,z) \cdot (2,-1,1) = 0$

$\Rightarrow 2x - y + z = 0$

para $u(1,1,-1) = 2 \cdot 1 - 1 - 1 = 0$ $\qquad$ 6: $u(1,1,-1)$

b) $|u| = \sqrt{1+1+1} = \sqrt{3}$

$\Rightarrow \frac{u}{|u|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

c) $4u = \frac{4}{\sqrt{3}}, \frac{4}{\sqrt{3}}, -\frac{4}{\sqrt{3}}$

23) $a \cdot b = 0$ $\qquad |a+b| = ?, \ |a-b| = ?$

$|a| = 6$

$|b| = 8$

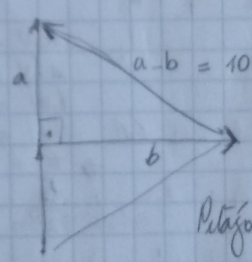
ou

$(a-b)^2 = a^2 + b^2 - 2\,b.c.\cos\theta$

ou $a \cdot a = 6$

$b \cdot b = 8$

$\Rightarrow a \cdot a = \frac{3}{4}\, b \cdot b$



Pitágoras

24) a) $|u+v|^2 = |u|^2 + |v|^2$ $\qquad$ b) igual

$|u+v| \cdot |u+v|$

$|u|^2 + u \cdot v + |v|^2$

$u \cdot v = 0$

$|u|^2 + |v|^2 = |u|^2 + |v|^2$

25) $u \cdot v = |u||v| \cos\theta$

a)
$u \cdot v = -2 + 1 - 2 = -3$
$|u| = \sqrt{4+1+1} = \sqrt{6}$
$|v| = \sqrt{6}$

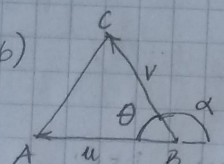
$\Rightarrow -3 = 6 \cos\theta = \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = 120^\circ$

b) $u \cdot v = -1 - 2 = -3$
$|u| = \sqrt{6}$
$|v| = \sqrt{3}$
$\Rightarrow -3 = \sqrt{12} \cdot \cos\theta \Rightarrow \cos\theta = -\frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 150$

26)

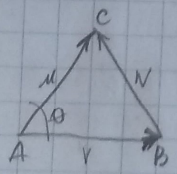


$u \cdot v = |u| \cdot |v| \cdot \cos\theta$

$u = BA = (1, 7, 0) \Rightarrow |u| = \sqrt{50} = 5\sqrt{2}$
$v = BC = (4, 3, 0) \Rightarrow |v| = \sqrt{25} = 5$

$u \cdot v = 4 + 21 = 25$
$\Rightarrow -17 = 5\sqrt{2} \cdot 5 \cdot \cos\theta$
$\frac{-17}{25\sqrt{2}} = \cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \theta_3 = 45^\circ ; \alpha = 135^\circ$

27)



$u = AC = (-3, 1, -2)$
$v = AB = (-1, -4, -4)$
$w = BC = u - v = (-2, 2, 2)$

$u \cdot v = |u||v| \cdot \cos\theta$
$u \cdot v = 3 - 1 + 8 = 10$
$|u| = \sqrt{9+1+4} = \sqrt{14}$
$|v| = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$
$\Rightarrow 10 = \sqrt{14} \cdot 3\sqrt{2} \cdot \cos\theta$
$\frac{5}{\sqrt{28}} = \cos\theta = \frac{5}{2\sqrt{7}} = \frac{5\sqrt{7}}{14}$?

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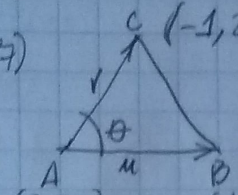
27)



$u = AB = B - A = (-1,-1,-4)$
$v = AC = C - A = (-3,1,-2)$

$u \cdot v = +3 - 1 + 8 = 10$
$|u| = \sqrt{1+1+16} = 3\sqrt{2}$
$|v| = \sqrt{9+1+4} = \sqrt{14}$

$u \cdot v = |u||v| \cdot \cos\theta$

$\Rightarrow 10 = 3\sqrt{2} \cdot \sqrt{14} \cdot \cos\theta = 3\sqrt{28} \cdot \cos\theta = 6\sqrt{7}\cos\theta = 10 \Rightarrow \cos\theta = \frac{10}{6\sqrt{7}} = \frac{5}{3\sqrt{7}}$

$= \frac{5\sqrt{7}}{21} = 0{,}63 \Rightarrow \theta \simeq 50{,}1^\circ$

28	2
14	2
7	7

28) $m = ?$ / $\cos\theta = -\frac{1}{2}$

$u(1,-3,1)$, $v(-2,1,m+1)$

$u \cdot v = |u||v|\cos\theta$

$u \cdot v = -2 - 3 + m + 1 = m - 3$
$|u| = \sqrt{1+4+1} = \sqrt{6}$
$|v| = \sqrt{4+1+m^2+2m+1} = \sqrt{m^2+2m+6}$

$m - 3 = \sqrt{6} \cdot \sqrt{m^2+2m+6} \cdot \left(-\frac{1}{2}\right) \Rightarrow 2m + 6 = \sqrt{6m^2+6m+36}$ $(\square^2)$

$4m^2 - 12m - 12m + 36 = 6m^2 + 6m + 36$
$-2m^2 - 30m = 0 \Rightarrow m^2 + 15m = 0 \Rightarrow m(m+15) = 0 \Rightarrow m_1 = 0$
$m_2 = -15$

$S = 0$ ou -18 ?

29) $n = ?$ / $\cos\theta_{u,v} = \frac{\sqrt{3}}{2}$

$u(-3,1,n)$, $v = j = (0,0,1)$

$u \cdot v = 0 + 0 + n$
$|u| = \sqrt{9+1+n^2}$
$|v| = 1$

$\Rightarrow n = 1 \cdot \sqrt{n^2+10} \cdot \frac{\sqrt{3}}{2} \Rightarrow 2n = \sqrt{3n^2+30} \Rightarrow 4n^2 = 3n^2 + 30$
$n^2 = 30 \Rightarrow n = \pm\sqrt{30}$

$S = +\sqrt{30}$?

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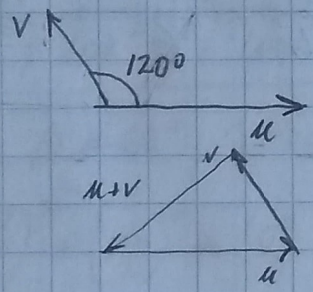
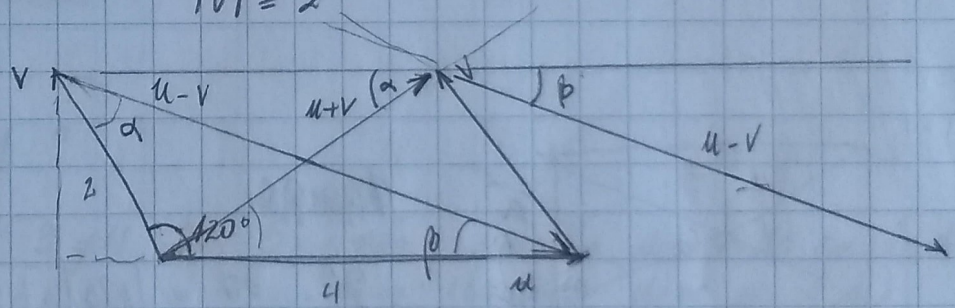
30)



$|u| = 4$
$|v| = 2$

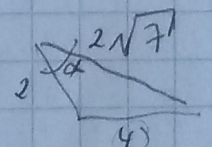
$u \cdot v = 4 \cdot 2 \cdot \left(-\frac{1}{2}\right) = -4$



$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\Theta$

$|u-v|^2 = 4 + 16 - 2 \cdot 4 \cdot 2 \cdot \left(-\frac{1}{2}\right)$

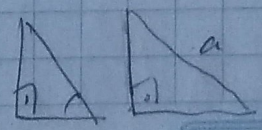
$|u-v|^2 = 20 + 8 \Rightarrow |u-v| = \sqrt{28} = 2\sqrt{7}$



$16 = 4 + 28 - 2 \cdot 2 \cdot 2\sqrt{7} \cdot \cos\alpha$

$16 = 32 - 8\sqrt{7} \cdot \cos\alpha$

$\cos\alpha = +\frac{16}{8\sqrt{3}} = +\frac{2\sqrt{3}}{3}$



3.1) a) 0

d) $d^2 = a^2 + a^2 \Rightarrow d^2 = 2a^2 \Rightarrow d = a\sqrt{2}$

$d_2^2 = 2a^2 + a^2 = 3a^2 \Rightarrow d = a\sqrt{3}$

$\Rightarrow OD \cdot OG = a\sqrt{2} \cdot a\sqrt{3} \cdot \frac{\sqrt{2}}{2}$

$= a^2\sqrt{3}$

3.4) a) 0 b) 0 c) 0 d) $|OB| = \sqrt{2a^2} = a\sqrt{2}$

$|OG| = \sqrt{a^2 + 2a^2} = \sqrt{3a^2} = a\sqrt{3}$

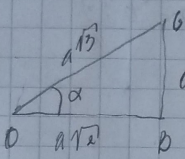
e) $EG \cdot CG \Rightarrow a^2$

$EG(a, a, 0)$

$CG(a, 0, a)$

f) $a^2 \cdot (a, a, a) = (a^3, a^3, a^3)$

g)



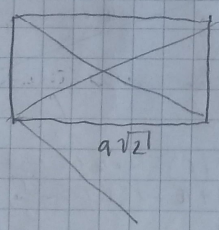
$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$

$= a^2 = 3a^2 + 2a^2 - 2 \cdot a\sqrt{3} \cdot a\sqrt{3} \cdot \cos\alpha$

$\Rightarrow -4a^2 = -2\sqrt{6}a^2 \cdot \cos\alpha$

$= \frac{+4a^2}{+2\sqrt{6}a^2} = \cos\alpha = \frac{2}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$?

h)



$OG \cdot BE = |OG| \cdot |BE| \cdot \cos\theta$

$(a, a, a) \cdot (a, a, -a)$

$a^2 + a^2 - a^2 =$

$a^2 = \sqrt{3} \cdot a\sqrt{3} \cdot \cos\theta$

$-a^2 a^3$

$\frac{a^2}{3a^2} = \cos\theta = \frac{1}{3}$

32) $\alpha, \beta, \gamma = ?$
$\vec{v}(6, -2, 3)$

$|\vec{v}| = \sqrt{36+4+9}$
$|\vec{v}| = 7$

$\cos\alpha = \frac{x}{|\vec{v}|} = \cos\alpha = \frac{6}{7} \Rightarrow \alpha = \arccos\frac{6}{7}$

$\cos\beta = \frac{y}{|\vec{v}|} = -\frac{2}{7} \Rightarrow \beta = \arccos -\frac{2}{7}$

$\cos\gamma = \frac{z}{|\vec{v}|} = \frac{3}{7} \Rightarrow \gamma = \arccos\frac{3}{7}$

33) $\vec{a} = ?$
$|\vec{a}| = 2$
$\alpha = 45°, \beta = 60, \gamma = 120$

$\cos\alpha = \frac{x}{|\vec{a}|} \Rightarrow \frac{\sqrt{2}}{2} = \frac{x}{2} \Rightarrow x = \sqrt{2}$

$\cdots \beta = \frac{y}{|\vec{a}|} \Rightarrow \frac{1}{2} = \frac{y}{2} \Rightarrow y = 1$

$\gamma \cdots \Rightarrow -\frac{1}{2} = \frac{z}{2} \Rightarrow z = -1$

$\Rightarrow \vec{a}(\sqrt{2}, 1, -1)$

34) $\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$,

$\alpha = 45°, \beta = 60°, \gamma = 90°$

$\Rightarrow \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (0)^2 = 1\ ?$

$\frac{2}{4} + \frac{1}{4} = 1\ ? \Rightarrow 3 \neq 4 \Rightarrow S: \{$ não, pois a soma dos quadrados dos cossenos deve ser $= 1$)

35) Similar a 34:

	30	45	60
sen	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$

$\cos^2 30° + \cos^2 60° + \cos^2 90 = 1$
$\frac{3}{4} + \frac{1}{4} + 0 = 1$

$\frac{\sqrt{3}}{2} = \frac{x}{10} \quad \frac{1}{2} = \frac{z}{10} \quad 0 = \frac{y}{10}$

$\Rightarrow (5\sqrt{3}, 0, 5)$

36) $\frac{\vec{v}}{|\vec{v}|} = ?$ $\boxed{\vec{v}(x, y, 0)}$ e $\frac{1}{4} + \cos^2\beta + 0^2 = 1 \Rightarrow \cos^2\beta = \frac{3}{4} \Rightarrow \boxed{\cos\beta = \pm\frac{\sqrt{3}}{2}}$

$\boxed{\cos\alpha = \frac{1}{2}} \Rightarrow \vec{v}\left(\frac{1}{2}, y, 0\right)$

$\Rightarrow \vec{v}\left(\frac{1}{2}, \pm\frac{\sqrt{3}}{2}, 0\right)$

37) $\bar{a} = ?$

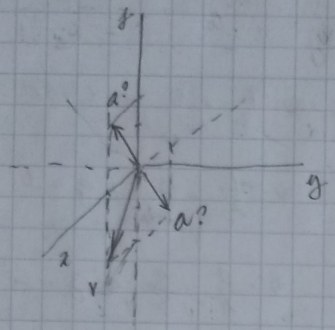
$|a| = 5$

$a(x, 0, z)$

$a \perp v(1, 0, -2) \Rightarrow \bar{a} \cdot V = 0 \Rightarrow (x, 0, z) \cdot (1, 0, -2) = 0$

$\bar{a} \cdot i < 0$

$\Rightarrow x - 2z = 0 \Rightarrow \boxed{z = \frac{x}{2}}$



$|a| = 5$

$\Rightarrow 5^2 = x^2 + \frac{x^2}{4} \Rightarrow \frac{5}{4}x^2 \Rightarrow x^2 = 20$

$\Rightarrow x = \pm 2\sqrt{5}$

$y = \pm \sqrt{5}$

$\Rightarrow \bar{a}(\pm 2\sqrt{5}, 0, \pm\sqrt{5})$

$\bar{a} \cdot \bar{i} < 0$

para $x = +2\sqrt{5}$, $\bar{a} \cdot \bar{i} > 0$

$x = -2\sqrt{5}$, $\bar{a} \cdot \bar{i} < 0$

$\Rightarrow \bar{a}(-2\sqrt{5}, 0, -\sqrt{5})$

38) $\bar{v} = ?$

a) $V(x, y, 0)$

$|V| = 8$

$\cos 30 = \frac{x}{|V|}$

$V \cdot (0, 1, 0) < 0$

$V \cdot i = |V| \cdot |i| \cdot \cos 30° = \boxed{V \cdot i = 8 \cdot 1 \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}}$

$\cos 30° = \frac{x}{|V|} \Rightarrow \frac{\sqrt{3}}{2} = \frac{x}{8} \Rightarrow \boxed{x = 4\sqrt{3}}$

$|V| = 8 \Rightarrow 8^2 = 16 \cdot 3 + y^2 \Rightarrow 16 \Rightarrow \boxed{y = \pm 4}$

$64 - 48 = 16$

$V \cdot i < 0 \Rightarrow (4\sqrt{3}, -4, 0)(0, 1, 0) = \boxed{-4}$

$\Rightarrow \boxed{V(4\sqrt{3}, -4, 0)}$

b) $V \perp Ox \Rightarrow v(0, y, z)$

$|V| = 2$

$\cos 60° = \frac{y}{|V|} \Rightarrow \frac{1}{2} = \frac{y}{2} \Rightarrow \boxed{y = 1}$

$\bar{V} \cdot \bar{k} > 0$

$|V| = 2 \Rightarrow 2^2 = 0^2 + 1^2 + z^2$

$\Rightarrow 4 = 1 + z^2 \Rightarrow z^2 = 3 \Rightarrow \boxed{z = \pm\sqrt{3}}$

$V \cdot k > 0 \Rightarrow (0, 1, \sqrt{3}) \cdot (0, 0, 1) = \sqrt{3} > 0$

S: $V(0, 1, \sqrt{3})$

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39) $v \perp u, w \Rightarrow v \cdot u = 0$ e $v \cdot w = 0$

$u(1,2,0)$, $w(2,0,1)$ — $(x,y,z)(1,2,0) = x + 2y = 0 \Rightarrow$

$(x,y,z)(2,0,1) = 2x + z = 0$

$-2x - 4y = 0$

$0 + z - 4y = 0 \Rightarrow \boxed{z = 4y}$

$v \cdot j > 0 \Rightarrow \boxed{\cos\beta > 0}$

$|v| = \sqrt{21}$

$|v| = \sqrt{21} \Rightarrow \sqrt{21}^2 = x^2 + y^2 + 16y^2$

$\Rightarrow 21 = x^2 + 17y^2 \Rightarrow \boxed{x^2 = -17y^2 + 21}$

Alternativa: $v = u \times w \Rightarrow$

i	j	k	i	j
1	2	0	1	2
2	0	1	2	0

$-4k + 0i - 1j + 2i + 0j + 0k$

$\boxed{v = (2, -1, -4)}$

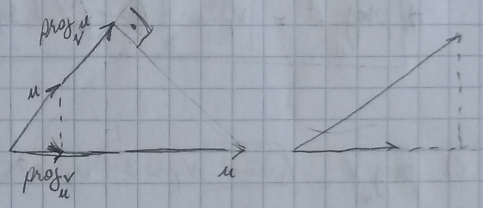
$\Rightarrow |v|^2 = 4 + 1 + 16 = 21 \Rightarrow \boxed{|v| = \sqrt{21}}$

$\cos\beta = \frac{-1}{\sqrt{21}} = -\frac{\sqrt{21}}{21} \Rightarrow \beta > 90°$

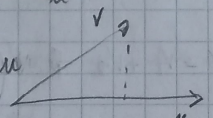
Assim $v = (-2, 1, 4)$, pois $\cos\beta = \frac{1}{\sqrt{21}} = \frac{\sqrt{21}}{21} \Rightarrow \beta < 90°$

40) $u(3,0,1)$, $v(-2,1,2)$

$\text{proj}_v u = ?$, $\text{proj}_u v = ?$



$\text{proj}_u v = \left(\frac{v \cdot u}{u^2}\right) u$



$\boxed{\text{proj}_u v = -\frac{4}{10} u = -\frac{2}{5} u}$

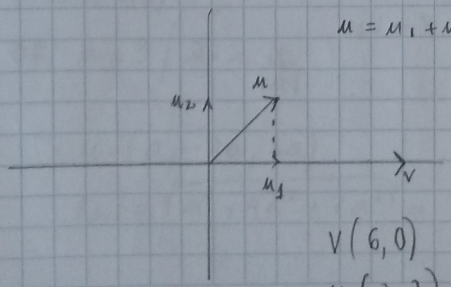
$\text{proj}_v u = \left(\frac{u \cdot v}{v^2}\right) v$

$\boxed{v \cdot u = -6 + 0 + 2 = -4}$

$\boxed{= \frac{-4}{9} \cdot v = \left(+\frac{8}{9}, -\frac{4}{9}, -\frac{8}{9}\right)}$

$\boxed{|u|^2 = 9 + 1 = 10}$

$|v|^2 = 4 + 1 + 4 = 9$



$u = u_1 + u_2 \Rightarrow \boxed{u_2 = u - u_1} = (2,2) - (2,0) = (0,2)$

$u_1 = \text{proj}_v u = \left(\frac{u \cdot v}{v^2}\right) v = \frac{1}{3} v = (2,0)$

$v(6,0)$ = $\boxed{u \cdot v = 12 + 0 = 12}$

$u(2,2)$ $\boxed{v \cdot v = 36}$

41) $v(4,-3,2)$ $\quad \text{proj}_i v = \left(\frac{v \cdot i}{i^2}\right) i = 4.(1,0,0) = \boxed{(4,0,0)}$ V

$i = (1,0,0)$
$j(0,1,0)$
$k(0,0,1)$

$\boxed{v \cdot i = (4,-3,2)(1,0,0) = 4}$ Ac.

$\boxed{i \cdot i = 1}$

42) $\text{proj}_u v = ?$, $v_1 = ?$, $v_2 = ?$

$\text{proj}_u v = v_1 = \left(\frac{v \cdot u}{|u|^2}\right) u$, $\quad v = v_1 + v_2 \Rightarrow \boxed{v_2 = v - v_1}$

a) $v \cdot u = 3 - 4 - 2 = -3$
$|u^2| = 1 + 4 + 4 = 9$

$\Rightarrow v_1 = -\frac{1}{3} u = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right) \Rightarrow v_2 = (3,-2,1) - \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

$\boxed{= \left(\frac{-10}{3}, \frac{-4}{3}, \frac{1}{3}\right)}$

b) $\boxed{u \cdot v = 3 + 1 - 1 = 3}$
$\boxed{u^2 = 3}$

$\Rightarrow \boxed{\text{proj}_u v = v_1 = \frac{3}{3} u = (1,1,1)} \Rightarrow \boxed{v_2 = (3,1,-1) - (1,1,1) =}$
$\boxed{v_2 = (2,0,-2)}$

c) $\text{proj}_u v = v_1 \quad v_1 = ?, v_2 = ? \Rightarrow v = v_1 + v_2 \Rightarrow v_2 = v - v_1$

$\boxed{\text{proj}_u v = \left(\frac{u \cdot v}{u^2}\right) u = \frac{3}{1} u = (3,0,0)}$

$\Rightarrow \boxed{v_2 = (3,5,4) - (3,0,0) = (0,5,4)}$

$u \cdot v = 6$
$u^2 = 4$

d) $\text{proj}_u v = v_1 = \left(\frac{u \cdot v}{u^2}\right) u \qquad v_2 = v - v_1$

$u \cdot v = 6 - 3 - 3 = 0$

tilibra

43) a) $ABC \in \Pi/2 \Leftrightarrow AB \cdot AC = 0$

$\Rightarrow (m-2, 2, 2) \cdot (-2, 3, -2) = 0 \Rightarrow -2m + 4 + 6 - 4 = 0 \Rightarrow -2m = -6 \Rightarrow \boxed{m = 3}$

b) $\text{proj}_{BC}^{AB} = BH = \left(\frac{AB \cdot BC}{BC \cdot BC}\right) BC = \frac{-9}{26}(BC)$

$\boxed{AB \cdot BC = -3 + 2 - 8 = -9}$

$BC = (-3, 1, -4)$

$AB = (1, 8, 2)$

$BC^2 = 9 + 1 + 16 = 26$

$\Rightarrow \boxed{|BH| = \frac{9}{26} \cdot |BC| = \frac{9}{26}\sqrt{26}}$

$|BC| = \sqrt{9 + 1 + 16} = \sqrt{26}$

$BH = \frac{-9}{26}(-3, 1, -4) = \boxed{\left(\frac{27}{26}, \frac{-9}{26}, \frac{36}{26}\right) \downarrow \frac{18}{13}}$

$BH = H - B \Rightarrow H = BH + B =$

$= \frac{27}{26} - 3, \frac{-9}{26} - 3, \frac{18}{13} - 5$

$$\boxed{\begin{aligned} x &= \frac{27 - 78}{26} = \frac{51}{26} \\ y &= \frac{-9 - 78}{26} = \frac{87}{26} \\ z &= \frac{18 - 65}{13} = \frac{47}{13} \end{aligned}}$$

44) a) $k = ? \;/\; u \cdot v = 0$

$(-2, 3) \cdot (k, -4) = 0 \Rightarrow -2k - 12 = 0 \Rightarrow 2k = 12 \Rightarrow \boxed{k = 6}$

b) $k = ? \;/\; \bar{u} = c\bar{v}$

$y: 3 \cdot \left(-\frac{4}{3}\right) = -4 \Rightarrow \boxed{c = -\frac{4}{3}}$

$\Rightarrow x_u \cdot c = x_v \Rightarrow -2 \cdot \left(-\frac{4}{3}\right) = \boxed{x_v = \frac{8}{3}}$

WINTERLE, p. 69

45)

a) $(4,3) = u$ $\quad v_1 = ?$, $v_2 = ?$

$v \perp u \Rightarrow v \cdot u = 0$

$v = \frac{v}{|v|}$

$v \cdot u = 0$ para $x = 1 \Rightarrow (4,3)\cdot(1,y) = 0$

$\Rightarrow 4 + 3y = 0 \Rightarrow \boxed{y = -\frac{4}{3}}$

$\Rightarrow v = (1, -\frac{4}{3}) \Rightarrow \boxed{|v| = \sqrt{1 + \frac{16}{9}} \Rightarrow \frac{5}{3}}$

$\Rightarrow v = \frac{v}{|v|} = \left(\frac{1}{5/3}, \frac{-\frac{4}{3}}{5/3}\right) = \left(\frac{3}{5}, -\frac{4}{5}\right) = v_1$

$\left(-\frac{3}{5}, \frac{4}{5}\right) = v_2$

b) $(-2,3) = u$

para $x = 1$, $u \cdot v = (-2,3)\cdot(1,y) = 0 \Rightarrow -2 + 3y = 0$

$\Rightarrow \boxed{y = \frac{2}{3}}$

$\Rightarrow v = (1, \frac{2}{3}) \Rightarrow |v| = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$

$\Rightarrow v_1 = \frac{v}{|v|} = \left(\frac{1}{\sqrt{13}/3}, \frac{\frac{2}{3}}{\sqrt{13}/3}\right) = \left(\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right)$

$v_2 = \left(-\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\right)$

c) $(-1,-1) = u$

para $x = 1$, $u \cdot v = (-1,-1)\cdot(1,y) = -1 - y = 0 \Rightarrow \boxed{y = -1}$

$\Rightarrow v = (1,-1) \Rightarrow |v| = \sqrt{2}$

$\Rightarrow v_1 = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$v_2 = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

WINTERLE, P. 67

46) $\frac{u}{|u|} = ?$, $\frac{v}{|v|} = ?$

$u \cdot v = 0$ para $x = 1$: $(6,8) \cdot (1, y) = 0 = 6 + 8y = 0 \Rightarrow 8y = -6$

$\Rightarrow \boxed{y = -\frac{3}{4}}$

$u = (6,8) \Rightarrow |u| = \sqrt{36+64} = 10$

$v = (1, -\frac{3}{4}) \Rightarrow |v| = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{16+9}{16}} = \frac{5}{4}$

$\Rightarrow \frac{u}{|u|} = (\frac{3}{5}, \frac{4}{5})$

$\frac{v}{|v|} = (\frac{4}{5}, -\frac{3}{5})$

40	2
20	2
10	2
5	

47) $u \cdot v = |u| \cdot |v| \cdot \cos\theta$

a) $(2,1)(4,-2) = \sqrt{4+1} \cdot \sqrt{16+4} \cdot \cos\theta$

$8 - 2 = 10 \cdot \cos\theta \Rightarrow \frac{3}{5} = \cos\theta$

$\boxed{S: \arccos \frac{3}{5}}$

b) $(1,-1)(-4,-2) = \sqrt{2} \cdot \sqrt{16+4} \cdot \cos\theta$

$-4 + 2 = \sqrt{40} \cdot \cos\theta \Rightarrow -2 = 2\sqrt{10} \cdot \cos\theta \Rightarrow -\frac{1}{\sqrt{10}} = \cos\theta$

$\boxed{\measuredangle = \arccos -\frac{1}{\sqrt{10}}}$

c) $(1,1) \cdot (-1,1) = \sqrt{2} \cdot \sqrt{2} \cdot \cos\theta$

$-1 + 1$

$\pi/2$

48) $\cos\alpha$ = cos dir a π = cosseno diretor

$\cos\alpha = \frac{\text{coordenada } x}{|v|}$,

a) $u(1,1)$, $|u| = \sqrt{2}$ $\Rightarrow \cos\alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \alpha = 45°$

b) $v(2,1)$ $\quad |v| = \sqrt{5}$ $\quad \cos_{dmi} = \frac{2}{\sqrt{5}} \Rightarrow \alpha = \arccos \frac{2}{\sqrt{5}}$

c) $u+v = (1,-1)+(2,1) = (3,0)$
$|u+v| = \sqrt{9} = 3$

$\cos\alpha = \frac{3}{3} = 1 \Rightarrow \alpha = 0$

d) $u-v = (-1,-2)$
$|u-v| = \sqrt{5} \Rightarrow \cos\alpha = -\frac{1}{\sqrt{5}} \Rightarrow \alpha = \arccos -\frac{1}{\sqrt{5}}$

e) $v-u = (2,1)-(1,-1) = (1,2)$
$\Rightarrow |v-u| = \sqrt{5} \Rightarrow \cos\alpha = \frac{1}{\sqrt{5}} \Rightarrow \alpha = \arccos \frac{1}{\sqrt{5}}$

49) $a = ?$
$\cos\theta = \frac{\sqrt{2}}{2}$
$u(2,1),\ v(1,a)$

$u \cdot v = |u| \cdot |v| \cdot \cos\theta$

$\Rightarrow (2,1)(1,a) = \sqrt{5} \cdot \sqrt{a^2+1} \cdot \frac{\sqrt{2}}{2}$

$\Rightarrow (4+2a)^2 = 10a^2+10 = 4a^2+16a+16$

$\Rightarrow 6a^2 - 16a - 6 = 0 = 3a^2 - 8a - 3 = 0$

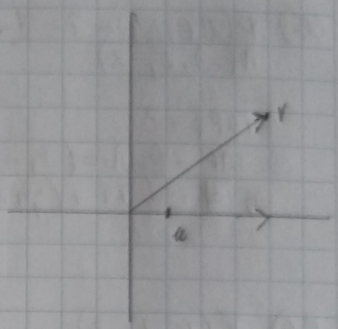
$\Delta = 64 - 4 \cdot 3 \cdot (-3) = 16$

$\Rightarrow a = \frac{8 \pm 4}{6}$

WINTERLE, P.67

50) $\text{proj}_u v = \frac{(u.v)}{u.u} . u$ $\quad v = v_1 + v_2$, $v_1 \parallel u$, $v_2 \perp u$

$\Rightarrow v_2 = v - v_1$



a) $\frac{4}{1} . (1,0) = \boxed{v_1 = (4,0)}$

$\boxed{v_2 = (4,3) - (4,0) = (0,3)}$

b) $\frac{(2,5).(1,1)}{2} = \frac{2+5}{2} = \frac{7}{2}(1,1) \Rightarrow \boxed{v_1 = \left(\frac{7}{2}, \frac{7}{2}\right)}$

$\Rightarrow \boxed{v_2 = (2,5) - \left(\frac{7}{2}, \frac{7}{2}\right) = \left(-\frac{3}{2}, \frac{3}{2}\right)}$

c) $\frac{(4,3)(1,2)}{16+9} = \frac{4+6}{25} = \frac{10}{25} = \frac{2}{5}(4,3)$

$\Rightarrow v_1 = \left(\frac{8}{5}, \frac{6}{5}\right)$, $v_2 = (1,2) - \left(\frac{8}{5}, \frac{6}{5}\right) = \left(-\frac{3}{5}, \frac{4}{5}\right)$

GEOMETRIA ANALÍTICA, ESTUDO + CONT. LISTA AB EX 8

p. 88

1) $u(3,-1,-2)$, $v(2,4,-1)$, $w(-1,0,1)$

a) $|u \times u| = 0$

b) 0

c) 0

d) 0

e) $(u-v) \times w = (1,-5,-1) \times (-1,0,1)$

i	j	k	i	j
1	-5	-1	1	-5
-1	0	1	-1	0

$-5k + 0i - 1j - 5i + 1j + 0k = -5i + 0j - 5k$

...

i) $u \times (v+w) = u \times (1,4,0) =$

i	j	k	i	j
3	-1	-2	3	-1
1	4	0	1	4

$+1k + 8i + 0j + 0i - 6j + 12k$

OK!!

