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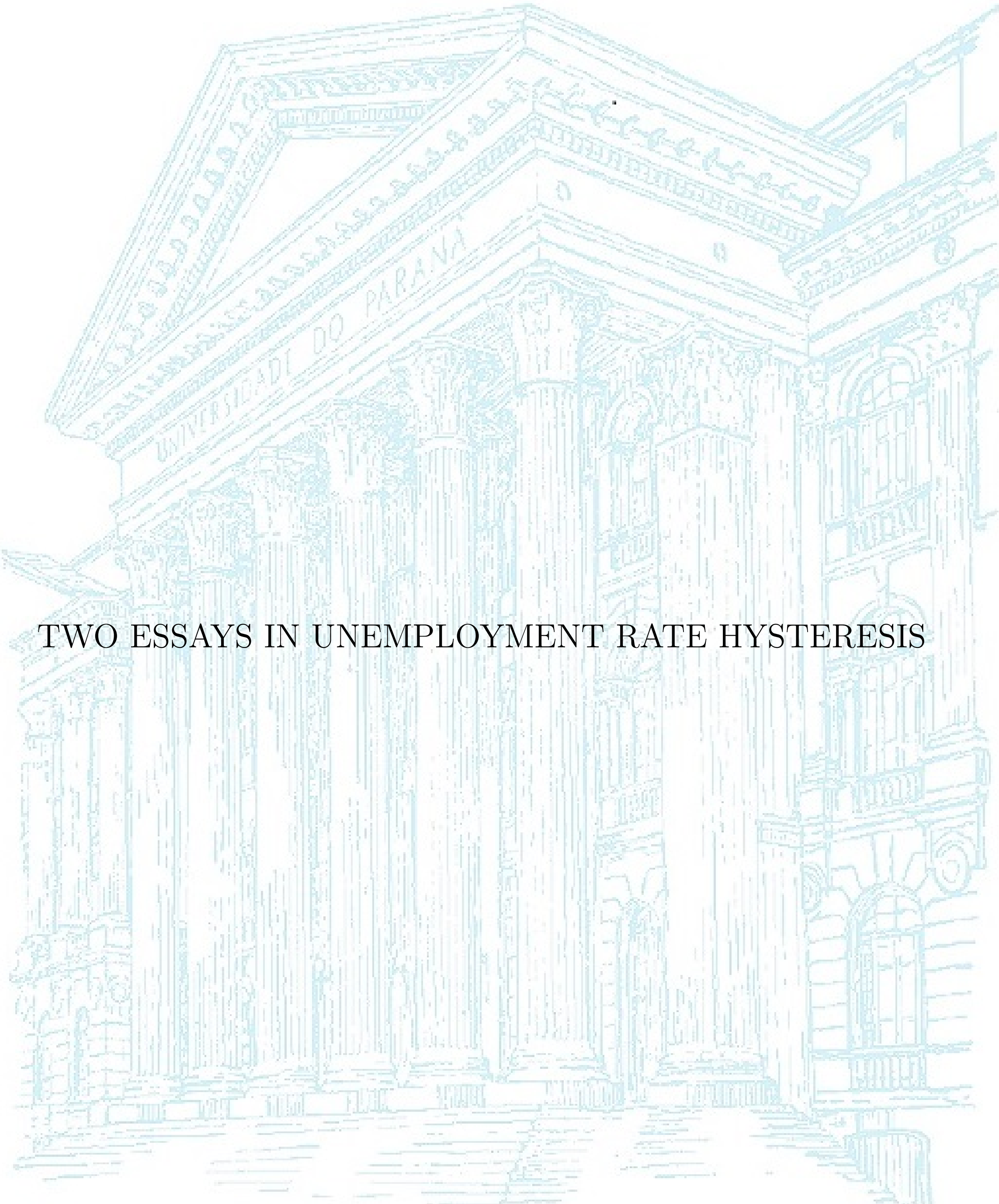
GUILHERME ALEXANDRE TOMBOLO



TWO ESSAYS IN UNEMPLOYMENT RATE HYSTERESIS

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TWO ESSAYS IN UNEMPLOYMENT RATE HYSTERESIS

Thesis presented as partial requirement for the degree of Doctor by the Graduate Program in Economic Development, Sector of Applied Social Sciences, Federal University of Paraná.

Advisor: Prof. Dr. João Basilio Pereima Neto

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TERMO DE APROVAÇÃO

Os membros da Banca Examinadora designada pelo Colegiado do Programa de Pós-Graduação em DESENVOLVIMENTO ECONÔMICO da Universidade Federal do Paraná foram convocados para realizar a arguição da tese de Doutorado de **GUILHERME ALEXANDRE TOMBOLO** intitulada: **Two Essays in Unemployment and Hysteresis**, após terem inquirido o aluno e realizado a avaliação do trabalho, são de parecer pela sua APROVAÇÃO.

Curitiba, 24 de Abril de 2017.

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DEDICATION

Dedico este trabalho a minha amada esposa Karime que foi fundamental para que eu superasse as dificuldades e concluísse esse trabalho; também o dedico aos meus pais, Aldivino e Célia, que sempre apoiaram meus estudos; e aos meus irmãos Fernando, Vinicius (*in memoriam*) e Matheus.

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RESUMO

O objetivo desta tese é analisar os efeitos da possível presença de histerese sobre a taxa de desemprego no Brasil. Vamos perseguir este objetivo através de dois ensaios ou artigos. No primeiro ensaio ou artigo "*Dynamic Effects of Hysteresis in Brazilian Unemployment*", testaremos a hipótese da presença de histerese total na taxa de desemprego brasileira por meio de um modelo de cointegração entre salário real médio, produto real per capita e taxa de desemprego proposta por Balmaseda et al. (2000). De acordo com a hipótese adequada dada pelo teste de cointegração [histerese parcial (fraca) ou histerese total (forte)], estimamos um modelo SVAR para identificar três choques: produtividade, demanda e oferta de trabalho. Estimado o modelo, analisamos a dinâmica do salário real médio, da produto real per capita e da taxa de desemprego e da variância dos erros de previsão. No segundo ensaio ou artigo, "*Hysteresis in a New Keynesian DSGE*", expandimos o modelo de desemprego de Galí (2011a,b) para considerar a hipótese de histerese na taxa de desemprego. Com histerese total, os vários choques que afetam a economia têm um efeito permanente sobre o emprego e a taxa de desemprego.Em uma economia deste tipo a taxa de desemprego não tende a uma certa média ou a uma "taxa natural" de desemprego no longo prazo. Neste artigo inserimos histerese no modelo Novo-Keynesiano padrão e estimamos dois DSGEs bayesianos, um com histerese e outro sem histerese, e comparamos seus comportamentos em relação às funções de resposta ao impulso e decomposição da variância do erro de previsão.

ABSTRACT

The aim of this thesis is to analyze the effects of the possible hysteresis presence on the Brazilian unemployment rate. We will pursue this objective through two essays or papers. In the first essay or paper, “Dynamic Effects of Hysteresis in Brazilian Unemployment”, we will test the hypothesis of total hysteresis presence in the Brazilian unemployment rate through a cointegration model between real wage, real output per capita and unemployment rate proposed by Balmaseda et al. (2000). According to the adequate hypothesis given by the cointegration test [partial (weak) hysteresis or total (strong) hysteresis], we estimated to SVAR model to identify three shocks: productivity, demand and labor-supply. With the SVAR model identified, we analyze the dynamics of real wage, real output and unemployment rate and the forecast errors variance (FEV). The sample we have covers the 1982Q3-2015Q4 period. In addition to estimating the model for the full period, we divide the sample into three parts to deal with the transformations suffered by the Brazilian economy in such period. The splits are: "before Real Plan” (1982Q3-1994Q2), "after Real Plan” (1994Q3-2015Q4) and "Inflation Targeting” regime (1999Q1-2015Q4). In the second essay or paper, “Hysteresis in a New Keynesian DSGE”, we expand the Galí (2011a,b) unemployment model to consider the hysteresis in unemployment rate hypothesis. With full hysteresis, the various shocks affecting the economy have a permanent effect on employment and unemployment rate. In an economy of this type the unemployment rate do not tend to a certain mean or to a “natural rate” of unemployment in the long-run. In this paper we insert hysteresis in the standard New Keynesian Model and estimate two Bayesian DSGEs, one with hysteresis and other without hysteresis, and compare their behaviors in regard to impulse responses and decomposition of forecast error variance.

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1 THESIS INTRODUCTION

Most of the modern theory of monetary policy takes into account some concept of the "natural rate of unemployment" or "NAIRU"[1] (Ball, 2009). According to the natural rate theory there is a point of equilibrium for the unemployment rate and output in the long-run that depend only on supply side factors such as technology, people's preferences, political and social institutions, etc. The natural unemployment rate would not be affected by aggregate demand factors in the long-run.

Expansionary monetary or fiscal policies could reduce the unemployment rate below the natural level for some time, which would also raise the price level in some amount. In the long-run, lower unemployment rate would be incompatible with the economy's "natural" equilibrium and therefore unemployment should rise to its "natural" level. After the adjustment process, the economy would return to the same levels of natural unemployment rate and output, but with higher inflation rate.

Proponents of the natural rate of unemployment concept admit that it can vary over time, but influenced by factors such as technology, minimum wages, unions, labor legislation, etc. Critics of the natural rate idea, meanwhile, say that unemployment rates seem to be more persistent over time (with delays to mean revert) than the natural rate theory might suggest. In some periods the unemployment rate would tend to remain low while in others it would tend to be high, that is, the unemployment rate would be strongly influenced by its own past values. This property of a process is called "hysteresis".

The concept of unemployment rate hysteresis was proposed by Blanchard and Summers (1986, 1987), Layard and Nickell (1987) and Lindbeck and Snower (1988a,b), who explain hysteresis with an insider-outsider theory of wage bargaining; and by Cross (1987) and Barro (1988), which explain hysteresis as a result of the "discouragement" of the long-term unemployed workers. Insider-outsider theories claim that workers already employed do not take into account the unemployed when negotiating wages and hiring conditions, which could lead to wage setting incompatible with lower unemployment rates. Dobbie (2004) makes a great review of the literature on hysteresis and theories of Insider-Outsiders labors markets.

[1]The concepts of "natural rate of unemployment" and NAIRU (Non-Accelerating Inflation Rate Of Unemployment) were developed by Friedman (1968) and Phelps (1968).

According to Ball (2009), although there is good empirical evidence for the existence of unemployment rates hysteresis, the same empirical evidence rejects Insider-Outsider theories. Still according to Ball a more promising explanation would have to do with long-term unemployment, i.e., with people unemployed long time. Workers unemployed for a long time lost their skills and productive experiences and become unattractive to employers. The unemployment for a long time can leave people discouraged and make them stop searching for a job with determination.

To explain the effect of hysteresis on inflation and the policy challenges that may emerge, we have developed a small model below.[2] In equation below, we make u_t^n represent the natural rate of unemployment, u_t is the unemployment rate, x_t are exogenous factors that affect the natural rate of unemployment, and α and β are parameters:

$$u_t^n = \alpha u_{t-1} + \beta x_t \tag{1.1}$$

Below we have a monetarist Phillips curve raised by expectations, where π is the inflation rate and δ is a parameter:

$$\pi_t = \pi_{t-1} + \delta(u_t - u_t^n) \tag{1.2}$$

Combining equation (1.1) with (1.2) we have:

$$\pi_t = \pi_{t-1} - \delta(1-\alpha)u_t - \delta\alpha(u_t - u_{t-1}) + \delta\beta x_t \tag{1.3}$$

If $\alpha = 1$, the model results in unemployment rate total hysteresis and is the first difference of the unemployment rate and not its level that matters for changes in the rate of inflation. Many authors (Lindbeck, 1991; Layard and Bean, 1989) consider that the total hysteresis hypothesis is very strong. They argue that realistic theories of unemployment should include mechanisms that sooner or later will bring unemployment rates to some "normal" level. Considering the previous equation (1.3), these authors consider the case of total hysteresis ($\alpha = 1$) as a special case, and models with "partial hysteresis" ($0 < \alpha < 1$) as the general case. When ($\alpha = 1$), the unemployment rate is a random walk. When

[2] This model was taken of Dobbie (2004), which was based in Hargraves-Heap (1980), Gordon (1989) and Franz (1987).

($0 < \alpha < 1$), the unemployment rate converges to a long-term average. The speed of convergence to the long-run level is inversely related to the size of α, the closer to 1 is α the longer will be the delay for unemployment rate convergence to the long-run average.

The implications of hysteresis for economic policy are important. Putting too much focus on inflation can be dangerous (Ball, 2009), because with hysteresis monetary policy shocks can leave unemployment rates excessively high for long periods or even indefinitely high if the hysteresis is total. With total hysteresis, the inflation target may be compatible with more than one level of the unemployment rate even in the long run. In the case of recessions, central banks could react more strongly to output and less to inflation for example. Ball (2009) finds that inadequate responses to recessions have contributed to high unemployment rates in some countries.

In relation to the Brazilian economy, some studies investigated the hysteresis hypothesis in the unemployment rate. Santos (2006) studies the Brazilian unemployment rate persistence using fractional integration models of the ARFIMA type (p, q, d), where q is the order of fractional integration. He concludes that the order of integration q is greater than one, implying an unemployment rate with full hysteresis. Gomes and Silva (2009) analyze the regional unemployment rates in six Brazilian metropolitan areas - São Paulo, Rio de Janeiro, Belo Horizonte, Porto Alegre, Salvador and Recife - by means of unit root tests with structural breaks. They found evidence of hysteresis in all regions except Rio de Janeiro.

Ayala et al. (2012) analyze the unemployment rate in 18 Latin American countries using unit root tests and ARFIMA models, both allowing endogenous structural break.s They found evidence that the unemployment rate is mean reverting in 16 countries including Brazil. Ferrari and Brasil (2015) using panel data including all Brazilian regions reject the unit root hypothesis in unemployment rates, which undermines the total hysteresis narrative. Santana et al. (2013) investigate the hysteresis hypothesis by means of unit root tests with structural breaks in six Brazilian metropolitan regions unemployment rates. In general, results indicate the existence of multiple breaks in the level and trend of unemployment rates in Brazil and its regions. For the period 1980:M6-2002:M12, tests reject the unit root hypothesis only for the unemployment rates of Brazil as whole and Rio de Janeiro. On the other hand, in the period 2003:M1-2013:M3, the results are favorable to the natural rate hypothesis in Porto Alegre, Rio de Janeiro and Salvador, and are favorable to the hysteresis hypothesis in Belo Horizonte, Recife and São Paulo.