$8i - 2j + 13k$ ✓

7) a) $OF \times OD =$

$= (OE + EF) \times (OE + ED)$

$= OE \times (EF + ED)$

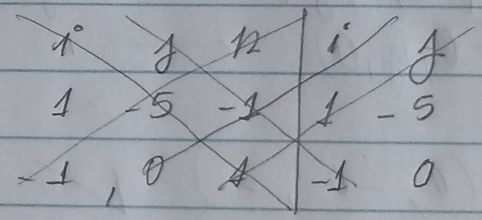
$= OE \times EG$

WINTERLE P. 88

1) $u(3,-1,-2)$, $v(2,4,-1)$, $w(-1,0,1)$

e) $(u-v)\cdot w$

$(1,-5,-1)\cdot(-1,0,1)$

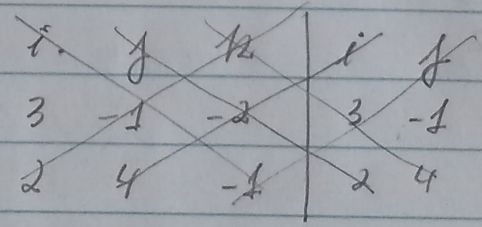


$-5k + 0i - 1j - 5i + 1j + 0k$

$= -5i + 0j - 5k$

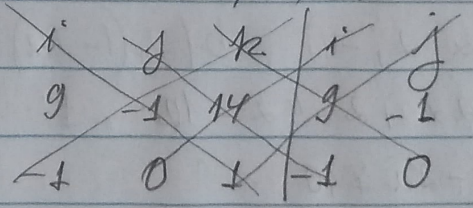
f) $(u \times v) \times w$

$u \times v$



$+2k + 8i + 3j + 1i - 4j + 12k$

$= 9i - 1j + 14k \quad (9,-1,14) \times w\,(-1,0,1)$



$-1k + 0i - 9j - 1i - 14j + 0k$

$= (-1,-23,-1)$

tilibra

1/1

i) $(u \times v) + (u \times w)$

$u \times (v+w)$ $\quad (v+w) = (1, 4, 0) = t$

$u \times t$ $\quad =$

i	j	k	i	j
3	-1	-2	3	-1
1	4	0	1	4

$+1k + 8i + 0j + 0i - 2j + 12k = (8, -2, 13)$

2)

3) $A(2,1,-1)$, $B(3,0,1)$, $C(2,-1,-3)$

$D = ?$

$AD = BC \times AC \Rightarrow D - A = (C-B) \times (C-A)$

$= (-1, -1, -4) \times (0, -2, -2)$

i	j	k	i	j
-1	-1	-4	-1	-1
0	-2	-2	0	-2

$+0k - 8i - 2j - 2i + 0j + 2k$

$= (-10, -2, +2) = D - (2, 1, -1)$

$= (-8, -1, +1) = D$

4) $x \cdot (1, 4, -3) = 7$

$x + 4y - 3z = -7$

$x + 4\left(\frac{1-x}{2}\right) - 3\left(\frac{5+x}{4}\right) = -7$

$x + 2 - 2x - \frac{15}{4} + \frac{3x}{4} = -7$

$-x - \frac{3x}{4} = -7 - 2 + \frac{15}{4}$

$\frac{-4x - 3x}{4} = \frac{-28 - 8 + 15}{4}$

$-7x = -21 \quad \Rightarrow x = 3$

i	j	k	i	j
x	y	z	x	y
4	-2	1	4	-2

$-4yk + 2zi - xj + yi + 4zj - 2xk$

$= (2z + y), (4z - x), (-4y - 2x)$

$\downarrow 3 \qquad \downarrow 5 \qquad \downarrow -2$

a) $2z + y = 3 \Rightarrow 2\left(\frac{5+x}{4}\right) + \frac{1-x}{2} = 3$ *

b) $4z - x = 5 \Rightarrow z = 5 + x/4$

c) $+4y + 2x = +2 \Rightarrow y = \frac{2 - 2x}{4} = \frac{1-x}{2}$

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WINTERLE, P. 88

5) a) ?

6) $x \cdot w = 0$ $\quad u(3,1,1), v(-4,1,3), w(1,2,0)$

$x \times u = v$

$(x,y,z)\cdot(1,2,0) = 0 = \boxed{x + 2y + 0z = 0}$

$x \cdot u = v \Rightarrow$

$$\begin{array}{ccc|cc} i & j & k & i & j \\ x & y & z & x & y \\ 3 & 1 & 1 & 3 & 1 \end{array}$$

$-3yk - zi - xj + yi + 3zj + xk =$

$(y-z), (3z-x), (x-3y) \Rightarrow y - z = -4 \Rightarrow \boxed{y = z - 4}$

$-4 \quad 1 \quad 3$

$x \cdot w = 0 =$

$\Rightarrow 0 = x + 2y$

$\Rightarrow x = -2y$

$\Rightarrow \boxed{\begin{array}{l} y - z = -4 \Rightarrow y = -4 + z \\ 3z - x = 1 \Rightarrow x = -1 + 3z \\ x - 3y = 3 \Rightarrow x = 3 + 3y \end{array}}$

$\rightarrow -2y^2 - 5y = 3 \Rightarrow -y = -\frac{3}{7}$

$\cancel{-x + 4y^2 = 0}$

$\cancel{-x + 3y = -3}$

$\cancel{0 + 4y^2 + 3y = -3}$

$\Rightarrow 4y^2 + 3y + 3 = 0$

$\Delta = b^2 - 4ac = 9 - 4.4.3 = -48 \nexists \mathbb{R}$

$\boxed{S = u \text{ e } v \text{ não são ortogonais}}$

$u \cdot v = -12 + 1 + 3 = \boxed{-8}$

7) a) $\vec{OF} \times \vec{OD}$

$(a, 0, a) \times (0, a, a)$

$\rightarrow$

i	j	k	i	j
a	0	a	a	0
0	a	a	0	a

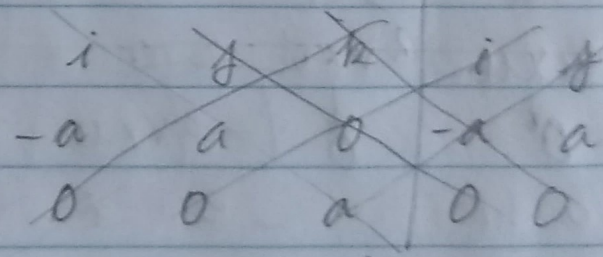
$+0k - a^2 i - a^2 j + 0ai + 0aj + a^2 k$

$= -a^2, -a^2, a^2$

WINTERLE P. 89

7b) AC x FA

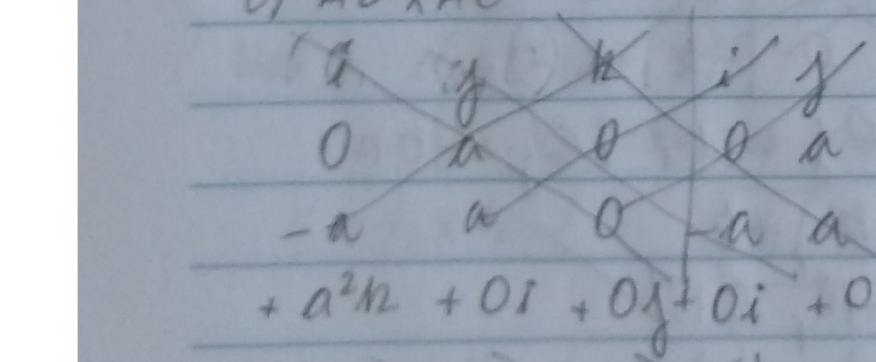
$(-a, a, 0) \times (0, 0, -a)$



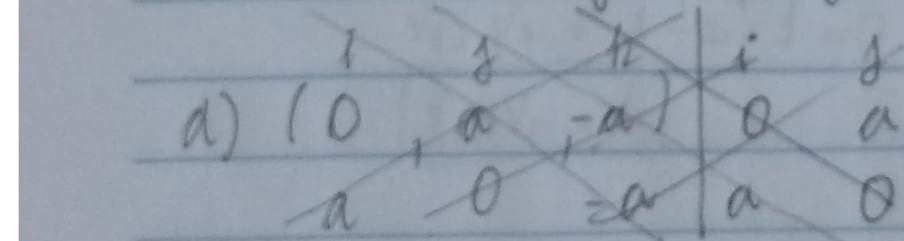
$0k + 0i + a^2 j + a^2 i + 0j + 0k$

$(a^2, a^2, 0)$

c) AB x AC



$+a^2 k + 0i + 0j + 0i + 0j + 0k = (0, 0, a^2)$

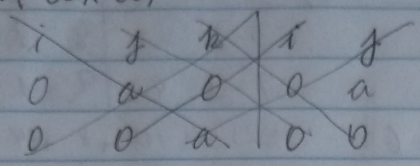


$-a^2 k + 0i + 0j - a^2 i - a^2 j + 0k = -a^2 i + a^2 j - a^2$

tilibra

e) $OA \cdot (OC \times OE)$

$$\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & a & 0 & 0 & a \\ 0 & 0 & a & 0 & 0 \end{array}$$



$0k + 0i + 0j + a^2 i + 0j + 0k$

$(a,0,0)(a^2,0,0) = \boxed{a^3}$

f) 0

8) $u(1,-2,1)$, $v(1,1,1)$, $w(1,0,-1)$

a) Dem: $u \cdot v = 0$ $\quad (1-2+1) = 0$

$v \cdot w = 0$ $\quad (1-1) = 0$

b) $u \times v = kw$

...

9) $u(-3,2,0)$, $v(0,-1,-2)$

$(u+2v) \times (v-u)$

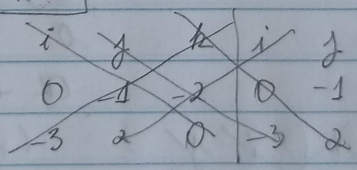
$u \times v - \cancel{|u|^2} + 2\cancel{|v|^2} - 2v \times u - v \times u$

$-u \times u \quad 2v \times v$

$0 \quad 0 \quad \boxed{-3 \, v \times u}$

-3

$$\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & -1 & -2 & 0 & -1 \\ -3 & 2 & 0 & -3 & 2 \end{array}$$



$-3j + 4i + 0j + 0i + 6j + 0k$

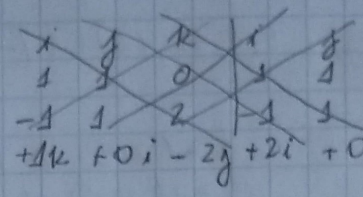
$= 4, +6, -3$

$= -12, +18, 9$ ✓

WINDERLE, 88

12) $u(1,1,0), v(-1,1,2)$

a) $u \times v = w =$



$+1k + 0i - 2j + 2i + 0j + 1k$

$= (2, -2, +2) \Rightarrow |w| = \sqrt{4+4+4} = 2\sqrt{3}$

$$\frac{w}{|w|} = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

b) $\frac{w}{|w|} \cdot 5$

13) igual

14) a) $|AB \times AD| = |AB| \cdot |AD| \cdot |\operatorname{sen} \theta| \quad (\theta = 60) = 2\sqrt{3} \Rightarrow |BD| =$

$2 \cdot 2 \cdot \frac{\sqrt{3}}{2}$

$2\sqrt{3} = S = \frac{|AM| \cdot |BD|}{2}$

b) $|BA \times BC| = |BA| \cdot |BC| \cdot |\operatorname{sen} 120| = 2\sqrt{3}$

$2 \cdot 2 \cdot \frac{\sqrt{3}}{2}$

c) 0

d) 0

e) $|BD \times AC| = 2 \times |AB \times AD| = 4\sqrt{3}$

Ou $|AB \times AD| = S_{\Delta} = \frac{b \cdot h}{2} =$

$\operatorname{sen} 30 = \frac{OP}{HIP} = \frac{OP}{2} \Rightarrow \frac{1}{2} = \frac{OP}{2} = 1 \Rightarrow |BD| = 2$

$2\sqrt{3} = \frac{|AM| \cdot 4}{2} \Rightarrow |AC| = 2\sqrt{3}$

WINTER CE, 88

14 f) $|BD \times CD| = |BD| \cdot |CD| \cdot \sin\theta$

$= 2 \cdot 2 \cdot \frac{\sqrt{3}}{2}$

16) $|u| = 2\sqrt{2}$, $|v| = 4$ $\sin\theta = 45°$

a) $|2u \times v| = ?$

$= 2|u \times v|$

$|u \times v| = |u||v| \sin\theta$

$= 2\sqrt{2} \cdot 4 \cdot \frac{\sqrt{2}}{2} = 8$

$\Rightarrow 2 \cdot 8 = 16$

b) $\left|\frac{2}{5}u \times \frac{1}{2}v\right| = ?$

$= \frac{1}{5}|u \times v| = \frac{8}{5}$

16) $u \cdot v = ?$

$|u \times v| = 12$

$|u| = 13$

$|v| = 1$

$|u \times v| = |u||v| \cdot \sin\theta$

$12 = 13 \cdot 1 \cdot \sin\theta \Rightarrow \frac{12}{13} = \sin\theta$

$u \cdot v = |u| \cdot |v| \cdot \cos\theta$

lagrange: $|u \times v|^2 = |u|^2 |v|^2 - (u \cdot v)^2$

$144 = 169 \cdot 1 - (u \cdot v)^2$

$\Rightarrow -25 = (u \cdot v)^2 \Rightarrow \boxed{u \cdot v = \pm 5}$

WINTERLE, P. 88

16) $u \cdot v = ?$
$|u \times v| = 12$
$|u| = 13$
$|v| = 1$

Lagrange: $|u \times v|^2 = |u|^2 \cdot |v|^2 - (u \cdot v)^2$

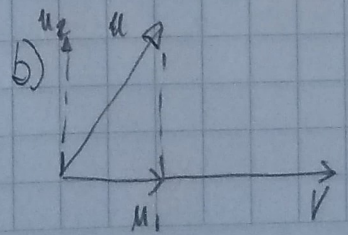
$\Rightarrow 12^2 = 13^2 \cdot 1^2 - (u \cdot v)^2$
$144 = 169 - (u \cdot v)^2$
$+25 = u \cdot v^2 \Rightarrow \boxed{u \cdot v = \pm 5}$

90 | 2
45 | 3
15 | 3
5 | 5

17) a) $A = |u \times v| = \begin{vmatrix} i & j & k & i & j \\ 3 & -1 & 2 & 3 & -1 \\ -2 & 2 & 1 & -2 & 2 \end{vmatrix}$

$= |-2k - 4i - 3j - 1i - 4j + 6k|$
$= |(-5, -7, 4)| = \sqrt{25 + 49 + 16} = \sqrt{90} = 3\sqrt{10}$

b)



alt 1) $u = u_1 + u_2$, $u_1 = \text{proj}_v u = \left(\frac{u \cdot v}{v \cdot v}\right) \cdot v = \frac{-6 - 2 + 2}{\sqrt{4+4+1}} \cdot v$

$\boxed{u_1 = -\frac{6}{3} v = -2v}$

$\boxed{u_1 = (4, -4, -2)}$

$\boxed{u_2 = u - u_1}$ ← $|u_2| = ?$

$u_2 = (-1, 3, 4) \rightarrow |u_2| = \sqrt{1 + 9 + 16} =$

WINTERLE, P. 88

16) $|u \times v|^2 = |u|^2 \cdot |v|^2 - (u \cdot v)^2$

$144 = 169 \cdot 1 - (u \cdot v)^2$

$\boxed{u \cdot v = \pm 5}$

$90 \mid 2$
$45 \mid 3$
$15 \mid 3$
$5 \mid$

17) $S_{\square} = |u \times v| \Rightarrow u \times v =$

i	j	k	i	j
3	-1	2	3	-1
-2	2	1	-2	2

$-2k - 4i - 3j - i - 4j + 6k$

$|(-5, -7, 4)| \Rightarrow \sqrt{25 + 49 + 16} = \sqrt{90} = \boxed{3\sqrt{10} = S_{\square}}$

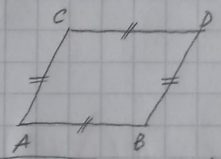
$S = b \cdot h$

$\boxed{b = |v| = 3}$

$\Rightarrow 3\sqrt{10} = 3 \cdot h \Rightarrow h = \sqrt{10}$

18) $S_{\square ABCD} = |AB \times AC|$ $AB \parallel CD$, $|AB| = |CD|$

$= ?$



$AB = B - A = (1, -1, -1)$

$CD = D - C = (-1, 1, 1) \Rightarrow AB = -1\,(CD)$, $\boxed{-1 = k} = AB \parallel CD$

$|AB| = \sqrt{3}$, $|CD| = \sqrt{3}$, $\Rightarrow$ ABCD é paralelogramo

$AC = C - A = (-5, 1, -4)$

$\Rightarrow |AB \times AC| =$

i	j	k	i	j
1	-1	-1	1	-1
-5	1	-4	-5	1

$|-5k + i + 4j + 4i + 5j + k|$

$\Rightarrow |(5, 9, -4)| = \sqrt{25 + 81 + 16} = \sqrt{122}$

17b) $|u \times v| = |u||v| \cdot \operatorname{sen}\theta$

$3\sqrt{10} = \sqrt{6+1+4} \cdot \sqrt{8} \cdot \operatorname{sen}\theta$

$\Rightarrow \frac{3\sqrt{10}}{\sqrt{14}} = \operatorname{sen}\theta = \frac{\sqrt{5}}{\sqrt{7}} \cdot \frac{\sqrt{35}}{7}$

$|u \times v| = A$

$A = b \times h$

$b = |v| = 3$

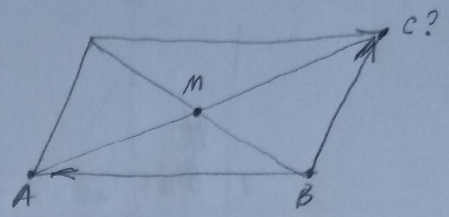
$3\sqrt{10} = 3 \cdot h \Rightarrow h = \sqrt{10}$

Unitate, p. 88

10) $M = \frac{A+C}{2} \Rightarrow 2M - A = C$

$\Rightarrow (6,4,-4) - (2,-4,0) = C$

$\Rightarrow \boxed{C = (4,8,-4)}$



$S_{\square} = |BA \times BC|$

$BA = A - B = (1,-1,1)$

$BC = C - B = (3,11,-3)$

$\Rightarrow BA \times BC =$

i	j	k	i	j
1	-1	1	1	-1
3	11	-3	3	11

$+3k - 11i + 3j + 3i + 3j + 11k$

$= (-18, 6, 14)$

$\Rightarrow |BA \times BC| = \sqrt{641+36+192} = \sqrt{296} = 2\sqrt{74}$

20) $S_{\square} = \sqrt{26}$

$S_{\square} = |u \times v|$ $\longrightarrow$ $u \times v =$

$m = ?$ / $S_{\square} = \sqrt{26}$

i	j	k	i	j
m	-3	1	m	-3
1	-2	2	1	-2

$+3k + 2i - 2mj - 6i + 1j - 2mk$

$= (-4, 1-2m, 3-2m)$

$|u \times v| = \sqrt{16 + (1-2m)^2 + (3-2m)^2}$

$\sqrt{26} = \sqrt{16 + 1 - 4m + 4m^2 + 9 - 12m + 4m^2}$

$26 = 26 - 16m + 8m^2$

$\Rightarrow 8m^2 - 16m = 0$

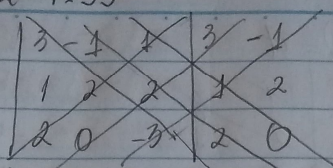
$\Rightarrow m^2 - 2m = 0$

$\Rightarrow m(m-2) = 0 \quad m = 0$ ou 2

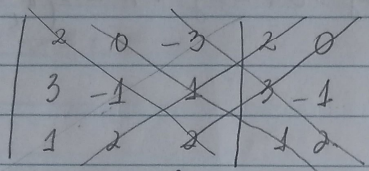
/ /

WINTERIE P.99

1) a)



$$\left|\begin{array}{ccc|cc} 3 & -1 & 1 & 3 & -1 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & 0 & -3 & 2 & 0 \end{array}\right.$$



$-4 + 0 - 3 \quad -18 - 4 + 0 = \boxed{-29}$

b)

$$\left|\begin{array}{ccc|cc} 2 & 0 & -3 & 2 & 0 \\ 3 & -1 & 1 & 3 & -1 \\ 1 & 2 & 2 & 1 & 2 \end{array}\right.$$

$-3 - 4 + 0 \quad -4 + 0 - 18 = -29$

2) $(u, v, w) = -5$

a) $(w, v, u) = 5$ b) $(v, u, w) = 5$

c) $(w, u, v) = -5$ d) $v \cdot (w \times u) = -5$

3)

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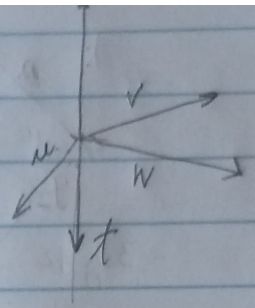
WINTERLE P. 100



3) $u \cdot (v \times w) = 2$

a) $u \cdot (w \times v) = -2$

b) $v \cdot (w \times u) = 2$

c) $(v \times w) \cdot u = 2$

d) -6

e) $u(2w \times v) = -4$

f) $(u+v) \cdot (u \times w) = u$?

4) $(u, w, x) = 2$, $(v, w, x) = 5$

a) $(u, x, -w)$

1 inversão

$= -2$

$-w = 2$

b) $(3u, 3w, -2x)$

$-18 \times 2 = -36$

c) $(2u, w, x) + (4v, w, x)$

$4 \quad + \quad 20$

d) $(5u - 3v, 2w, x) = (5u, 2w, x) - (3v, 2w, x)$

$20 \quad - \quad 30 \quad = -10$

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5) $(u, v, w) = 0$?

a) $\begin{matrix} 1 & -1 & 2 \\ 2 & 2 & 1 \\ -2 & 0 & -4 \end{matrix}$ $\begin{matrix} 1 & -1 \\ 2 & 2 \\ -2 & 0 \end{matrix}$

$+8 + 0 + 8 \quad -8 - 2 + 0 = 6$

b) $\begin{matrix} 2 & -1 & 3 \\ 3 & 1 & -2 \\ 7 & -1 & 4 \end{matrix}$ $\begin{matrix} 2 & -1 \\ 3 & 1 \\ 7 & -1 \end{matrix}$

$-24 - 4 + 12 + 8 + 14 - 9 = 0$

6) $k = ?$ / $(u, v, w) = 0$

a) $\begin{matrix} 2 & -1 & k \\ 1 & 0 & 3 \\ k & -3 & k \end{matrix}$ $\begin{matrix} 2 & -1 \\ 1 & 0 \\ k & -3 \end{matrix}$

$0k - 12 + k + 0k - 2k + 3k = 0$

$2k = 12 \Rightarrow k = 6$

b) $\begin{matrix} 2 & k & 1 \\ 1 & 2 & k \\ 3 & 0 & -3 \end{matrix}$ $\begin{matrix} 2 & k \\ 1 & 2 \\ 3 & 0 \end{matrix}$

$-6 + 0 + 3k + 12 + 3k^2 + 0 = 0$

$3k^2 + 3k - 18 = 0$

$\Delta = b^2 - 4ac = 9 - 4 \cdot 3 \cdot (-18) = 225$

$k_1 = \frac{-3 + 15}{6} = \boxed{+2}$

$k_2 = \qquad = \boxed{-3}$

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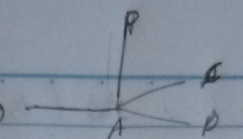
Winterle, 99

7) $AB \times AC = AP \mid AP \cdot AD = 0$

a) ou $(AB \times AC) \cdot AD = 0$?



$AB = (-3, 0, -6)$ | $-3\ \ 0$

$AC = (-2, 1, -1)$ | $-2\ +2$

$AD = (1, -2, -4)$ | $+1\ -2$

$+16 + 6 + 0 \quad + 12 + 0 = 24 = 0$

b) $AB = -2\ \ 0\ \ -4 \mid -2\ \ 0$

$AC = -1\ \ -1\ \ -5 \mid -1\ \ -1$

$AD = 1\ \ 0\ \ -4 \mid 1\ \ 0$

$-4 + 0 + 0 \quad -8 + 0 + 0 = -4$, não

8) $m = ? \mid (AB, AC, AD) = 0$

$AB \quad 2-m\ \ -3\ \ -5 \mid 2-m\ \ -3$

$AC \quad 5-m\ \ -2\ \ -1 \mid 5-m\ \ -2$

$AD \quad 3-m\ \ -3\ \ -4 \mid 3-m\ \ -3$

$-30 + 10m - 6 + 3m \quad +16 - 8m + 9 - 3m + 75 - 15m = 0$

$-60 + 12m$ ←

$-7m + 64 = 0 \Rightarrow m =$

$-5m + 4 = 0 \Rightarrow m = 4$

9)

$1\ \ 0\ \ 0 \mid 1\ \ 0$

$0\ \ 1\ \ 0 \mid 0\ \ 1$

$0\ \ 0\ \ 1 \mid 0\ \ 0$

$1 + 0 + 0 = 1$

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WINTERLE P. 100

7) a) $(AB \times AC) \cdot AD = 0$?

$AB\ (-3, 0, -6)$
$AC\ (-2, 1, -1)$
$AD\ (1, -2, -4)$

i	j	k	i	j
-3	0	-6	-3	0
-2	1	-1	-2	1
1	-2	-4	1	-2

$+6 + 6 - 0 \quad +12 + 0 - 24 = 0$ ✓

S: Sim

~~também: ñ existe k tal que AB = k AC~~
~~AB = k AD~~

-2	-2	-6	-2	-2
-1	0	-2	-1	0
-3	-1	-7	-3	-1

$+0 + 4 + 14 \quad +0 - 12 - 6 = 0$

b)

AB (-2, 0, -4)	-2	0
AC (-1, -1, -1)	-1	-1
AD (1, 0, -4)	1	0

$-4 + 0 + 0 \quad -8 + 0 + 0 = -12$

S: não

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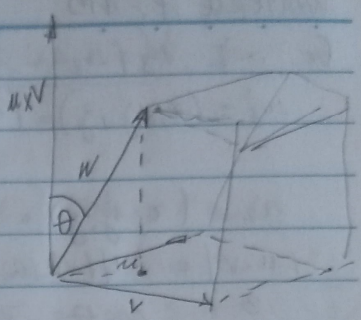
WINTELLE P. 100

10) $\begin{matrix} 3 & -1 & 4 & 3 & -1 \\ 2 & 0 & 1 & 2 & 0 \\ -2 & 1 & 3 & -2 & 1 \end{matrix}$

$+0 - 3 + 10 \quad +0 + 2 + 8 = \boxed{17 = V}$



$h_{u \times v} = |w| \cdot |\cos\theta| \Rightarrow \boxed{h = \frac{\sqrt{30} \cdot 17}{30}}$

$\cos\theta = \frac{u \cdot v}{|u| \cdot |v|} = \frac{w \cdot u \times v}{|w| \cdot |u \times v|} = \frac{2 + 5 + 10 = 17}{30}$

$u \times v = \begin{matrix} i & j & k & i & j \\ 3 & -1 & 4 & 3 & -1 \\ 2 & 0 & 1 & 2 & 0 \end{matrix}$

$+2k + 0i - 3j - 1i + 8j + 0k \Rightarrow -1, +5, +2$

$\boxed{|u \times v| = \sqrt{1 + 25 + 4} = \sqrt{30}}$

$\boxed{|w| = \sqrt{4 + 1 + 25} = \sqrt{30}}$

A Livro diz: $h = 17/30$

Упражнение 99

11) $m = ?$ / $\vec{v_1} \cdot \vec{v_2} \times \vec{v_3} = 33$

$\vec{v_1} \times \vec{v_2} \cdot \vec{v_3}$

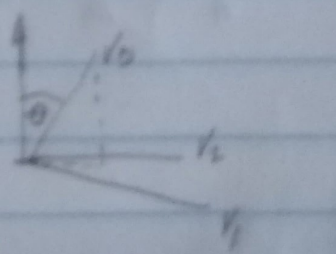
0	-1	2	0	-1	
-4	2	-3	-4	2	
3	m	-2	3	m	

$-12 + 0 + 8 + 0 + 3 - 8m = 33$

$-8m = 34 \Rightarrow m = \frac{34}{8} = -\frac{17}{4}$

$h = |\vec{w}| \cdot \cos\theta$

$\vec{v_1} \times \vec{v_2}$



$|\vec{v_3}| = \sqrt{9 + m^2 + 4}$

$= \sqrt{13 + m^2}$

$\vec{v_1} \times \vec{v_2} = (-3, -8, -4)$

$\Rightarrow |\vec{v_1} \times \vec{v_2}| = \sqrt{9 + 64 + 16} = \sqrt{89}$

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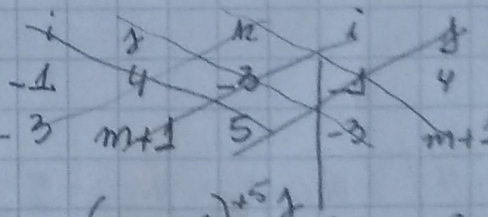
WINTERLE, 99

12) $m = ?$ / $|AB \times CD| = 20$

$AB = B - A =$ $-1 \quad 4 \quad -3$

$CD = D - C =$ $-3 \quad m+1 \quad 5$



$+12k - (3m+3)i + 20i + 9j + (-m-1)k$

$(3m+23),\ 14j,\ (-m+11)$

$|AC \times BD| = \sqrt{9m^2 + 138m + 529 + 196 + m^2 + 22m + 121} = 20$

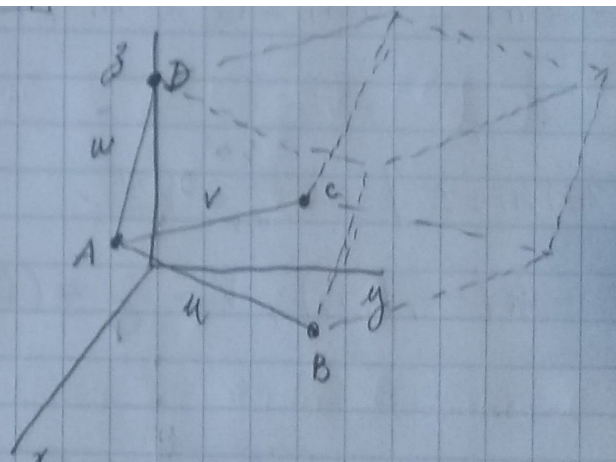
$\sqrt{10m^2 + 146m + 725} = 20$

WIMERIE, 99

13) $(u, v, w) = 25$

$D(0, 0, z)$, $D = ?$

$u = AB = B - A = (-3, -1, 0)$
$v = AC = C - A = (1, 1, -3)$
$w = AD = D - A = (-2, -1, z-1)$

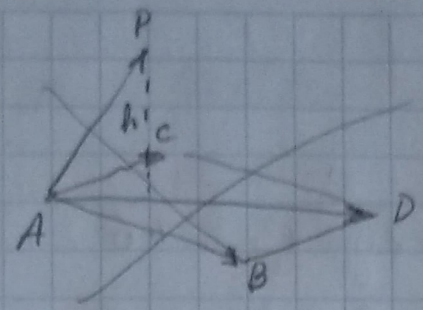


$$\Rightarrow \begin{array}{ccc|cc} -3 & -1 & 0 & -3 & -1 \\ 1 & 1 & -3 & 1 & 1 \\ -2 & -1 & z-1 & -2 & -1 \end{array}$$

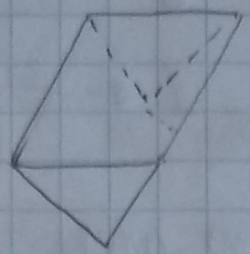
$+0 + 9 + z - 1 - 3z + 3 - 6 + 0$
$= -2z + 5 = 25 \Rightarrow z = -10$

14) ...

15)



$V_{ABCP} = ?$

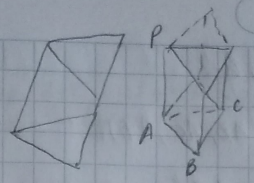


alternativa: a) ~~soma AB+AC=AD~~
b) ~~|AD×AP| = área~~
c) ~~área~~ = A. |AD|

Winterle, p. 99

15) $V_{\triangle} = |\langle AB, AC, AP\rangle| / 6$

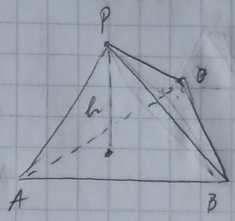
$V_{\square} = b \cdot h / \quad b = |AB \times AC|$



$AB = B - A = (0, 4, 0)$
$AC = C - A = (-3, 3, 0)$
$AP = P - A = (0, -3, 8)$

$= 72$ u.v. $= V_{\square}$

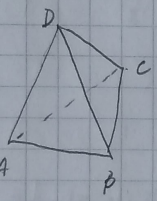
6 · tetraedru = 2 tetraedre = 1 paralelogram
$\Rightarrow V_{\triangle} = 12$ u.v.



$b = |(4, 0, 0) \times (3, 3, 0)| = |(0, 0, 12)| = 12$

$|(0, 0, 8)| = \sqrt{64} = 8 \Rightarrow h_{\square} = h_{\triangle} = \frac{6}{b} = \frac{72}{8} = 9$

16)



$\text{Vol}_{\text{tetra}} = \frac{1}{6} \text{Vol}_{\text{paralel.}}$

$\text{Vol}_{\text{paral}} = \langle AB, AC, AD\rangle$

$\text{Vol}_{\text{tetra}} = 3$

$m = ?$

$\Rightarrow \frac{\langle AB, AC, AD\rangle}{6} = 3$

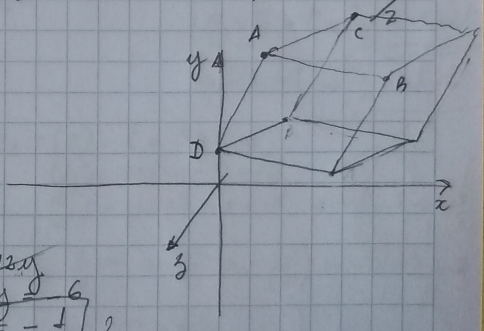
$\langle AB, AC, AD\rangle = \begin{vmatrix} 2 & 1 & -4 \\ m & -1 & 3 \\ -3 & 1 & -2 \end{vmatrix} \begin{matrix} 2 & 1 \\ m & -1 \\ -3 & 1 \end{matrix}$

$+12 - 6 + 3m + 4 - 9 - 4m \Rightarrow \frac{-2m + 1}{6} = 3$

$\Rightarrow -2m = +17 \Rightarrow m = -17$

17) $D(0, y, 0)$

$V_{\square} = 6 = (AB, AC, AD)$



$AB = B - A = (-1, -2, 4)$
$AC = C - A = (3, -6, 0)$
$AD = D - A = (2, y-4, 1)$

$+48 + 6 \quad +6 - 48 + 12y$

$\Rightarrow 12y + 12 = 6 \Rightarrow 12y = -6$

$y = -\frac{1}{2}$?

$|12y + 12| = 6$

$\Rightarrow 12y + 12 = 6 \Rightarrow y = -1/2$

sau $12y + 12 = -6 \Rightarrow y = -\frac{18}{12} = -\frac{3}{2}$?

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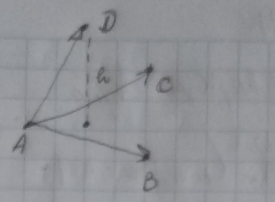
11) $V_{\square} = b \cdot h$

$V_{\square} = |(AB, AC, AD)|$

$b = |AB \times AC|$



$AB = B - A = (-3, -3, 0)$

$AC = C - A = (-3, 0, 3)$

$AD = D - A = (2, 5, -1)$

$+45 + 9 - 18$

$\Rightarrow V = |36|$

$b: |AB \times AC| = \begin{vmatrix} i & j & k \\ -3 & -3 & 0 \\ -3 & 0 & 3 \end{vmatrix}$

$= -9k - 9i - 9i$

$= |(-9, -9, -9)| = \sqrt{243} = 9\sqrt{3}$

$\Rightarrow 36 = 9\sqrt{3} \cdot h$

$\Rightarrow \frac{4}{\sqrt{3}} = h = \frac{4\sqrt{3}}{3} = d(D, \pi_{ABC})$ ✓

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WINTERLE P. 119 $\boxed{AB = B - A = (-1, +2, -2)}$

E1) $r: (x, y, z) = (2, -3, 4) + t(AB)$

a) $C \in r$?

$C\left(\frac{5}{2}, -4, 5\right)$

$z \Rightarrow 5 = 4 - 2t \Rightarrow 1 = -2t \Rightarrow t = -\frac{1}{2}$

$y \Rightarrow -4 = -3 + 2t \Rightarrow -1 = 2t \Rightarrow t = -\frac{1}{2}$

$2 - t = \frac{5}{2} \Rightarrow -t = \frac{5}{2} - 2 = \frac{5-4}{2} = -\frac{1}{2}$

b) $D \in r$?

$D(-1, 3, 4)$

x) $2 + t = -1 \Rightarrow \boxed{t = -3}$

y) $-3 + 2t = 3 = \boxed{t = 0}$ não existe t / $D \in r$

z ...

2) $r: \begin{cases} x = -1 + 2t \\ y = 2 - 3t \\ z = 3 \end{cases}$

3) $r: \begin{cases} x = 1 \\ y = 2 \\ z = 3 + t \end{cases}$

4) a) $(x, 6, z) \Rightarrow 3 - t = 6 \Rightarrow t = -3$

$\Rightarrow -1 \quad -10$

b) $2 + t = 3 - t \Rightarrow 2t = 1 \Rightarrow t = 1/2$

$2 + \frac{1}{2} = \frac{4+1}{2} = \frac{5}{2}$

$\Rightarrow P = (5/2, 5/2, -3)$

$3 - \frac{1}{2} = \frac{6-1}{2} = \frac{5}{2}$

c) $-4 + 2t - 4 \cdot (2 + t) = 8 + 4t \Rightarrow -2t = 12 \Rightarrow t = -6$

$Q(-4, \ldots)$

5) $r: (x, y, z) = (4, -3, -2) + t(3, -4, -1)$

Pen

$P(m, n, -5) \Rightarrow -2 - t = -5 \Rightarrow \boxed{t = 3}$

$m = ?$

$n = ?$

$\Rightarrow m = 4 + 9 = 13$

$n = -3 - 12 = -15$

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WINTERLE p. 119

6) $r: (x, y, z) = (x, y, z) + t(a, b, c)$

a) $AB = B - A = (1, +2, -2)$

$= (1, -1, 2) + t(1, 2, -2) \Rightarrow x = 1 + t$

$y = -1 + 2t$

$z = 2 - 2t$

d) $AB(0, 1, 0)$ $\Rightarrow x = 0$

$y = t$

$z = 0$

7) a) $A(2, 0, 4)$, $B(0, 0, 4)$ $\Rightarrow r: \begin{cases} x = 2 - 2t \\ y = 0 \\ z = 4 \end{cases}$ $+?$

$AB = B - A = (-2, 0, 0)$

b) $C(0, 3, 0)$, $D(2, 3, 0)$

$CD = (2, 0, 0)$ $r: \begin{cases} x = 2t \\ y = 3 \\ z = 0 \end{cases}$

c) $AD = (0, 3, -4)$ $r: \begin{cases} x = 2 \\ y = 3t \\ z = 4 - 4t \end{cases}$

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7 d) $B(0,0,4)$, $C(0,3,0)$

$BC = C - B = (0, 3, -1)$

$r: \begin{cases} x = 0 \\ y = 3t \\ z = 4 - t \end{cases}$

e) $D(2,3,0)$, $E(2,0,0)$

$DE = (0, -3, 0)$

$r: \begin{cases} x = 2 \\ y = 3 - 3t \\ z = 0 \end{cases}$

8) $AB = (+1, -2, -5) \Rightarrow r: (x, y, z) = (3, -1, 4) + t(+1, -2, -5)$

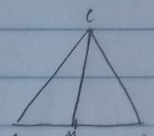
$P(2, 1, 9)$

$-1 - 2t = 1 \Rightarrow \boxed{t = -1}$

$x = 3 - 1 = 2$

$z = 4 - 5 \cdot -1 = 9$

9)



$M = \frac{A+B}{2} = \frac{2}{2}, \frac{1}{2}, \frac{4}{2} = (1, \frac{1}{2}, 2)$

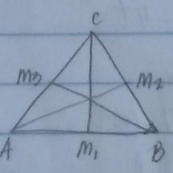
$CM = M - C = (-1, \frac{3}{2}, -2)$

$x = 2 - t$

$y = -1 + \frac{3}{2}t$

$z = 4 - 2t$

10)



$M_2 = \frac{B+C}{2} \Rightarrow 2M_2 = B + C$

$M_1 = \frac{A+B}{2} \Rightarrow 2M_1 = A + B$

$M_3 = \frac{A+C}{2} = 2M_3 = A + C$

$\Rightarrow A = 2M_3 - C$

$B = 2M_2 - C$

$2M_1 = 2M_3 - C + 2M_2 - C$

$\Rightarrow 2M_1 - 2M_3 - 2M_2 = -2C$

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$\Rightarrow B + C = 2M_3 + M_2 - M_1$

$C \equiv (4, -4, -8)$

$\Rightarrow 2M_2 = B + C \Rightarrow (2, -6, 0) - (1, -1, -8) \Rightarrow B = (1, -5, 8)$

$\Rightarrow 2M_1 - B = A \Rightarrow (4, -2, 6) - (1, -5, 8) \Rightarrow A = (3, +2, -2)$

$AB = B - A = (-2, -7, 10) \Rightarrow r : (x, y, z)$

$$\begin{cases} x = 3 - 2t \\ y = 2 - 7t \\ z = -2 + 10t \end{cases}$$

$2M_2 - C$

C

tilibra

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12) $P_1 \in r$?
$P_2 \in r$?

$r: (x,y,z) = (+3,-1,+2) + t(-1,2,-2)$

$P_1 = (5,-5,6) \in r$

$x = 3 - t = 5 \Rightarrow t = -2$
$-1 + 2t = -5 \Rightarrow 2t = -4 \Rightarrow t = -2$
$2 - 2t = 6 \Rightarrow -2t = 4 \Rightarrow t = -2$

$P_2 = (4,-1,12) \in r$

$3 - t = 4 \Rightarrow t = -1$
$-1 + 2t = -1 \Rightarrow 2t = -2 = -1$
$2 - 2t = 12 \Rightarrow -t = 5$

13) $r: (x,y,z) = (1,-3,0) + t(2,-1,4)$

a) $P \in r \mid x = 5$ $\Rightarrow 1 + 2t = 5 \Rightarrow \boxed{t = 1}$
$(5,-4,4)$

b) $P \in r \mid y = 2$ $\Rightarrow -3 - t = 2 \Rightarrow \boxed{t = -5}$
$(-9,2,-20)$

14) $r: (x,y,z) = (-1,+2,-4) + t\boxed{(3,2,1)}$

$P(1,?,?) \in r$ $\Rightarrow -1 + 3t = 1 \Rightarrow t = 2/3$

$2 + 2 \cdot \frac{2}{3} = 2 + \frac{4}{3} = \frac{6+4}{3} = \frac{7}{3}$ ✗

$-4 + \frac{2}{3} = \frac{-12+2}{3} = 10/3$ ✗

$3t = 2x + 1 \Rightarrow x = -\frac{1}{2} + \frac{3}{2}t$
$2t = 3y - 2 \Rightarrow y = \frac{2}{3} + \frac{2}{3}t$
$z + 4 = t \Rightarrow z = t - 4$

$r: (x,y,z) = (-\frac{1}{2}, \frac{2}{3}, 4) + t(\frac{3}{2}, \frac{2}{3}, -1)$

$P(1, 4/3, -3)$

$1 = -\frac{1}{2} + \frac{3}{2}t \Rightarrow 2 = -1 + 3t \Rightarrow t = 1$

$y = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

$z = 4 - 1 = 3$

$V_2 = ? \mid (x,4,z) = (\frac{9}{2}, 2, +3$

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15) reduzidas:

a) $(4,0,-3)+t(2,4,5) \Rightarrow \begin{cases} x = 4+2t \\ y = 4t \\ z = -3+5t \end{cases} \Rightarrow \begin{matrix} x-4/2 \\ y/4 \\ 3+z/5 \end{matrix}$

$\Rightarrow \frac{x-4}{2} = \frac{y}{4} \Rightarrow y = 2x-8$

$\Rightarrow \frac{x-4}{2} = \frac{3+z}{5} \Rightarrow \frac{5x-20}{2} - 3 = z \Rightarrow z = \frac{5x}{2} - 13$

b) $A(1,-2,3), B(3,-1,-1)$

$AB = B-A = (2,1,-4)$

$\begin{cases} x = 1+2t \\ y = -2+t \\ z = 3-4t \end{cases} \Rightarrow \begin{matrix} t = x-1/2 \\ t = y+2 \\ t = -z+3/4 \end{matrix}$

$\frac{x-1}{2} = y+2 \Rightarrow x-1 = 2y+4 \Rightarrow x-5 = 2y$

$y = \frac{x}{2} - \frac{5}{2}$

$\frac{x-1}{x} = \frac{-z+3}{x2} \Rightarrow 2x-2 = -z+3$ ✓

$2x+5 = -z$

16) $A(-1,6,3), B(2,2,1)$

$AB = (3,-4,-2) \Rightarrow r: \begin{cases} x = -1+3t \\ y = 6-4t \\ z = 3-2t \end{cases} \Rightarrow \begin{matrix} x+1/3 = t \\ -y+6/4 = t \\ -z+3/2 = t \end{matrix}$

$\frac{-z+3}{2} = \frac{x+1}{3} = -3z+9 = 2x+2$

$= -3z+7 = 2x$

$\Rightarrow x = -\frac{3}{2}z + \frac{7}{2}$

$-\frac{y+6}{x2} = \frac{-z+3}{x} = -2z+6$

$\Rightarrow +y = +2z$

17) $r: \begin{cases} x = x \\ y = 2x+3 \\ z = x-1 \end{cases}$

a) $P(3,9,2) \quad 9 = 2x+3 \Rightarrow 2x = 6 \Rightarrow x = 3$

b) $Q(2(x-1), ?, x-1)$

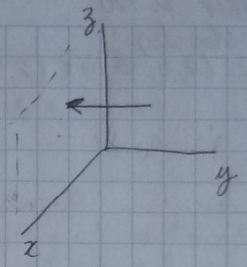
$x = 2x-2 \Rightarrow -x = -2 \Rightarrow x = 2$

$\Rightarrow Q(2,7,1)$

$= 3x-3 \Rightarrow -x = -6, x = 6$

19) a) $A(3,-2,4) + t(1,0,0)$ $\quad r: \begin{cases} x = 3+t \\ y = -2 \\ z = 4 \end{cases}$

b) $A(2,2,4) + t(0,1,0)$ $\quad r \begin{cases} x = 2 \\ y = 2+t \\ z = 4 \end{cases}$



c) $A(-2,3,4) + t(0,0,1)$ $\quad r: \begin{cases} x = -2 \\ y = 3 \\ z = 4+t \end{cases}$

d) $A(4,-1,3) + t(3,-2,0)$ $\quad r: \begin{cases} x = 4+3t \\ y = -1-2t \\ z = 3 \end{cases}$

e) $(3,-1,3)$

21) $\measuredangle = ?$

a) $r_1: \begin{cases} x = -2-t \\ y = t \\ z = 3-2t \end{cases}$ $\quad r_2:$

$u = (-1,1,-2)$ $\quad v = (2,1,1)$

$|u| = \sqrt{1+1+4} = \sqrt{6}$ $\quad |v| = \sqrt{6}$

$u \cdot v = -2+1-2 = -3$

$u \cdot v = |u| \cdot |v| \cdot \cos\theta$

$\Rightarrow -3 = 6 \cdot \cos\theta \Rightarrow \cos\theta = -\frac{1}{2}$

$\theta = 150°$

60°?

b) $r: \begin{cases} y = -2x+3 \\ z = x-2 \end{cases}$ $\quad s: y = \frac{z+1}{-1}, x = 4$

$x = \frac{y-3}{-2} = z+2$

$(1,-2,1)$ $\quad (0,1,-1)$

$u \cdot v = -2-1 = -3$

$|u| = \sqrt{6}$

$|v| = \sqrt{2}$

$\Rightarrow -3 = 2\sqrt{3} \cdot \cos\theta$

$= \frac{-3}{2\sqrt{3}} = \cos\theta = -\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 120°$? 30°

$y = t$

$\frac{z+1}{-1} = t \Rightarrow z+1 = -t$

$\Rightarrow z = -1-t$

c) $V_r(\sqrt{12}, 1, -3)$, $V_s(0,0,1)$ $\quad u \cdot v = |u| \cdot |v| \cdot \cos\theta$

$u \cdot v = -3$

$|u| = \sqrt{2+1+9} = \sqrt{12} = 2\sqrt{3}$

$|v| = 1$

$\Rightarrow -3 = 2\sqrt{3} \cdot \cos\theta \Rightarrow \cos\theta = -\frac{3}{2\sqrt{3}}$

$= -\frac{3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2}$

$\theta = 120°$

30°?

WINTERLE, P. 121

E21, CONTD.

d) $\vec{v}_{n_1}(2,-1,-2)$, $\vec{v}_{n_2}=(0,4,3)$

$u.v = |u|.|v|.\cos\theta$

$u.v = 0+-4-6 = -10$

$|u| = \sqrt{9} = 3$

$|v| = \sqrt{16+9} = 5$

$\Rightarrow -10 = 3.5.\cos\theta$

$\Rightarrow \cos\theta = \frac{-10}{15} = -\frac{2}{3}$

$\frac{y}{4} = t \rightarrow y = 4t$

$\frac{z-2}{3} = t \Rightarrow z = 2+3t$

E22) a) $n = ?$ / $\theta = \pi/6$

a) $\vec{v}_{n_1} = u$, $\vec{v}_{n_2} = v$

$u(4,5,3)$, $v(1,n,2)$

$u.v = |u||v|.\cos\theta$

$u.v = 4+5n+6 = 5n+10$

$|u| = \sqrt{16+25+9} = \sqrt{50} = 5\sqrt{2}$

$|v| = \sqrt{n^2+5}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$y-5 = nx \Rightarrow x = \frac{y-5}{n}$

$z+2 = 2x \Rightarrow x = \frac{z+2}{2}$

$\Rightarrow x = \frac{y-5}{n} = \frac{z+2}{2}$

$\Rightarrow v = (1,n,2)$

$\Rightarrow 5n+10 = 5\sqrt{2}.\sqrt{n^2+5}.\frac{\sqrt{3}}{2} = \frac{10n+20}{5} = \sqrt{6}.\sqrt{n^2+5}$

$= 2n+4 = \sqrt{6n^2+30}$

$= 4n^2+16n+16 = 6n^2+30 \Rightarrow -2n^2+16n-14 = 0$

$= 2n^2-16n+14 = 0$

$= n^2-8n+7 = 0$

$\Delta = b^2-4ac = 64-4.1.7 = 36$

$n = \frac{-b \pm \sqrt{\Delta}}{2a}$

$n_1 = \frac{8+6}{2} = 7$

$n_2 = \frac{8-6}{2} = 1$

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22 b) $n = ?$ / $\theta = 30°$

$\theta = 30° \Leftrightarrow \boxed{V_{r_1} \cdot V_{r_2} = |V_{r_1}||V_{r_2}| \cdot \frac{\sqrt{3}}{2}}$ (A)

$r_1 = \begin{cases} x = y + 1/n \\ x = 3/2 \end{cases} \Rightarrow V_{r_1} = (1, n, 2)$, pois $\frac{x}{1} = \frac{y+1}{n} = \frac{3}{2}$

$r_2 = Oy \Rightarrow V_{r_2} = (0,0,1)$

Em (A):

$0+0+2 = \sqrt{1+n^2+4} \cdot 1 \cdot \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{4}{\sqrt{3}} = \sqrt{n^2+5} \Rightarrow \frac{4\sqrt{3}}{3} = \sqrt{n^2+5} \Rightarrow \frac{16 \cdot 3}{3 \cdot 3} = n^2 + 5$

$\frac{16 - 15}{3} = \frac{1}{3}$

$\Rightarrow \frac{16}{3} - 5 = n^2$

$\Rightarrow n = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

$\boxed{S = \pm\sqrt{15}\ ?}$

23) $V_{r_1} \cdot V_{r_2} = 0$, $m = ?$

a) $V_{r_1} = (2m, 3, -4)$, $V_{r_1} \Rightarrow r: \frac{y}{1} = \frac{x+1}{2} = \frac{-z+4}{1}$

$-y = -z + 4$

$\Rightarrow 4m + 3 + 4 = 0$

$4m = -7 \Rightarrow m = -\frac{7}{4}$

$\Rightarrow V_{r_2} = (2, 1, -1)$

b) $\frac{x}{1} = \frac{y-3}{m} = \frac{z+1}{1} \Rightarrow V_{r_1} = (1, m, 1)$

$V_{r_2} = (-3, 2m, m)$

$V_{r_1} \cdot V_{r_2} = 0 = -3 + 2m^2 + m = 0$

$2m^2 + m - 3 = 0 \quad \Delta = 1 - 4 \cdot 2 \cdot (-3) = 25$

$m = \frac{-1 \pm 5}{4} = -\frac{3}{2}$ ou 1

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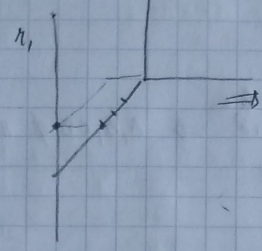
24) r: ? / $A \subset r$ e $v_r = v_{r_1} \times v_{r_2}$

a) $A(3,2,-1)$ $\quad v_{r_1} = (0,0,1)$ $\quad v_{r_2} = \begin{cases} x = x \\ x = y+3 \\ x = \frac{-z+3}{-2} \end{cases} \Rightarrow v_{r_2} = (1,1,-2)$



$$\Rightarrow v_r = \begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 1 & 1 \end{array}$$

$-1i \quad +1j$

$\boxed{v_r = (-1, 1, 0)}$

$$\Rightarrow r: \begin{cases} x = 3 - t \\ y = 2 + t \\ z = -1 \end{cases}$$

b) $A(0,0,0)$ $\quad v_{r_1} = (2,1,2)$ $\quad v_{r_2} = (3,-1,0)$

$$v_r = \begin{array}{ccc|cc} i & j & k & i & j \\ 2 & 1 & 2 & 2 & 1 \\ 3 & -1 & 0 & 3 & -1 \end{array}$$

$-3k + 2i \quad +6j - 2k$

$v_r = (2, 6, -5)$

$$\Rightarrow r: \begin{cases} x = 2t \\ y = 6t \\ z = -5t \end{cases}$$

c) $A \in r_1, r_2$

$$r_1: \begin{cases} t = x - 2 \Rightarrow x = 2 + t \\ t = \frac{y+1}{2} \Rightarrow y = -1 + 2t \\ t = \frac{z}{3} \Rightarrow z = 3t \end{cases}$$

$v_{r_1} = (1, 2, 3)$

$$r_2 \begin{cases} y = -x - 1 \\ y = y \\ y = \frac{z-2}{2} \end{cases} \qquad v_{r_2}(-1,1,2)$$

$\boxed{B, p/\ y = 1 = (-2, 1, -\frac{1}{2})}$

$$\Rightarrow r_2 \begin{cases} x = -2 - t \\ y = 1 + t \\ z = -\frac{1}{2} + 2t \end{cases}$$

$x = x \Rightarrow 2 + t = -2 - t \Rightarrow 2t = -4$

$\Rightarrow \boxed{t = -2}$

$\Rightarrow A = (0, -5, -6)$

$v_r = v_{r_1} \times v_{r_2} = \quad i$

$$\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & 2 & 3 & 1 & 2 \\ -1 & 1 & 2 & -1 & 1 \end{array}$$

$+2k - 3i - 2j \quad +4i - 3j + 1k$

$= (1, -5, 3)$

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24) a) $v_{r_1} = (0,0,1)$ $\quad v_{r_2}$: $x = x$, $x = y + 3$, $x = \frac{z-3}{2}$ $\Rightarrow v_{r_2}(1,1,-2)$

$v_r = v_{r_1} \times v_{r_2}$ =

i	j	k	i	j
0	0	1	0	0
1	1	-2	1	1

$-1\,i$ $\quad +1\,j$

$v_r = (-1,1,0)$

$\Rightarrow r = \begin{cases} x = 3 - t \\ y = 2 + t \\ z = -1 \end{cases}$

b) $v_{r_1}(2,1,2)$, $v_{r_2}(3,-1,0)$

i	j	k	i	j
2	1	2	2	1
3	-1	0	3	-1

$-3k + 2i \quad +6j - 2k$

$v_r (2,6,-5)$

$A (0,0,0) \Rightarrow r \{$

c) $A = r_1 \cap r_2 = ?$

$r_1: \begin{cases} t = x - 2 \Rightarrow x = 2 + t \\ t = y + 1/2 \Rightarrow y = -1 + 2t \\ t = z/3 \Rightarrow z = 3t \end{cases}$ $\quad \boxed{v_{r_1} = (1,2,3)}$ $\quad \boxed{P_{a_1} = (2,-1,0)}$

r_2: $y = x - 1 \,/ \cdot (-1)$ $\Rightarrow r_2 \begin{cases} t = \frac{x-1}{-1} \Rightarrow x = 1 - t \\ t = y \quad y = t \\ t = \frac{z-2}{2} \quad z = 2 + 2t \end{cases}$ $\quad \boxed{v_{r_2}(-1,1,2)}$

$y = y$

$y = z - 2/2$

$\Rightarrow v_{r_2}(-1,1,2)$

$A = r_1 \cap r_2 \Rightarrow \begin{cases} 2 + t = 1 - h \\ -1 + 2t = h \\ 3t = 2 + 2h \\ \text{3.0} \quad 0 = 2 + 2(-1) \\ 0 = 0 \end{cases}$ $\quad \begin{cases} t + h = -1 \\ 2t - h = 1 \end{cases}$ $\quad 3t = 0 \Rightarrow \boxed{t = 0}$ $\quad \Rightarrow 0 + \boxed{h = -1}$

$\Leftarrow \boxed{r_1 \cap r_2}$

$\Rightarrow A = r_1, t=0 = \boxed{(2,-1,0)}$ $\quad v_r = v_{r_1} \times v_{r_2} \Rightarrow$

i	j	k	i	j
1	2	3	1	2
-1	1	2	-1	1

$+2k - 3i - 2j + 4i - 3j + 1k$

$\Leftarrow \boxed{v_r = (1,-5,3)}$

$r: \{ \ldots$

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WINTERLE, p. 122

25) a) $x = x$

$2x - 3 = -3x + 7 \Rightarrow 5x = 10 \Rightarrow x = 2$

$-x + 5 = x + 1 \Rightarrow -2x = -4 \Rightarrow x = 2$ > ok, I E / $I = r_1 \cap r_2$

$I = (x, y, z) = (2, 2 \cdot (2) - 3, -(2) + 5) = (2, 1, 3)$

b) $r_1 \begin{cases} h = x - 3/2 \\ h = y + 1 / -3 \\ h = z - 2/4 \end{cases} \Rightarrow \begin{aligned} x &= 3 + 2h \\ y &= -1 - 3h \\ z &= 2 + 4h \end{aligned}$

$r_1 \cap r_2 = \begin{aligned} -1 + t &= 3 + 2h \Rightarrow t - 2h = 4 \\ 4 - t &= -1 - 3h \Rightarrow -t + 3h = -5 \\ -8 + 3t &= 2 + 4h \end{aligned}$

$h = -1 \rightarrow t - 2(-1) = 4 \Rightarrow t = 2$

$3t - 4h = 10$

$\Rightarrow 6 - 4(-1) = 10$ ✓ I E

$\Rightarrow I(1, 2, -2)$

c) $r_1 \begin{cases} y = 2x - 3 \\ z = -x - 10 \end{cases}$ $r_2 \begin{cases} y = 3x + 4 \\ z = -2x - 1 \end{cases}$

$I = ? \; / \; I = r_1 \cap r_2 \Rightarrow x = x$

$y = 2x - 3 = 3x + 4 \Rightarrow -x = 7 \Rightarrow x = -7$

$z = -x - 10 = -2x - 1 \Rightarrow x = 9$

$x_y \neq x_z \Rightarrow I \nexists$

d) $\begin{aligned} 2 - t &= -3 + 6h \\ 3 - 5t &= 1 + 7h \\ 6 - 6t &= -1 + 13h \end{aligned}$

$\hookrightarrow -6t - 13h = 7$

$6 - 13 = 7$ ✓ I E

$\Rightarrow (\cdot(-5)) \Rightarrow \begin{aligned} 5t + 30h &= +25 \\ -5t - 7h &= -21 \end{aligned}$

$23h = 23 \Rightarrow h = 1 \Rightarrow t = -1$

$I(3, 8, 12)$

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25 e) $r_1: \begin{cases} x = 2 + t \\ y = 4 - 2t \\ z = 1 + 3t \end{cases}$ $r_2: \begin{cases} x = -1 + 4h \\ y = 2 + 3h \\ z = 5 - 2h \end{cases}$ $\Rightarrow 2 + t = -1 + 4h \Rightarrow t - 4h = -3$ $\Rightarrow 4 - 2t = 2 + 3h \Rightarrow -2t - 3h = -2$

$2t - 8h = -6$

$-9h = -8$

$\Rightarrow \boxed{h = 8/9}$

$t - 4\left(\frac{8}{9}\right) = -3$

$t = -3 + \frac{32}{9}$

$\boxed{t = \frac{-27 + 32}{9} = \frac{5}{9}}$

(z): $1 + 3t = 5 - 2h$

$1 + 3\left(\frac{5}{9}\right) = 5 - 2\left(\frac{8}{9}\right)$

$1 + \frac{5}{3} = 5 - \frac{16}{9}$

$\frac{3+5}{3} = \frac{45 - 16}{9}$

$\frac{8}{3} = \frac{29}{9}$ ← não verifica a coordenada z $\Rightarrow$ sem interseção

25 f) $r_2: x = -y + 6 = -z + 2$

$\Rightarrow t = x$

$t = -y + 6$

$t = -z + 2$

$\Rightarrow r_2: \begin{cases} x = t \\ y = +6 - t \\ z = +2 - t \end{cases}$ $r_1: \begin{cases} x = 2 + h \Rightarrow \\ y = 4 - h \Rightarrow \\ z = -h \Rightarrow \end{cases}$

(x): $t = 2 + h \Rightarrow t - h = 2$

(y): $6 - t = 4 - h \Rightarrow -t + h = -2$

(z): $2 - t = -h \Rightarrow -t + h = 2$

$r_1 = (x, y, z) = \boxed{(2, 4, 0)} + h(1, -1, -1)$

$r_2 = (x, y, z) = (0, 6, 2) + t(1, -1, -1)$

existe em r_2 também?

sim: $\boxed{h = 0}$

$(h = 0) \Rightarrow x = 2$

$y = 4$

$z = 0$

Portanto, as retas são coincidentes !!