It is difficult to work with the unemployment rate in Brazil because of some issues. One issue is the short period of data available, beginning in the 1980s generally. Monthly Employment Survey [*Pesquisa Mensal do Emprego* (PME) in Portuguese], the main monthly indicator of unemployment in Brazil, began to be collected in 1980 and covers only 6 metropolitan regions of the country (Recife, Salvador, Belo Horizonte, Rio de Janeiro, São Paulo and Porto Alegre) the working age population (WAP) sum in these regions is about 25% of the Brazilian (WAP). The GDP share of these metropolitan regions in Brazilian GDP was around 33% in the late 2000s, i.e., these metropolitan regions produce about a third of the Brazilian GDP and contain about a quarter of the Brazilian (WAP). In addition, the PME underwent a methodological reform in 2002 and was closed in February 2016, when it was replaced by the Continuous National Household Sample Survey [*Pesquisa Nacional por Amostra de Domicílios Contínua* (PNAD) in Portuguese][3].

The aim of this thesis is to analyze the effects of the possible hysteresis presence on the Brazilian unemployment rate. We will pursue this objective through two essays or papers. In the first essay or paper, "Dynamic Effects of Hysteresis in Brazilian Unemployment", we will test the hypothesis of total hysteresis presence in the Brazilian unemployment rate through a cointegration model between real wage, real output per capita and unemployment rate proposed by Balmaseda et al. (2000). According to the adequate hypothesis given by the cointegration test [partial (weak) hysteresis or total (strong) hysteresis], we estimated to SVAR model to identify three shocks: productivity, demand and labor-supply. With the SVAR model identified, we analyze the dynamics of real wage, real output and unemployment rate and the forecast errors variance (FEV). The sample we have covers the 1982Q3-2015Q4 period. In addition to estimating the model for the full period, we divide the sample into three parts to deal with the transformations suffered by the Brazilian economy in such period. The splits are: "before Real Plan" (1982Q3-1994Q2), "after Real Plan" (1994Q3-2015Q4) and "Inflation Targeting" regime (1999Q1-2015Q4).

[3]The Monthly Employment Survey and the Continuous National Household Sample Survey are carried out by the Brazilian Institute of Geography and Statistics [Instituto Brasileiro de Geografia e Estatística (IBGE) in Portuguese], an agency of the Brazilian federal government.

In the second essay or paper, "Hysteresis in a New Keynesian DSGE", we expand the Galí (2011a,b) unemployment model to consider the hysteresis in unemployment rate hypothesis. With full hysteresis, the various shocks affecting the economy have a permanent effect on employment and unemployment rate. In an economy of this type the unemployment rate do not tend to a certain mean or to a "natural rate" of unemployment in the long-run. In this paper we insert hysteresis in the standard New Keynesian Model and estimate two Bayesian DSGEs, one with hysteresis and other without hysteresis, and compare their behaviors in regard to impulse responses and decomposition of forecast error variance.

There are two main justifications for identifying if hysteresis is an important feature of the Brazilian unemployment rate. The first is to discover a characteristic of the Brazilian labor market. The second is related to the role that an unemployment rate with hysteresis should have in the development of public policies such as monetary and fiscal ones. As pointed out by Ball (1999, 2009), ignoring the presence of hysteresis in the unemployment rate when developing public policies can lead to socially harmful outcomes as high unemployment rates for long periods.

2 DYNAMIC EFFECTS OF HYSTERESIS IN BRAZILIAN UNEMPLOYMENT

ABSTRACT

In this paper we investigate hysteresis presence in Brazilian unemployment rate measured by the IBGE's PME. To this end, we estimate four SVAR models for the period 1982Q3-2015Q, and the subperiods 1982Q3-1994Q2 (before Real Plan), 1994Q3-2015Q4 (after Real Plan) and 1999Q1-2015Q4 (Inflation Targeting). We use the model of Balmaseda et al. (2000) to identify three shocks (productivity, demand and labor-supply). Our many findings are: first, Brazilian unemployment rate measured by IBGE's PME can be considered a process with full hysteresis (unemployment rate is a I(1) variable) in all the sub-periods. Second, demand shocks play a similar role in explaining the unemployment rate forecast error variance (FEV) in all sub-periods. Third, productivity shocks was more important in explaining the unemployment rate (FEV) in "before Real Plan" period, while labor-supply shocks were more important in the "after Real Plan" period. Third, real wages seemed be more flexible in "before Real Plan" period compared with subsequent periods, mainly compared with "Inflation Targeting" period.

2.1 INTRODUCTION

There are three theories that seek to explain the phenomenon of persistence in economic variables such as output and unemployment. These theories are known as "physical capital", "human capital" and "insider-outsider" theories or hypothesis to explain the unemployment persistence. "The physical capital story simply holds that reductions in the capital stock associated with the reduced employment that accompanies adverse shocks reduce the subsequent demand for labor and so cause protracted unemployment." (Blanchard and Summers, 1986, p.27). To support this view in the European case, frequently one quotes that despite the substantial increase in the unemployment rate, capacity utilization rate remained in normal levels on average. Thus it is argued that the existing capital stock is insufficient to employ the labor force at level of full employment.

In Neudorfer et al. (1990), is supposed that negative demand expectations can reduce investment and capital formation leading to a long lasting low labor demand. If the capital-output ratio is relatively fixed in the short term, the capital stock may be a limit to increase employment. So to Neudorfer et al. (1990) *apud* (Santos, 2006, p.20), "high investment is a necessary precondition to stimulate the labor market conditions". In Roed (1997), capacity utilization reduction to a level below what is given by a oligopolistic market can lead to high unemployment that can be reduced only through a slow capital accumulation. Still in Roed (1997), asymmetric investment decisions can lead persistent unemployment; in recessions, investments could be orientated to cut costs (with saving labor technologies), while in expansions investments could be oriented to capacity expansion.

Blanchard and Summers (1986) are skeptical about the physical capital theory of unemployment hysteresis. They give two reasons. First, there are some possibilities for labor-capital substitution in the medium-long run, so reductions in capital stock affect labor demand in the same way as a adverse supply shock. Second, substantial disinvestment during the 1930s did not prevent the recovery associated with rearmament in various countries.The same way, the civilian capital destruction in WWII did not prevent the full employment in many countries after the war. A third critique that we could add to those of Blanchard and Summers (1986), is the fact that physical capital theory is concentrated in low labor demand as a cause for persistent unemployment.

But labor supply (people willing work at prevailing average wages) also seems to play an important role in persistence of unemployment rate. The workforce seems to follow employment over the economic cycles in many countries, decreasing below normal in recessions and increasing above in expansions, although the labor force decline less than the employment leading to a rising unemployment rate in recessions. Of course a combination of the physical capital theory of persistence with other theories of hysteresis, as those that mark the skills loss of those who are unemployed as a cause of their exit from labor force, could weaken this third criticism.

A second theory or hypothesis to explain the unemployment persistence is the "Human Capital" channel. According to this theory, proposed by Phelps (1972) and Hargraves-Heap (1980) among others, prolonged unemployment can deteriorate the accumulation of human capital by workers. As stated by Blanchard and Summers (1986), "unemployed workers lose the opportunity to maintain and update their labor skills." Long lasting unemployment can also lead to a detachment from the labor market both through a discouragement to job seeking or demolition of the individual's social network, leaving him increasingly isolated and away from the formal labor market. Another channel that human capital could affect employability would be the preference of employers for short term unemployed because they have their skills intact (Roberts and Morin, 1999).

Finally, a high unemployment rate leads to a large human capital depreciation of the workers. This depreciation causes unemployment to remain high which could explain the unemployment rate persistence or hysteresis. However, once again Blanchard and Summers (1986) are skeptical that the human capital hypothesis alone can explain the persistence of unemployment rates. They say that, according to the arguments of human capital hypothesis, labor force participation should decrease rather than the unemployment rate increase after adverse shocks. Of course this observation is not valid for all the arguments in human capital hypothesis. The point of (Blanchard and Summers, 1986, p. 29) is "that to the extent that there is some irreversibility associated with unemployment shocks, it becomes more difficult to explain why temporary shocks have such large short-run effects."

The third and more popular hypothesis to explain the unemployment persistence is called "insider-outsider" theory. Insiders-outsiders models of labor market emphasize the market division between insiders and outsiders, and how insiders prevail in the process of wage determination. Initial papers in this issue, besides Blanchard and Summers (1986), are Gottfries and Horn (1987) and Lindbeck and Snower (1988c). These authors emphasize an asymmetry in the wage setting process, where outsiders are disregarded and insiders set wages in view to preserve their own jobs. Recessions reduce the number of employees and, in turn, reduces the number of insiders. This process can generate hysteresis in wages, employment and unemployment rates.

In relation to the Brazilian economy, some studies investigated the hysteresis hypothesis in the unemployment rate. Santos (2006) studies the Brazilian unemployment rate persistence using fractional integration models of the ARFIMA type (p, q, d), where q is the order of fractional integration. He concludes that the order of integration q is greater than one, implying an unemployment rate with full hysteresis. Gomes and Silva (2009) analyze the regional unemployment rates in six Brazilian metropolitan areas - São Paulo, Rio de Janeiro, Belo Horizonte, Porto Alegre, Salvador and Recife - by means of unit root tests with structural breaks. They found evidence of hysteresis in all regions except Rio de Janeiro.

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This paper will follow a different route from the ones mentioned above. We will test the hypothesis of total hysteresis presence in the Brazilian unemployment rate through a cointegration model between real wage, real output and unemployment rate proposed by Balmaseda et al. (2000). According to the adequate hypothesis given by the cointegration test [partial (weak) hysteresis or total hysteresis (strong)], we estimated a SVAR model to identify three shocks: productivity, demand and labor-supply. With the SVAR model identified, we will analyze the dynamics of real wage, real output and unemployment rate and the forecast variance errors (FEV). The sample we have covers the 1982Q3-2015Q4 period. In addition to estimating the model for the full period, we divide the sample into three parts to deal with the transformations suffered by the Brazilian economy in such period. The splits are: "before Real Plan" (1982Q3-1994Q2), "after Real Plan" (1994Q3-2015Q4) and "Inflation Targeting" regime (1999Q1-2015Q4).

The remainder of this paper is structured as follows. Section (2.2) presents the insider-outsider model of hysteresis of Balmaseda et al. (2000). Section (2.3) reviews the long-run constraint identification methodology of Blanchard and Quah (1989). Section (2.4) presents the data and its sources and apply conventional univariate unit root tests. Section (2.5) estimates the models and shows the main results: impulse responses, decomposition of the forecast variance error (FEV), and calculates a real wage rigidity index proposed by Layard et al. (1991) and compares it among the subsamples. Finally section (2.6) concludes.

2.2 BALMASEDA, DOLADO AND LÓPEZ-SALIDO (2000) MODEL

We adopt the model of Balmaseda et al. (2000) to represent the relation between labor market and economy. They outlined a small stylized model composed of five equations to identify three types of shocks: demand, productivity and labor supply. The three first equations are (in logs):

$$y = \phi\,(m - p) + \alpha a \tag{2.1}$$

$$y = a + n \tag{2.2}$$

$$p = w - a \tag{2.3}$$

where y is the log real output, $(m - p)$ is the real aggregated demand, m represents shift factors in nominal aggregated demand elements (fiscal and monetary policies for example), p is the log of price level, n represents log employment, a is a shift factor in productivity (technical progress and capital accumulation), and w represents the log of nominal wage. Equation (2.1) is a aggregate demand function with $\phi > 0$ and $\alpha > 0$ what permits that productivity factors affect the demand through consumption and investment decisions (what is implied by the permanent income theory). Equation (2.2) is a long-run constant return to scale (CRS) production function. Finally equation (2.3) is a price-setting rule as function of wages and productivity.

The following two equations describes the labor market supply side of a hysteretic perspective, one of the distinct contributions of Balmaseda et al. (2000).

$$l = \varphi\,(w - p) - \delta u + \chi \tag{2.4}$$

$$w = \arg\left[n^e = \lambda l_{-1} + (1 - \lambda)\,n_{-1}\right] \tag{2.5}$$

where l is the log of labor supply, n^e is the log of expected employment, u is the unemployment rate, e χ is a labor supply shift (reflecting preferences and institutional factors). Equation (2.4) is a labor supply function. The parameter δ is expected to be greater than zero if the discouragement effect of long term unemployment dominates the offsetting effect of secondary households members who decide participate more in labor market when the family's head becomes discouraged and leaves the labor market, otherwise $\delta \leq 0$. Balmaseda et al. (2000) taken equation (2.5) from Blanchard and Summers (1986), this equation characterizes the wage setting behavior. Targeted nominal wages w are chosen one period in advance in order to equate expected employment to a weighted average of past labor supply and employment.