26 a) $r_1 \cap r_2 \Rightarrow m = ?$

$r_1 = x = x$

$y = 2x - 5 \Rightarrow \boxed{\frac{y+5}{2} = x}$

$z = -x + 2 \Rightarrow \boxed{-z + 2 = x}$

$\Rightarrow x = \frac{y+5}{2} = -z + 2 \Rightarrow r_1 = \begin{cases} x = t \\ \frac{y+5}{2} = -t \Rightarrow \boxed{y = -5 - 2t} \\ -z + 2 = t \Rightarrow \boxed{z = 2 - t} \end{cases}$

$r_2 = \begin{cases} t = x - 5 \Rightarrow x = 5 + t \\ t = y/m \Rightarrow y = mt \\ t = z + 1 \Rightarrow z = -1 + t \end{cases}$

$r_1 \cap r_2$: (x): $t = 5 + t \Rightarrow$

(y): $mt = -5 - 2t$

(z): $-1 + t = +2 - t \Rightarrow 2t = 3 \Rightarrow t = \frac{3}{2}$

$\frac{3}{2}m = -5 + 2\left(\frac{3}{2}\right) = -5 + 3 = -2$

$\Rightarrow m = -\frac{4}{3}$

266) $r_2: \begin{cases} \frac{x-1}{3} = t \Rightarrow x = 1+3t \\ \frac{y+2}{1} = t \Rightarrow y = -2+t \\ \frac{z}{-2} = t \Rightarrow z = -2t \end{cases}$

$\Rightarrow m-t = 1+3t$

$\Rightarrow 1+t = -2+t \Rightarrow$?

$\Rightarrow -2t = 2t \Rightarrow 4t = 0$

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1) $\pi = 3x + y - z - 4 = 0$

a) $P_\pi(1, 3, ?) \Rightarrow 3 \cdot 1 + 3 - z = 4 \Rightarrow \boxed{z = +2}$

b) $P_\pi(0, y, 2) \Rightarrow 0 + y - 2 - 4 = 0 \Rightarrow \boxed{y = 6}$

c) $k \mid P(k, 2, k-1) \in \pi$

$\Rightarrow 3k + 2 - k + 1 - 4 = 0 \Rightarrow 2k = 1 \Rightarrow \boxed{k = 1/2}$

d) $P = ? \mid P = (2, 2z, z)$

$\Rightarrow 6 + 2z - z - 4 = 0 \Rightarrow z = -2 \Rightarrow \boxed{P = (2, -4, -2)}$

e) $k = ? \mid \pi_1 = kx - 4y + 4z - 7 = 0 \parallel \pi = 3x + y - z - 4 = 0$

$n_1 = (k, -4, 4)$ $\qquad n = (3, 1, -1)$

$k = \boxed{-12}$ $\qquad n' = -4 \, (n_1)$

2) eq. geral de π_1: ? $\mid \pi_1 \parallel \pi \Rightarrow \vec{n}_1 = k\,\vec{n}$

$\pi_1 \in A(4,-2,1)$

$\pi \in \vec{n}(2,-3,-1) \Rightarrow \pi_1 \in n_1(2,-3,-1)$

$\Rightarrow \pi_1 = 2(4) - 3(-2) - 1(1) + d = 0 \Rightarrow \boxed{d = -13}$

$\Rightarrow$ eq geral de $\pi_1 = 2x - 3y - z - 13 = 0$

3) eq. g. π = ? $\mid \pi \perp r$ e $\pi \in A(-1,2,3)$

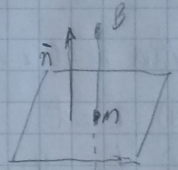
$\Rightarrow \vec{n} = k\vec{v}$

$\vec{v} = (2,-3,4) = \vec{n}$

$\Rightarrow 2(-1) - 3(2) + 4(3) + d = 0 \Rightarrow d = -4$

$\Rightarrow$ eq. g $\pi = 2x - 3y + 4z - 4 = 0$

4) eq. g. π = ? $\mid \pi \in \frac{A+B}{2}$, $\vec{n} = k \cdot \vec{AB}$



$\frac{A+B}{2} = \boxed{(2,-4,5/2)} = M$?

$\vec{AB} = B - A = \boxed{(-6,-6,-3)} \Rightarrow \boxed{(-3,-3,-1)}$

$\Rightarrow$ eq. g. $\pi = 2(-3) - 4(-3) + \frac{5}{2}(-1) + d = 0 = -6 + 12 - 5/2 =$

$\Rightarrow d = -6 + \frac{5}{2} = \frac{-12+5}{2} = -\frac{7}{2}$

5) eq. paramétricas $_{\pi}$ = ? $\mid \pi = 3x - 2y - z - 6 = 0$

a) determinar 3 pontos A, B, C:

para $x = y = 0 \Rightarrow -z - 6 = 0 \Rightarrow z = -6$ $\quad A(0,0,-6)$

para $x = 1$, $y = 0 \Rightarrow 3 - z - 6 = 0 \Rightarrow +z = -3$ $\quad B(1,0,-3)$

para $x = 0$, $y = 1 \Rightarrow -2 - z - 6 = z = -8$ $\quad C(0,1,-8)$

$\Rightarrow AB = B - A = (1,0,3)$

$AC = C - A = (0,1,-2)$

$\Rightarrow$ eq vet $\pi = (x,y,z) = (0,0,-6) + h(1,0,3) + t(0,1,-2)$

$\Rightarrow \boxed{\pi \begin{cases} x = h \\ y = t \\ z = -6 + 3h - 2t \end{cases}}$

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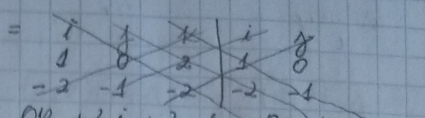
6) $A(1,1,4)$

$u(1,0,2)$, $v(-2,-1,-2)$ $\Rightarrow \vec{n} = u \times v =$



$0i + 2j + 2k + 4i + 0j - 1k - 2k - 4j + 1k$

$\vec{n} = (2, -2, -1)$

$\Rightarrow 2x - 2y - z + d = 0$

$\Rightarrow 2(1) - 2(1) - 1(4) + d = 0 \Rightarrow \boxed{d = 4}$

$\Rightarrow$ eq. g. π $\boxed{2x - 2y - z + 4 = 0}$

Eq. g. e paramétricas de π = ?

7) $AB(-3, 2, -3)$

$AC(0, 1, -3)$

$0i + 3j - 6k - 6i + 0j - 2k$

$AB \times AC = (-3, -6, -3)$

$\pi: \begin{cases} x = 1 - 2h \\ y = 2h + t \\ z = 2 - 3h - 3t \end{cases}$

$\Rightarrow -3(1) - 6(0) - 3(2) + d = 0 = -3 - 4 + d \Rightarrow \boxed{d = 7}$

8) $AB(1, 1, 5)$

$AC(-1, 1, 2)$

$+1k - 5i - j + 1i - 5j + 1k$

$= (-4, -6, 2)$

$A(0,0,0)$

$\pi: \begin{cases} x = h - t \\ y = h + t \\ z = 5h + t \end{cases} = -4x - 6y + 2z = 0 = (2x + 3y - z = 0)$

9) $AB(-4, 6, 4)$

$AC(-2, 3, 5)$

$+12k - 12i + 20j + 30i - 8j - 12k$

$AB \times AC = (18, 12, 0)$

$A(2, 0, -1)$

$\pi: \begin{cases} x = 2 - 4h - 2t \\ y = 6h + 3t \\ z = -1 + 4h + 5t \end{cases} = 3x + 2y + d = 0$

$3(2) + 2(0) + d = 0 \Rightarrow \boxed{d = -6}$

10) $AB(-6, -3, -1)$

$AC(-2, -1, 1)$

$AB \times AC = -6k - 1i + 6j - 3i + 2j + 6k$

$= (-4, 8, 0)$

$A(2, 1, 0)$

$\pi: \begin{cases} x = 2 + 6h - 2t = \boxed{-4x - 8y + d = 0} \\ y = 1 - 3h - t \\ z = -h - t = x - 2y = 0 \end{cases}$

$\Rightarrow -4(2) + 8(1) + d = 0 \Rightarrow \boxed{d = 0}$

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11) $AB\ (-5,-2,0)$ $\quad \begin{array}{ccc|cc} i & j & k & i & j \\ -5 & -2 & 0 & -5 & -2 \\ 2 & 1 & 0 & 2 & 1 \end{array}$ $\quad \pi: \begin{cases} x = 2 - 5h + 2t \\ y = 1 - 2h + t \\ z = 3 \end{cases} = -z + 3 = 0$

$AC\ (2,1,0)$

$4k + 0i + 0j + 0i + 0j - 5k$ $\quad = \boxed{z - 3 = 0}$

$n = (0,0,-1)$

$3(-1) + d = 0 \Rightarrow \boxed{d = 3}$

12) $\alpha = ?$ / $(AB, AC, AD) = 0$

$AB = B - A = 2-\alpha \quad 2 \quad -5 \quad | \quad 2-\alpha \quad 2$

$AC = C - A = -4-\alpha \quad -2 \quad -3 \quad | \quad -4-\alpha \quad -2$

$-\alpha \quad 1 \quad -5 \quad | \quad -\alpha \quad 1$

$+10\alpha + 6 - 3\alpha - 40 - 10\alpha + 20 - 10\alpha + 6\alpha + 20 + 5\alpha = 0$

$-2\alpha + 6 = 0 \Rightarrow \alpha = 3$

13) – 18) eq. geral do π: ?

13) $\begin{array}{c|ccc|cc} & i & j & k & i & j \\ u & 1 & -1 & 1 & 1 & -1 \\ v & 2 & 3 & 0 & 2 & 3 \end{array}$ $\quad$ eq. geral π: $\boxed{-3x + 2y + 5z + 16 = 0}$

$+2k - 3i + 0j + 0i + 2j + 3k = \bar{n} = (-3, 2, 5)$

$A(2,0,-2)$

$\Rightarrow -3(2) + 2(0) + 5(-2) + d = 0 \Rightarrow \boxed{d = 16}$

14) $V_r = (2,0,-3)$

$\overline{AB}(2,1,3)$

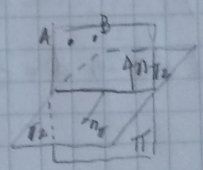
$n = V_r \times \overline{AB} = \begin{array}{ccc|cc} i & j & k & i & j \\ 2 & 0 & -3 & 2 & 0 \\ 2 & 1 & 3 & 2 & 1 \end{array}$ $\quad$ eq. geral π: $\boxed{3x - 12y + 2z + 25 = 0}$

$0k + 3i - 6j + 0i - 6j + 2k$

$\bar{n}(3, -12, 2)$

$A(-3, 1, -2) \Rightarrow 3(-3) - 12(1) + 2(-2) + d = 0 \Rightarrow \boxed{d = 25}$

15) $\pi \in \overline{AB}$ e n_{π_2} e A



$AB(-4, 3, -4)$

$\bar{n}_{\pi_2}(2, 1, -1)$

$\bar{n}_\pi = AB \times n_{\pi_2} = \begin{array}{ccc|cc} i & j & k & i & j \\ -4 & 3 & -4 & -4 & 3 \\ 2 & 1 & -1 & 2 & 1 \end{array}$

$-6k + 4i - 4j \quad | \quad -3i - 8j - 4k$

$\bar{n}_\pi(1, -12, -10) \longrightarrow x - 12y - 10z - 5 = 0$

$A(1, -2, 2)$

$\rightarrow 1(1) - 12(-2) - 10(2) + d = 0 \Rightarrow \boxed{d = -5}$

16) $\pi \in AB, k, A$

$\bar{n}_\pi = AB \times k$

$A(2,1,2)$

$k(0,0,1)$

$\overline{AB}(-1,-2,2)$

$\bar{n}_\pi = \begin{matrix} i & j & k & i & j \\ 0 & 0 & 1 & 0 & 1 \\ -1 & -2 & 2 & -1 & -2 \end{matrix}$

$+0k + 2i + 0j + 0i - 1j + 0k$

$\bar{n}_\pi = (2, -1, 0)$

$A(2,1,2)$

$\Rightarrow 2(2) - 1(1) + 1(0) + d = 0 \Rightarrow \boxed{d = -3}$

$\Rightarrow$ eq. g. $\pi = 2x - y - 3 = 0$

17) $\pi \in V_k, \bar{n}_{\pi_1}, A$

$A \in r \mid t = 1 \Rightarrow A(3,0,5)$

$n_{\pi_1} = (2, 2, -3)$

$V_k (1, -1, +2)$

$n_\pi = V_k \times \bar{n}_{\pi_1}$

$\begin{matrix} i & j & k & i & j \\ 1 & 1 & 2 & 1 & -1 \\ 2 & 2 & -3 & 2 & 2 \end{matrix}$

$+2k - 4i + 3j \quad +3i + 4j + 2k$

$\bar{n}_\pi = (-1, 7, 4)$

$A = (3, 0, 5)$

$\Rightarrow -1(3) + 7(0) + 4(5) + d = 0 \Rightarrow \boxed{d = -17}$

$\boxed{\pi: +x - 7y - 4z + 17 = 0}$

18) $\pi \in A(4,1,1), \bar{n}_{\pi_1}, n_{\pi_2}$

$n_{\pi_1} = (2, 1, -3)$

$n_{\pi_2} = (1, 1, -2)$ $\Big\rangle \bar{n}_{\pi_1} \perp \bar{n}_{\pi_2}$

$\bar{n}_\pi = n_{\pi_1} \times n_{\pi_2}$

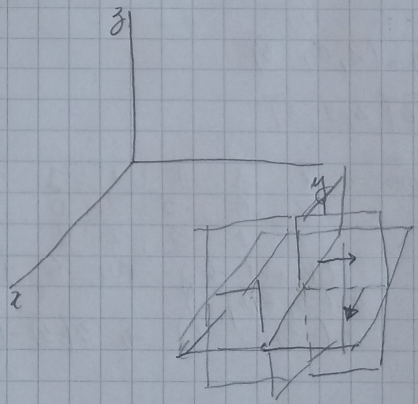
$\begin{matrix} i & j & k & i & j \\ 2 & 1 & -3 & 2 & 1 \\ 1 & 1 & -2 & 1 & 1 \end{matrix}$

$-1k + 3i + 4j - 2i - 3j + 2k$

$\bar{n}_\pi = (1, 1, 1)$

$A = (4, 1, 1)$

$\Rightarrow 4 + 1 + 1 + d = 0 \Rightarrow d = -6 \Rightarrow$ eq. g $\pi: \boxed{x + y + z - 6 = 0}$



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16) $\pi \in AB,\ k,\ A$

$\bar{n}_\pi = AB \times k$

$A(2,1,2)$

$k(0,0,1)$

$\overline{AB}(-1,-2,2)$

$\bar{n}_\pi = (2,-1,0)$

$A(2,1,2)$

$\Rightarrow 2(2) - 1(1) + 1(0) + d = 0 \Rightarrow \boxed{d = -3}$

$\Rightarrow$ eq. g. $\pi = 2x - y - 3 = 0$

17) $\pi \in V_k,\ \bar{n}_{\pi_1},\ A$

$A \in r \mid t = 1 \Rightarrow A(3,0,5)$

$n_{\pi_1} = (2,2,-3)$

$V_k(1,-1,+2)$

$n_\pi = V_k \times \bar{n}_{\pi_1}$

$+2k - 4i + 3j \qquad +3i + 4j + 2k$

$\bar{n}_\pi = (-1,7,4)$

$A = (3,0,5)$

$\Rightarrow -1(3) + 7(0) + 4(5) + d = 0 \Rightarrow \boxed{d = -17}$

$\boxed{\pi: +x - 7y - 4z + 17 = 0}$

18) $\pi \in A(4,1,1),\ \bar{n}_{\pi_1},\ n_{\pi_2}$

$n_{\pi_1} = (2,1,-3)$

$n_{\pi_2} = (1,1,-2) \Rightarrow \bar{n}_{\pi_1} \perp \bar{n}_{\pi_2}$

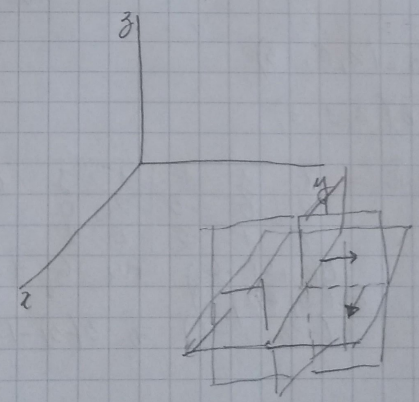
$\bar{n}_\pi = n_{\pi_1} \times n_{\pi_2}$

$-1k + 3i + 4j - 2i - 3j + 2k$

$\bar{n}_\pi = (1,1,1)$

$A = (4,1,1)$

$\Rightarrow 4 + 1 + 1 + d = 0 \Rightarrow d = -6 \Rightarrow$ eq. g $\pi: \boxed{x + y + z - 6 = 0}$



19-22: $r_1 \parallel r_2$ ou $r_1 \times r_2$

19) $r_2 \in A,C \mid x_A = 1, x_C = 2$

$r_2 \subset B \mid$

$\pi \in A, B, V_{r_2}$

$A(1,-1,1)$

$C(2,1,0)$

$\overrightarrow{AC}(1,2,-1)$

$r_2 = \begin{cases} t = \frac{x-1}{3} \Rightarrow x = 1+3t \\ y = -1 \\ t = \frac{z-1}{-1} \Rightarrow z = 1-t \end{cases}$

$\Rightarrow B(1,-1,1)$

$V_{r_2}(3,0,-1)$

$n_\pi = AC \times V_{r_2}$

$\begin{matrix} i & j & k & i & j \\ 1 & 2 & -1 & 1 & 2 \\ 3 & 0 & -1 & 3 & 0 \end{matrix}$

$-6k + 0i + 1j - 2i - 3j + 0k$

$\vec{n}_\pi = (-2,-2,-6)$

$B = (1,-1,1)$

$\Rightarrow -2(1) - 2(-1) - 6(1) + d = 0 \Rightarrow d = 6$

eq. π: $-2x - 2y - 6z + 6 = 0$

$= x + y + 3z - 3 = 0$

20) $A(1,-2,3)$

$\vec{n} = \begin{matrix} i & j & k & i & j \\ 3 & 5 & -1 & 3 & 5 \\ -2 & -1 & 2 & -2 & -1 \end{matrix}$

$+6k - 1i - 4j + 6i + 2j - 2k$

$= (5,-3,4)$

$\Rightarrow 5(1) - 2(-2) + 4(3) + d = 0 \Rightarrow d = -21$

$\Rightarrow \pi: 5x - 2y + 4y - 21 = 0$

21) $A(-2,0,-3) \in \pi$

$V_{r_1} = (1,-1,0)$

$V_{r_2} = (1,-1,0)$

$V_{r_2}: \begin{cases} x = x/1 \\ x = \frac{+y+1}{-1} \\ z = 0x + 3 \end{cases} \Rightarrow B = (1,-2,3)$

$\Rightarrow r_1 \parallel r_2$

$B \in r_2 \mid x = 1 \Rightarrow B(1,-2,3)$

$\pi \subset V_{r_1}, \overrightarrow{AB}, A$

$\overrightarrow{AB}(3,-2,6)$

$\Rightarrow \vec{n} = V_{r_1} \times \overrightarrow{AB}$

$\begin{matrix} i & j & k & i & j \\ 1 & -1 & 0 & 1 & -1 \\ 3 & -2 & 6 & 3 & -2 \end{matrix}$

$+3k + 0i - 6j - 6i + 0j - 2k$

$\vec{n}(-5,-6,1)$

$\Rightarrow -5(-2) - 6(0) + 1(-3) + d = 0 \Rightarrow d = -7$

$\Rightarrow \pi = 5x + 6y - z + 7 = 0$ X

22) $\pi_1 \begin{cases} x = x \\ z = x \\ y = -3 \end{cases} \Rightarrow x = z$

$V_{\pi_1}(1, 0, 1)$

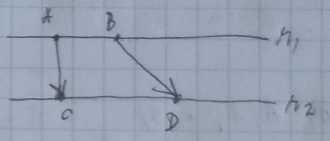
$V_{\pi_2}(-1, 0, -1)$

$C(0, 1, 8)$

$\bar{n}_{\pi} = V_{\pi_1} \times V_{\pi_2} = \begin{matrix} i & j & k & i & j \\ 1 & 0 & 1 & -1 & 0 \\ -1 & 0 & -1 & -1 & 0 \end{matrix}$

$Ac: A_{\pi_1}(1, -3, 1) \; p \; x = 1$ $\quad C_{\pi_2}(0, 1, 2)$

$B_{\pi_1}(2, -3, 2) \; p \; x = 2$ $\quad D_{\pi_2}(-1, 1, 1) \quad A = 1)$



$\bar{n} = AC \times BD$

$AC = (-1, 4, 1)$

$BD = (-3, 4, -1)$

$\Rightarrow \bar{n} = \begin{matrix} i & j & k & i & j \\ -1 & 4 & 1 & -1 & 4 \\ -3 & 4 & -1 & -3 & 4 \end{matrix}$

$+4k - 4i - 1j \quad -4i - 3j - 4k$

$= (-8, -4, 8)$

$= (-2, -1, 2)$

$C(0, 1, 2)$

$\Rightarrow -2(0) - 1(1) + 2(2) + d = 0 \Rightarrow d = -3$

$\Rightarrow \boxed{2x + y - 2z + 3 = 0}$

23) $A(4, 3, 2)$ $\quad B_{\pi}(0, 2, 3)$

$V_n(1, -1, 2)$

$\Rightarrow AB_{\pi}(-4, -1, 1)$

$\bar{n} = AB_{\pi} \times V_n = \begin{matrix} i & j & k & i & j \\ -4 & -1 & 1 & -4 & -1 \\ 1 & -1 & 2 & 1 & -1 \end{matrix}$

$+1k + 1i + 8j - 2i + 1j + 4k$

$\bar{n} = (-1, 9, 5)$

etc ...

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23) $\pi \subset A, r$ $\qquad \pi = ?$

$r \subset BC$

$B_{t_0} = (0,2,3)$

$C_{t_1} = (1,1,5)$

$A(4,3,2) \Rightarrow AB(-4,-1,1)$

$AC(-3,-2,-3)$

$$\bar{n}_\pi = AB \times AC = \begin{array}{ccc|cc} i & j & k & i & j \\ -4 & -1 & 1 & -4 & -1 \\ -3 & -2 & -3 & -3 & -2 \end{array}$$

$-3k + 2i - 12j + 3i - 3j = 8k$

$\bar{n} = (5,-15,5)$

$\pi: 20 - 45 + 10 + d = 0 \Rightarrow \boxed{d = 15}$

$\Rightarrow \pi: 5x - 15y + 5z + 15 = 0$

$\pi: x - 3y + z + 3 = 0$?

24) $A(1,-1,2)$ $\qquad r = (0,0,z) \Rightarrow r \subset B, C$

$B = (0,0,1)$

$C = (0,0,2)$

$\pi \subset A, r$

$\Rightarrow \pi \subset AB, AC$

$AB(-1,1,-1)$

$AC(-1,1,0)$

$$AB \times AC = n_\pi = \begin{array}{ccc|cc} i & j & k & i & j \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & 0 & -1 & 1 \end{array}$$

$+1k + 1i \qquad +1j - 1k$

$\pi \subset (0,0,z) \Rightarrow d = 0$ $\qquad \bar{n}_\pi = (1,1,0)$

$\boxed{\pi \Rightarrow x + y = 0}$

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25) equação geral: $\begin{cases} \pi \\ A \end{cases} = \boxed{ax+by+cz+d=0}$

$r: \begin{cases} B(0,2,3) \\ V_r(1,-1,2) \end{cases}$ $\Rightarrow AB \in \pi$, $V_r \in \pi$

$AB = B - A = (-4,-1,1)$

$A(4,3,2)$ $\Rightarrow \vec{n} = V_r \times AB \Rightarrow$

$-4k + 2i - 1j - 1i - 8j - 1k$

$\Rightarrow n(1,-9,-5)$

$\Rightarrow \boxed{\pi: 1x - 0y - 5z + d = 0}$

$4 - 27 - 100 + d = 0 \Rightarrow d = 33$

24) $\boxed{O_z(0,0,1) = V_r}$ $B \in O_z$ $A, B, O_z \in \pi$

$B(0,0,1)$

$A(1,-1,2)$

$AB \in \pi$, $\boxed{AB = B - A = (-1,1,-1)}$ $n = V_r \times AB =$

$\vec{n} \Rightarrow -1i - 1j$

$(-1,-1,0)$

$\Rightarrow \pi: \begin{cases} \vec{n} \\ A \end{cases} \Rightarrow -1x - 1y + d = 0$

$-1(1) - 1(-1) + d = 0$

$d = 0 \Rightarrow \boxed{\pi: -x - y = 0}$

27) $V_r(0,0,1)$ $\boxed{\pi \in V_r, AB, A}$

$AB = B - A = (2,-3,-6)$

$A(0,3,4)$ $\Rightarrow \vec{n} = V_r \times AB$

$+3i \quad +2j \Rightarrow \vec{n}(3,2,0)$

$\Rightarrow 3.(0) + 2(3) + d = 0 \Rightarrow \boxed{d = -6}$

$\Rightarrow \boxed{\pi: 3x + 2y - 6 = 0}$

26) $V_r(1,0,0)$ $V_r, AB, A \in \pi$

$\pi: \begin{cases} A \\ AB \times V_r = \vec{n} \end{cases}$

$AB = B - A = (2,-2,-1) \Rightarrow V_r \times AB =$

$A(-2,0,2)$

$+1j \quad -2k \Rightarrow \vec{n} = (0,1,-2)$

$\pi: 0(-2) + 1(0) - 2(2) + d = 0$

$\Rightarrow \boxed{d = 4}$

$\Rightarrow \boxed{\pi: y - 2z + 4 = 0}$

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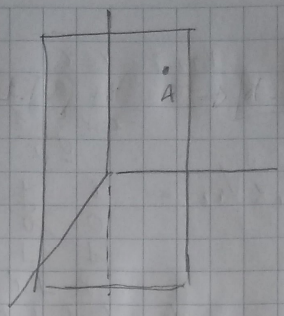
24) $\pi = ?$

$$\begin{cases} A(1,-1,2) \\ \pi = ax + bx + 0 = 0 \end{cases}$$

$\pi \subset O(0,0,0)$

$\rightarrow a \cdot 1 + b \cdot (-1) = 0$

para $a = 1$ $\boxed{b = 1}$



$$\boxed{\begin{array}{l} \bar{n}(x,y,0) \\ \pi \subset Oz = \pi \subset (0,0,0) \end{array}} \rightarrow \bar{n}(a,b,0)$$

$\Rightarrow \boxed{x + y = 0} = \pi$

Alternativa

$A(1,-1,2)$

$$Oz = r = \begin{cases} x = 0 \\ y = 0 \\ z = t \end{cases} \Rightarrow B_{t=0} = (0,0,0)$$

$C_{t=1} = (0,0,1)$

$$\Rightarrow \bar{n} = AB \times AC = \begin{array}{ccc|cc} i & j & k & i & j \\ -1 & 1 & -2 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \end{array}$$

$AB(-1,+1,-2)$

$AC(-1,+1,-1)$

$+1k + 2i + 1j - 1i + 2j - 1k$

$\bar{n} = (1,3,0)$

27) $\pi \begin{cases} A(2,3,0) \\ \vec{n} \end{cases}$ $\quad V_1 = (0,1,0)$

$V_2 = \overline{AO} = O - A(-2,-1,-1)$

$n = V_1 \times V_2$

i	j	k	i	j
0	1	0	0	1
-2	-1	-1	-2	-1

$+2k \qquad +1i \Rightarrow \vec{n}(1,0,2)$

$\Rightarrow \pi: 1(2) + 0(3) + 2(0) + d = 0$

$\Rightarrow \boxed{d = -2}$ $\Rightarrow \boxed{\pi: x + 2z - 2 = 0}$

28) $\pi \parallel xOy \Rightarrow \vec{n}(0,0,1)$

$\pi \begin{cases} A(3,-2,3) \\ \vec{n} \end{cases} \Rightarrow \pi: 0(3) + 0(-2) + 1(3) + d = 0 \Rightarrow \boxed{d = -3}$

$\Rightarrow \pi: z - 3$

29) $\pi \perp Oy$, $A(3,4,-1)$ $\Rightarrow \vec{n}_\pi = (0,1,0)$

$\Rightarrow \pi: 0(3) + 1(4) + 0(-1) + d = 0 \Rightarrow \boxed{d = -4}$

$\Rightarrow \boxed{\pi: y = 4}$ ✓

30) $\pi \subset Ox \Rightarrow \pi \subset \vec{i}(1,0,0)$, $\pi \subset O(0,0,0)$

$\pi \subset A(1,-2,1)$

$\Rightarrow \pi \subset i, \overline{OA}$

$\Rightarrow \vec{n}_\pi = \vec{i} \times \overline{OA}$

$OA = A - O = (1,-2,1)$

$\vec{i} = (1,0,0)$

i	j	k	i	j
1	-2	1	1	-2
1	0	0	1	0

$+2k \qquad +1j$

$\Rightarrow \vec{n}(0,1,2)$

$\pi: \begin{cases} A \\ \vec{n} \end{cases} \Rightarrow 0(1) + 1(-2) + 2(1) + d = 0$

$\Rightarrow \boxed{d = 0}$ $\Rightarrow \boxed{\pi: y + 2z = 0}$ ✓

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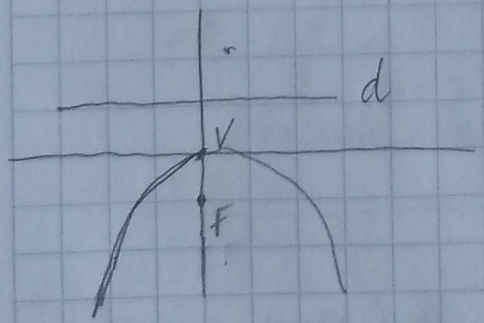
Winterle, p. 181 1-10: gráfico, F, d $\boxed{x^2 = 2py}$

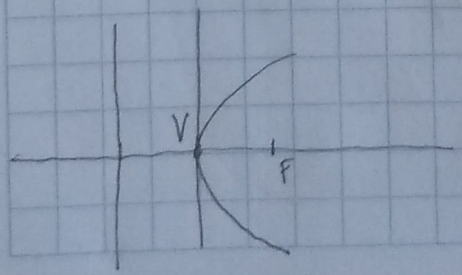
1) $2p = -4 \Rightarrow p = -2 \Rightarrow \boxed{p/2 = -1}$



$\boxed{F(0, -1)}$ $\boxed{d: y = 1}$

2) $2p = 6 \Rightarrow p = 3$ $\boxed{p/2 = 1,5}$ $\Rightarrow F\left(\frac{3}{2}, 0\right)$