The microfoundations of equation (2.5) follow an insider-outsider framework in the original formulation of Blanchard and Summers (1986), but it can represent other characteristics and theories of wage bargaining. Insiders-outsiders models of labor market emphasize the market division between insiders and outsiders and how insiders prevail in the process of wage determination. In this arrangement outsiders are disregarded and insiders set wages in view to preserve their own jobs. Recessions reduce the number of employees and, in turn, reduces the number of insiders. This process, in turn, can generate hysteresis in wages, employment and unemployment rates. Parameter $\lambda \in [0, 1]$ in equation (2.5) measures the magnitude that insiders affect the wage determination. The parameterization of equation (2.5) yields partial (weak) hysteresis if $0 < \lambda < 1$ and full (strong) hysteresis if $\lambda = 0$. The system of equations (2.1) to (2.5) is complemented by the definition of unemployment (2.6) below

$$u = l - n \tag{2.6}$$

To close the model, Balmaseda et al. (2000) consider that the shift factors in nominal aggregated demand elements m (fiscal and monetary policies for example), the shift factor in productivity a (technical progress and capital accumulation), and the labor supply shift χ (reflecting preferences and institutional factors) follow random walks like bellow

$$\Delta m = \varepsilon_d \tag{2.7}$$

$$\Delta a = \varepsilon_s \tag{2.8}$$

$$\Delta \chi = \varepsilon_l \tag{2.9}$$

Solving the system of equations (2.1)-(2.9) to $\Delta(w - p) = \Delta\omega$, Δy and u, where ω is the real wage rate, one obtains:

$$\Delta\omega = \varepsilon_s \tag{2.10}$$

$$\begin{aligned}(1 - \rho L)\Delta y = {} & \phi\Delta\varepsilon_d + [\phi + \alpha - (1 + c)(1 - \rho)]\,\Delta\varepsilon_s \\ & - (1 - \rho)\Delta\varepsilon_l + (1 + \varphi)(1 - \rho)\varepsilon_s + (1 - \rho)\varepsilon_l \end{aligned} \tag{2.11}$$

$$(1 - \rho L)u = (1 + \delta)^{-1}\left[(1 + \varphi - \phi - \alpha)\,\varepsilon_s - \phi\varepsilon_d + \varepsilon_l\right] \tag{2.12}$$

where L is the lag operator (for example $LX_t = X_{t-1}$), and $\rho = (1+\delta)^{-1}(1+\delta-\lambda)$. The parameter ρ measures the unemployment persistence in this model of partial (weak) hysteresis. These persistence is a increasing function of both the discouragement parameter (δ) and the weight of lagged labor force on wage determination λ. In this way, the model of Balmaseda et al. (2000) nest the both partial (weak) and (strong) full hysteresis. To see this note that to $b < \infty$, $\rho = 1$ if $\lambda = 0$, in this case the unemployment rate has a unit root (u is a I(1) variable) and therefore displays full (strong) hysteresis. The output and real wages are I(1) variables em both cases.

Making $L = 1$, $\Delta\varepsilon_d = \Delta\varepsilon_s = \Delta\varepsilon_l = 0$, and $0 < \lambda < 1$ (which implies $0 < |\rho| < 1$) in the system of equations (2.10)-(2.12) we obtain the log-run restrictions embedded in equations (2.10)-(2.12):

$$\begin{bmatrix} \Delta\omega \\ \Delta y \\ u \end{bmatrix} = \begin{bmatrix} c_{11}(1) & 0 & 0 \\ c_{21}(1) & c_{22}(1) & 0 \\ c_{31}(1) & c_{32}(1) & c_{33}(1) \end{bmatrix} \begin{bmatrix} \varepsilon_s \\ \varepsilon_l \\ \varepsilon_d \end{bmatrix} \tag{2.13}$$

where $c_{11}(1) = 1$, $c_{21}(1) = (1+\varphi)$, $c_{22}(1) = 1$, $c_{31}(1) = (1+\delta)(1-\rho)(1-\varphi-\phi-\alpha)$, $c_{32}(1) = [(1+\delta)(1-\rho)]^{-1}$, $c_{33}(1) = -\phi[(1+\delta)(1-\rho)]^{-1}$.

Observing equation (2.13) one can see that demand shocks are restricted to have no permanent effects on both the levels of real output and wages. This is a Blanchard and Quah (1989) type of VAR identification. Other characteristic of equation (2.13) is that it conforms with the theory of natural rate of unemployment because the unemployment rate follows a stationary process (u is a $I(0)$ variable), though with possible large persistence (weak or partial hysteresis). However, as commented earlier, the model of Balmaseda et al. (2000) encompasses the case of full (strong) hysteresis on the unemployment rate. We can permit for full (strong) hysteresis making $\lambda = 0$ in equation (2.5); $\Delta\varepsilon_d = \varepsilon_d$, $\Delta\varepsilon_s = \varepsilon_s$, $\Delta\varepsilon_l = \varepsilon_l$ in the system of equations (2.10)-(2.12), $L = 0$ in (2.11) and $L = 1$ in (2.12). These considerations implies $\rho = 1$ and u (unemployment rate) becomes a $I(1)$ rather than a $I(0)$ process; in other words, the unemployment rate has a unit root in this case. With full (strong) hysteresis, the matrix of long-run multipliers becomes:

$$\begin{bmatrix} \Delta\omega \\ \Delta y \\ \Delta u \end{bmatrix} = \begin{bmatrix} c_{11}(1) & 0 & 0 \\ c_{21}(1) & c_{22}(1) & 0 \\ c_{31}(1) & c_{32}(1) & c_{33}(1) \end{bmatrix} \begin{bmatrix} \varepsilon_s \\ \varepsilon_d \\ \varepsilon_l \end{bmatrix} \tag{2.14}$$

where, $c_{11}(1) = 1$, $c_{21}(1) = \phi + \alpha$, $c_{22}(1) = \phi$, $c_{31}(1) = (1+\delta)^{-1}(1+\varphi-\phi-\alpha)$,

$c_{32}(1) = -(1+\delta)^{-1}\phi$, $c_{33}(1) = (1+\delta)^{-1}$.

In addition to making the unemployment rate a $I(1)$ variable, the full (strong) hysteresis switches the role of aggregated demand (ϵ_d) and labor supply ϵ_l shocks as regards their permanent effect on real output. Whereas with partial (weak) hysteresis, demand shocks have not permanent effect on real output, under full (strong) hysteresis is the labor supply shock that have not permanent effect on real output. Thus considering unemployment rate stationary or non-stationary is crucial to choose the appropriate way of identifying the model of Balmaseda et al. (2000).

2.3 VAR IDENTIFICATION METHODOLOGY

We use the methodology of Blanchard and Quah (1989) to identify the three shocks contained in the model (2.10)-(2.12). A simplified version of the both reduced form models (partial and full hysteresis) can be written without loss of generality as a VAR of order 1 [VAR(1)] [1] and ignoring deterministic terms (constants and trends):

$$\begin{bmatrix}\Delta\omega_t\\ \Delta y_t\\ u_t\end{bmatrix} = \begin{bmatrix}\phi^p_{11} & \phi^p_{12} & \phi^p_{13}\\ \phi^p_{21} & \phi^p_{22} & \phi^p_{23}\\ \phi^p_{31} & \phi^p_{32} & \phi^p_{33}\end{bmatrix}\begin{bmatrix}\Delta\omega_{t-1}\\ \Delta y_{t-1}\\ u_{t-1}\end{bmatrix} + \begin{bmatrix}b^p_{11} & b^p_{12} & b^p_{13}\\ b^p_{21} & b^p_{22} & b^p_{23}\\ b^p_{31} & b^p_{32} & b^p_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{s,t}\\ \varepsilon_{l,t}\\ \varepsilon_{d,t}\end{bmatrix}$$

$$\begin{bmatrix}\Delta\omega_t\\ \Delta y_t\\ \Delta u_t\end{bmatrix} = \begin{bmatrix}\phi^h_{11} & \phi^h_{12} & \phi^h_{13}\\ \phi^h_{21} & \phi^h_{22} & \phi^h_{23}\\ \phi^h_{31} & \phi^h_{32} & \phi^h_{33}\end{bmatrix}\begin{bmatrix}\Delta\omega_{t-1}\\ \Delta y_{t-1}\\ \Delta u_{t-1}\end{bmatrix} + \begin{bmatrix}b^h_{11} & b^h_{12} & b^h_{13}\\ b^h_{21} & b^h_{22} & b^h_{23}\\ b^h_{31} & b^h_{32} & b^h_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_{s,t}\\ \varepsilon_{d,t}\\ \varepsilon_{l,t}\end{bmatrix}$$

The VARs above can be rewritten in compact form:

$$Y_{k,t} = \Phi_k Y_{k,t-1} + B_k\varepsilon^k_t, \qquad E(\epsilon_t\epsilon'_t) = \Sigma_\epsilon \tag{2.15}$$

$$Y_{k,t} = \Phi_k Y_{k,t-1} + e_{k,t}, \qquad E(e_{k,t}e'_{k,t}) = B_k\Sigma_\epsilon B'_k = \Sigma_{ek} \tag{2.16}$$

where $k = p$ for the model with partial (weak) hysteresis and $k = h$ for the model with full (strong) hysteresis; $Y_{p,t} = [\Delta\omega_t, \Delta y_t, u_t]'$, $Y_{h,t} = [\Delta\omega_t, \Delta y_t, \Delta u_t]'$, $\varepsilon^p_t = [\varepsilon_{s,t}, \varepsilon_{l,t}, \varepsilon_{d,t}]'$, $\varepsilon^h_t = [\varepsilon_{s,t}, \varepsilon_{d,t}, \varepsilon_{l,t}]'$, Φ_k and B_k are 3x3 matrices of parameters, $e_{k,t} = B_k\varepsilon^k_t$. Equation (2.15) is a SVAR and equation (2.16) is a reduced form VAR. From structural VAR (2.15), one can note that are fifteen unknowns to identify. There are nine elements in

[1] The discussion below can be extended easily to VAR(j) processes with $j > 1$, where j is the lag length, because any VAR(j) process can be written in VAR(1) form. See (Lütkepohl, 2005, p.15)

matrix B_k linking the VAR residuals of equation (2.15) with structural innovations ε_t, three variances of structural shocks $(\sigma_s^2, \sigma_l^2, \sigma_d^2)$ and three covariances $(\sigma_{s,l}, \sigma_{s,d}, \sigma_{l,d})$ in the variance-covariance matrix of structural shocks Σ_ϵ. The identification of matrix B_k is fundamental to identify the primitive shocks ε^k of the model, matrix B_k measures the contemporaneous effects of the ε shocks over the model variables. Taking the long run responses of the (2.15)

$$\begin{aligned} E\left[\sum_{s=0}^{\infty} Y_{k,t} \mid \varepsilon_t\right] &= \left(I + \Phi_k + \Phi_k^2 + \Phi_k^3 + ... + \Phi_k^{\infty}\right) B_k \varepsilon_t^k \\ &= (I - \Phi_k)^{-1} B_k \varepsilon_t^k \end{aligned} \tag{2.17}$$

and calculating the reduced form estimations of $\hat{\Phi}_k$ and $\hat{\Sigma}_e$

$$\hat{\Phi}_k = \sum Y_t Y_{t-1}' \left[\sum Y_t Y_{t-1}'\right]^{-1} \tag{2.18}$$

$$\hat{\Sigma}_{ek} = \frac{1}{T-1} \sum \left(Y_t - \hat{\Phi}_k Y_{t-1}'\right) \left(Y_t - \hat{\Phi}_k Y_{t-1}'\right)^{-1} \tag{2.19}$$

We have what is need to impose the Blanchard and Quah (1989) identifying assumption as indicated in equations (2.13) and (2.14), this assumption is that $(I - \Phi_k)^{-1} B_k = C(1)$ is lower triangular. Such assumption will help us to identify the matrix B_k of the contemporary shock effects. We want to find a matrix B_k such that

$$(I - \Phi_k)^{-1} B_k = C_k(1) = \begin{bmatrix} c_{11}(1) & 0 & 0 \\ c_{21}(1) & c_{22}(1) & 0 \\ c_{31}(1) & c_{32}(1) & c_{33}(1) \end{bmatrix} \qquad \text{and} \qquad B_k B_k' = \Sigma_\epsilon$$

As in Clarida and Galí (1994), a way of finding B_k it is through a Choleski Decomposition. For this propose, let is define a matrix Q such that

$$\begin{aligned} Q &= (1 - \Phi_k)^{-1} B_k B_k' \left[(1 - \Phi_k)^{-1}\right]' \\ &= (1 - \Phi_k)^{-1} \Sigma_{ek} \left[(1 - \Phi_k)^{-1}\right]' \end{aligned}$$

therefore

$$\text{chol}\,(Q) = (I - \Phi_k)^{-1} B_k \qquad \text{or} \qquad B_k = (I - \Phi_k)\,\text{chol}(Q) \tag{2.20}$$

where chol(Q) is the lower triangular Choleski decomposition of the matrix Q which can be obtained from estimated VAR (2.16).

Remembering that there are fifty unknowns to identify. Models (2.13) and (2.14) provide three long-run restrictions (of Blanchard and Quah (1989) type), the matrix of variance-covariance $\Sigma_{ek} = B_k \Sigma_\epsilon B_k'$ of model (2.16) provides more six restrictions (three variances and three covariances). Balmaseda et al. (2000) make two more assumptions: $\sigma^2_{\varepsilon_s} = \sigma^2_{\varepsilon_l} = \sigma^2_{\varepsilon_d} = 1$ and $\sigma_{\varepsilon_{s,l}} = \sigma_{\varepsilon_{s,d}} = \sigma_{\varepsilon_{l,d}} = 0$, that is, the variances of structural shocks are equal to one, and the covariances between them are zero (structural shocks are orthogonal). The shocks normalization may seem innocuous, but is now recognized that normalization can affect statistical inference in a SVAR, especially the confidence intervals for impulses responses. (Waggoner and Zha, 2003; Hamilton et al., 2004).

From the model (2.1)-(2.5) of Balmaseda et al. (2000), we can see that this normalization problem only appears in equation (2.1) where a unit shock on m (through ϵ_d from equation (2.7)) as ϕ effect on the output demand y_d. In equations (2.2) and (2.3), a unit shock on a (through ϵ_s from equation (2.8)) as an unit effect on the output supply y_d and price p, respectively. And in equation (2.5) a unit shock in χ (through ϵ_l from equation (2.9)) as a unit effect in labor supply l.

2.4 DATA SOURCES, TRANSFORMATIONS AND UNIT ROOT TESTS

The unemployment rate was taken from the IBGE's [2] Monthly Employment Survey (MES or PME in Portuguese). The PME was initiated in 1980, having undergone a complete revision in 1982 and two partial ones in 1988 and 1993. The research underwent a new and extensive process of methodological revision in 2001. Between 1980 and 2001, the research considered as members in Working Age Population (WAP) people aged 15 years or over. In 2002, the survey considered people with 10 years of age or older as Working Age Population members. The survey was monthly and included 6 Brazilian metropolitan regions: Recife, Salvador, Belo Horizonte, Rio de Janeiro, São Paulo and Porto Alegre. The sum of the working age population (WAP) in these regions is about 25% of the Brazilian (WAP).The GDP share of these metropolitan regions on Brazilian

[2]IBGE is the acronym for the Brazilian Institute of Geography and Statistics. IBGE is a statistical agency of the Brazilian federal government.

GDP was about 33% at the end of the 2000s, i.e., these metropolitan regions produce about one-third of Brazil's GDP and employ about a quarter of Brazilian (WAP). The (PME) was closed in February 2016 to be replaced by the Continuous National Household Sample Survey (Continuous NHSS or PNAD Contínua in Portuguese). The PNAD is also carried out by the IBGE foundation.

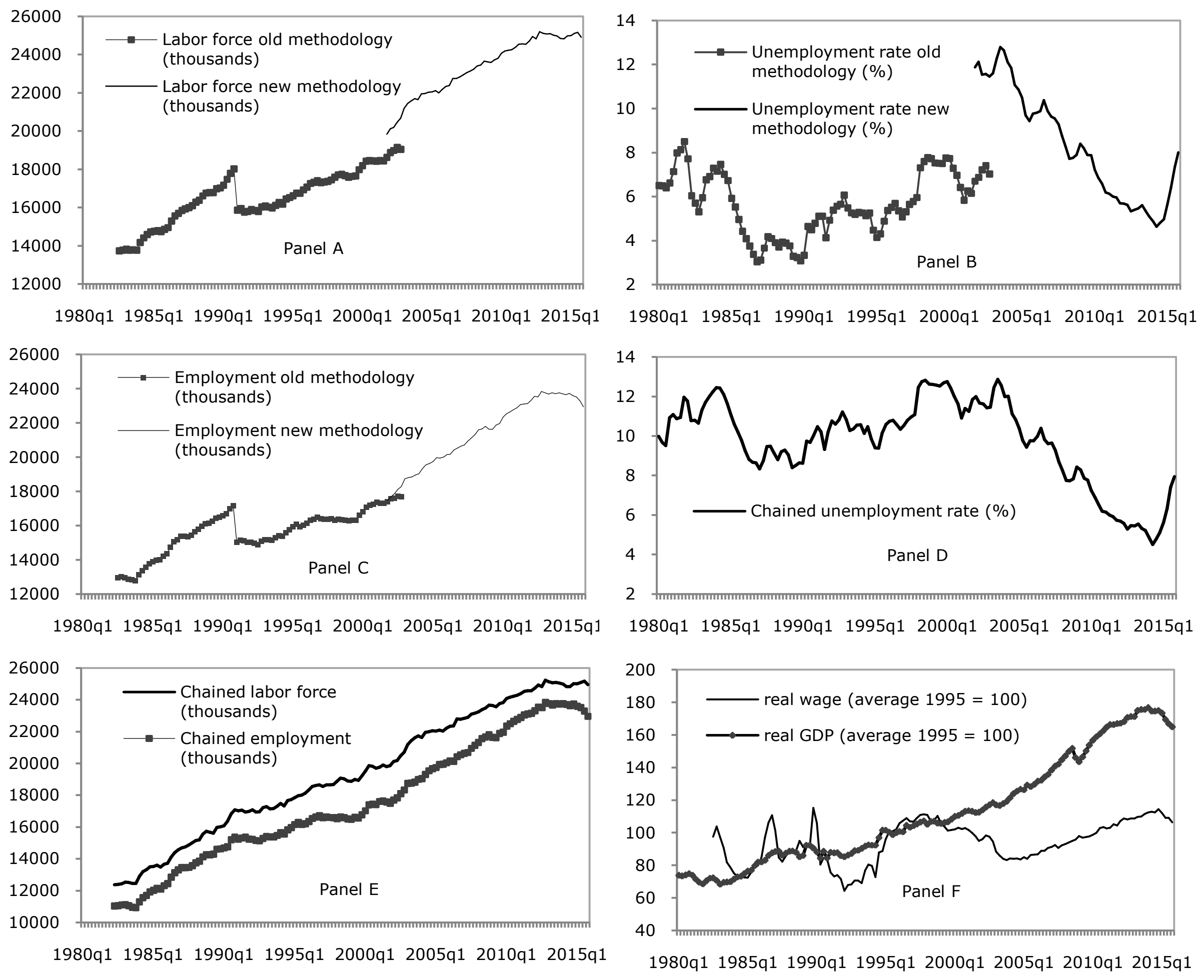

FIGURE 2.1 – LABOR FORCE, EMPLOYMENT, UNEMPLOYMENT RATE, REAL GDP AND REAL WAGE IN BRAZIL - 1980Q1 - 2015Q4

NOTE: Seasonally adjusted series (X-13 ARIMA)
SOURCE: IBGE Foundation

The labor and employment data of the PME are plotted by the old and new methodology in (FIGURE 2.1, PANEL A and C), which were aggregated for the quarterly frequency by averaging the months within each quarter. On the other hand, the unemployment rate by the old methodology did not have a break in 1991. The unemployment measured by the old methodology has a lower level compared to the unemployment measured by the new methodology. Part of this difference can be explained by the higher Working Age Population considered in the new methodology (people 10 years of age or older). The unemployment rate data by the old and new PME methodology are plotted in (FIGURE 2.1, PANEL B). To construct our database, we chain the data from the PMEs and correct a level break in the labor force and employment variables that occurred in 1991 due to sample modifications of the survey. Chained Labor force and employment data are in (FIGURE 2.1, PANEL E). Labor force and employment appear to move closely at high frequencies, but they began to move sharply in the early 1990s and then re-approximated in the mid-2000s. Chained Unemployment rate seasonally adjusted data are shown in (FIGURE 2.1, PANEL D).

The values of nominal wages were also taken from IBGE's PME and chained between the old and new methodologies to obtain a continuous series of nominal wages covering the entire period 1982Q3-2015Q4. Then the chained series of nominal wages was deflated by a consumer price index (INPC from IBGE) to create a series of real wages covering the period 1982Q3-2015Q4. Real GDP values were also taken from IBGE and were divided by a series of population to generate the real GDP per capita. In order to obtain a population series in the quarterly frequency a cubic interpolation method was applied on the decennial series of population from the Brazilian National Census (data also taken from IBGE). The graph with the series of real GDP per capita and real wage is shown in (FIGURE 2.1, PANEL F).

Before estimate the model itself, we apply unit root tests on the three modeled variables [log of real wage (ω), log of per capita GDP (y) and unemployment rate (u)]. The unit root tests of DF-GLS and PP in (TABLE 2.1) state that the three variables have a unit root with significance above 10%, which is in accordance with the consensus on these variables in the Brazilian economic literature. In the KPSS test on the unemployment rate one can not reject the null hypothesis of stationarity at 5% of significance, although this result is not corroborated by the DF-GLS and PP tests, and therefore is not robust. In the language we are using, the unemployment rate has total hysteresis (strong) according to the DF-GLS and PP tests, but on the other hand the unemployment rate has partial (weak) hysteresis according to the KPSS tests.

TABLE 2.1 – UNIT-ROOT TESTS (LOG REAL GDP, LOG REAL WAGE, UNEMPLOYMENT RATE): 1980Q1-2015Q4

Dickey-Fuller GLS[1] (DF-GLS) test						
Variable	Sample size	Constant	Trend	Lags	Test ratio	p-value
Log real wage (ω)	131	yes	yes	2	-2.2373	0.2436
Log real GDP (y)	143	yes	yes	0	-1.7699	0.4932
Unemployment rate (u)	142	yes	no	1	-1.5495	0.1525
Phillips-Perron[1] (PP) test						
Variable	Sample size	Constant	Trend	Lags	Test ratio	p-value
Log real wage (ω)	133	yes	yes	-	-2.9848	0.1404
Log real GDP (y)	146	yes	yes	-	-2.7482	0.2192
Unemployment rate (u)	143	yes	no	-	-1.4108	0.5755
KPSS[2] test						
Variable	Sample size	Constant	Trend	Lags	Test ratio	p-value
Log real wage (ω)	134	yes	yes	2	0.1483	0.0500
Log real GDP (y)	147	yes	yes	0	1.4577	0.0000
Unemployment rate (u)	144	yes	no	1	2.6901	0.0000

NOTES: (1) In the tests of Dickey-Fuller GLS and Phillips-Perron the null hypothesis is of unit root presence. (2) In the KPSS test the null hypothesis is of unit root absence.
SOURCE: Calculations made by the Eviews econometric software.

2.5 MODEL ESTIMATION AND RESULTS

Instead of applying a low power univariate unit root test to check if u is a stationary variable, Balmaseda et al. (2000) use Johansen (1995) cointegration approach which has more power due to the use of covariates. It is well known that if we have n variables, there may be at least $r \leq n-1$ cointegration vectors. In the model under analysis, $n = 3$, and $r = 0$, or $r = 1$, or $r = 2$. In order to apply the Johansen procedure, it is necessary to write the models (2.15) and (2.16) in form of a error correction model (ECM), without loss of generality we write a ECM(1) that is the ECM of a VAR(2):

$$
\begin{bmatrix} \Delta\omega_t \\ \Delta y_t \\ \Delta u_t \end{bmatrix} = \begin{bmatrix} \delta_{10} & \delta_{11} \\ \delta_{20} & \delta_{21} \\ \delta_{30} & \delta_{31} \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} \mu_{10} & \mu_{11} & \beta_{11} & \beta_{21} & \beta_{31} \\ \mu_{20} & \mu_{21} & \beta_{12} & \beta_{22} & \beta_{32} \end{bmatrix} \begin{bmatrix} 1 \\ t \\ \omega_{t-1} \\ y_{t-1} \\ u_{t-1} \end{bmatrix}
$$
$$
+ \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} \begin{bmatrix} \Delta\omega_{t-1} \\ \Delta y_{t-1} \\ \Delta u_{t-1} \end{bmatrix} + \begin{bmatrix} e_{w,t} \\ e_{y,t} \\ e_{u,t} \end{bmatrix}
$$

the above system can be rewritten in compact form

$$
\Delta X_t = \delta_0 + \delta_1 t + \alpha \left\{ \mu_0 + \mu_1 t + [\beta_1', \beta_2']' X_{t-1} \right\} + \Lambda \Delta X_{t-1} + e_t \tag{2.21}
$$

where $X_t = [\omega_t, y_t, u_t]'$; $\delta_0 = [\delta_{10}, \delta_{20}, \delta_{30}]'$ is a vector of unrestricted constants; $\delta_1 = [\delta_{11}, \delta_{21}, \delta_{31}]'$ is a vector of unrestricted trend coefficients; $\mu_0 = [\mu_{10}, \mu_{20}]'$ is a vector of restricted constants; $\mu_1 = [\mu_{11}, \mu_{21}]'$ is a vector of restricted trend coefficients; $\alpha = [\alpha_1, \alpha_2]$ is a matrix with the vectors of adjustment α_1 and α_2, with $\alpha_1 = [\alpha_{11}, \alpha_{21}, \alpha_{31}]'$ and $\alpha_2 = [\alpha_{12}, \alpha_{22}, \alpha_{32}]'$; $\beta = [\beta_1, \beta_2]$ is a matrix with the cointegration vectors β_1 and β_2, with $\beta_1 = [\beta_{11}, \beta_{21}, \beta_{31}]'$ and $\beta_2 = [\beta_{12}, \beta_{22}, \beta_{32}]'$; and $e_t = [e_{w,t}, e_{y,t}, e_{u,t}]$.

Balmaseda et al. (2000) propose the following scheme to test the unit root hypothesis on the unemployment rate. If one models a VAR in levels including (ω, y, u) and an unrestricted linear trend (*time*) and one can not reject the null hypothesis that $r = 1$ while the null hypothesis of $r = 0$ is rejected, and additionally one can not reject the hypothesis that the cointegration vector is $\beta_1 = (0, 0, 1)$, this will mean that u is a $I(0)$ variable while ω and y are I(1) processes without cointegration between the three variables. In this case the model (2.13) specification with partial (weak) hysteresis is the appropriate specification. On the other hand, if the null hypothesis of presence of at least one cointegration vector $r = 1$ is rejected, or if it is not rejected, but the null hypothesis that the cointegration vector is $\beta_1 = (0, 0, 1)$ is rejected, then the appropriate specification is (2.14) which describes the model with total (strong) hysteresis. For a better understanding of what the constraint on the β_1 cointegration vector implies, we will write the ECM (2.21) with this constraint:

$$\begin{bmatrix}\Delta\omega_t\\ \Delta y_t\\ \Delta u_t\end{bmatrix} = \begin{bmatrix}\delta_{10} & \delta_{11}\\ \delta_{20} & \delta_{21}\\ \delta_{30} & \delta_{31}\end{bmatrix}\begin{bmatrix}1\\ t\end{bmatrix} + \begin{bmatrix}\alpha_{11}\\ \alpha_{21}\\ \alpha_{31}\end{bmatrix}\begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}\omega_{t-1}\\ y_{t-1}\\ u_{t-1}\end{bmatrix}$$
$$+ \begin{bmatrix}\lambda_{11} & \lambda_{12} & \lambda_{13}\\ \lambda_{21} & \lambda_{22} & \lambda_{23}\\ \lambda_{31} & \lambda_{32} & \lambda_{33}\end{bmatrix}\begin{bmatrix}\Delta\omega_{t-1}\\ \Delta y_{t-1}\\ \Delta u_{t-1}\end{bmatrix} + \begin{bmatrix}e_{w,t}\\ e_{y,t}\\ e_{u,t}\end{bmatrix}$$

the system above can be rewritten in separated equations:

$$\begin{aligned}\Delta\omega_t &= \delta_{10} + \delta_{11}t + \lambda_{11}\Delta\omega_{t-1} + \lambda_{12}\Delta y_{t-1} + (\alpha_{11} + \lambda_{13})u_{t-1} - \lambda_{13}u_{t-2}\\ \Delta y_t &= \delta_{20} + \delta_{21}t + \lambda_{21}\Delta\omega_{t-1} + \lambda_{22}\Delta y_{t-1} + (\alpha_{21} + \lambda_{23})u_{t-1} - \lambda_{23}u_{t-2}\\ u_t &= \delta_{30} + \delta_{31}t + \lambda_{31}\Delta\omega_{t-1} + \lambda_{32}\Delta y_{t-1} + (\alpha_{31} + \lambda_{33})u_{t-1} - \lambda_{33}u_{t-2}\end{aligned}$$

The system of equations above describe a common VAR and not an ECM, note that the unemployment rate in this system is represented in levels, which is compatible with the partial (full) hysteresis model (2.13).