$d: x = -\frac{3}{2}$



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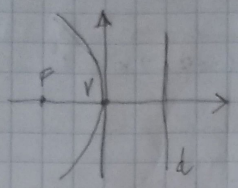
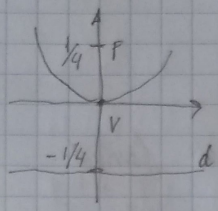
3) $2p = -8 \Rightarrow p = -4 \Rightarrow \boxed{p/2 = -2}$



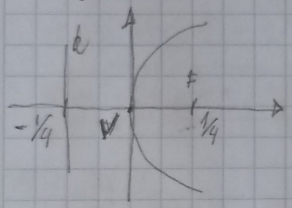
$\boxed{F(-2,0),\ d: x = 2}$

4) $x^2 = y \Rightarrow 2p = 1,\ p = 1/2,\ \boxed{p/2 = 1/4}$



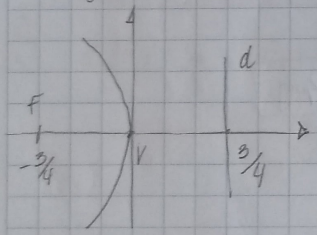
$F(0, 1/4),\ d: y = -1/4$

5) $y^2 = x \Rightarrow 2p = 1,\ p = 1/2,\ \boxed{p/2 = 1/4}$



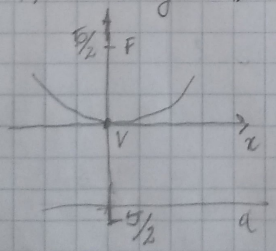
$\boxed{F(1/4, 0),\ d: x = -1/4}$

6) $y^2 = -3x \Rightarrow 2p = -3,\ p = -3/2,\ \boxed{p/2 = -3/4}$



$\boxed{F(-3/4, 0),\ d: x = 3/4}$

7) $x^2 = 10y \Rightarrow 2p = 10 \Rightarrow p = 5\ \boxed{p/2 = 5/2}$



$\boxed{F(0, 5/2)}$

$\boxed{d: y = -5/2}$

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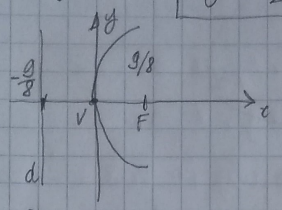
GA - Winterle, p. 181

8) $2y^2 = 9x \Rightarrow \boxed{y^2 = \frac{9}{2}x} \Rightarrow 2p = \frac{9}{2} \Rightarrow p = \frac{9}{4} \Rightarrow \boxed{p/2 = \frac{9}{8}}$

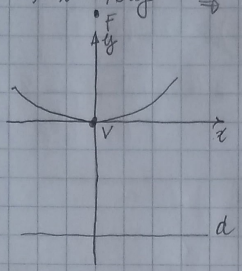
$\boxed{V(0,0)}$



$F(\frac{9}{8}, 0)$

$d: x = -\frac{9}{8}$

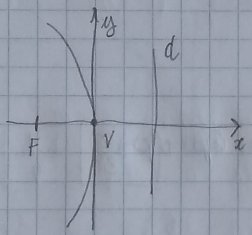
9) $x^2 = 16y \Rightarrow 2p = 16,\ p = 8,\ p/2 = 4$



$F(0, 4)$

$d: \boxed{y = -4}$

10) $y^2 = -8x \Rightarrow 2p = -8,\ p = -4, \boxed{p/2 = -2}$

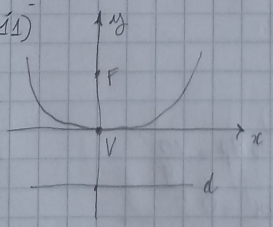


$F(-2, 0)$

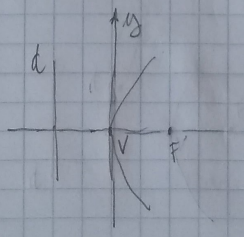
$d: x = 2$

$\boxed{2p = 8}$ $x^2 = 8y$

11)

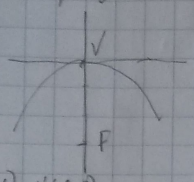


12)



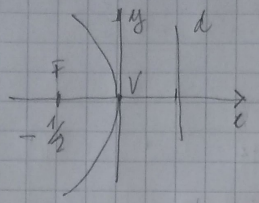
$y^2 = 8x$

13) V(0,0)
F(0,-3)



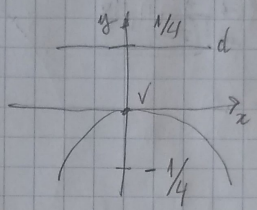
$p=6$ $\Rightarrow x^2=-12y$

14) V(0,0)
F(-1/2,0)



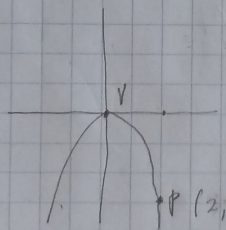
$y^2=-2x$

15) F(0,-1/4)
d: $y=1/4$



$x^2=-y$

16) V(0,0)



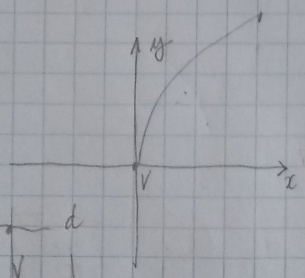
$\Rightarrow x^2=-2py$

$4=-2p\cdot(-3)=6p \Rightarrow p=\frac{2}{3}$ $-\frac{1}{4}$

$2p=\frac{1}{3}$

$\Rightarrow x^2=-\frac{1}{3}y$

17)

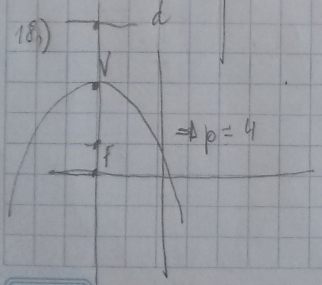


$y^2=2px$

$25=2p\,4 \Rightarrow 8p=25 \Rightarrow 2p=\frac{25}{4}$

$y^2=\frac{25}{4}x \Rightarrow 4y^2-25x=0$

18)



$\Rightarrow p=4$

$x^2=-2py$

$\Rightarrow x^2=-8y$

$(x+2)^2=-8(y-3)$

$\Rightarrow x^2+4x+8y-24+4=0$

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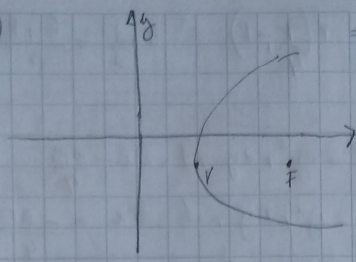
19)



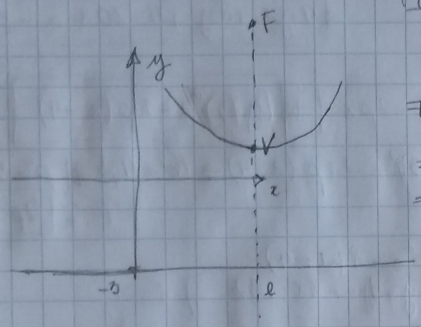
$= \boxed{y^2 = 2px}$

$\frac{p}{2} = 3 \Rightarrow p = 6 \quad \boxed{2p = 12}$

$\Rightarrow \boxed{(y+1)^2 = 12(x-2)}$

$= \boxed{y^2 + 2y - 12x + 1 + 24 = 0}$

20)

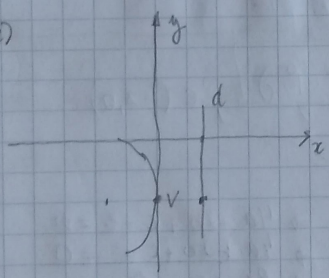


$\Rightarrow p = 8, \boxed{2p = 16}$

$\Rightarrow x^2 = 2py$

$\Rightarrow \boxed{(x-1)^2 = 16(y-1)} = x^2 - 8x - 16y + 16 + 16 = 0$

21)



$3x - 3 = 0 \Rightarrow x = \frac{3}{2}$

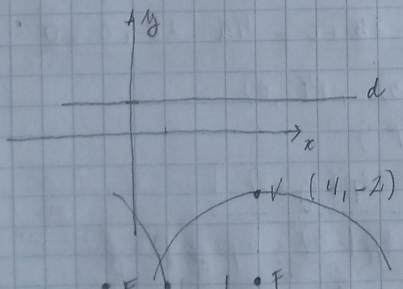
$\Rightarrow p = 3 \quad \boxed{2p = 6}$

$V(0, -2)$

$y^2 = 2px$

$(y+2)^2 = -6x \Rightarrow \boxed{y^2 + 4y + 6x + 4 = 0}$

22)



$\boxed{p = 6}$

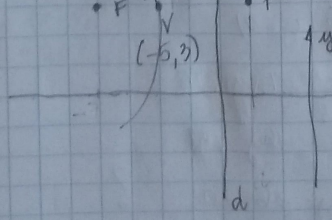
$\Rightarrow \boxed{x^2 = -2py}$

$(x-4)^2 = -12(y+2) \Rightarrow$

$\Rightarrow x^2 - 8x + 16 = -12y - 24$

$\Rightarrow x^2 - 8x + 12y + 40 = 0$

23)

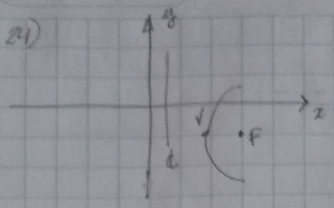


$\Rightarrow \boxed{y^2 = -2px} \quad \boxed{p = 4}$

$\boxed{(y-3)^2 = -8(x+5)}$

$\Rightarrow y^2 - 6y + 9 = -8x - 40$

24)



$p = \frac{5}{2} \Rightarrow \boxed{2p = 5}$ $\quad V\left(\frac{7}{4}, -1\right)$ $\quad 3 + \frac{1}{2} = \frac{6+1}{2} = \frac{7}{2}$

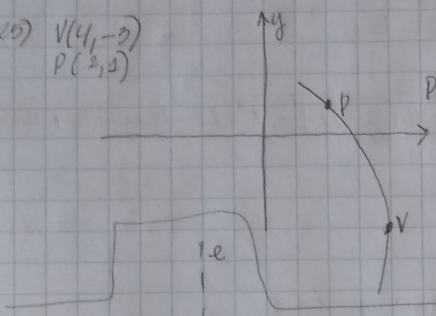
$\boxed{y^2 = 2px}$

$(y+1)^2 = 5\left(x - \frac{7}{4}\right) = y^2 + 2y + 1 = 5x - \frac{35}{4}$

$-y^2 + 2y + 1 - 5x = -\frac{35}{4}$

$\Rightarrow \boxed{4y^2 + 8y + 4 - 20x = -35}$

25) $V(4,-3)$
$P(2,1)$



$\Rightarrow \boxed{y^2 = -2px} \Rightarrow \boxed{(y+3)^2 = -2p(x-4)}$

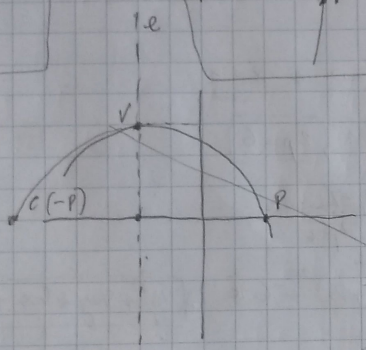
$P(2,1): (1+3)^2 = -2p(2-4) \Rightarrow 16 = 4p \Rightarrow \boxed{p = 4}$

$\boxed{2p = 8}$

$\Rightarrow (y+3)^2 = -8(x-4)$

$\Rightarrow \boxed{y^2 + 6y + 9 = -8x + 32}$

26)



$V(-2,3)$ $\quad \boxed{x^2 = -2py}$ $\quad C(-2,0)$

$D(2,0)$ $\quad \frac{}{4 = ?}$

$y = ax^2 + bx + c$

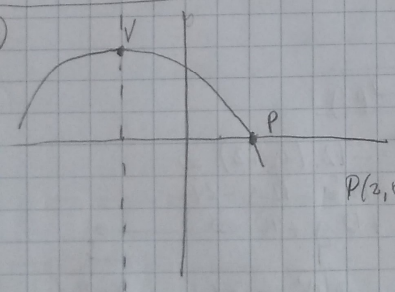
$\Rightarrow 3 = a \cdot 4 - b \cdot 2 + c \Rightarrow 4a - 2b + c - 3 = 3$

$\Rightarrow 0 = 4a + 2b + c \Rightarrow 4a + 2b + c = 0$

$\Rightarrow 8a + c = 3$

$\Rightarrow 0 = 4a - 2b + c = 4a - 2b + c = 0$

26)



$V(-2,3)$, $P(2,0)$

$\Rightarrow x^2 = -2py$

$\Rightarrow \boxed{(x+2)^2 = -2p(y-3)}$

$P(2,0) \Rightarrow 16 = -2p(-3) = 6p \Rightarrow p = \frac{8}{3}$

$\Rightarrow (x+2)^2 = -\frac{16}{3}(y-3)$

$\Rightarrow x^2 + 4x + 4 = -\frac{16y + 48}{3}$

$\Rightarrow 3x^2 + 12x + 12 = -16y + 48$

$\Rightarrow 3x^2 + 12x + 16y - 36 = 0$?

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Eq. reduzida V, F, d, e, gráfico

27) $x^2+4x+4 = -8y-12+4 = -8y+16$

$-8(y+2)$

$\rightarrow (x+2)^2 = -8(y+1)$ $\rightarrow V(-2,-1)$

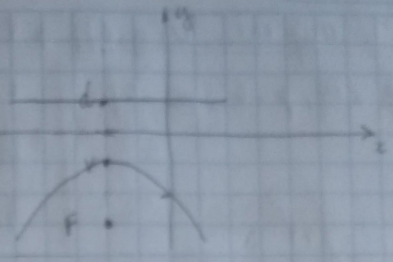
$d: x = -2$

$2p = -8 \rightarrow p = 4$

$p/2 = -2$

$F(-2,-3)$

$d: y = -1$



28) $x^2-2x+1 = 20y+21+1$

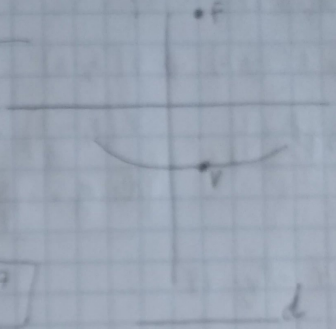
$(x-1)^2 = 20(y+2)$ $\rightarrow V(1,-2)$

$2p = 20, p = 10, p/2 = 5$

$F(1,3)$

$d: y = -7$

$e: x = 1$



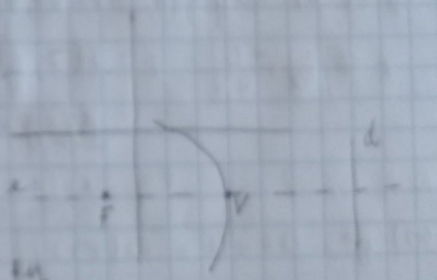
29) $y^2+4y+4 = -16x+44+4$

$(y+2)^2 = -16(x-3)$ $\rightarrow d(3,-2)$

$2p = -16, p = -8$ $p/2 = -4$

$F = (-1,-2)$

$d: x = 7$



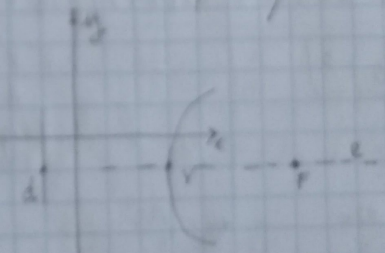
30) $y^2+2y+1 = 16x-48$

$(y+1)^2 = 16(x-3)$ $V(3,-1)$

$2p = 16, p = 8, p/2 = 4$ $F(7,-1)$

$d: x = -1$

$e: y = -1$



31) $4y = x^2-8x$ $\rightarrow x^2-8x+16 = 4y+4+16$

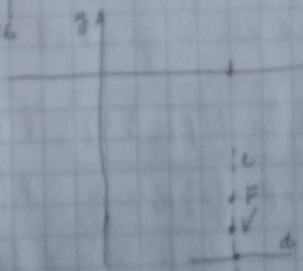
$(x-4)^2 = 4(y+5)$ $\rightarrow V(4,-5)$

$2p = 4, p = 2, p/2 = 1$

$F(4,-4)$

$d: y = -6$

$e: x = 4$



32) $x^2 = 12y - 72 = 12(y-6)$ $\Rightarrow V(0,6)$

$2p = 12,\ p = 6 \quad \boxed{p/2 = 3}$ $\quad F(0,9)$

$\boxed{d: y = 3}$

$\boxed{e: x = 0}$

33) $x^2 - 4x \overset{+4}{=} y - 2 + 4$

$\Rightarrow (x-2)^2 = 1(y+2) \Rightarrow V(2,-2)$

$2p = 1,\ p = \frac{1}{2} \quad \boxed{\frac{p}{2} = \frac{1}{4}}$ $\quad F\left(2, -\frac{7}{4}\right) \Rightarrow \boxed{e: x = 2}$

$\boxed{d: y = -\frac{9}{4}}$

$\frac{1}{4} - 2 = \frac{1-8}{4} = \frac{7}{4}$

34) $x^2 - 4x + 4 = y + 4$

$\Rightarrow (x-2)^2 = 1(y-4) \quad V = (2,4)$

$\Rightarrow \boxed{p/2 = \frac{1}{4}}$ $\quad F\left(2, \frac{17}{4}\right)$

$d: y = \frac{15}{4}$

$e: x = 2$

$4 + \frac{1}{4} = \frac{17}{4}$

35) $y^2 = 12x + 12 = 12(x+1) \Rightarrow V(-1,0)$

$\Rightarrow \boxed{p/2 = 3}$ $\quad F(2,0)$

$d: x = -4$

$e: y = 0$

36) $2x^2 - 12x = y - 14$

$\Rightarrow x^2 - 6x + 9 = \frac{y}{2} - 7 + 9$

$\Rightarrow \boxed{(x-3)^2 = \frac{1}{2}(y+4)} \quad V(3,-4)$

$2p = \frac{1}{2},\ p = \frac{1}{4} \quad \boxed{p/2 = \frac{1}{8}}$ $\quad F\left(3, -\frac{31}{8}\right)$

$d: y = -\frac{33}{8}$

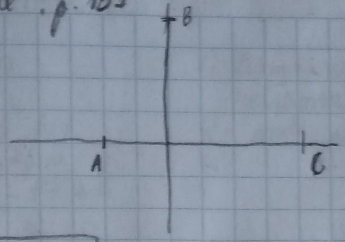
$e: x = 3$

$\frac{1}{8} - 4 = \frac{1-32}{8} = -\frac{31}{8}$

GA - ESTUDO, Winterle . p. 131

37) $x^2 = 2py$

$y = ax^2 + bx + c$



(A) $\Rightarrow 0 = 4a - 2b + c$

(B) $\Rightarrow 4 = 0 - 0 + c \Rightarrow \boxed{c = 4}$

(C) $\Rightarrow 0 = 16a + 4b + 4$

$\Rightarrow 4b = -16a - 4$

$\Rightarrow b = -4a - 1$

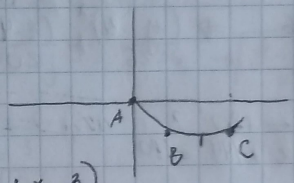
$\Rightarrow 0 = 4a - 2(-4a - 1) + 4$

$\Rightarrow 0 = 12a + 2 + 4 \Rightarrow \boxed{a = \frac{-6}{12} = -\frac{1}{2}}$

$\Rightarrow b = \frac{2}{+1} \times \left(-\frac{1}{2}\right) - 1 = 1$

$\Rightarrow \boxed{PAR: y = -\frac{1}{2}x^2 + x + 4}$

38)



(A) $\boxed{c = 0}$

(B) $-1 = 1a + b + 0$

(C) $-1 = 9a + 3b + 0$

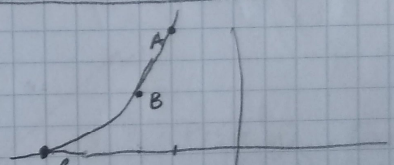
(B') $+3 = -3a - 3b + 0$ (B' : x -3)

$2 = 6a \Rightarrow \boxed{a = \frac{1}{3}}$

$\Rightarrow$ (B) $-1 = \frac{1}{3} + b$

$\Rightarrow \boxed{b = \frac{-4}{3}} \Rightarrow \boxed{PAR: y = \frac{1}{3}x^2 - \frac{4}{3}x}$

39) $\exists C$, $\boxed{y^2 = 2px}$, $\boxed{x = ay^2 + by + c}$



(A) $-2 = 16a + 4b + c \Rightarrow 4 = 16a + 4b$

(B) $-3 = 4a + 2b + c \Rightarrow 1 = 4a + b$

(C) $-6 = c \Rightarrow \boxed{c = -6}$ $\Rightarrow \boxed{b = 1 - 4a}$

$\Rightarrow \boxed{b = 2}$

$3 = 4a + 2(1 - 4a) \Rightarrow 3 = 4a + 2 - 8a$

$\Rightarrow 1 = -4a \Rightarrow \boxed{a = -\frac{1}{4}}$

$\Rightarrow \boxed{PAR: x = -\frac{y^2}{4} + 2y - 6}$

EA - Winterle p. 181

40) $y = -x^2+4x+5 \quad = (y=0) \Rightarrow 0 = +x^2-4x-5$

$\Delta = b^2-4ac = 16-4\cdot+1\cdot5 = 36$

$\Rightarrow x_1 = \frac{4+6}{2} = 5$

$x_2 = -1$

$\Rightarrow -x^2+4x = y-5$

$\Rightarrow x^2-4x = -y+5$

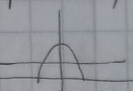
$\Rightarrow x^2-4x+4 = -y+9$

$(x-2)^2 = -1(y-9)$

$\Rightarrow \boxed{V(2,9)}$

$2p = -1, \quad p = -1/2, \quad \boxed{p/2 = -1/4}$

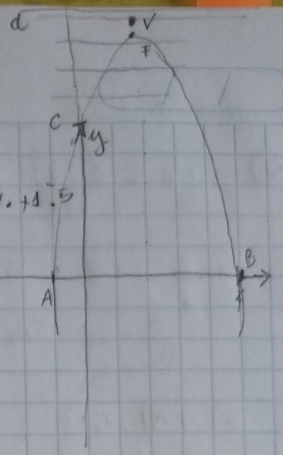
$\Rightarrow$



$\boxed{A(-1,0), B(5,0), C(0,5)}$

$F\left(2, 9-\frac{1}{4}\right)$

$d: y = \frac{37}{4}$



$9 - \frac{1}{4} = \frac{36-1}{4} = \frac{35}{4}$

41) PAR: $\begin{cases} x = -\frac{1}{4}t^2 \\ y = t \end{cases}$

42) PAR: $\begin{cases} x = t \\ y = \frac{t^2}{2} \end{cases}$

43) PAR: $\begin{cases} x+4 = t \Rightarrow x = t-4 \\ \frac{-(t-4+4)^2}{2} = y-1 \Rightarrow -\frac{t^2}{2} = y-1 \Rightarrow y = -\frac{t}{2}+1 \end{cases}$

44) $y^2-4y+4 = -x-1+4$

$\Rightarrow (y-2)^2 = -1(x-3) \Rightarrow$ PAR: $\begin{cases} -t^2 = x-3 \Rightarrow \boxed{x = -t^2+3} \\ t = y-2 \Rightarrow \boxed{y = t+2} \end{cases}$

45) $\begin{cases} t = x-1 \\ y = \frac{(x-1)^2}{3} - 2 \end{cases} \Rightarrow 3y+2 = (x-1)^2 = x^2-2x+1$

$\Rightarrow x^2-2x-3y-1 = 0$

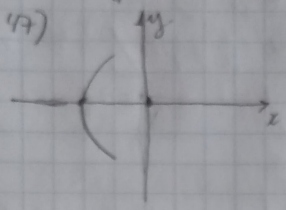
$\uparrow -5?$

Unitate, p. 181

46) ec. gen. PAR: $x^2 = 2py$

$x = \frac{y^2}{8} + 4 \Rightarrow 4(x-4) = y^2 \Rightarrow \boxed{y^2 - 4x + 16 = 0}$

47)



$y^2 = 2px$

$\Rightarrow y^2 = 8x$ $\quad p/2 = 2 \Rightarrow p = 4 \mid 2p = 8$

mas, $V(-2,0)$

$\Rightarrow y^2 = 8(x+2) \Rightarrow y^2 = 8x + 16$

$PAR \cap Oy \Leftrightarrow x = 0$

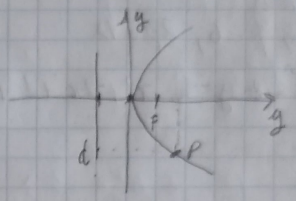
$\Rightarrow y^2 = 16 \Rightarrow \boxed{y = \pm 4} \Rightarrow PAR \cap Oy = (0,4), (0,-4)$

48) $\boxed{y^2 = 4x} \Rightarrow 2p = 4, p = 2, \boxed{p/2 = 1}$

$V(0,0)$

$F(1,0)$

$d: x = -1$



$PAR: d(P,F) = d(P,d)$

$d(P,d) = 3 = d(P,F)$

$\begin{cases} d(P,F) = |PF| \\ PF = F - P = (1,0) - (x,y) = (1-x, -y) \Rightarrow |PF| = \sqrt{1 - 2x + x^2 + y^2} \end{cases}$

$\Rightarrow x^2 - 2x + 1 + y^2 = 9 \Rightarrow x^2 - 2x + y^2 = 8$

$\Rightarrow x^2 - 2x + 1x - 8 \Rightarrow x^2 + 2x - 8 = 0$

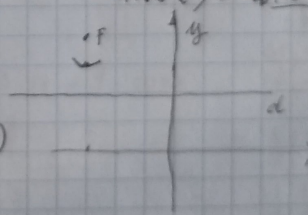
$\Delta = b^2 - 4ac = 4 - 4 \cdot 1 \cdot 8 = 36$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm 6}{2} = -4$ sau 2

$y^2 = 4x \Rightarrow y^2 = 8 \Rightarrow y = \pm\sqrt{8}$

$\Rightarrow PAR \cap Oy = (2, \pm\sqrt{8})$

mas $x > 0 \Rightarrow \boxed{x = 2}$



49) a) $p = 2$

$V(-3,3) \Rightarrow (x+3)^2 = 4(y-3)$

$\Rightarrow PAR: x^2 + 6x + 9 - 4y - 12$

$\Rightarrow x^2 - 4y + 6x + 21 = 0$

49b) $d(F,P) = d(d,P)$

$P(x,y)$

$d(P,P) = |FP| = \sqrt{x^2 + y^2 - 6y + 9}$

$FP = P - F = (x-0, y-3)$

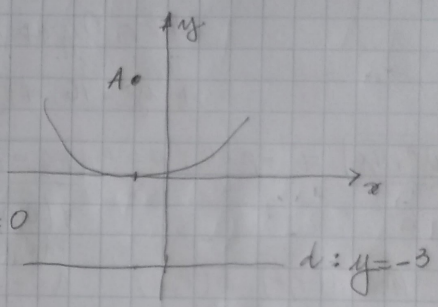
$d(d,P) \dots ?$

50) $V(-1,0)$

$x^2 = 2py$

$\frac{p}{2} = 3$, $p = 6$, $2p = 12$

$(x+1)^2 = 12y \Rightarrow x^2 + 2x + 1 - 12y = 0$

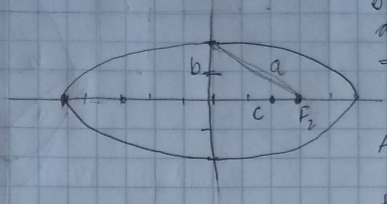


Unitarie, p. 199

4 - 10 : gráfico, vértices A_1, A_2, F_1, F_2, e

10)

1) $\frac{x^2}{25} + \frac{y^2}{9} = 1$



$b = 3$

$a = 5$

$\Rightarrow 5^2 = 3^2 + c^2 \Rightarrow c^2 = 21$

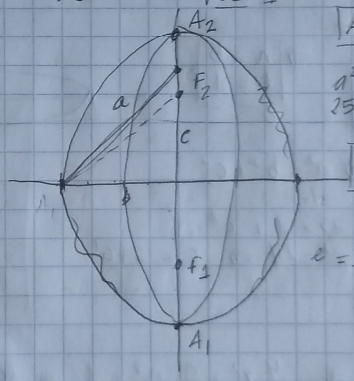
$c = \pm\sqrt{21}$

$A_2(5,0), A_1(-5,0)$

$F_1(-\sqrt{21},0), F_2(+\sqrt{21},0)$

$e = \frac{c}{a} = \frac{\sqrt{21}}{5}$

2) $25x^2 + 4y^2 = 100 \Rightarrow \frac{x^2}{4} + \frac{y^2}{25} = 1$ $\quad a = 5,\ b = 2$



$A_1(0,-5), A_2(0,5)$

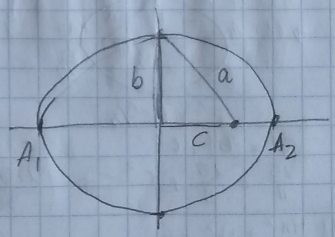
$a^2 = b^2 + c^2$

$25 = 4 + c^2 \Rightarrow c = \sqrt{21}$

$F_1(0,-\sqrt{21}), F_2(0,+\sqrt{21})$

$e = \frac{c}{a} = \frac{\sqrt{21}}{5}$

3) $9x^2 + 16y^2 = 144 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$ $\quad a = 4,\ b = 3$



$a^2 = b^2 + c^2$

$16 = 9 + c^2 \Rightarrow c = \pm\sqrt{7}$

$A(\pm 4, 0)$

$F(\pm\sqrt{7}, 0)$

$e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

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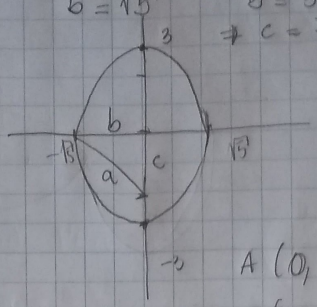
4) $9x^2 + 5y^2 = 45 \Rightarrow \frac{x^2}{5} + \frac{y^2}{9} = 1 \Rightarrow$ $a = 3$, $b = \sqrt{5}$

$\Rightarrow a^2 = b^2 + c^2$
$9 = 5 + c^2$
$\Rightarrow c = \pm 2$



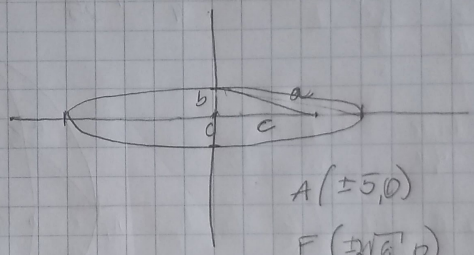
$A(0, \pm 3)$

$F(0, \pm 2)$

$e = \frac{c}{a} = \frac{2}{3}$

5) $x^2 + 25y^2 = 25 =$

$= \left[\frac{x^2}{25} + \frac{y^2}{1} = 1\right] \Rightarrow \left[\begin{matrix} a = 5 \\ b = 1 \end{matrix}\right]$



$\Rightarrow a^2 = b^2 + c^2$

$25 = 1 + c^2$

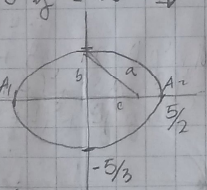
$\Rightarrow c^2 = 24 \Rightarrow c = 2\sqrt{6}$