2.5.1 Cointegration tests

The sample we considered in the model estimation covers the period 1982Q3-2015Q4. The Brazilian economy underwent several transformations and economic policy regimes in this period. To account for these transformations we estimate four models within this period (1982-2015). The first estimated model covers the full sample (1982Q3-2015Q4). The second model covers the ante Real Plan period (1982Q3-1994Q2). The third model covers the Post-Real Plan period (1994Q3-2015Q4). And, finally, the fourth model covers the Inflation Targeting Regime (1999Q1-2015Q4).

Between 1980 and 1994, the Brazilian economy alternated periods of very high inflation with times of hyperinflation. In 1994 the government implemented an inflation stabilization plan called “Plano Real” (Real Plan). This plan consisted in exchange the old currency (Cruzeiro) for a new called Real. This exchange was made in a gradual way in order to deindex the economy. The currency was changed in July 1994. Months later, the Real Plan also adopted a Crawling Peg regime in relation to the US dollar exchange rate.

TABLE 2.2 – P-VALUES OF THE JOHANSEN COINTEGRATION TESTS: 1982Q3-2015Q4

Johansen Test 1982Q3 – 2015Q4 - full period Lags = 5 (lags of first differences in cointegration equation)										
	No Constant		Restrict Constant		Unrestricted Constant		Restrict Trend		Unrestricted Trend	
order	Trace	Lmax	Trace	Lmax	Trace	Lmax	Trace	Lmax	Trace	Lmax
0	0.00	0.00	0.01	0.01	0.42	0.38	0.16	0.25	0.05	0.21
1	0.33	0.27	0.39	0.28	0.67	0.78	0.39	0.27	0.11	0.22
2	0.75	0.75	0.77	0.77	0.22	0.22	0.80	0.80	0.06	0.06
c. vectors	1	1	1	1	0	0	0	0	1	0
Johansen Test 1982Q3 – 1994Q3 – Ante Real Plan Lags = 0 (lags of first differences in cointegration equation)										
	No Constant		Restrict Constant		Unrestrict Constant		Restrict Trend		Unrestrict Trend	
order	Trace	Lmax	Trace	Lmax	Trace	Lmax	Traco	Lmax	Trace	Lmax
0	0.57	0.84	0.73	0.66	0.71	0.59	0.27	0.24	0.07	0.17
1	0.33	0.37	0.82	0.77	0.87	0.90	0.60	0.48	0.21	0.35
2	0.33	0.33	0.80	0.80	0.36	0.36	0.82	0.82	0.08	0.08
c. vectors	0	0	0	0	0	0	0	0	1	0
Johansen Test 1994Q4 – 2015Q4 - Post Real Plan Lags = 1 (lags of first differences in cointegration equation)										
	No Constant		Restrict Constant		Unrestricted Constant		Restrict Trend		Unrestricted Trend	
order	Trace	Lmax	Trace	Lmax	Trace	Lmax	Trace	Lmax	Trace	Lmax
0	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.02	0.02	0.02	0.02	0.48	0.62	0.29	0.30	0.07	0.21
2	0.37	0.37	0.27	0.27	0.16	0.16	0.53	0.53	0.03	0.03
c. vectors	2	2	2	2	1	1	1	1	3	1
Johansen Test 1999Q1 – 2015Q4 – Inflation Targeting Lags = 1 (lags of first differences in cointegration equation)										
	No Constant		Restrict Constant		Unrestricted Constant		Restrict Trend		Unrestricted Trend	
order	Trace	Lmax	Trace	Lmax	Trace	Lmax	Trace	Lmax	Trace	Lmax
0	0.00	0.01	0.01	0.04	0.06	0.05	0.09	0.18	0.30	0.57
1	0.04	0.16	0.09	0.13	0.47	0.67	0.30	0.30	0.28	0.44
2	0.03	0.03	0.28	0.28	0.12	0.12	0.54	0.54	0.09	0.09
c. vectors	3	1	2	1	1	1	1	0	0	0

SOURCE: Calculations made by the Eviews econometric software.

Foreign currency crises that affected emerging countries in the late of 1990s (Asian countries, Russia and even Brazil) forced the Brazilian authorities to abandon the Crawling Peg regime in favor of a Inflation Targeting with floating exchange rate and primary surplus targets for government accounts. This so-called tripod regime (Inflation Targeting, Floating Exchange Rate and Primary Surplus Targeting) has been adopted up to now by the Brazilian authorities. In (TABLE 2.2), we performed cointegration tests between the three variables - real wage (ω), real per capita GDP (y) and unemployment rate (u) - for the four periods considered. In most of the results there is evidence of at least one cointegration vector among the three variables in all periods considered, except for the post Real Plan period (1994Q3-2015Q4) where the Johansen test identified three (3) cointegration vectors at 5% of significance and one (1) cointegration vector at 1% of significance both in the "Unrestricted Trend Case". In the Inflation Targeting period (1999Q1-2015Q4) in three cases ("No Constant", "Restrict Constant", and "Unrestricted Constant") the Johansen test identified two (2) cointegration vectors at 5% of significance.

Based on these results, we now test the constraint proposed by Balmaseda et al. (2000) on the cointegration vector β_1 of the unrestricted trend case. The unrestricted trend case (or quadratic deterministic trend) is the fifth case of Johansen's cointegration schemes. Recalling that if the null hypothesis that the cointegration vector is $\beta_1 = (0, 0, 1)$ is rejected, then the appropriate specification is (2.14) and the unemployment rate has total (strong) hysteresis. On the other hand, if the null hypothesis that the cointegration vector is $\beta_1 = (0, 0, 1)$ is not rejected, then the appropriate specification is (2.13) and the unemployment rate has partial (weak) hysteresis. The constraints imposed on the cointegration vector β_1 are tested and the results are shown in (TABLE 2.3).

TABLE 2.3 – TESTS ON THE VECTOR β_1 RESTRICTIONS

Cointegration case: 5 - unrestricted trend (quadratic deterministic trend) Null Hypothesis $\beta_1 = (0, 0, 1)$					
Period	Range	Lags	Obs.	$\chi^2(2)$	p-value
Full period	1982Q3-2015Q4	5	128	13.0284	0.0015
Ante Real Plan	1982Q3-1994Q2	0	47	16.7390	0.0002
Post Real Plan	1994Q4-2015Q4	3	82	20.5948	0.0000
Inflation Targeting	1999Q1-2015Q4	0	67	40.4416	0.0000

SOURCE: Calculations made by the Eviews econometric software.

According to the tests results shown in (TABLE 2.3), we can not reject the hypothesis that Brazilian unemployment rate measured by PME has a unit root in all the four periods analyzed. Thus, we can not reject the hypothesis that Brazilian unemployment rate shows

TABLE 2.4 – VAR MODELS ESTIMATED FOR THE MODELS “FULL PERIOD (1982Q3-2015Q4)” AND “BEFORE REAL PLAN (1982Q3-1994Q2)

	1982Q3-2015Q4 Full Period (Lags = 5)[1,2,3]			**1982Q3-1994Q2 Before Real Plan (Lags = 5)[1,2,3]**		
	$\Delta\omega_t$	Δy_t	Δu_t	$\Delta\omega_t$	Δy_t	Δu_t
const.	−1.195 (−2.30)**	0.668 (4.07)***	0.093 (2.31)***	−2.193 (−1.83)**	– (–)	0.078 (1.20)
$\Delta\omega_{t-1}$	– (–)	– (–)	−0.009 (−1.68)*	0.200 (1.78)*	– (–)	– (–)
Δy_{t-1}	0.519 (2.20)**	– (–)	−0.070 (−3.99)***	– (–)	– (–)	– (–)
Δu_{t-1}	– (–)	−0.677 (−1.92)*	0.145 (1.67)*	– (–)	– (–)	0.464 (5.82)***
$\Delta\omega_{t-2}$	−0.239 (−3.03)***	– (–)	0.010 (1.92)*	−0.419 (−3.19)***	– (–)	0.012 (2.46)**
Δy_{t-2}	0.840 (3.4)***	– (–)	−0.035 (−2.01)**	– (–)	– (–)	−0.065 (−4.22)***
Δu_{t-2}	– (–)	– (–)	– (–)	−8.414 (−3.37)***	– (–)	−0.530 (−5.19)***
$\Delta\omega_{t-3}$	– (–)	– (–)	– (–)	−0.018 (−4.14)***	– (–)	−0.018 (−4.14)***
Δy_{t-3}	– (–)	−0.186 (−2.57)***	– (–)	– (–)	– (–)	– (–)
Δu_{t-3}	– (–)	– (–)	– (–)	– (–)	– (–)	0.391 (4.74)***
$\Delta\omega_{t-4}$	– (–)	−0.077 (−2.93)***	0.022 (3.74)***	– (–)	−0.091 (−2.31)**	0.030 (5.34)***
Δy_{t-4}	– (–)	– (–)	−0.044 (−2.73)***	−1.241 (−3.49)***	– (–)	– (–)
Δu_{t-4}	– (–)	– (–)	– (–)	– (–)	– (–)	– (–)
$\Delta\omega_{t-5}$	– (–)	0.079 (3.28)***	– (–)	−0.232 (−2.10)**	– (–)	– (–)
Δy_{t-5}	0.760 (3.12)***	0.163 (1.91)*	−0.041 (−2.13)**	1.082 (2.99)***	0.317 (3.25)***	v – (–)
Δu_{t-5}	– (–)	– (–)	−0.199 (−2.73)***	– (–)	– (–)	– (–)
LM test for autocorrelation	lags = 5; df = 45; t. st. = 34.66; pval. = 0.87			lags = 5; df = 45; t. st. = 45.31; pval. = 0.46		
Multivariate ARCH LM test	lags = 5; df = 180; t. st. = 220; pval = 0.02			lags = 5; df = 180; t. st. = 187; pval. = 0.34		
Test for no normality	Doornik and Hansen (1994) t. st. = 196; pval. = 0.0000			Doornik and Hansen (1994) t. st. = 2.36; pval. = 0.88		
Identified long run impact matrix[4]	5.584 [0.981]	0.000 [0.000]	0.000 [0.000]	5.751 [1.205]	0.000 [0.000]	0.000 [0.000]
	1.402 [0.404]	1.252 [0.193]	0.000 [0.000]	1.293 [0.745]	2, 354 [0.551]	0.000 [0.000]
	−0.283 [0.113]	−0.347 [0,072]	0.259 [0.035]	−0.335 [0.148]	−0.331 [0.105]	0.218 [0.049]

SOURCE: VAR models calculated by the JMulTi times series software (Lütkepohl and Krätzig, 2004).
NOTES: (1) Lags selected by the Akaike criterion. (2) No significant lags were excluded from the model (considering a t-value of |1.62| as threshold) through the “system down procedure” function available in the JMulti software and explained in (Lütkepohl and Krätzig, 2004). (3) Calculated t-values are in parenthesis “(...)”, superscript (***) means significance at 1%, (**) means significance at 5% and (*) means significance at 10% . (4) Values in brackets “[...]” indicates bootstrap standard errors.

total (strong) hysteresis, and therefore the appropriate model to estimate the effects of productivity (ϵ_s), demand (ϵ_d) and labor supply (ϵ_l) shocks on the modeled variables (ω, y and u) is the model (2.14).

We estimated the four VAR models using the JMulti time series software (Lütkepohl and Krätzig, 2004). In (TABLE 2.4) we show the models estimated for the "full period" (1982Q3-2015Q4) and for the "before Real Plan" (1982Q3-1994Q2). Five lags selected by the Akaike criterion were used in both models. Non-significant lags were excluded from the model (considering a t-value of |1.62| as threshold) through the "system down procedure" function available in the JMulti software and explained in (Lütkepohl and Krätzig, 2004). In (TABLE 2.5) we show the models estimated for the "after Real Plan" period (1994Q3-2015Q4) and for "Inflation Targeting" period (1999Q1-2015Q4). The two models of (TABLE 2.5) were estimated with one lag selected by the Akaike criterion.

TABLE 2.5 – VAR MODELS ESTIMATED FOR THE MODELS "AFTER REAL PLAN" (1994Q3-2015Q4) AND "INFLATION TARGETING" (1999Q1-2015Q4)

	1994Q3-2015Q4 After Real Plan (Lags = 1)[1,2]			**1999Q1-2015Q4 Inflation Targeting (Lags = 1)[1,2]**		
	$\Delta\omega_t$	Δy_t	Δu_t	$\Delta\omega_t$	Δy_t	Δu_t
const.	−0.069 (−0.29)	0.406 (2.62)***	0.077 (1.88)*	−0.158 (−0.68)	0.403 (2.55)***	0.058 (1.37)
$\Delta\omega_{t-1}$	0.142 (1.85)*	0.083 (1.62)*	−0.013 (−0.94)	0.304 (2.60)***	0.095 (1.20)	−0.045 (−2.12)**
Δy_{t-1}	0.293 (1.71)*	0.267 (2.35)**	−0.135 (−4.51)***	0.209 (1.09)	0.372 (2.85)***	−0.176 (−5.00)***
Δu_{t-1}	−0.646 (−1.15)	0.173 (0.47)	0.206 (2.10)**	−0.359 (−0.60)	−0.136 (−0.33)	0.113 (1.02)
LM test for autocorrelation	lags = 5; df = 45; t. st. = 65.04; pval. = 0.03			lags = 5; df = 45; t. st. = 52.94; pval. = 0.19		
Multivariate ARCH LM	lags = 5; df = 180; t. st. = 165; pvalue = 0.78			lags = 5; df = 180; t. st. = 186; pval. = 0.36		
Test for nonormality	Doornik and Hansen (1994) t. st. = 47; pval. = 0.0000			Doornik and Hansen (1994) t. st. = 112; pval. = 0.0000		
Identified long run impact matrix[3]	2.576 [0.405]	0.000 [0.000]	0.000 [0.000]	2.830 [0.676]	0.000 [0.000]	0.000 [0.000]
	0.840 [0.383]	1.522 [0.253]	0.000 [0.000]	1.216 [0.645]	1.723 [0.412]	0.000 [0.000]
	−0.300 [0.138]	−0.379 [0.107]	0.386 [0.057]	−0.473 [0.212]	−0.426 [0.133]	0.316 [0.046]

SOURCE: VAR models estimated by the JMulTi software (Lütkepohl and Krätzig, 2004).
NOTES: (1) Lags selected by the Akaike criterion. (2) Calculated t-values are in parenthesis "(...)", superscript (***) means significance at 1%, (**) means significance at 5% and (*) means significance at 10% . (3) Values in brackets "[...]" indicates bootstrap standard errors.

2.5.2 Impulse Responses

In (FIGURE 2.2) the model three variables impulse response functions are shown in relation to the three identified shocks: productivity, demand and labor supply. In general, the real wage responses to the three shocks appears to be satisfactory. The real wage responds pro-cyclically to (positive) shocks of productivity and labor supply, while responds counter-cyclically to a (positive) shock of demand. This countercyclical response

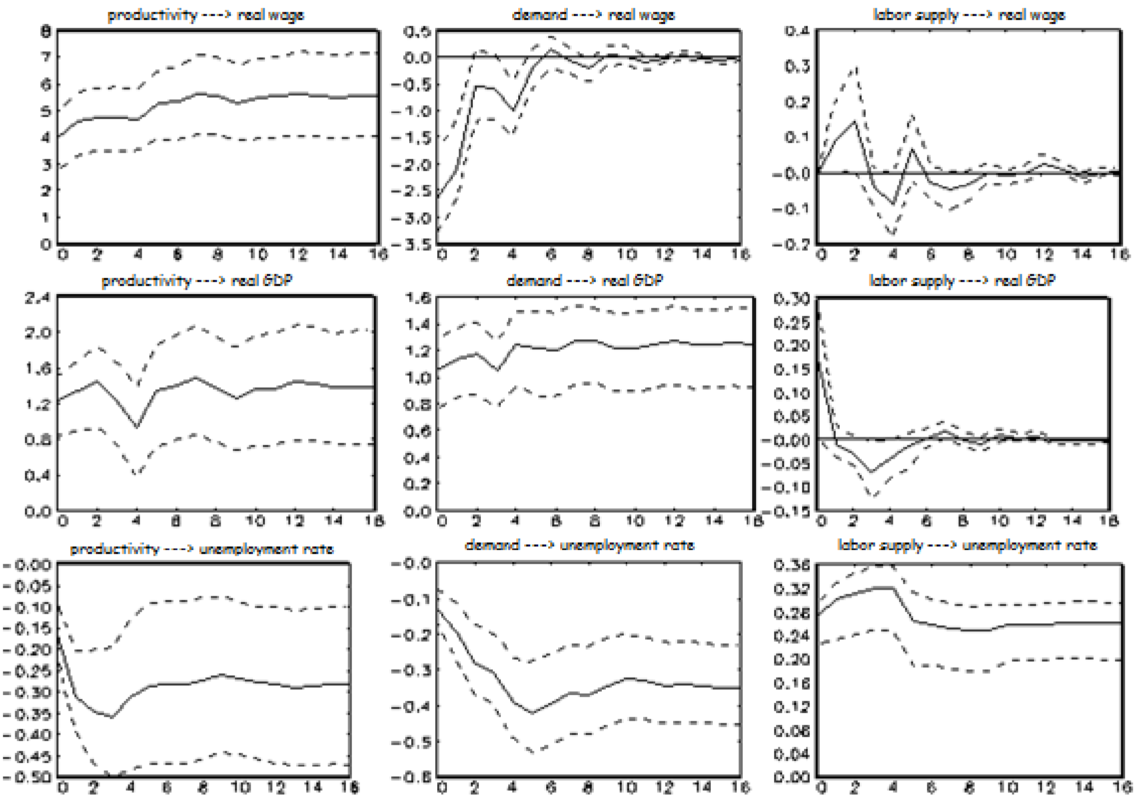

FIGURE 2.2 – ACCUMULATED IMPULSE RESPONSES (IN %) - FULL PERIOD 1982Q3-2015Q4

NOTE: Dashed lines depict approximate 90% confidence intervals computed using 10,000 bootstrap replications according to method of Efron (1979).
SOURCE: VAR model and impulse responses calculated by the JMulTi times series software (Lütkepohl and Krätzig, 2004).

of real wages also occurs in continental Europe, while wages in the United States appear to react pro-cyclically to demand shocks (Balmaseda et al., 2000). The real wage response to a labor supply shock seems to be non-significant (confidence interval passes over the zero axis). Continuing in (FIGURE 2.2), the three shocks on real per capita GDP have the expected shapes although the labor supply shock on per capita output appears to be non-significant (confidence interval passes over the zero axis). GDP per capita responds pro-cyclically to the three shocks. Still in (FIGURE 2.2), unemployment rate responses to the three shocks have the expected shapes. Unemployment rate responds counter-cyclically to shocks of productivity and demand, and reacts in a pro-cyclical way to labor supply shocks.

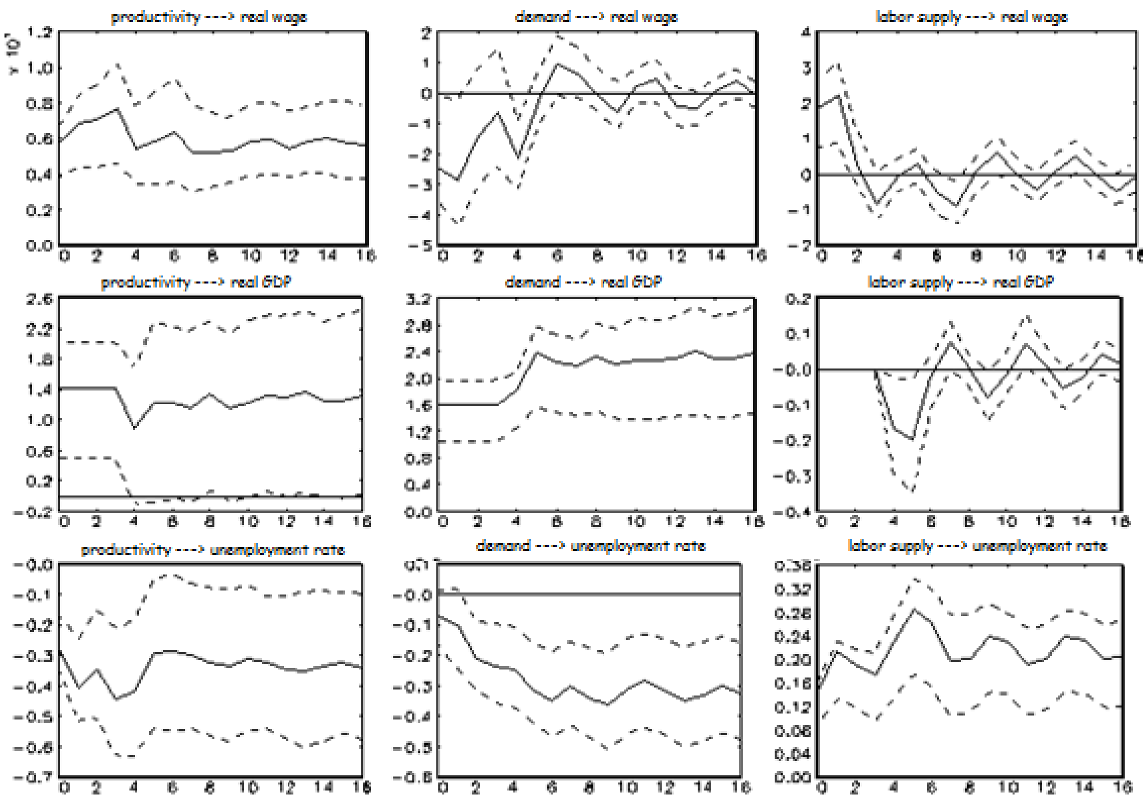

FIGURE 2.3 – ACCUMULATED IMPULSE RESPONSES (IN %) - BEFORE REAL PLAN 1982Q3-1994Q2

NOTE: Dashed lines depict approximate 90% confidence intervals computed using 10,000 bootstrap replications according to method of Efron (1979).
SOURCE: VAR model and impulse responses calculated by the JMulTi times series software (Lütkepohl and Krätzig, 2004).

The long-run (accumulated) response of unemployment rate to a productivity shock (of one standard deviation) is about -0.23%, while the long-run response of unemployment rate to a demand shock (of one deviation Standard) is about -0.35%. These two long-run multipliers can be identified as coefficients of a modified "Okun" (Okun, 1962) relationship that separates supply from demand shocks.

In (FIGURE 2.3) impulse responses generated by the model “before Real Plan” are shown. Variables reactions to the shocks have expected shape and are similar to those responses of the full period model (1982Q3-2015Q4, FIGURE 2.2). It should be noted that in the model “before Real Plan”, the real wage response to a (positive) demand shock is not significant at 10% of significance (confidence interval passes over the zero axis), although it has the countercyclical response as expected. How occurs in the full period model (1982Q3-2015Q4, FIGURE 2.2), the real wage and real GDP per capita

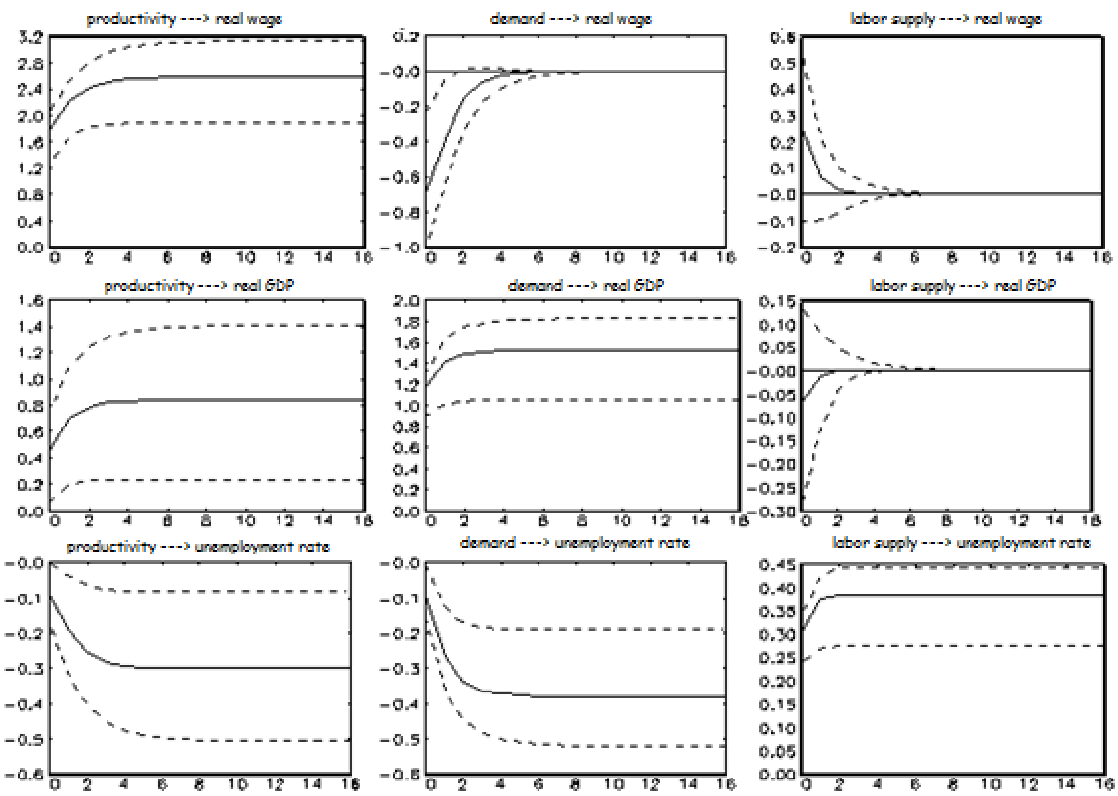

FIGURE 2.4 – ACCUMULATED IMPULSE RESPONSES (IN %) - AFTER REAL PLAN 1994Q3-2015Q4

NOTE: Dashed lines depict approximate 90% confidence intervals computed using 10,000 bootstrap replications according to method of Efron (1979).
SOURCE: VAR model and impulse responses calculated by the JMulTi times series software (Lütkepohl and Krätzig, 2004).

responses to a (positive) labor supply shock are not significant at 10% level of significance (confidence interval passes over the zero axis).

In (FIGURE 2.4) we show the impulse responses of the “after Real Plan” model (1994Q3-2015Q4). Again, responses have the shapes as expected. Comparing with the full period model (1982Q3-2015Q4, FIGURE 2.2) and "before Real Plan" model (1982Q3-1994Q2, FIGURE 2.3), one can note that the short and long-run impacts of supply shocks on real wages and on real GDP per capita falls by about one half. Like occurs in the two previous models (FIGURES 2.2 and FIGURE 2.3), the real wage and real GDP per capita responses to a (positive) labor supply shock are not significant at 10 % level of significance axis) in the "after Real Plan" model (1994Q3-2015Q4, FIGURE 2.3).