$A(\pm 5, 0)$

$F(\pm 2\sqrt{6}, 0)$

$e = \frac{c}{a} = \frac{2\sqrt{6}}{5}$

6) $4x^2 + 9y^2 = 25 \Rightarrow \frac{4}{25}x^2 + \frac{9}{25}y^2 = 1 = \frac{x^2}{25/4} + \frac{y^2}{25/9} \Rightarrow \left[\begin{matrix} A = \frac{5}{2} \\ B = \frac{5}{3} \end{matrix}\right]$



$A(\pm \frac{5}{2}, 0)$

$F(\quad, 0)$

$a^2 = b^2 + c^2$

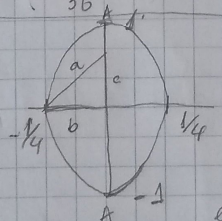
$\frac{25}{4} = \frac{25}{9} + c^2 \Rightarrow \frac{225 - 100}{36} = c^2 \Rightarrow \left[c = \pm \frac{5\sqrt{5}}{6}\right]$

$e = \frac{c}{a} = \frac{5\sqrt{5}}{6} : \frac{5}{2}$

$\Rightarrow e = \frac{\sqrt{5}}{3}$

7) $4x^2 + y^2 = 1$

$\Rightarrow \frac{x^2}{1/4} + \frac{y^2}{1} = 1$



$a = 1$
$b = 1/2$

$A(0, \pm 1)$

$a^2 = b^2 + c^2$
$1 = \frac{1}{4} + c^2 \Rightarrow c^2 = \frac{3}{4}$

$\Rightarrow c = \frac{\pm\sqrt{3}}{2}$

$e = \frac{c}{a}$ $\quad F(0, \pm \frac{\sqrt{3}}{2})$

$\hookrightarrow e = \frac{\sqrt{3}}{2}$

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8) $4x^2 + 25y^2 = 1 \Rightarrow \boxed{\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{25}} = 1}$

$\boxed{a = 1/2 \quad b = 1/5}$

$A(\pm 1/2, 0)$

$F(\pm \frac{\sqrt{21}}{10}, 0)$

$\left(\frac{25-4}{100}\right)$

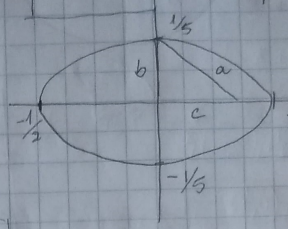
$a^2 = b^2 + c^2$

$\frac{1}{4} = \frac{1}{25} + c^2$

$\Rightarrow c^2 = \frac{21}{100}$

$\Rightarrow c = \frac{\sqrt{21}}{10}$

$\Rightarrow \boxed{e = \frac{c}{a} = \frac{\frac{\sqrt{21}}{10}}{\frac{1}{2}} = \frac{\sqrt{21}}{5}}$



9) $x^2 + 2y^2 = 5 \Rightarrow \frac{x^2}{5} + \frac{y^2}{\frac{5}{2}} = 1 \Rightarrow \boxed{a = \sqrt{5} \quad b = \sqrt{\frac{5}{2}}}$

$a^2 = b^2 + c^2$

$\Rightarrow 5 = \frac{5}{2} + c^2$

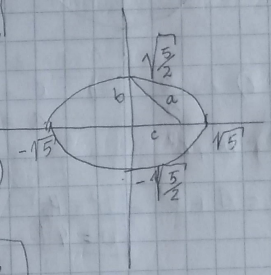
$\Rightarrow c^2 = \frac{10-5}{2} = \frac{5}{2}$

$\Rightarrow c = \pm\sqrt{\frac{5}{2}}$

$\Rightarrow e = \frac{c}{a} = \frac{\sqrt{\frac{5}{2}}}{\sqrt{5}} = \sqrt{\frac{1}{2}} = \boxed{\frac{\sqrt{2}}{2} = e}$

$A(\pm\sqrt{5}, 0)$

$F(\pm\sqrt{\frac{5}{2}}, 0)$



10) $9x^2 + 25y^2 = 25 \Rightarrow \frac{x^2}{\frac{25}{9}} + y^2 = 1 \Rightarrow \boxed{a = \frac{5}{3} \quad b = 1}$

$a^2 = b^2 + c^2$

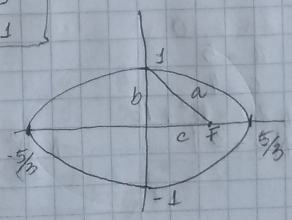
$\Rightarrow \frac{25}{9} = 1 + c^2$

$\Rightarrow \frac{25-9}{9} = c^2 = \frac{16}{9} = c^2$

$\Rightarrow \boxed{c = \pm\frac{4}{3}} \Rightarrow e = \frac{c}{a} = \frac{\frac{4}{3}}{\frac{5}{3}} = \frac{4}{5}$

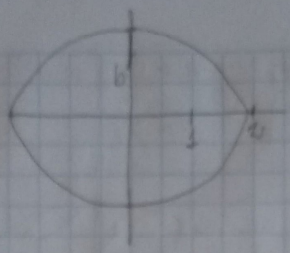
$A(\pm\frac{5}{3}, 0)$

$F(\pm\frac{4}{3}, 0)$



10) a) $e = \frac{1}{2}$

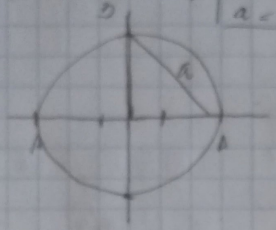
$\Rightarrow c = 1$
$a = 2$



$a^2 = b^2 + c^2$

$\Rightarrow 4 - 1 = b^2 = 3$
$\Rightarrow b = \pm\sqrt{3}$

b) $e = \frac{1}{5} \Rightarrow \boxed{\begin{matrix} c = 1 \\ a = 5 \end{matrix}}$

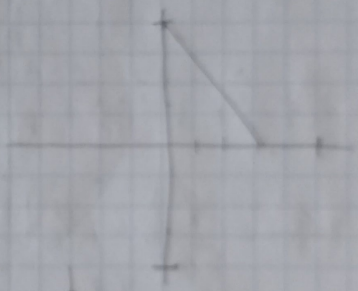


c) $a^2 = b^2 + c^2$
$9 = b^2 + 1$
$\Rightarrow \boxed{b = \pm\sqrt{2}}$

d) $e = \frac{3}{5} \Rightarrow \begin{matrix} c = 3 \\ a = 5 \end{matrix}$

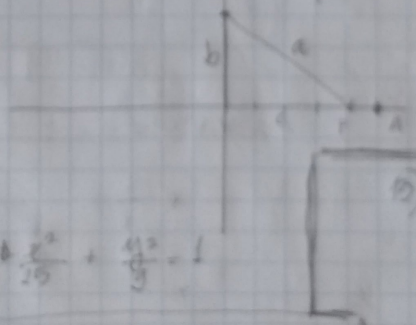
$a^2 = b^2 + c^2$

$25 = 9 + b^2 \Rightarrow b^2 = 16$
$\boxed{b = 4}$



12) $F(\pm 4, 0)$

$2a = 10$
$a = 5$
$c = 4$
$\Rightarrow 25 = b^2 + 16$
$\Rightarrow \boxed{b = 3}$



$\Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$

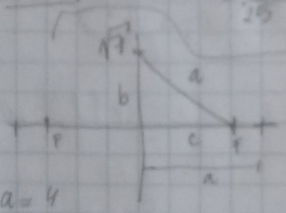
13)

$\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow \boxed{\begin{matrix} b = 3 \\ c = 5 \end{matrix}}$

$a^2 = b^2 + c^2$
$a^2 = 50$
$\boxed{a = 5\sqrt{2}}$

$\boxed{\frac{x^2}{25} + \frac{y^2}{50} = 1}$

14)



$a = 4$
$c = 3$
$a^2 = b^2 + c^2$
$16 = b^2 + 9$
$\Rightarrow b^2 = 7 \Rightarrow \boxed{b = \pm\sqrt{7}}$

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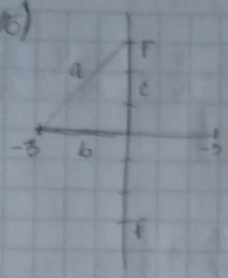
15)



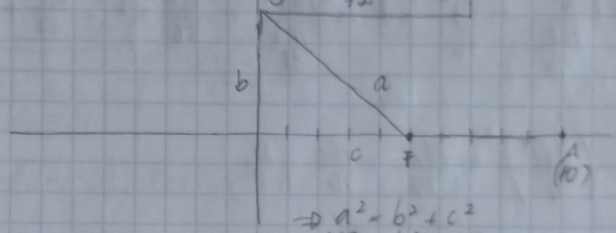
$F=(0,\pm 3) \Rightarrow \boxed{c=3}$

$\frac{c}{a}=\frac{\sqrt{3}}{2} \Rightarrow \frac{3}{a}=\frac{\sqrt{3}}{2} \Rightarrow a\sqrt{3}=6 \Rightarrow \boxed{a=\frac{6}{\sqrt{3}}=2\sqrt{3}}$

$a^2=b^2+c^2 \Rightarrow 12=b^2+9 \Rightarrow \boxed{b=\pm\sqrt{3}}$

$\boxed{\frac{x^2}{3}+\frac{y^2}{12}=1}$

16)



$\boxed{a=10}$

$e=\frac{c}{a}=\frac{1}{2}=\frac{c}{10}$

$\Rightarrow \boxed{c=5}$

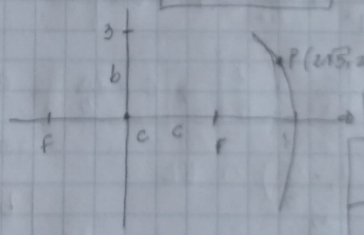
$\Rightarrow a^2=b^2+c^2$

$100=b^2+25$

$\Rightarrow \boxed{b=\pm 5\sqrt{3}}$, $\Rightarrow \boxed{\frac{x^2}{100}+\frac{y^2}{75}=1}$

75 | 3
25 | 5
5 | 5
1 |

17) $C(0,0)$



$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$

$\boxed{b=3}$

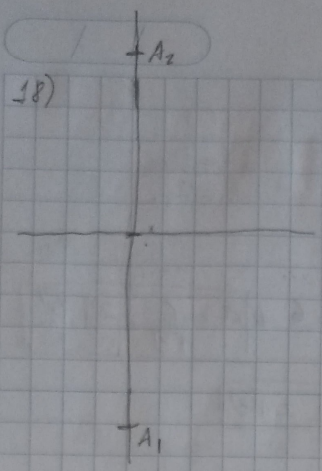
$P(2\sqrt{5},2) \rightarrow \frac{20}{a^2}+\frac{4}{9}=1$

$\Rightarrow \frac{20}{a^2}=1-\frac{4}{9}=\frac{5}{9}$

$\Rightarrow 180=5a^2 \Rightarrow \boxed{a=\pm 6}$

$\Rightarrow \boxed{\frac{x^2}{36}+\frac{y^2}{9}=1}$

18)



$\Rightarrow \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 = EL$

$\boxed{a = 6}$

$P(3,2) \in$ elipse

$\Rightarrow \frac{9}{b^2} + \frac{4}{36} = 1 \Rightarrow \frac{9}{b^2} = \frac{32}{36} = \frac{16}{18} = \frac{8}{9}$

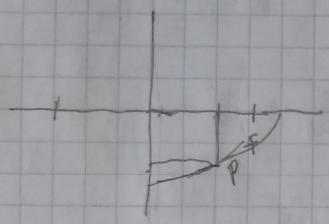
$\Rightarrow 8b^2 = 81 \Rightarrow b^2 = \frac{81}{8} \Rightarrow \boxed{b = \frac{9}{2\sqrt{2}} = \frac{9\sqrt{2}}{4}}$

$1 - \frac{4}{36} = \frac{36-4}{36} = \frac{32}{36} = \frac{8}{9}$

$\Rightarrow EL: \frac{x^2}{\frac{162}{16}} + \frac{y^2}{36} = 1 = \frac{16x^2}{\frac{162}{81}} + \frac{y^2}{36} = \boxed{1 = \frac{8x^2}{81} + \frac{y^2}{36}}$

$\left(\frac{9\sqrt{2}}{4}\right)^2 = \frac{81 \cdot 2}{16} = \frac{162}{16} = \boxed{\frac{81}{8}}$

19)



$\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 = EL}$ *

$e = \boxed{\frac{c}{a} = \frac{2}{3}} \Rightarrow \boxed{c = \frac{2a}{3}}$

$a^2 = b^2 + \left(\frac{2a}{3}\right)$

$a^2 = b^2 + \frac{4a^2}{9}$

$\boxed{b^2 = \frac{5}{9}a^2}$

$P\left(2, -\frac{5}{3}\right) \cap EL$

* $\Rightarrow \frac{4}{a^2} + \frac{\frac{25}{9}}{\frac{5}{9}a^2} = 1$

$\Rightarrow \frac{4}{a^2} + \frac{5}{a^2} = 1 \Rightarrow a^2 = 9 \Rightarrow \boxed{a = \pm 3} \Rightarrow b^2 = 5 \Rightarrow \boxed{b = \sqrt{5}}$

$\Rightarrow \boxed{\frac{x^2}{9} + \frac{y^2}{5} = 1}$

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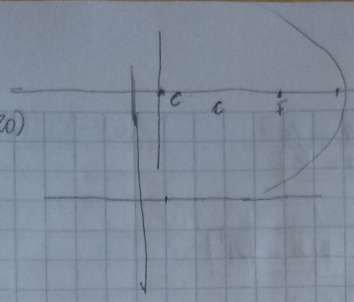
20)



$C(1,4)$

$\Rightarrow \boxed{c = 4}$

$e = \frac{c}{a} = \frac{2}{3} \Rightarrow C = \frac{2}{3}a \Rightarrow 4 = \frac{2}{3}a$

$\Rightarrow \boxed{a = 6}$

$a^2 = b^2 + c^2$

$\Rightarrow 36 = b^2 + 16 \Rightarrow b^2 = 20 \Rightarrow \boxed{b = 2\sqrt{5}}$

$\Rightarrow EL = \frac{(x-1)^2}{36} + \frac{(y-4)^2}{20} = 1$

$= 20(x-1)^2 + 36(y-4)^2 = 720$

$= 10(x-1)^2 + 18(y-4)^2 = 360$

$= 5(x-1)^2 + 9(y-4)^2 = 180$

$\Rightarrow 5x^2 - 10x + 5 + 9y^2 - 72y + 144 = 180$

$\Rightarrow \boxed{5x^2 - 10x + 9y^2 - 72y - 31 = 0}$

21) $\boxed{a = 5}$

$F_1(2,-1), F_2(2,5) \Rightarrow \boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$: EL

$\Rightarrow \boxed{c = 3}$

$\Rightarrow \boxed{C = (2,2)}$

$a^2 = b^2 + c^2$

$25 = b^2 + 9 \Rightarrow \boxed{b = 4}$

$\Rightarrow \boxed{\frac{(x-2)^2}{16} + \frac{(y-2)^2}{25} = 1} = 25(x-2)^2 + 16(y-2)^2 - 400 = 0 =$

$= \boxed{25x^2 - 100x + 100 + 16y^2 - 64y + 64 - 400 = 0}$ -236

22) EL: ?

$F_1(-1,-0), F_2(-1,5)$

$\Rightarrow \boxed{C(-1,2)}$

$\Rightarrow \boxed{c = 4}$

$e = \frac{c}{a} = \frac{2}{3} \Rightarrow a = \frac{3c}{2}$

$\Rightarrow \boxed{a = 6}$ $a^2 = b^2 + c^2$

$\Rightarrow 36 = b^2 + 16 \Rightarrow \boxed{b = 2\sqrt{5}}$

$\Rightarrow EL: \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\Rightarrow EL: \frac{(x+1)^2}{20} + \frac{(y-1)^2}{36} = 1 \Rightarrow 9(x+1)^2 + 5(y-1)^2 - 180 = 0$

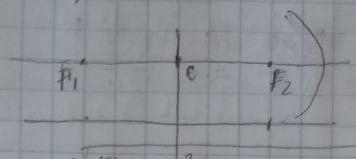
$\Rightarrow \boxed{9x^2 + 18x + 9 + 5y^2 - 10y + 5 - 180 = 0}$ -166

23) EL: ?
$F_1(-3,2)$, $F_2(3,2)$

$\Rightarrow C = (0,2)$
$\Rightarrow c = 3$

$e = \frac{c}{a} = \frac{1}{2} \Rightarrow a = 2c$
$\Rightarrow \boxed{a = 6}$

$a^2 = b^2 + c^2$
$36 = b^2 + 9 \Rightarrow \boxed{b = \sqrt{27}}$



$\Rightarrow \boxed{EL: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \Rightarrow EL: \frac{x^2}{36} + \frac{(y-2)^2}{27} = 1$

$= 27x^2 + 36y^2 - 144y + 144 - 972 = 0$
-828
$\Rightarrow \boxed{3x^2 + 9y^2 - 16y - 92 = 0}$

24) EL: ?
$A_1(-7,2)$, $A_2(-1,2)$

$\boxed{b = 1}$

$\Rightarrow \boxed{a = 3}$
$\Rightarrow C(-4,2)$

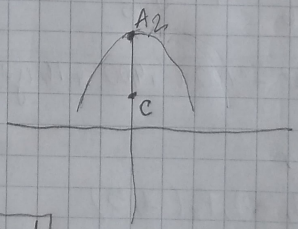
$a^2 = b^2 + c^2$
$\Rightarrow 9 = 1 + c^2 \Rightarrow \boxed{c = 2\sqrt{2}}$

$\Rightarrow EL: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{(x+4)^2}{9} + \frac{(y-2)^2}{1} = 1$

$\Rightarrow (x+4)^2 + 9(y-2)^2 - 9 = 0 = \boxed{x^2 + 8x + 16 + 9y^2 - 36y + 36 - 9 = 0}$
$+43$

25) EL: ?
$C(0,1)$
$A_2(0,2)$

$e = \frac{c}{a} = \frac{\sqrt{3}}{2}$
$\Rightarrow \boxed{c = \frac{a\sqrt{3}}{2} = \sqrt{3}}$



$\Rightarrow \boxed{a = 2}$

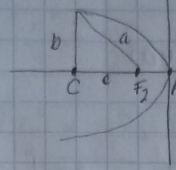
$a^2 = b^2 + c^2$
$4 = b^2 + 3 \Rightarrow \boxed{b = 1}$

$\Rightarrow \boxed{\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1}$

$\Rightarrow EL: \frac{x^2}{1} + \frac{(y-1)^2}{4} = 1 \Rightarrow \boxed{4x^2 + y^2 - 2y + 1 - 4 = 0}$
-3

26) $C(-3,0)$

$\Rightarrow \boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1}$



$\Rightarrow \boxed{\begin{matrix} a = 3 \\ c = 2 \end{matrix}}$) $a^2 = b^2 + c^2$

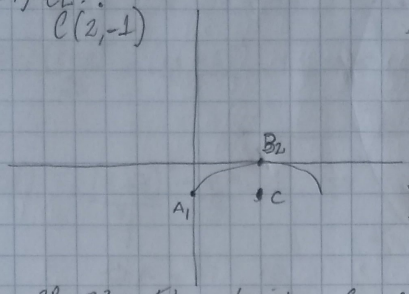
$9 = b^2 + 4 \Rightarrow \boxed{b = \sqrt{5}}$

$\Rightarrow \boxed{\frac{(x+3)^2}{9} + \frac{y^2}{5} = 1} \Rightarrow 5(x+3)^2 + 9y^2 = 45$

$\Rightarrow \boxed{5x^2 + 30x + 45 + 9y^2 = 45}$

27) EL: ?

$C(2,-1)$



$\Rightarrow \boxed{\begin{matrix} b = 1 \\ a = 2 \end{matrix}}$

$\Rightarrow \frac{(x-2)^2}{4} + \frac{(y+1)^2}{1} = 1$

$\Rightarrow (x-2)^2 + 4(y+1)^2 = 4$

$\Rightarrow \boxed{x^2 - 4x + 4 + 4y^2 + 8y + 4 = 4}$

28 – 33: EL reduzida, C, A_1, A_2, F_1, F_2, e, gráfico

28) $9x^2 - 36x + 16y^2 + 96y = -36 + 36 + 144$

$+36 \qquad +144$

$\Rightarrow (3x-6)^2 + (4y+12)^2 = 144 \Rightarrow 9(x-2)^2 + 16(y+3)^2 = 144$

$\Rightarrow \boxed{\frac{(x-2)^2}{16} + \frac{(y+3)^2}{9} = 1}$

$C(2,-3)$

$A_1(-2,-3)$

$A_2(6,-3)$

$A: x^2 - 4x + 4 = 16 \qquad \Delta = 16 - 4.1.-12 = 64$

$\Rightarrow x^2 - 4x - 12 = 0 \qquad \boxed{x = \frac{4 \pm 8}{2}} \Rightarrow x_1 = -2$

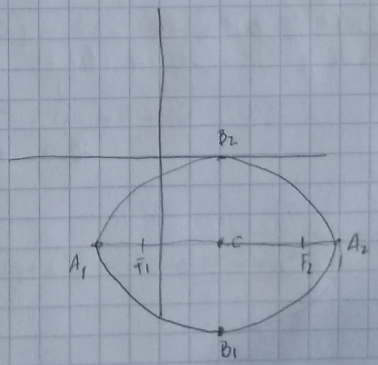
$x_2 = 6$

$a = 4$

$b = 3 \Rightarrow 16 = 9 + c^2 \Rightarrow c = \pm\sqrt{7}$

$\Rightarrow \boxed{F = (2 \pm \sqrt{7}, -3)}$

$\boxed{e = \frac{c}{a} = \frac{\sqrt{7}}{4}}$



29) $25x^2 + 50x \overset{+25}{\uparrow} + 16y^2 + 64y \overset{+64}{\uparrow} = 311 + 25 + 64 = 400$

$(5.2.5)$ $(4.2.8)$

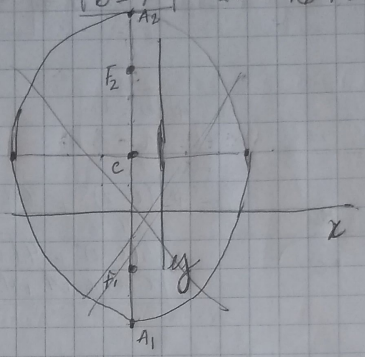
$\Rightarrow (5x+5)^2 + (4y+8) = 400 = 25(x+1)^2 + 16(y-2)^2 = 400$

$\Rightarrow \boxed{\frac{(x+1)^2}{16} + \frac{(y+2)^2}{25} = 1} \Rightarrow \boxed{\begin{matrix} a = 5 \\ b = 4 \end{matrix}}$ $\circlearrowleft$ $a^2 = b^2 + c^2$

$25 = 16 + c^2 \Rightarrow \boxed{c = \pm 3}$

$\Rightarrow \boxed{C(-1,-2)}$

$\Rightarrow \boxed{A_1A_2 \parallel Oy}$



$A_1(-1,-7), A_2(-1,3)$

$F_1(-1,-5), F_2(-1,1)$

$e = \frac{c}{a} = \frac{3}{5}$

30) $4x^2 - 24x + 36 + 9y^2 + 18y + 9 = -\cancel{9} + 36 + \cancel{9}$

$2.2.6$ $2.3.3$

$\Rightarrow (2x-6)^2 + (3y+3)^2 = 36 \Rightarrow \boxed{\cancel{4}\frac{(x-3)^2}{9} + \cancel{9}\frac{(y+1)^2}{4} = \cancel{36}\ 1} \Rightarrow C(3,-1)$

$\Rightarrow A_1A_2 \parallel Ox$

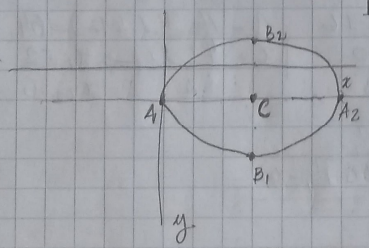
$\Rightarrow a = 3$

$b = 2$

$\circlearrowleft$ $a^2 = b^2 + c$

$9 = 4 + c^2$

$\Rightarrow c = \sqrt{5}$



$A(3 \pm 3, -1)$

$F(3 \pm \sqrt{5}, -1)$

$\Rightarrow e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

31) $16x^2+64x+64+y^2-4y+4 = -52+64+4 = 16$

$2\cdot 4\cdot 8 \qquad 2\cdot 1\cdot 2$

$\Rightarrow (4x+8)^2+(y-2)^2 = 16$

$\Rightarrow 16(x+2)^2+(y-2)^2 = 16 \Rightarrow \boxed{\frac{(x+2)^2}{1}+\frac{(y-2)^2}{16}=1}$

$\Rightarrow C=(-2,2)$

$\boxed{\begin{matrix}a=4\\b=1\end{matrix}}$ $A_1A_2 \parallel Oy$

$\Delta\ a^2=b^2+c^2 \Rightarrow 16=1+c^2 \Rightarrow c=\sqrt{15}$

$F(-2, 2\pm\sqrt{15})$

$A(-2, 2\pm 4)$

$\Rightarrow \boxed{e=\frac{c}{a}=\frac{\sqrt{15}}{4}}$

32) $16x^2-96x+144+9y^2+72y+144 = -144+144$

$2\cdot 4\cdot 12 \qquad 2\cdot 3\cdot 12$

$\Rightarrow (4x-12)^2+(3y+12)^2 = 144 \Rightarrow 16(x-3)^2+9(y+4)^2 = 144$

$\Rightarrow \boxed{\frac{(x-3)^2}{9}+\frac{(y+4)^2}{16}=1} \Rightarrow \boxed{C(3,-4)}$

$\Rightarrow \boxed{\begin{matrix}a=4\\b=3\end{matrix}} \Rightarrow \Delta\ a^2=b^2+c^2$

$16=9+c^2 \Rightarrow \boxed{c=\sqrt{7}}$

$\Rightarrow A_1A_2 \parallel Oy$

$\Rightarrow \boxed{A(3,-4\pm 4)}$

$\boxed{F(3,-4\pm\sqrt{7})}$ $\qquad \boxed{e=\frac{c}{a}=\frac{\sqrt{7}}{4}}$

33) $4x^2-8x+4+9y^2-36y+36 = -4+4+36$

$2\cdot 2\cdot 2 \qquad 2\cdot 3\cdot 6$

$(2x-2)^2+(3y-6)^2 = 36$

$\Rightarrow 4(x-1)^2+9(y-2)^2 = 36 \Rightarrow \boxed{\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1} \Rightarrow C(1,2)$

$A(1\pm 3, 2)$

$F(1\pm\sqrt{5}, 2)$

$\Rightarrow \boxed{\begin{matrix}a=3\\b=2\end{matrix}}$ $\Delta\ a^2=b^2+c^2$

$9=4+c^2$

$\Rightarrow \boxed{c=\sqrt{5}}$

$e=\frac{c}{a}=\frac{\sqrt{5}}{3}$

tilibra

34) $x^2 + 4y^2 = 4$

$\Rightarrow \boxed{\frac{x^2}{4} + \frac{y^2}{1} = 1}$ $\Rightarrow a = 2$, $b = 1$ $= EL: \begin{cases} x = 2\cos\theta \\ y = \operatorname{sen}\theta \end{cases}$

35) $\frac{x^2}{36} + \frac{y^2}{36} = 1$ $\Rightarrow a = 6$, $b = 6$ $\Rightarrow EL: \begin{cases} x = 6\cos\theta \\ y = 6\operatorname{sen}\theta \end{cases}$

36) $\frac{x^2}{1/9} + \frac{y^2}{1/16} = 1$ $\Rightarrow a = 1/3$, $b = 1/4$ $\Rightarrow EL: \begin{cases} x = \frac{1}{3}\cos\theta \\ y = \frac{1}{4}\operatorname{sen}\theta \end{cases}$

37) $\frac{(x-1)^2}{25} + \frac{(y+1)^2}{9} = 1$ $\Rightarrow a = 5$, $b = 3$, $C(1,-1)$ $\Rightarrow EL: \begin{cases} x = 1 + 5\cos\theta \\ y = -1 + 3\operatorname{sen}\theta \end{cases}$

38) $\frac{(x+7)^2}{1/7} + \frac{y^2}{7} = 1$ $\Rightarrow EL: \begin{cases} x = -7 + \frac{\sqrt{7}}{7}\cos\theta \\ y = \sqrt{7}\operatorname{sen}\theta \end{cases}$

$\Rightarrow a = \sqrt{7}$

$b = \sqrt{1/7} = \frac{\sqrt{7}}{7}$

39) $4x^2 + 9y^2 - 54y + 81 = -45 + 81 \Rightarrow 4x^2 + (3y-9)^2 = 36 \Rightarrow 4x^2 + 9(y-3)^2 = 36$

2.3.

$\Rightarrow \boxed{\frac{x^2}{9} + \frac{(y-3)^2}{4} = 1}$ $\Rightarrow EL: \begin{cases} x = 3\cos\theta \\ y = 3 + 2\operatorname{sen}\theta \end{cases}$

40) Eq. geral EL: ? $a = 5$, $b = 5$ $\Rightarrow \frac{x^2}{25} + \frac{y^2}{25} = 1 = \boxed{x^2 + y^2 - 25 = 0}$

41) $a = 3$, $b = 1$ $\Rightarrow \frac{x^2}{9} + \frac{y^2}{1} = 1 \Rightarrow \boxed{9x^2 + y^2 - 9 = 0}$

42) $C(2,3)$, $a = 4$ (x), $b = 2$ (y), $A_1A_2 \parallel Ox$ $\Rightarrow \frac{(x-2)^2}{16} + \frac{(y-3)^2}{4} = 1 \Rightarrow (x-2)^2 + 4(y-3)^2 = 16$

$\Rightarrow x^2 - 4x + 4 + 4y^2 - 24y + 36 = 16$? 24?

EX. 31 - 33

45) $\frac{x^2}{2} + \frac{(y+1)^2}{1} = 1 = x^2 + 2(y+1)^2 = 2 \Rightarrow \boxed{x^2 + 2y^2 + 4y + 2 = 2}$

Winterle, 199

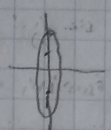
44) $F = ?$

Eli: $\begin{cases} x = 4 + 3\cos t \\ y = -2 + 5\sin t \end{cases}$

$C(4, -2)$

$\boxed{a = 5}$

$\boxed{b = 3}$



$a^2 = b^2 + c^2$, $d(F, C) = c$

$25 = 9 + c^2 \Rightarrow \boxed{c = 4}$

$\Rightarrow F(4, -2 \pm 4)$

45) curva = Eli

$F_1(4, -1)$, $F_2(4, 7)$ $\Rightarrow$  $\Rightarrow C(4, 3)$

$\boxed{a = 6}$

$\boxed{c = 4}$

$a^2 = b^2 + c^2 \Rightarrow 36 = b^2 + 16 \Rightarrow \boxed{b = \pm 2\sqrt{5}}$ $\Rightarrow \frac{(y-3)^2}{36} + \frac{(x-4)^2}{20} = 1$

$\Rightarrow 5(y-3)^2 + 9(x-4)^2 = 180$

$\Rightarrow 5y^2 - 30y + 45 + 9x^2 - 72x + 144 = 180$

$\Rightarrow 9x^2 + 5y^2 - 72x - 30y + 9 = 0$

36, 20	2
18, 10	2
9, 5	3
3, 5	3
1, 5	5
,	

tilibra

$2 - \frac{8}{3} = \frac{6-8}{3} = -\frac{2}{3}$

Winterle, p. 199

46)

$d(P,A) = \frac{d(P,r)}{2}$

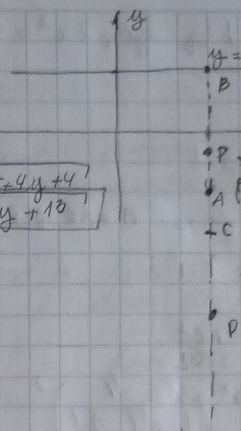
$d(P,A) = |AP| \Rightarrow AP = P - A = (x,y) - (3,-2)$
$= (x-3, y+2)$

$\Rightarrow |AP| = \sqrt{x^2 - 6x + 9 + y^2 + 4y + 4}$

$|AP| = \sqrt{x^2 - 6x + y^2 + 4y + 13}$

$d(P,r) = \frac{8}{3}$

$r: \begin{cases} x = t \\ y = 2 \end{cases}$



$d(P,r) = d((x,y), r)$

$\Rightarrow x^2 - 6x + 9 + y^2 + 4y + 4 + 13 = \frac{64}{9} - 13 + 9 + 4$

2.1.3 2.1.2

$(x-3)^2 + (y+2)^2 = \frac{64}{9}$

$x^2 - 6x + 9 + y^2$

$\Rightarrow \boxed{C(3, -10/3)}$

$\Rightarrow A_2(3, -2/3), A_1(3, -6)$

$\Rightarrow F_2(3, -2) \Rightarrow \boxed{c: 4/3, \quad a: 8/3}$

$PB \cdot u(1,0) = 0$

$(PB = B - P = (t, 2) - (x, y)$
$= (t-x, 2-y)$

$\Rightarrow (t-x, 2-y) \cdot (1,0) = 0$

$\Rightarrow t - x = 0 \Rightarrow \boxed{t = x}$

$\frac{4}{3} \cdot 2 = \frac{8}{3}$

$C: \frac{10}{3} + 2$

$\boxed{c = \frac{-10+6}{3} = \frac{4}{3}}$

$-\frac{2}{3} - 6 = \frac{2-18}{3}$

$= -\frac{20}{3} \cdot \frac{1}{2}$

$a = -\frac{10}{3} + \frac{2}{3} = \left|\frac{8}{3}\right| = -\frac{10}{3}$

$\Rightarrow a^2 = b^2 + c^2 \Rightarrow \frac{64}{9} = b^2 + \frac{16}{9}$

$\Rightarrow b^2 = \frac{48}{9} \Rightarrow \boxed{b = \frac{4\sqrt{3}}{3}}$

84	2
42	2
21	3
7	7

48	2
24	2
12	2
6	2
3	3
1	

16
x3
48

$\Rightarrow 9\frac{(y+10/3)^2}{64} + \frac{9(x-3)^2}{48} = 1$

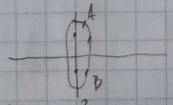
$\Rightarrow 9(y+10/3)^2 + 9(x-3)^2 = 3072$

$\Rightarrow 9y^2 + 9 \cdot 2 \cdot \frac{10}{3} \cdot y + \frac{100}{9} + 9x^2 - 2 \cdot 9 \cdot 3x + 9 = 3072$

64, 48	2
32, 24	2
16, 12	2
8, 6	2
4, 3	2
2, 3	2
1, 3	3
1, 1	

47) Eli: ?

C(0,0) $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$



(A) $\frac{14}{a^2} + \frac{1}{b^2} = 1 \Rightarrow \frac{14}{a^2} + \frac{1}{b^2} = \frac{8}{a^2} + \frac{4}{b^2}$

$\Rightarrow \frac{14}{a^2} - \frac{8}{a^2} = \frac{4}{b^2} - \frac{1}{b^2} \Rightarrow \frac{6}{a^2} = \frac{3}{b^2} \Rightarrow \boxed{2b^2 = 3a^2}$

(B) $\frac{2}{a^2} + \frac{4}{b^2} = 1$

$\Rightarrow \frac{48}{3b^2} + \frac{4}{b^2} = 1 \Rightarrow \frac{8}{b^2} = 1 \Rightarrow b^2 = 8 \Rightarrow \boxed{b = 2\sqrt{2}}$

$\frac{8}{a^2} + \frac{4}{8} = 1 \Rightarrow \frac{8}{a^2} = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow a^2 = 16$

$\boxed{a = 4}$

$\frac{y^2}{16} + \frac{x^2}{8} = 1 \Rightarrow y^2 + 2x^2 = 16$

48) $C(0,0)$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$e = \frac{c}{a} = \frac{1}{2} \Rightarrow a = 2c \Rightarrow \boxed{c = 2a}$

$A(2,3)$ e Eli

I) $a^2 = b^2 + c^2$

$\Rightarrow \frac{4}{1 - \frac{9}{b^2}} = b^2 + \left(\frac{4}{\sqrt{1 - \frac{9}{b^2}}}\right)^2$

$\Rightarrow \frac{4}{1 - \frac{9}{b^2}} = b^2 + \frac{16}{1 - \frac{9}{b^2}} \Rightarrow b^2 = -\frac{12}{1 - \frac{9}{b^2}} = -\frac{12 \cdot b^2}{(b^2 - 9)} \Rightarrow \frac{b^2}{b^2} = -\frac{12}{b^2 - 9}$

$\Rightarrow b^2 - 9 = -12 \Rightarrow b^2 = -3$ X

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{4}{a^2} + \frac{9}{b^2} = 1$

$\Rightarrow \frac{4}{a^2} = 1 - \frac{9}{b^2} = \frac{2}{a} = \sqrt{1 - \frac{9}{b^2}}$

$\Rightarrow 2 = a\sqrt{1 - \frac{9}{b^2}}$

$\boxed{\Rightarrow \frac{2}{\sqrt{1 - \frac{9}{b^2}}} = a}$

$1 - \frac{9}{b^2} = \frac{b^2 - 9}{b^2}$

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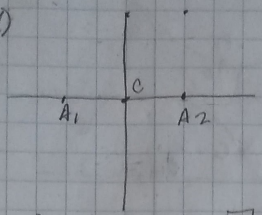
WINTERLE p. 214 (HIPER)

1)



$\Rightarrow a = 2$
$b = 3$
$\Rightarrow C(0,0)$
$A(\pm 2, 0)$
$F(\pm\sqrt{13}, 0)$

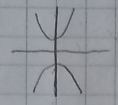
$e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

$r: y = \frac{b}{a}x = \frac{3}{2}x$

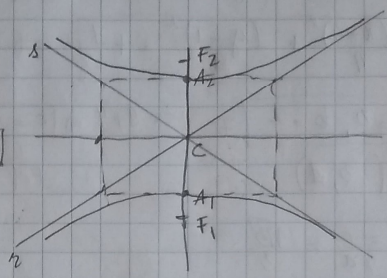
$s: y = \frac{b}{a} = -\frac{3}{2}x$

$c^2 = 4 + 9 \Rightarrow c = \sqrt{13}$

2)



$a = 2$
$b = 3$
$c^2 = a^2 + b^2$
$\Rightarrow c^2 = 4 + 9 \Rightarrow \boxed{c = \sqrt{13}}$



$C(0,0)$
$A(0, \pm 2)$
$F(0, \pm\sqrt{13})$
$e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

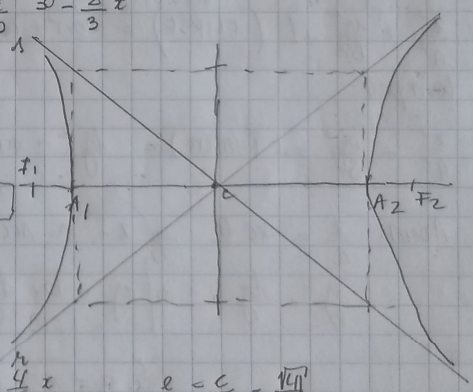
$r: y = +\frac{a}{b} = \frac{2}{3}x \quad s: -\frac{a}{b} = y - \frac{2}{3}x$

3) $16x^2 - 25y^2 = 400$

$\Rightarrow \boxed{\frac{x^2}{25} - \frac{y^2}{16} = 1} \Rightarrow \boxed{\begin{matrix} a = 5 \\ b = 4 \end{matrix}}$

$\Rightarrow c^2 = 25 + 16$

$\boxed{c = \pm\sqrt{41}}$



$C(0,0)$
$A(\pm 5, 0)$
$F(\pm\sqrt{41}, 0)$

$r: \frac{b}{a} \Rightarrow y = \frac{4}{5}x \qquad e = \frac{c}{a} = \frac{\sqrt{41}}{5}$

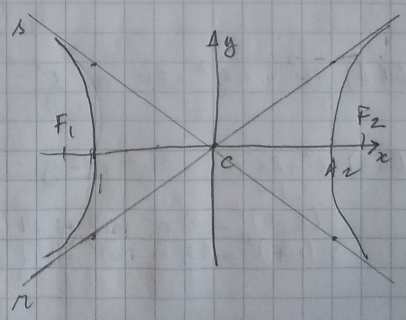
$s: -\frac{b}{a} \Rightarrow y = -\frac{4}{5}x$

4) $\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$

$\Rightarrow a = 4$
$b = 3$
$\Rightarrow c^2 = 16 + 9 \Rightarrow \boxed{c = 5}$

$C(0,0)$
$A(\pm 4, 0)$
$F(\pm 5, 0)$

$e: \frac{c}{a} = \frac{5}{4}$



$r: \frac{b}{a} \Rightarrow y = \frac{3}{4}x$

$s: -\frac{b}{a} \Rightarrow y = -\frac{3}{4}x$

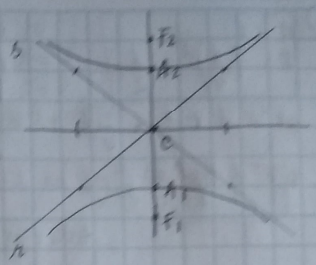
5) $5y^2 - 4x^2 = 20$

$\Rightarrow \boxed{\frac{y^2}{4} - \frac{x^2}{5} = 1} \Rightarrow \begin{cases} a = 2 \\ b = \sqrt{5} \\ c^2 = 4 + 5 \\ c = 3 \end{cases}$

$C(0,0)$
$A(0, \pm 2)$
$F(0, \pm 3)$

$e = \frac{c}{a} = \frac{3}{2}$

Ass: $y = \pm \frac{a}{b} x = \pm \frac{2}{\sqrt{5}} x = \pm \frac{2\sqrt{5}}{5}$



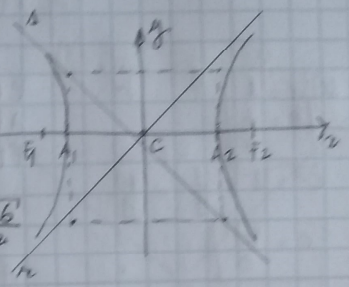
6) $x^2 - 2y^2 = 8$

$\Rightarrow \boxed{\frac{x^2}{8} - \frac{y^2}{4} = 1} \Rightarrow \begin{cases} a = 2\sqrt{2} \\ b = 2 \\ c^2 = 8 + 4 \\ c = 2\sqrt{3} \end{cases}$

$C(0,0)$
$A(\pm 2\sqrt{2}, 0)$
$F(\pm 2\sqrt{3}, 0)$

$e = \frac{c}{a} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

Ass: $y = \pm \frac{b}{a} x = \pm \frac{\sqrt{2}}{2} x$



7) $4y^2 - x^2 = 16$

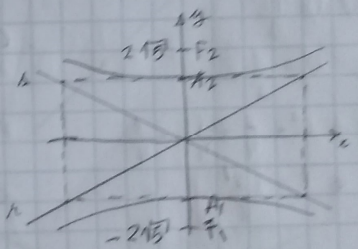
$\Rightarrow \frac{y^2}{4} - \frac{x^2}{16} = 1$

$\Rightarrow \begin{cases} a = 2 \\ b = 4 \end{cases}$
$c^2 = 20$
$\Rightarrow \boxed{c = 2\sqrt{5}}$

$C(0,0)$
$A(0, \pm 2)$
$F(0, \pm 2\sqrt{5})$

$e = \frac{c}{a} = \sqrt{5}$

Ass: $y = \pm \frac{a}{b} x = \pm \frac{2}{4} x$



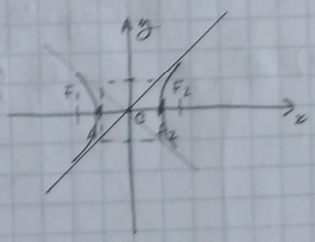
8) $x^2 - y^2 = 1$

$\Rightarrow \begin{cases} a = 1 \\ b = 1 \\ c = \sqrt{2} \end{cases}$

$C(0,0)$
$A(\pm 1, 0)$
$F(\pm \sqrt{2}, 0)$

$e = \sqrt{2}$

Ass.: $y = \pm \frac{b}{a} x = \pm x$

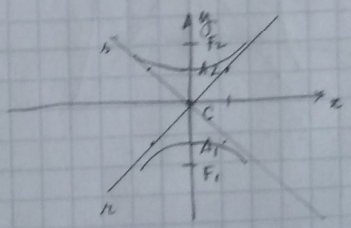


9) $\frac{y^2}{2} - \frac{x^2}{2} = 1 \Rightarrow \begin{cases} a = \sqrt{2} \\ b = \sqrt{2} \\ c = 2 \end{cases}$

$C(0,0)$
$A(0, \pm \sqrt{2})$
$F(0, \pm 2)$

$e = \frac{c}{a} = \sqrt{2}$

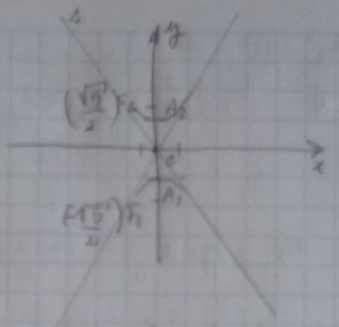
Ass.: $y = \pm \frac{a}{b} x = \pm x$



10) $a = 1$
$b = \left(\frac{1}{4}\right)^2 = \frac{1}{2}$
$\Rightarrow c^2 = 1 + \frac{1}{4} = \frac{5}{4}$
$\Rightarrow c = \frac{\sqrt{5}}{2}$

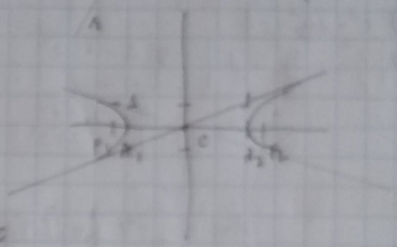
$C(0,0)$
$A(0, \pm 1)$
$F(0, \pm \frac{\sqrt{5}}{2})$
$e = \frac{c}{a} = \frac{\sqrt{5}}{2}$

Ass.: $y = \pm \frac{a}{b} x = \pm 2x$



11) $a = 1$
$b = \frac{1}{3}$
$c^2 = 1 + \frac{1}{9} = \frac{10}{9}$
$c = \frac{\sqrt{10}}{3}$

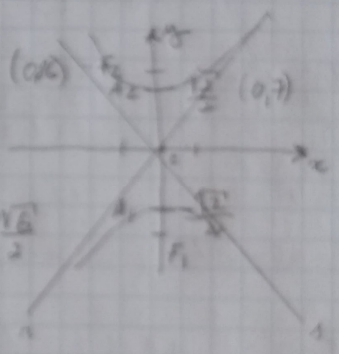
$C(0,0)$
$A(\pm 1, 0)$
$F(\pm \frac{\sqrt{10}}{3}, 0)$
$e = \frac{c}{a} = \frac{\sqrt{10}}{3}$
Ass.: $y = \pm \frac{b}{a} = \pm \frac{1}{3} x$



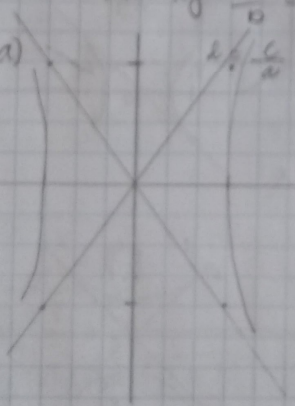
12) $a = \frac{\sqrt{2}}{2}$
$b = \frac{1}{2}$
$\Rightarrow c^2 = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
$\Rightarrow c = \frac{\sqrt{3}}{2}$

$C(0,0)$
$A(0, \pm \frac{\sqrt{2}}{2})$
$F(0, \pm \frac{\sqrt{3}}{2})$
$e = \frac{c}{a} = \frac{\sqrt{3}}{4} \cdot \frac{2}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

Ass.: $y = \pm \frac{a}{b} = \pm \sqrt{2} x$

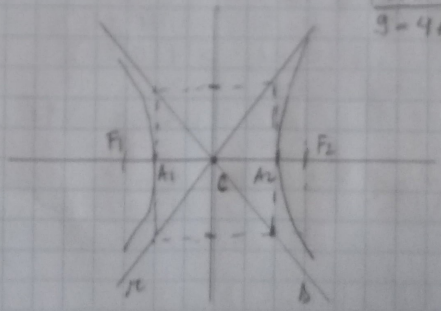


13) a)



$e = \frac{3}{2} = \frac{c}{a} \Rightarrow c = 3$
$a = 2$
$\Rightarrow b^2 = c^2 - a^2 = 3^2 - 2^2$
$b^2 = 5$
$\Rightarrow b = \sqrt{5}$

b) $e = \frac{3}{2} = \frac{c}{a} \Rightarrow \begin{matrix} c = 3 \\ a = 2 \end{matrix} \Rightarrow b = \sqrt{5}$
$9 = 4 + b^2$



14) $C(0,0)$

$c = 5$, $a = 3$

$25 = 9 + b^2$

$\Rightarrow b = 4$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

$\Rightarrow 16x^2 - 9y^2 = 144$

15) $C(0,0)$

$a = 2$

$c = 3$

$9 = 4 + b^2$

$\Rightarrow b = \sqrt{5}$

$\Rightarrow \frac{y^2}{4} - \frac{x^2}{5} = 1 \Rightarrow 5y^2 - 4x^2 = 20$

16)

$c = 4$

$a = 1$

$\Rightarrow 16 = 1 + b^2$

$\Rightarrow b = \sqrt{15}$

$\Rightarrow \frac{y^2}{1} - \frac{x^2}{15} = 1$

$\Rightarrow 15y^2 - x^2 = 15$

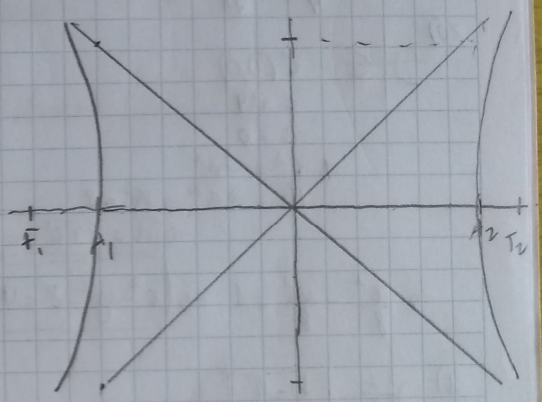
17) $c = 8$, $e = \frac{4}{3} = \frac{c}{a}$

$\Rightarrow \frac{4}{3} = \frac{8}{a} \Rightarrow 4a = 24 \Rightarrow a = 6$

$c^2 = a^2 + b^2$

$\Rightarrow 64 = 36 + b^2$

$\Rightarrow b^2 = 28 \Rightarrow b = 2\sqrt{7}$



$\Rightarrow \frac{x^2}{36} - \frac{y^2}{28} = 1 \Rightarrow \frac{x^2}{9} - \frac{y^2}{7} = 4$

$= \frac{7x^2 - 9y^2 = 252}{63}$

18) $a = 5$

$e = \frac{c}{a} = \frac{2}{1} \Rightarrow \frac{c}{5} = 2 \Rightarrow c = 10$

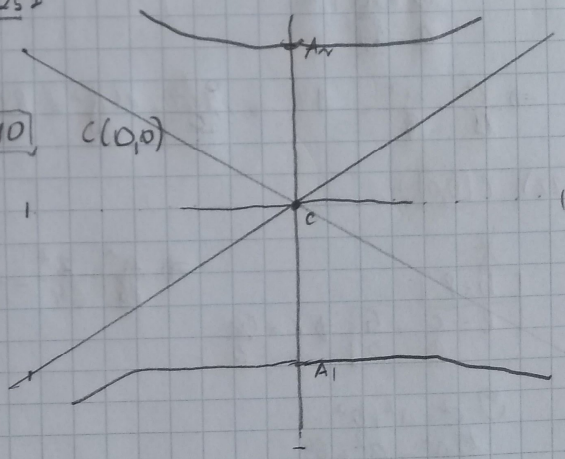
$\Rightarrow$ P) $c^2 = a^2 + b^2$

$100 = 25 + b^2 \Rightarrow b^2 = 75$

$b = 5\sqrt{3}$

75	3
25	5
5	5
1	

$C(0,0)$



HIP: $\frac{y^2}{25} - \frac{x^2}{75} = 1$

$\Rightarrow 3y^2 - x^2 = 75$

19)

$C(0,0)$

$a = 2$

c (distância focal) $= \sqrt{11}$

$c^2 = a^2 + b^2$

$11 = 4 + b^2 \Rightarrow b^2 = 7$

$\boxed{b = 2\sqrt{7}}$

hip: $\frac{y^2}{4} - \frac{x^2}{7} = 1 \Rightarrow 7x^2 - 4y^2 = 28$

40	2
20	2
10	2
5	5

20)

$C(0,0)$

$c = \pm 4$

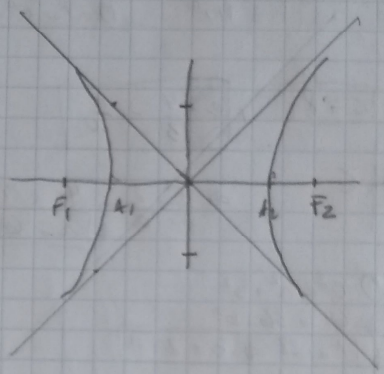
$a = b$

$\Rightarrow c^2 = 2a^2$

$16 = 2a^2 \Rightarrow a^2 = 8$

$\Rightarrow \boxed{a = 2\sqrt{2}} = b$

$\Rightarrow \frac{x^2}{8} - \frac{y^2}{8} = 1 \Rightarrow \boxed{x^2 - y^2 - 8 = 0}$



21)

$\boxed{b = 2}$

$\boxed{c = 5}$

$c^2 = a^2 + b^2$

$25 = a^2 + 4$

$a = \sqrt{21}$

$\Rightarrow \frac{x^2}{21} - \frac{y^2}{4} = 1 \Rightarrow 4x^2 - 21y^2 = 84$

22) $C(0,0)$

$\boxed{b = 8}$

$\boxed{e = \frac{c}{a} = \frac{5}{3}} \Rightarrow c = \frac{5a}{3}$

$c^2 = a^2 + b^2$

$\frac{25a^2}{9} = a^2 + 64$

$\Rightarrow \frac{25a^2}{9} = \frac{9a^2 + 576}{9}$

$\Rightarrow 16a^2 = 576$

$\Rightarrow \boxed{a = 6}$

$\Rightarrow \frac{y^2}{36} - \frac{x^2}{64} = 1 \Rightarrow \boxed{16y^2 - 9x^2 = 576}$

36, 64	2
18, 32	2
9, 16	2
9, 8	2
9, 4	2
9, 2	2
3, 1	3
1, 1	3
1	

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23)

C(0,0)

$P(8,2) \in$ hip

$\boxed{a = \pm 4}$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{64}{16} - \frac{4}{b^2} = 1 \Rightarrow \frac{4}{b^2} = 3 \Rightarrow \boxed{b = \frac{2\sqrt{3}}{3}}$

$\frac{x^2}{16} - \frac{y^2}{\frac{4}{3}} = 1$

$\text{Hip}: \frac{x^2}{16} - \frac{3y^2}{4} = 1 \Rightarrow \frac{x^2 - 12y^2 = 16}{16}$

24) $\boxed{\text{hip} \in C: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1} \Rightarrow \frac{x^2}{9} - \frac{y^2}{36} = 1 \Rightarrow \boxed{4x^2 - y^2 = 36}$

$\boxed{a = 3}$

$y = \frac{b}{a}x \Rightarrow \frac{b}{a} = \pm 2 \Rightarrow \boxed{b = \pm 2a}$

$\Rightarrow \boxed{b = \pm 6}$

25) $\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1} \Rightarrow \frac{y^2}{4} - 4x^2 = 1 \Rightarrow y^2 - 16x^2 = 4$

$\boxed{a = 2}$

$\text{Ass}: y = \pm \frac{1}{4}x = \pm \frac{b}{a}x \Rightarrow \frac{b}{a} = \frac{1}{4}$

$\Rightarrow b = \frac{a}{4}$

$\Rightarrow \boxed{b = \frac{1}{2}}$

26) $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1} \Rightarrow \frac{2x^2}{18} - \frac{2y^2}{18} = \frac{18}{18}$

$c = 3$

$\frac{b}{a} = 1 \Rightarrow a = b$

$\Rightarrow 9 = 2a^2 \Rightarrow \boxed{a = \frac{3\sqrt{2}}{2}} = b$

27) $C(3,2)$

$\Rightarrow \boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1} \Rightarrow \frac{x^2}{4} - \frac{y^2}{12} = 1$

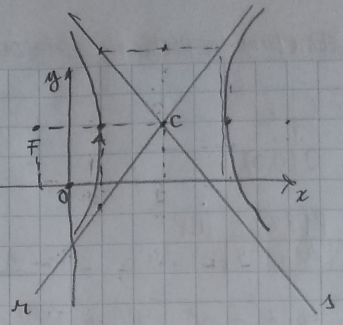
$\Rightarrow \boxed{3x^2 - y^2 = 12}$ *

$\Rightarrow \boxed{a = 2}$

$\boxed{c = 4}$

$c^2 = a^2 + b^2$

$16 = 4 + b^2 \Rightarrow b^2 = 12 \Rightarrow \boxed{b = 2\sqrt{3}}$



* $3(x-3)^2 - (y-2)^2 = 12 \Rightarrow \boxed{3x^2 - 18x + 27 - y^2 + 4y - 4 = 12x - 11}$

28) $\nearrow C \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$C(4,-2) \Rightarrow \frac{(x-4)^2}{1} - \frac{(y+2)^2}{8} = 1$

$\boxed{a = 1}$

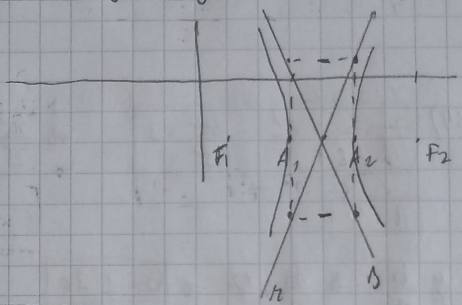
$\boxed{c = 3}$

$\Rightarrow 8(x-4)^2 - (y+2)^2 = 8$

$c^2 = a^2 + b^2$

$\Rightarrow 9 = 1 + b^2, \ b^2 = 8 \ \boxed{b = 2\sqrt{2}}$

$\boxed{8x^2 - 64x + 128 - y^2 - 4y - 4 = 8}$



29) $\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$

$C(2,-2)$

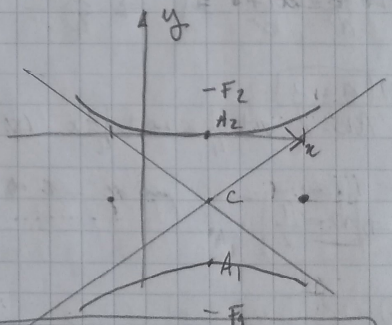
$a = 2$

$c = \sqrt{13}$

$13 = 4 + b^2, \ \boxed{b = 3}$

$\frac{(y+2)^2}{4} - \frac{(x-2)^2}{9} = 1$

$9(y+2)^2 - 4(x-2)^2 = 36 \Rightarrow \boxed{9y^2 + 36y + 36 - 4x^2 + 16x - 16 - 36 = 0}$



30) $C(5,2)$

$a = 3$

$e = \frac{c}{a} = 2 \Rightarrow \boxed{c = 2a = 6}$

$\Rightarrow 36 = 9 + b^2 \Rightarrow \boxed{b = 3\sqrt{3}}$

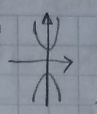
$\Rightarrow \frac{(y-2)^2}{9} - \frac{(x-5)^2}{27} = 1 \Rightarrow 3(y-2)^2 - (x-5)^2 = 27$

$\Rightarrow \boxed{3y^2 - 12y + 12 - x^2 + 10x - 25 - 27 = 0}$ 40

9	25	3
3	25	3
1	25	5
	5	5
	1	

225

9	27	3
3	9	3
1	3	3

31) 

$C(3,1)$

$c = 3$

$e = \frac{c}{a} = 2 \Rightarrow \boxed{a = \frac{c}{2} = \frac{3}{2}}$

$\Rightarrow 9 = \frac{9}{4} + b^2$

$\Rightarrow b^2 = 9 - \frac{9}{4} = \frac{36-9}{4} = \frac{27}{4}$

$\Rightarrow \boxed{b = \pm \frac{3\sqrt{3}}{2}}$

$\frac{4(y-1)^2}{9} - \frac{4(x-3)^2}{27} = 1$

$\Rightarrow 12(y-1)^2 - 4(x-3)^2 = 27$

$\Rightarrow \boxed{12y^2 - 24y + 12 - 4x^2 + 24x - 36 - 27 = 0}$

-51

32) $C(-3,1)$

$c = 3$

$a = 2$

$\Rightarrow 9 = 4 + b^2 \Rightarrow \boxed{b = \sqrt{5}}$

$\Rightarrow \frac{(x+3)^2}{4} - \frac{(y-1)^2}{5} = 1$

$\Rightarrow 5(x+3)^2 - 4(y-1)^2 = 20$

$\Rightarrow \boxed{5x^2 + 30x + 45 - 4y^2 + 8y - 4 - 20 = 0}$

$+21$

33) $C(5,1)$

$c = 4$

$\boxed{b = 2\sqrt{2}}$

$\Rightarrow 16 = a^2 + 8 \Rightarrow \boxed{a = 2\sqrt{2}}$

$(x-5)^2 - (y-1)^2 = 8$

$\Rightarrow \boxed{x^2 - 10x + 25 - y^2 + 2y - 1 - 8 = 0}$

$+16$

34) $a = b$

$HiP \parallel Oy$

$C(-3,0)$

$\boxed{a = 4 = b}$

$\Rightarrow \frac{y^2}{} - \frac{(x+3)^2}{} = 16$

$\Rightarrow \boxed{y^2 - x^2 - 6x - 9 - 16 = 0}$

35) ~~$\Rightarrow$~~ c

$C(2,-5)$

$\boxed{c = 3}$

$\boxed{a = b}$ $\Rightarrow 9 = 2a \Rightarrow \boxed{a = \frac{9}{2} = b}$

$\Rightarrow (x-2)^2 - (y+5)^2 = \frac{9}{2}$

$\Rightarrow x^2 - 4x + 4 - y^2 - 10y - 25 = \frac{9}{2}$

$\Rightarrow 2x^2 - 8x + 8 - 2y^2 - 20y - 50 = 9$

$-51 = 0$

225

Winterle, p. 214

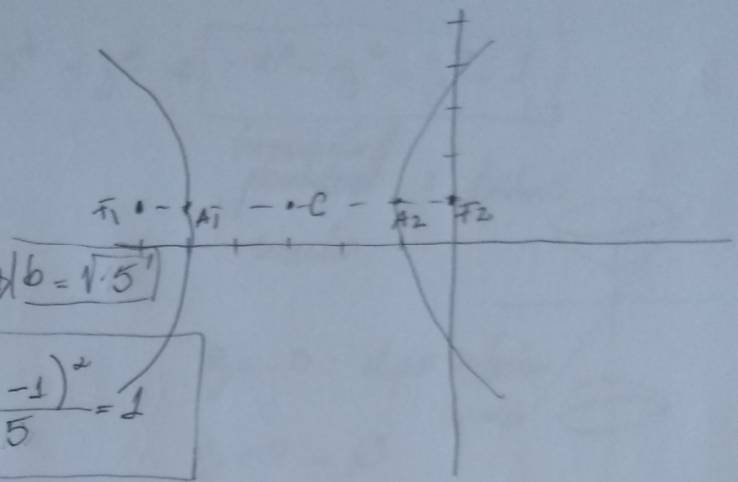
32) $C(-3,1)$

$a = 2$

$c = 3$

*) $c^2 = a^2 + b^2$

$9 = 4 + b^2 \Rightarrow \boxed{b = \sqrt{5}}$



$$\boxed{\frac{(x+3)^2}{4} - \frac{(y-1)^2}{5} = 1}$$

$\Rightarrow 5(x+3)^2 - 4(y-1)^2 = 20 \Rightarrow \boxed{5x^2 + 30x + 45 - 4y^2 + 8y - 4 - 20 = 0}$