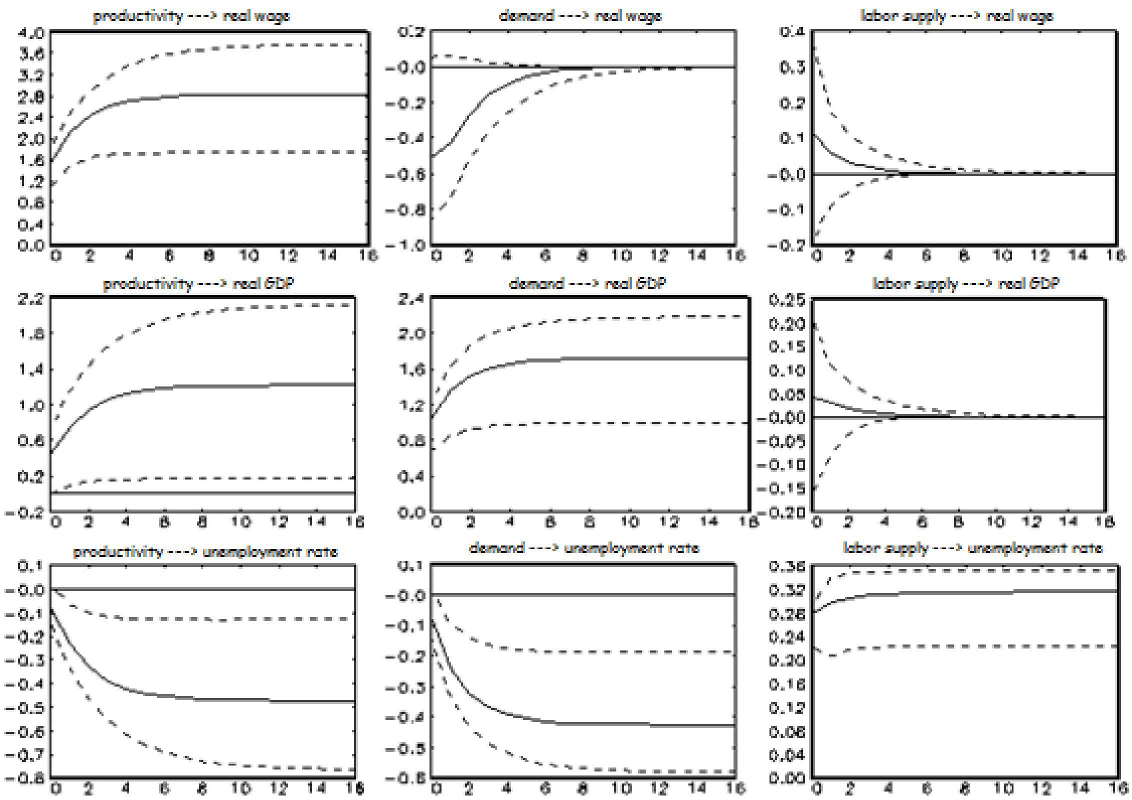

FIGURE 2.5 – ACCUMULATED IMPULSE RESPONSES (IN %) - INFLATION TARGETING 1999Q1-2015Q4

NOTE: Dashed lines depict approximate 90% confidence intervals computed using 10,000 bootstrap replications according to method of Efron (1979).
SOURCE: VAR model and impulse responses calculated by the JMulTi times series software (Lütkepohl and Krätzig, 2004).

Finally, in (FIGURE 2.5), we have the inflation target model (1999Q1-2015Q4) impulse responses. The graphics are similar to what occurs in the "after Real Plan" model with the expected shapes. But in this Inflation Targeting period (1999Q1-2015Q4), we see that the long-run real GDP per capita response to a (positive) productivity shock is twice that of the "after Real Plan" model (1994Q3-2015Q4) one (1.33% vs. 0.62%) and equal to what occurs in the “full period” model (1982Q3-2015Q4). It is also seen that in this period (1999Q1-2015Q4), the long-run response of the unemployment rate to a (positive) productivity shock is twice that of the “after Real Plan” model (1994Q3-2015Q4) and “full period” model (1982Q3-2015Q4) ones (respectively -0.23% and -0.27 % vs. -0.50%).

2.5.3 Variance Decomposition

Analyzing the forecast errors variance decomposition (FEVD) in (TABLE 2.6), for real wages, productivity shocks dominate the (FEVD) in the short and long-run in the four models considered, although the models ”Full period” (1982Q3-2015Q4) and ”before Real Plan” (1982Q3-1994Q2) the share of long-run (FEVD) explained by productivity shocks is lower while demand shocks increase their share. In the real output case, demand shocks dominate short and long-run (FEVD) in the models “before Real Plan” (1982Q3-1994Q2), “after Real Plan” (1994Q3-2015Q4) and “Inflation Targeting” (1999Q1- 2015Q4), although this dominance is slight in the ”before real Plan” model (1982Q3-1994Q2). In the ”Full period” model (1982Q3-2015Q4), productivity shocks dominate (FEVD) slightly.

The period covered by the “before Real Plan” model (1982Q3-1994Q2) was marked by very high inflation (with episodes of hyperinflation) and many nominal shocks from the external sector and government budget. This combination of nominal shocks can be responsible for this share difference in productivity and demand shocks in explaining the real wage and output (FEVD’s) when one compares the models ”Full Period” and ”before Real Plan” with the other two models.

In the case of our main focus of interest, the unemployment rate, except in the “before Real Plan” model (1982Q3-1994Q2), labor-supply shocks dominate the short and long-run (FEVD). In relation to demand shocks, these explain between 5% and 8% of the (FEVD) at one quarter horizon in all models, except for the “Full Period” model (1982Q3-2015Q4) where the percentage is 14%; in the long-run demand shocks explain between 19% and 26% of (FEVD) in all models. With regard to productivity shocks, these explain about 75% of the (before Real Plan) model (1982Q3-1994Q2) (FEVD) at one quarter horizon and only 7% in the “after Real Plan” model (1994Q3 -2015Q4). In the ”Full Period” model (FEVD), productivity shocks account for about 23% of (FEVD) at one quarter horizon. In the infinite (long-run) horizon, productivity shocks account for about 59% of

TABLE 2.6 – FORECAST ERROR VARIANCE DECOMPOSITION

Model	Variable	Horizon	Shocks type		
			productivity	demand	labor supply
Full period (1982Q3-2015Q4)	real wage	1	0.69	0.31	0.00
		4	0.62	0.38	0.00
		∞	0.60	0.39	0.01
	real output (GDP)	1	0.58	0.41	0.01
		4	0.57	0.40	0.03
		∞	0.60	0.39	0.01
	unemploy-ment rate	1	0.23	0.14	0.63
		4	0.32	0.19	0.49
		∞	0.31	0.23	0.46
Before Real Plan (1982Q3-1994Q2)	real wage	1	0.78	0.14	0.08
		4	0.67	0.17	0.16
		∞	0.54	0.27	0.19
	real output (GDP)	1	0.43	0.57	0.00
		4	0.43	0.57	0.00
		∞	0.44	0.54	0.02
	unemploy-ment rate	1	0.75	0.05	0.20
		4	0.70	0.12	0.18
		∞	0.59	0.19	0.22
After Real Plan (1994Q3-2015Q4)	real wage	1	0.85	0.13	0.02
		4	0.82	0.15	0.03
		∞	0.82	0.15	0.03
	real output (GDP)	1	0.13	0.87	0.00
		4	0.16	0.84	0.00
		∞	0.16	0.84	0.00
	unemploy-ment rate	1	0.08	0.08	0.84
		4	0.14	0.26	0.60
		∞	0.15	0.26	0.59
Inflation Targeting (1999Q1-2015Q4)	real wage	1	0.90	0.10	0.00
		4	0.90	0.09	0.01
		∞	0.90	0.09	0.01
	real output (GDP)	1	0.15	0.85	0.00
		4	0.22	0.78	0.00
		∞	0.22	0.78	0.00
	unemploy-ment rate	1	0.07	0.06	0.87
		4	0.26	0.25	0.49
		∞	0.27	0.26	0.47

SOURCE: VAR model and forecast variance error decomposition calculated by the JMulTi times series software (Lütkepohl and Krätzig, 2004).

the ”before Real Plan” model (FEVD), 30% in the “Inflation Targeting” model and only 13% in the ”after Real Plan” model (1994Q3-2015Q4).

These differences in the role of productivity shocks in explaining the unemployment rate (FEVD) among the models are difficult to understand, a possible cause for such differences may be the demographic transformation that Brazil has been experiencing since the mid-1990s with a demographic pyramid flattening. This flattening implies more people entering in the working age population. This demographic transformation may have transferred from productivity to labor supply shocks a part of the Brazilian unemployment rate (FEVD) explanation.

2.5.4 Measuring the Wage Rigidity

As a final exercise we compute a real wage rigidity index (RWR) from our estimated models. As Balmaseda et al. (2000), we take the approach of Layard et al. (1991) to compute a real wage rigidity index computed as the reciprocal of the real wage estimated responses to the unemployment rate responses. In our estimated models, productivity shock is the only one that have a permanent effect in real wages, so like in Balmaseda et al. (2000), a possible definition of RWR is given by

$$RWR = \left| \lim_{k \to \infty} \frac{\partial \sum_{k=0}^{\infty} \partial \Delta u_{t+k} / \partial \epsilon_{s,t}}{\partial \sum_{k=0}^{\infty} \partial \Delta \omega_{t+k} / \partial \epsilon_{s,t}} \right| \tag{2.22}$$

The interpretation of equation (2.22) is as follows: the larger the RWR value (in modulus), the greater is the real wage rigidity. In (TABLE 2.7) are the RWR indexes calculated for the four estimated models. The RWR index is larger in the "after Real Plan" model than it is in the "before Real Plan" (0.12 vs 0.06 respectively). In the "Inflation Targeting" model, the RWR index is 0.17, a wage rigidity almost three times higher than that of the "before Real Plan" model (RWR = 0.06). One explanation for this different RWR indexes would be that the inflation stabilization allowed the nominal wages setting for longer periods, which increased the real wages rigidity.

TABLE 2.7 – REAL WAGE RIGIDITY (RWR) INDICES

Model	RWR
"Full Period" (1982Q3-2015Q4)	0.05
"Before Real Plan" (1982Q3-1994Q2)	0.06
"After Real Plan" (1994Q3-2015Q4)	0.12
"Inflation Targeting" (1999Q1-2015Q4)	0.17

2.6 CONCLUDING REMARKS

Analizing the Brazilian economy (1982Q3-2015Q4 period) in this paper, we have showed evidences that the unemployment rate can be a I(1) variable and, therefore, follow a full hysteresis process. In addition, we have presented empirical evidences about dynamic responses of real wages, real output per capita and unemployment rate to three identified shocks, which are productivity, demand and labor supply. To identify these shocks, we have made use of restrictions stemming from a simple insider-outsider model of the labor market proposed by Balmaseda et al. (2000).

This model allow us to recover the structural shocks (productivity, demand and labor supply) through a recursive set of long-run restrictions on the long-run matrix of the system multipliers [a type of Blanchard and Quah (1989) identification methodology]. As the Brazilian economy underwent several transformations between 1980 e 2015, we split our sample in three periods: "before Real Plan" (1982Q3-1994Q2), "After Real Plan" (1994Q3-2015Q4) and "Inflation Targeting" (1999Q1-2015Q4).

Our many findings are: first, the Brazilian unemployment rate measured by IBGE's PME can be considered a process with full hysteresis (unemployment rate is a I(1) variable) in all the sub-periods. Second, demand shocks play a similar role in explaining the forecast error variance (FEV) of the unemployment rate in all sub-periods. Third, productivity shocks was more important in explaining the (FEV) of the unemployment rate in the "before Real Plan" period, while labor-supply shocks were more important in the "after Real Plan" period. Third, real wages seemed be more flexible in the "before Real Plan" period compared with subsequent periods, mainly compared with the "Inflation Targeting" period.

3 HYSTERESIS IN A NEW KEYNESIAN DSGE

ABSTRACT

We expand the Galí (2011a,b) unemployment model to consider the hysteresis in unemployment rate hypothesis. With full hysteresis, the various shocks affecting the economy have a permanent effect on employment and unemployment rate. In an economy of this type the unemployment rate do not tend to a certain mean or to a "natural rate" of unemployment in the long-run. In this paper we insert hysteresis in the standard New Keynesian Model and estimate two Bayesian DSGEs, one with hysteresis and other without hysteresis, and compare their behaviors in regard to impulse responses and decomposition of forecast error variance.

3.1 INTRODUCTION

The behavior differences between the unemployment rates in Europe (and Latin America) and the United States since the 1980s began to draw the attention of economists. Since the 1980s, unemployment rates in Europe and Latin America have been higher and more persistent (with high autocorrelation) than those observed in the United States. The term "unemployment rate with hysteresis" was used to describe this phenomenon of high and persistent unemployment rates observed in these places. There are three theories that seek to explain the phenomenon of persistence in economic variables such as output and unemployment. These theories are known as "physical capital", "human capital" and "insider-outsider" theories or hypothesis to explain the unemployment persistence.

"The physical capital story simply holds that reductions in the capital stock associated with the reduced employment that accompanies adverse shocks reduce the subsequent demand for labor and so cause protracted unemployment." (Blanchard and Summers, 1986, p.27). A second theory or hypothesis to explain the unemployment persistence is the "Human Capital" channel. According to this theory, proposed by Phelps (1972) and Hargraves-Heap (1980) among others, prolonged unemployment can deteriorate the accumulation of human capital by workers. As stated by Blanchard and Summers (1986), "unemployed workers lose the opportunity to maintain and update their labor skills."

Long lasting unemployment can also lead to a detachment from the labor market both through a discouragement to job seeking or demolition of the individual's social network, leaving him increasingly isolated and away from the formal labor market. Another channel that human capital could affect employability would be the preference of employers for short term unemployed because they have their skills intact (Roberts and Morin, 1999).

Finally, insiders-outsiders models of labor market emphasize the market division between insiders and outsiders, and how insiders prevail in the process of wage determination. The seminal papers in this issue were Blanchard and Summers (1987), Gottfries and Horn (1987) and Lindbeck and Snower (1985, 1988c). These authors emphasize an asymmetry in the wage setting process, where outsiders are disregarded and insiders set wages in view to preserve their own jobs. Recessions reduce the number of employees and, in turn, reduces the number of insiders. This process can generate hysteresis in wages, employment and unemployment rates.

In the last 10 years or so, many authors have expanded the New Keynesian model into microfounded and estimated stochastic general equilibrium models (DSGE), examples of pioneers works are Smets and Wouters (2003, 2007), Dib (2003), Edge et al. (2005), Fernandez-Villaverde and Rubio-Ramirez (2006), Fukac and Pagan (2006), among others. Standard versions of New-Keynesian DSGE models abstract the unemployment rate and only model worked hours varying in intensive margin, i.e., these models only consider the variation of hours worked and not the total change of employed individuals (extensive margin). Recently some models have explicitly included the unemployment rate in DSGE models, as examples we have Gertler and Trigari (2009), Christoffel et al. (2009), Blanchard and Galí (2010), Christiano et al. (2010), Galí (2011a,b, 2015, 2016), Galí et al. (2012), Liu (2014) and others.

As unemployment is included in the DSGE model varies from paper to paper. Gertler and Trigari (2009), Christoffel et al. (2009), Christiano et al. (2010) and Blanchard and Galí (2010) introduce the unemployment rate in the DSGE model through labor market search and matching frictions *à la* Mortensen and Pissarides (1994). In these models involuntary unemployment is due to the (inefficient) equilibrium of a search and matching process. By other side, Galí (2011a,b) and Galí et al. (2012) reinterpret the standard New-Keynesian model in terms of extensive margin (number of employees) and indivisible work instead of intensive margin (hours worked). Interpreted in this way, according to the authors, the standard New-Keynesian DSGE model would have an inherent theory of unemployment. According to this theory, involuntary unemployment results from the combination of imperfect substitution between the different types of labor with nominal wage rigidity that give market power to occupations or unions to set wages above the level of full employment (level of involuntary unemployment absence).