$+21$

LÜBES EMFOCO

33) $C(5,1)$

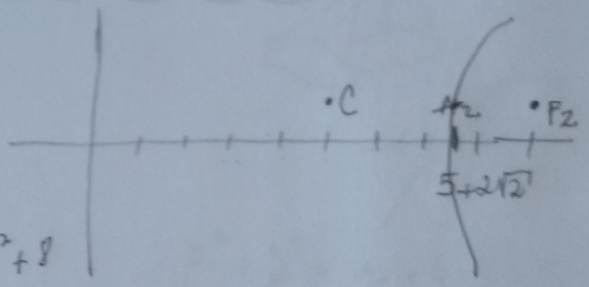
$c = 4$

$b = 2\sqrt{2}$

$a =$

*) $c^2 = a^2 + b^2 = 16 = a^2 + 8$

$\Rightarrow \boxed{a = 2\sqrt{2}}$



$$\frac{(x-5)^2}{8} - \frac{(y-1)^2}{8} = 1 \Rightarrow \boxed{x^2 - 10x + 25 - y^2 + 2y - 1 - 8 = 0}$$

$+16$

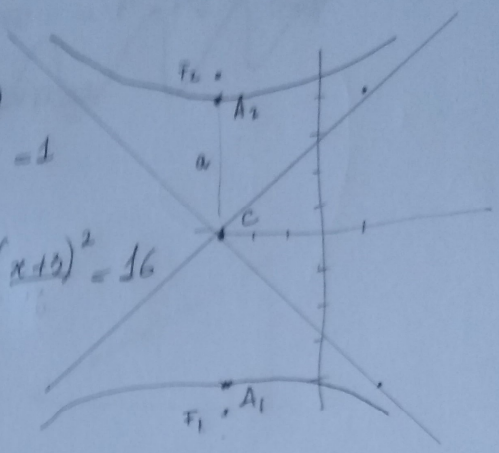
8 | 2
4 | 2
2 | 2
1

Unitário p. 214

34) $a = b$

$\Rightarrow C(-3, 0)$

$A(-3, 4) \Rightarrow \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$



$a = 4 \Rightarrow b = 4 \Rightarrow y^2 - (x+3)^2 = 16$

$c^2 = a^2 + b^2$

$\Rightarrow c^2 = 32 \Rightarrow c = 4\sqrt{2}$

$y^2 - x^2 - 6x - 9 = 16$

$\boxed{y^2 - x^2 - 6x - 25 = 0}$

LUBES EM FOCO

36) $C(2,-3)$



$A(3,-1), B(-1,0) \in$ HIP

$(A) \Rightarrow \frac{(\overset{2}{\cancel{-1+3}})^2}{a^2} - \frac{(\overset{1}{\cancel{3-2}})^2}{b^2} = 1 \Rightarrow \frac{4}{a^2} - \frac{1}{b^2} = 1$

$(B) \Rightarrow \frac{(3)^2}{a^2} - \frac{(-3)^2}{b^2} = 1$

$\Rightarrow$

$$\boxed{\begin{array}{l} \Rightarrow \frac{9}{a^2} - \frac{9}{b^2} = 1 \\ \hline -\frac{36}{a^2} + \frac{9}{b^2} = -9 \quad (-9) \end{array}}$$

$+\frac{27}{a^2} = +8 \Rightarrow a = \frac{3\sqrt{3}}{2\sqrt{2}}$

$\Rightarrow \boxed{\frac{3\sqrt{6}}{4} = a}$

$\Rightarrow \frac{\cancel{9}}{\frac{\cancel{54}}{\cancel{16}}} \frac{8}{3} - \frac{9}{b^2} = 1$

$\Rightarrow \boxed{+\frac{9}{b^2} = 1 - \frac{8}{3} = +\frac{5}{3}}$

$\Rightarrow 5b^2 = 27$

$\Rightarrow \boxed{b = \frac{3\sqrt{3}}{\sqrt{5}} = \frac{3\sqrt{15}}{5}}$

$\Rightarrow \frac{(y+3)^2}{\frac{27}{8}\frac{\cancel{54}}{\cancel{16}}} - \frac{(x-2)^2}{\frac{27}{5}\frac{\cancel{135}}{\cancel{25}}} = 1$

$\Rightarrow 8(y+3)^2 - 5(x-2)^2 = 27 \Rightarrow \boxed{8y^2 + 48y + \cancel{72} - 5x^2 + 20x - \cancel{20} - \cancel{27} = 0 \quad +25}$

37) $\boxed{C(-2,1)}$

$\boxed{\exists / \in}$

$\boxed{A(0,2), B(-5,6) \in \text{HIP}}$

$(A) \Rightarrow \frac{(0+2)^2}{a^2} - \frac{(2-1)^2}{b^2} = 1 \Rightarrow \frac{2}{a} - \frac{1}{b} = 1$

$(B) \Rightarrow \frac{(-3)^2}{a^2} - \frac{(6-1)^2}{b^2} = 1 \Rightarrow \frac{3}{a} - \frac{5}{b} = 1$

$\Rightarrow \frac{2}{a} - \frac{1}{b} = \frac{3}{a} - \frac{5}{b} \Rightarrow +\frac{1}{a} = +\frac{4}{b} = \boxed{b = 4a}$

$\Rightarrow \frac{2}{a} - \frac{1}{4a} = 1 = \frac{8-1}{4a} = 1 \Rightarrow 4a = 7 \Rightarrow \boxed{a = \frac{7}{4}}$

$\Rightarrow \frac{(x+2)^2}{\frac{49}{16}} - \frac{(y-1)^2}{49} = 1 \Rightarrow 16(x+2)^2 - (y-1)^2 = 49$

$\Rightarrow \boxed{b = 7}$

Winterle, p. 236

1) a) $(x-2)^2 + (y+3)^2 + (z-1)^2 = 16$

$\Rightarrow x^2 - 4x + 4 + y^2 + 6y + 9 + z - 2z + 1 - 16 = 0$

-2

b) $CP = P - C = (-2, 4, 1)$

$|CP| = r = \sqrt{4+16+1} = \sqrt{21} \Rightarrow r^2 = 21$

$(x-4)^2 + (y+1)^2 + (z+2)^2 = 21$

$\Rightarrow x^2 - 8x + 16 + y^2 + 2y + 1 + z^2 + 4z + 4 - 21 = 0$

c) $M = C = \frac{A+B}{2}$, $|\overline{CA}| = r$

$\Rightarrow C = (2, 1, -4)$

$\Rightarrow CA = A - C = (-3, 2, -1)$

$\Rightarrow |CA| = \sqrt{9+4+1} = \sqrt{14} \Rightarrow r^2 = 14$

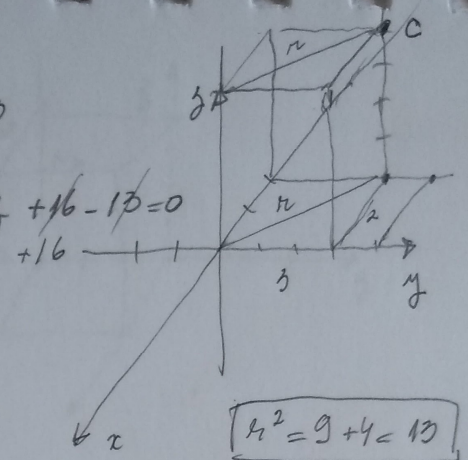
$\Rightarrow QUAD_{Eq}: (x-2)^2 + (y-1)^2 + (z+4)^2 = 14$

$= x^2 - 4x + 4 + y^2 - 2y + 1 + z^2 + 8z + 16 - 14 = 0$

$+7$

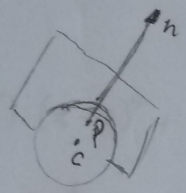
d) $(x+2)^2+(y-3)^2+(z-4)^2=13$

$= x^2+4x+4+y^2-6y+9+z^2-8z+16-13=0$



$r^2 = 9+4 < 13$

e)



$\vec{n}_\pi = (1,2,-2)$ ← ?

$P = QUAD_{EU} \cap \pi$

$P \in r \begin{cases} \mathbb{C} \\ \vec{n} \end{cases}$, $r = \begin{cases} x = t \\ y = -4+2t \\ z = 3-2t \end{cases}$

$r = |CP|$

$C(0,-4,3)$

$\pi = x+2y-2z+2=0$

$= t+2(-4+2t)-2(3-2t)+2=0$

$= t+4t-8-6+4t+2=0 \Rightarrow 9t=16 \Rightarrow t=\frac{16}{9}$

$\Rightarrow P(\frac{16}{9}, -\frac{4}{9}, -\frac{5}{9})$

$\Rightarrow CP = P-C = (\frac{16}{9}, \frac{32}{9}, -\frac{32}{9}) = |CP|^2 = \frac{256}{81}+\frac{1024}{81}+\frac{1024}{81}$

$= ?$

$-4+\frac{32}{9} = \frac{-36+32}{9}$

$3-\frac{32}{9} = \frac{27-32}{9}$

$-\frac{4}{9}+4 = \frac{-4+36}{9}$

$-\frac{5}{9}-3 = \frac{-5-27}{9}$

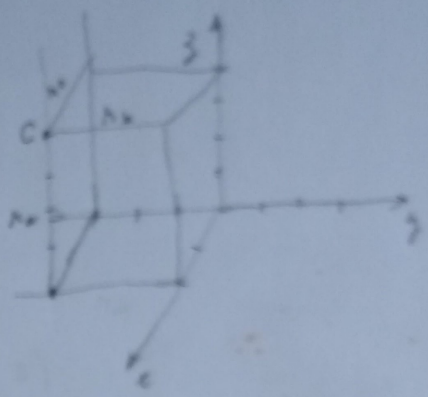
2) $QUAD_{eli} = ?$

$C(2,-3,4)$

a) $r_a = 4$

b) $r_b = |-3|$

c) $r_c = 2$



a) $(x-2)^2 + (y+3)^2 + (z-4)^2 = 16$

$\Rightarrow x^2 - 4x + 4 + y^2 + 6y + 9 + z^2 - 8z + 16 = 26$

$+13$

b) ... $= 9$

c) ... $=$

3) $\pi : ?$ / $\pi \subset P$, $P \subset QUAD_{eli}$

a) $\pi \begin{cases} \bar{n} \\ P \end{cases}$

$\bar{n} = \overline{CP} = P - C = (2,1,-2) \Rightarrow \pi = 2(x) + y + 2z + d = 0$

$C(0,0,0)$ $\Rightarrow 2 \cdot 2 + 1 + 2(2) + d = 0$

$P(2,1,-2)$ $\Rightarrow d = -9$

$\Rightarrow \pi : 2x + y - 2z - 9 = 0$

b) $C(3,-1,2)$

$P(1,-3,4)$

$P = QUAD_{Eu} \cap \pi$

$|CP| = \vec{n}_\pi$

$CP = P - C = (-2,-2,2)$

$\pi: \begin{cases} \vec{n} \\ P \end{cases} \Rightarrow -2x - 2y + 2z + d = 0$

$(P) \Rightarrow -2(1) - 2(-3) + 2(4) + d = 0$

$\Rightarrow \boxed{d = -12}$

$\Rightarrow \pi: -2x - 2y + 2z - 12 = 0$

$\Rightarrow x + y - z + 6 = 0$

c) $x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 - 6z + 9 = 11 + 4 + 1 + 9$

1, 2, 3

$(x-2)^2 + (y+1)^2 + (z-3)^2 = 25$

$C(2,-1,3)$

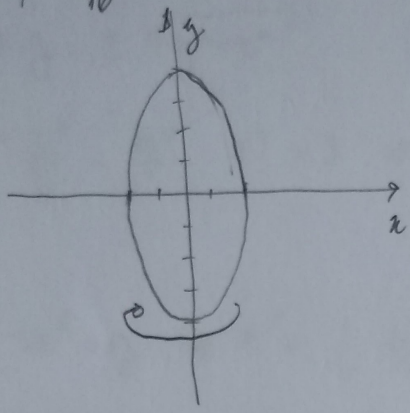
$\boxed{\vec{n} = \vec{CP} = P - C = (0,-4,3)}$

$P(2,-5,6)$

$\pi: \begin{cases} \vec{n} \\ P \end{cases} \Rightarrow 0(2) - 4(-5) + 3(6) + d = 0 \Rightarrow \boxed{d = -38}$

$\Rightarrow \pi: -4y + 3z + 38 = 0$

4) a) $\frac{x^2}{4} + \frac{y^2}{16} = 1$, $z = 0$, eixo maior



$\Rightarrow x = \pm\sqrt{x^2+z^2}$

$= \frac{x^2+z^2}{4} + \frac{y^2}{16} = 1$

$\Rightarrow \boxed{\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{4} = 1}$

b) $y = \pm\sqrt{y^2+z^2}$

$\Rightarrow \frac{x^2}{4} + \frac{y^2+z^2}{16} = 1 \Rightarrow \boxed{\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{16} = 1}$

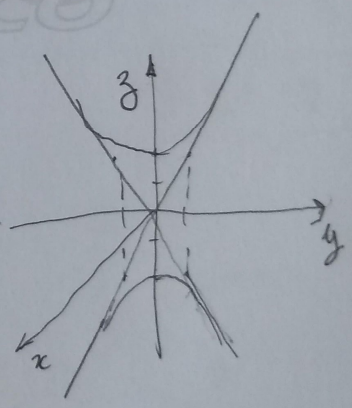
c) $\boxed{x^2 + y^2 + z^2 = 9}$

d) $\frac{1}{4}z^2 - y^2 = 1$

$z = \pm\sqrt{x^2+z^2}$

$\Rightarrow \frac{x^2+z^2}{4} - y^2 = 1$

$\boxed{\frac{x^2}{4} - y^2 + \frac{z^2}{4} = 1}$



e) $\frac{z^2}{4} - y^2 = 1$, $x = 0$

$\Rightarrow y = \pm\sqrt{y^2 + x^2}$

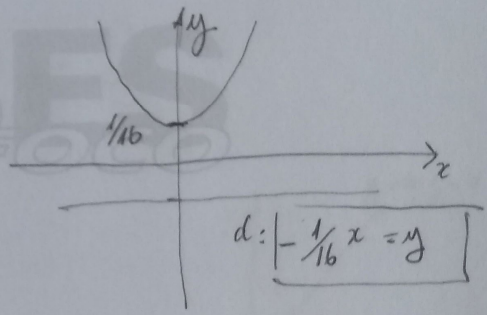
$= \frac{z^2}{4} - y^2 - x^2 = 1 \Rightarrow \boxed{-x^2 - y^2 + \frac{z^2}{4} = 1}$

f) $4x^2 = y \Rightarrow x^2 = \frac{1}{4} y$

$\Rightarrow \frac{1}{4} = 2p \Rightarrow p = \frac{1}{8}$, $\frac{1}{2} p = \frac{1}{16}$

$x = \pm\sqrt{x^2 + z^2}$

$\Rightarrow \boxed{4x^2 + 4z^2 = y}$



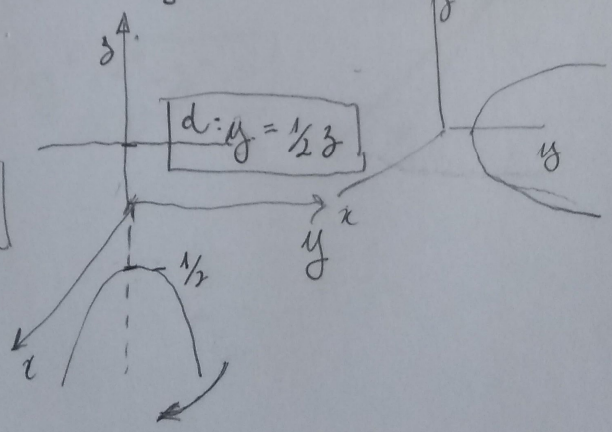
$d: \boxed{-\frac{1}{16} x = y}$

ou $z = -2y^2$

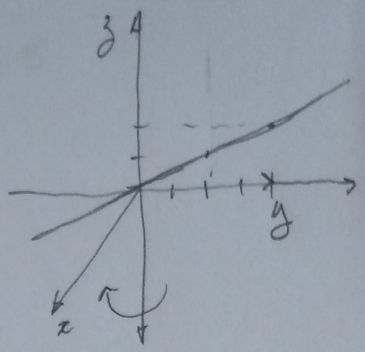
$\Rightarrow y^2 = -\frac{1}{2} z \Rightarrow p/2 = 1/8$

$y = \sqrt{y^2 + x^2}$

$\Rightarrow \boxed{y^2 + x^2 = -\frac{1}{2} z}$



h) $z = 2y$



$y = \pm\sqrt{y^2 + x^2}$

$\Rightarrow z = \pm 2\sqrt{y^2 + x^2}$

$\Rightarrow z^2 = 4y^2 + 4x^2$

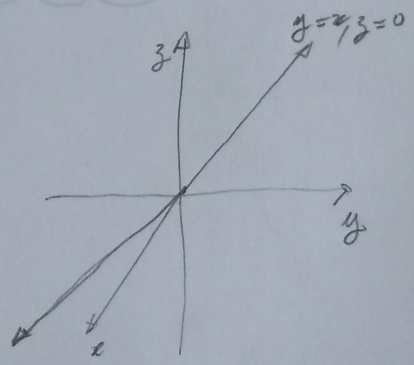
$\Rightarrow y^2 + x^2 - \frac{z^2}{4} = 0 \Rightarrow \boxed{x^2 + y^2 - \frac{z^2}{4} = 0}$

i) $z = 2y$, $\circlearrowleft Oy$

$z = \pm\sqrt{z^2 + x^2}$

$\Rightarrow z^2 + x^2 = 4y^2 \Rightarrow \boxed{x^2 - 4y^2 + z^2 = 0}$

j) $y = x$, $z = 0$, $\circlearrowleft Oy$



$y = \pm\sqrt{x^2 + z^2}$

$\Rightarrow y^2 = x^2 + z^2 \Rightarrow \boxed{y^2 - x^2 - z^2 = 0}$

5) a) $\frac{x^2}{5^2} + \frac{y^2}{5^2} + \frac{z^2}{5^2} = 1$, esfera $C(0,0)$, $r = 5$

b) $2x^2 + 4y^2 + z^2 = 16$

$\Rightarrow \frac{x^2}{8} + \frac{y^2}{4} + \frac{z^2}{16} = 1$

c) $36x^2 + 16y^2 + 9z^2 = 144$

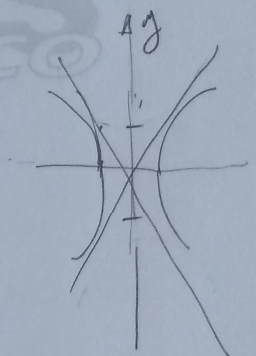
$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$

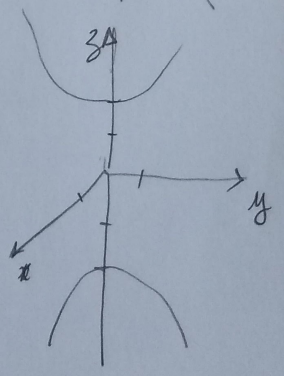
d) $36x^2 + 16y^2 - 9z^2 = 144$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$

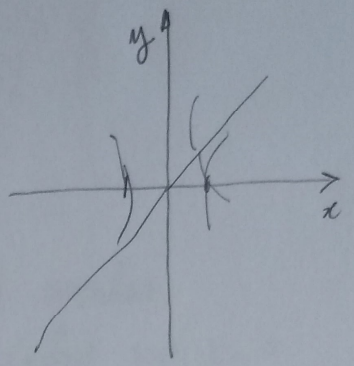
e) $x^2 - \frac{y^2}{4} + 1z^2 = 1$

f) $\frac{z^2}{4} - x^2 - y^2 = 1$ $\Rightarrow$ Oz





g) $-x^2 + \frac{y^2}{4} - \frac{z^2}{2} = 1$



h) $4x^2 + z^2 = y$

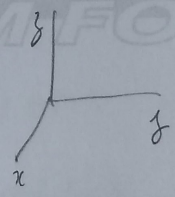
$\Rightarrow \frac{x^2}{1/4} + \frac{z^2}{1} = y$

i) $9x^2 + 4y^2 + 9z = 0$

$\Rightarrow \boxed{\frac{x^2}{1} - \frac{y^2}{9/4} = z}$

LUBES EMFOCO

j) $y^2 + 4z^2 = x$



$\boxed{\frac{y^2}{1} + \frac{z^2}{1/4} = x}$

k)

l) $36x^2 + 9z^2 = 4y^2 \Rightarrow y^2 = \frac{x^2}{1/9} + \frac{z^2}{4/9}$

m) $4x^2 + 4y^2 = z^2$ → sup cônica
$\frac{1}{4}$ $\frac{1}{4}$

n) ✓

o) $x^2 + y^2 - z + 2 = 0$ → parabolóide circular,
$= \frac{x^2}{2} - \frac{y^2}{2} + \frac{z}{2} = 1$ mas c/ vértice em $z = 2$

p) $x^2 + y^2 = -z$ → parabóloide circular virado p/
z negativo, vértice (0,0)

q) $x^2 + y^2 = -z + 6 = -(z - 6)$

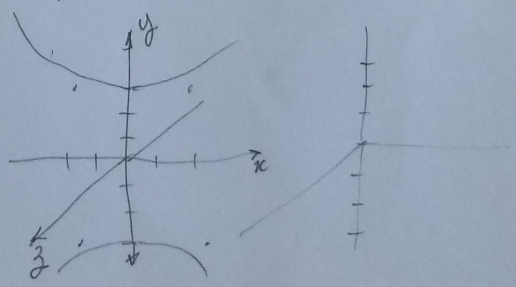
r) $x^2 + z^2 = +1(y + 2)$

s) círculo, ou cilindro $r : 3$

t) $x^2 = z$, $\boxed{p/2 = 1/4}$ ~~parabolóide circular~~
sup. cilíndrica parabólica

u) $x^2 = -z + 4$
$\Rightarrow x^2 = -(z - 4)$ ⇒ Sup. cilíndrica parabólica

v)



6) a) $(-3)\ \frac{x^2}{1} + \frac{y^2}{4} = 1$

$(-2)\ \frac{x^2}{1} + \frac{y^2}{4} = +\frac{2}{3}$

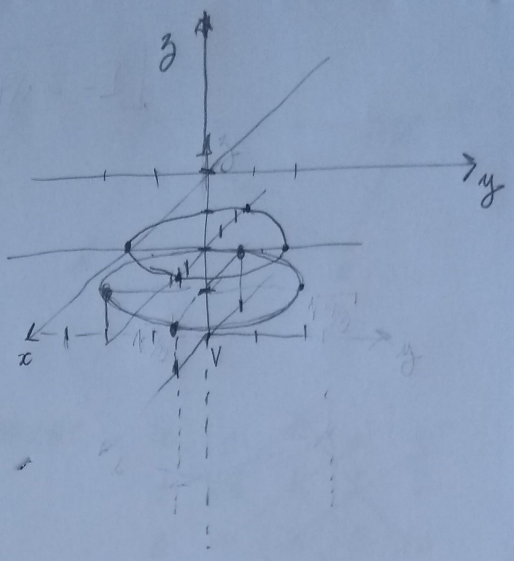
$\Rightarrow \frac{12x^2 + 3y^2 = 8}{\times 2}$

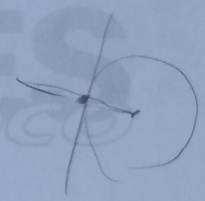
$\Rightarrow \frac{3}{2}x^2 + \frac{3}{8}y^2 = 1$

$= \frac{x^2}{\frac{2}{3}} + \frac{y^2}{\frac{8}{3}} = 1$

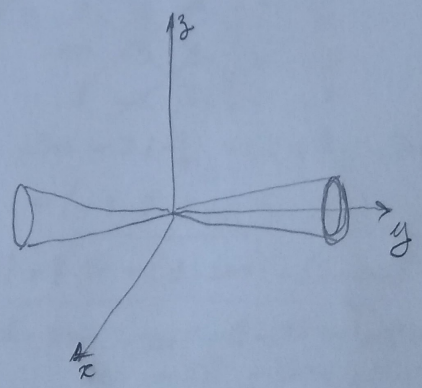
$\frac{2\sqrt{3}}{3}$ ↑ 1.15

$\frac{2\sqrt{6}}{3}$ ↑ 1,6





b)

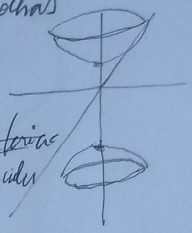


Winterle, p. 237

6c) $-1 = x^2 + y^2 - z^2 \Rightarrow \boxed{-x^2 - y^2 + z^2 = 1}$

hiperboloide ~~parabólico~~ 2 folhas

circular

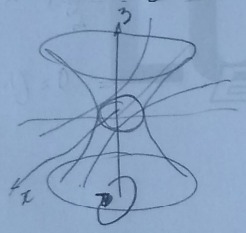


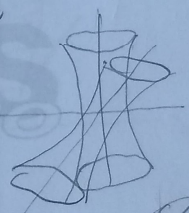
$z = -3$: elipse ~~reta~~ circular

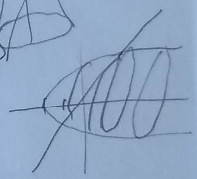
$z = 0 = \emptyset$

$z = 3$ elipse ~~reta~~ circular

d) $1 = x^2 + y^2 - z^2$ → ~~parabolóide~~ 1 folha hiperboloide







e) $y = -2$

$x^2 + \frac{z^2}{2} = 0$ duas retas?

$x^2 + \frac{z^2}{2} = y + 2$

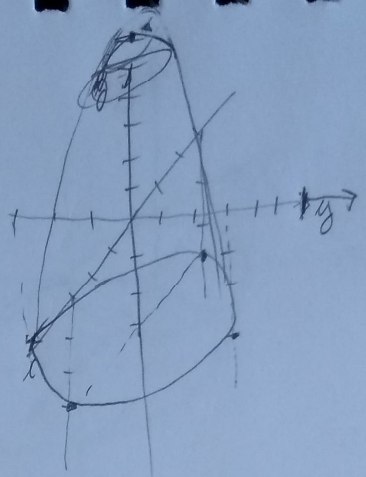
$y = 0 \Rightarrow x^2 + \frac{z^2}{2} = 2 =$ ~~sup. cilíndrica~~ elipse

~~ou sup~~

f) $y - 6 = -x^2 - z^2$

$\Rightarrow x^2 + z^2 = 6 - y$

$= -1(-6 + y)$



$(-3) = x^2 + z^2 = 9$

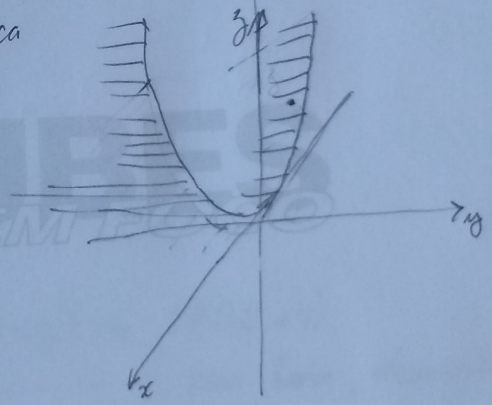
$(5) = x^2 + z^2 = 1$

parabolóide circular

g) sup cilíndrica parabólica

$x^2 = 2z$

(

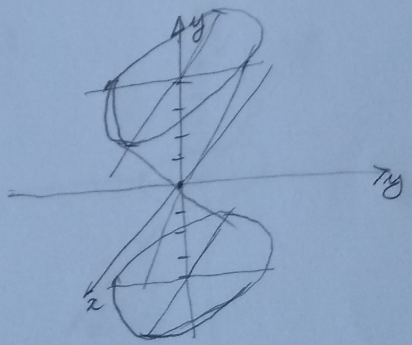


h) $x^2 - y^2 + z^2 = 0$

$(-4) = x^2 + z^2 = 16$

$(0) x^2 + z^2 = 0$

(4)



cônica circular

i) $\frac{y^2}{2} + z^2 = 4 + x$

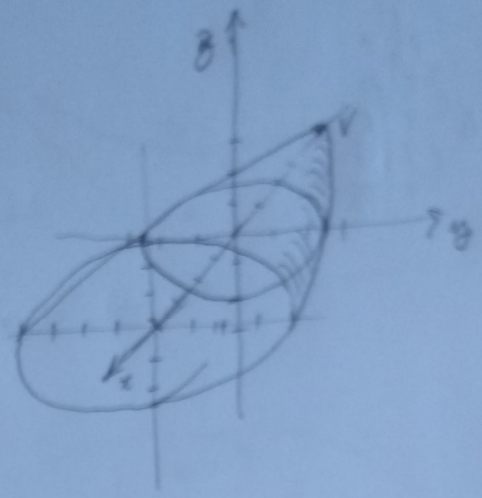
$(x=-4) \Rightarrow \frac{y^2}{2} + z^2 = 0$

$(x=0) \frac{y^2}{2} + z^2 = 4$

$\Rightarrow \frac{y^2}{8} + \frac{z^2}{4} = 1$

$(x=4) \Rightarrow \frac{y^2}{2} + z^2 = 8$

$= \frac{y^2}{16} + \frac{z^2}{8} = 1$



paraboloide elíptico

LUBES

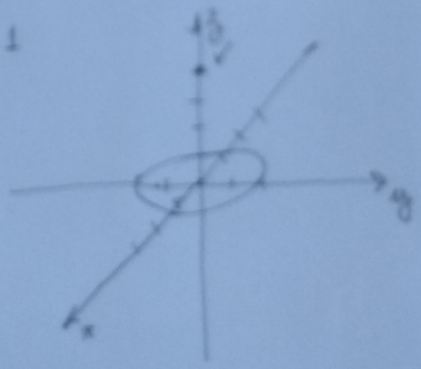
j) $-2x^2 - y^2 = -4 + z$

$\Rightarrow 2x^2 + y^2 = +4 - z$

$\Rightarrow 2x^2 + y^2 = -1(-4+z) \Rightarrow V(0,0,+4)$

para baixo, decrescente

(z=0) $2x^2 + y^2 = 4 \Rightarrow \frac{x^2}{2} + \frac{y^2}{4} = 1$



paraboloide elíptico