Galí (2015) expands the models in Galí (2011a,b) in order to consider the presence of a unit root in unemployment rate. He considers three hypothesis: the natural rate hypothesis, the log-run trade-off hypothesis and the hysteresis hypothesis. In the natural rate hypothesis, Galí supposes that the natural wage markup follows a random walk. In this way, the natural rate of unemployment also would be a random walk as well as the unemployment rate. Under the long run trade-off hypothesis, the unit root in the unemployment rate results from the presence of a unit root in the wage inflation since both variables are linked through the wage Phillips Curve. In this case, unit root in wage inflation is assumed to be inherited, in turn, from a unit root in the central bank inflation target. Finally, in the hysteresis hypothesis, Galí adapt the insider-outsider model of unemployment of Blanchard and Summers (1987) into the New Keynesian Model. He concludes that none of the three hypothesis can, by itself, account for the evidence on unemployment and wage inflation for the period 1970-2014 in Europe Union, though both the long run trade-off and hysteresis hypothesis (insider-outsider model) can help interpret certain aspects of the joint behavior of the unemployment rate and wage inflation.

Following the spirit of Galí (2015), our objective is introduce unemployment rate with hysteresis into a small New Keynesian DSGE, but through a different mechanism. We suppose that the labor supply path performs in a non-stationary way and is non-cointegrated with the employment growth. This combination produce a New-Keynesian model with a non-stationary unemployment rate, i.e., a unemployment rate with full (or strong) hysteresis. To demonstrate the implications of our hysteresis model, we estimated two small Bayesian DSGEs with closed economy and no government applied to the Brazilian economy. One of them adopts the standard model of Galí (2011a,b) and Galí et al. (2012) without hysteresis, while the other model adopts the hypothesis of unemployment rate hysteresis according to the assumption of non-stationary labor supply path that we adopt.

Beyond this introduction, the remainder of this paper is structured as follows. Section (3.2) presents the models with and without hysteresis; section (3.3) derives the relations of consumption, labor-supply, wage-setting and unemployment; section (3.4) drives the firms behavior in relation to output, hiring and prices setting; section (3.5) presents a summary of both models; in section (3.6) we estimate both models applying Bayesian techniques and analyze results like impulse response functions and forecast error variance decompositions; finally section (3.7) concludes.

3.2 MODELS CHARACTERIZATION

It is assumed the existence of a large number of identical households, each of them composed by a continuum of members. The family members are indexed by the pair $i, j \in [0,1] \times [0, N_t(i)]$. The index i represents the type of labor the household member is specialized. The j determines the labor disutility which is given by $[(X_t^{1-\sigma}\theta_t)/(z_t^{l\varphi} {H_t}^{1+\varphi})] \int_0^{N_t(i)} j^{\varphi} dj$ if the individual is employed and zero if he is unemployed, $N_t(i) = \int_0^{N_t(i)} j dj$ is the "effective" labor supply or employment of workers specialized in labor type i. Household utility function is separable in consumption and labor and consumption is a CES aggregate of various goods. Like Galí (2011a,b), we follow Hansen (1985) and Rogerson (1988) in assuming that labor is indivisible. Indivisible labor means that in each period an individual works a fixed number of hours or does not work at all. As observed by (Galí, 2011b, p. 7), extensive margin variations (changes in employment) dominates the observed fluctuations in total hours of work [product of intensive margin (hours) by extensive margin (employment)]. So the assumption of indivisible labor can be a good approximation. We also assume there is full risk sharing among workers. Full risk sharing in association with separable preferences implies the same level of consumption to all individuals, employed or not. This is a strong assumption, mainly from the welfare viewpoint, but it is useful for simplify the model. We add external habits formation in consumption as a additional model feature. The household i utility U in period t is given by:

$$
\begin{aligned}
U[C_t^{\#}, N_t(i)] =& \frac{(C_t^{\#})^{1-\sigma}}{1-\sigma} - \frac{X_t^{1-\sigma}\theta_t}{(z_t^l)^{\varphi} H_t^{1+\varphi}} \int_0^1 \int_0^{N_t(i)} j^{\varphi} dj di \\
=& \frac{(C_t^{\#})^{1-\sigma}}{1-\sigma} - \frac{X_t^{1-\sigma}\theta_t}{(z_t^l)^{\varphi} H_t^{1+\varphi}} \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di
\end{aligned} \tag{3.1}
$$

where t is time in quarters; $C_t^{\#} = C_t - hC_{t-1}$, is the consumption adjusted by habits[1] like in Boldrin et al. (2001); $C_t = [\int_0^1 C_t(v)^{(\epsilon_{p,t}-1)/\epsilon_{p,t}} di]^{\epsilon_{p,t}/(\epsilon_{p,t}-1)}$ is a basket of differentiated goods with elasticity of substitution ($\epsilon_{p,t}$); h is the parameter measuring external habit in consumption; σ is the inverse elasticity of intertemporal substitution of consumption; $N_t(i)$ is the "effective" labor supply or employment of workers specialized in labor type i; φ is the Frisch inverse elasticity of labor supply in relation to the real wage (Frisch, 1932); z_t^l is an exogenous preference shifter that decreases the disutility of labor and affects the labor supply with $z_t^l = (z_0^l)^{(1-\rho_l)}(z_{t-1}^l)^{\rho_l} e^{\varepsilon_t^l}$, $z_0^l = 1$, $\rho_l \in [0,1]$ and $\varepsilon_t^l \sim i.i.d.(0, \sigma_l^2)$ is a temporary labor supply shock; X_t is a variable which guarantees a balanced growth path for the model and will be set soon; $H_t = (N_{t-1})^{d_l}(H_{t-1})^{(1-d_l)} e^{g_{n,t}}$ is a variable governing the labor growth rate, d_l is a dummy variable that assumes the value zero (0) if the labor supply is cointegrated with the employment in a such way that makes unemployment rate stationary, or assumes the value one (1) if the labor supply is no cointegrated with employment and therefore unemployment rate is not stationary, $g_{n,t} = (1-\rho_n)g_n + \rho_n g_{n,t-1} + \varepsilon_t^n$ is the labor supply permanent growth where g_n is the steady-state growth rate of the labor supply and employment, $\rho_n \in [0,1]$ and $\varepsilon_t^n \sim i.i.d.(0, \sigma_n^2)$ is a permanent labor supply shock; θ_t is an endogenous preference shifter given by $\theta_t = (Z_t^s / C_t^{\#})^{\sigma}$ and $Z_t^s = (C_t^{\#})^{\gamma_c}(Z_{t-1}^s)^{1-\gamma_c}$.

It is assumed the representative household seeks to maximize the present value of utility,

$$
\max E_0 \sum_{k=0}^{\infty} \beta^k \left[\frac{(C_{t+k}^{\#})^{1-\sigma}}{1-\sigma} - \frac{X_{t+k}^{1-\sigma}\theta_{t+k}}{(z_{t+k}^l)^{\varphi} H_{t+k}{}^{1+\varphi}} \int_0^1 \frac{N_{t+k}(i)^{1+\varphi}}{1+\varphi} di \right] \tag{3.2}
$$

[1]According (Dennis, 2009, p. 1016): "Consumption habits have a long history in macroeconomics. Duesenberry (1949) argues that habit formation can arise through a desire to advance socially or to acquire high quality goods, desires prompted by the '...*inferiority feelings that are aroused by unfavorable comparisons between living standards.*' Similarly, Ryder and Heal (1973) argue that the '... *satisfaction that a man derives from consuming a given bundle of goods depends not only on that bundle, but also on his past consumption and on his general social environment.*'" (Dennis, 2009, p. 1016)

subject to a sequence of flow budget constraints given by

$$P_tC_t + \frac{P_t^b B_t^n}{z_t^c} \leq B_{t-1}^n + W_t^n(i)N_t(i) + T_t^n$$

where the superscript n denotes nominal variables; P_t is the CPI index; $P_t^b = 1/(1+r_t)$ is the market price of government bonds; r_t is the nominal interest rate; B_t^n is the amount of government nominal riskless bonds of one period paying a unit of account (money); W_t^n, is the nominal wage rate; N_t is the "effective" labor supply or employment; and T_t^n represents lump-sum transfers which include dividends paid by firms and unemployment insurance earned by workers; z_t^c, according Smets and Wouters (2007), represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households, a positive shock to this wedge decreases the required return on assets and reduces current consumption. This wedge also can be seen like a risk premium shock. In our model this shock is interpreted as generic demand shock with $\log z_t^c = \tilde{z}_t^c = \rho_l \tilde{z}_{t-1}^c + \epsilon_t^c$, $\rho_c \in [0,1]$ e $\epsilon_t^c \sim i.i.d.(0, \sigma_c^2)$.

The budget constraint can be rewritten as

$$\begin{aligned} C_t + \frac{P_t^b}{z_t^c}\frac{B_t^n}{P_t} &\leq \frac{P_{t-1}}{P_t}\frac{B_{t-1}^n}{P_{t-1}} + \frac{W_t^n(i)}{P_t}N_t(i) + \frac{T_t^n}{P_t} \\ C_t + \frac{P_t^b}{z_t^c}B_t^r &\leq \frac{1}{(1+\pi_t)}B_{t-1}^r + W_t^r(i)N_t(i) + T_t^r \end{aligned} \tag{3.3}$$

where the superscript r indicates real variables; $B_t^r = B_t^n/P_t$; $W_t^r = W_t^n/P_t$; and $(1+\pi_t) = P_t/P_{t-1}$, where π_t is the inflation rate.

The firms production function is

$$Y_t(v) = z_t^a A_t N_t(v)^{1-\alpha} + \frac{\Phi - 1}{\Phi} y \tag{3.4}$$

where $Y_t(v)$ is the real output for the firm (v), z_t^a is a temporary productivity shifter, with $z_t^a = (z_0^a)^{(1-\rho_{za})}(z_{t-1}^a)^{\rho_{za}} e^{\varepsilon_t^{za}}$, $z_0^a = 1$, $\rho_{za} \in [0,1]$ and $\varepsilon_t^{za} \sim i.i.d.(0, \sigma_{za}^2)$ is a temporary productivity shock; A_t is a permanent productivity shifter, with $A_t = A_{t-1}e^{g_{a,t}}$, $g_{a,t} = (1-\rho_a)g_a + \rho_a g_{a,t-1} + \varepsilon_t^a$, $\rho_a \in [0,1]$ and $\varepsilon_t^a \sim i.i.d.(0, \sigma_a^2)$ is a permanent productivity shock; N_t is the employment; $(1-\alpha)$ is the labor elasticity in the production function;

and $[(\Phi-1)/\Phi]y$ is a fixed cost as percentage of the steady-state output $y = \bar{Y}_t/(A_t H_t^{1-\alpha})$.

The firm (v) demand for the labor type (i) is given by

$$N_t(i,v) = \left[\frac{W_t^n(i)}{W_t^n}\right]^{-\epsilon_{w,t}} N_t(v)$$

where $W_t^n = [\int_0^1 W_t(i)^{1-\epsilon_{w,t}} di]^{1/(1-\epsilon_{w,t})}$ is the economy aggregated nominal wage and $N_t(v) = [\int_0^1 N_t(i,v)^{(\epsilon_{w,t}-1)/\epsilon_{w,t}} di]^{\epsilon_{w,t}/(\epsilon_{w,t}-1)}$. Integrating the equation above in both sides over (v) we obtain the economy's labor demand for the labor type (i):

$$\begin{aligned} \int_0^1 N_t(i,v)dv &= \left[\frac{W_t^n(i)}{W_t^n}\right]^{-\epsilon_{w,t}} \int_0^1 N_t(v)dv \\ N_t(i) &= \left[\frac{W_t^n(i)}{W_t^n}\right]^{-\epsilon_{w,t}} N_t \end{aligned} \tag{3.5}$$

where (ϵ_w) is the elasticity of substitution among the labor types (i). We can define the gross wage markup as $\mu_t^w = \epsilon_{w,t}/(\epsilon_{w,t}-1)$ and the log wage markup as $\log \mu_t^w = \tilde{\mu}_t^w = (1-\rho_w)\bar{\mu}^w + \rho_w \tilde{\mu}_{t-1}^w + \varepsilon_t^w$ with $\rho_w \in [0,1]$, and $\varepsilon^w \sim i.i.d.(0, \sigma_w^2)$.

The demand for the good (v) from the firm (v) is:

$$Y_t(v) = \left[\frac{P_t(v)}{P_t}\right]^{-\epsilon_{p,t}} Y_t \tag{3.6}$$

where $P_t = [\int_0^1 P_t(v)^{1-\epsilon_{p,t}} dv]^{1/(1-\epsilon_{p,t})}$ is the economy aggregated price level; (ϵ_p) is the elasticity of substitution among the goods types (v). We can define the gross price markup as $\mu_t^p = \epsilon_{p,t}/(\epsilon_{p,t}-1)$ and the log wage markup as $\log \mu_t^p = \tilde{\mu}_t^p = (1-\rho_p)\bar{\mu}^p + \rho_p \tilde{\mu}_{t-1}^p + \varepsilon_t^p$ with $\rho_p \in [0,1]$, and $\varepsilon^p \sim i.i.d.(0, \sigma_p^2)$.

In order to solve and estimate a DSGE model it is necessary that it has a well-defined steady-state, i.e., the variables are constant at the model equilibrium. Here, all variables except for P_t^b and π_t are growing over time due to A_t and/or H_t growing (at a balanced growth path N_t grows solely because of H_t). To see this differentiate the production function above in relation to time and divide by Y

$$\begin{aligned} \frac{\dot{Y}}{Y} &= \frac{\dot{z^a}}{z^a} + \frac{\dot{A}}{A} + (1-\alpha)\frac{\dot{N}}{N} \\ g_y &= g_a + (1-\alpha)g_n \end{aligned}$$

where $(\dot{Y}/Y) = g_y$ is the output balanced path growth rate; $(\dot{z^a}/z^a) = 0$ is the temporary productivity growth in the balanced growth path; $(\dot{A}/A) = g_a$ is the balanced path growth rate of the permanent productivity shifter A_t; and $(\dot{N}/N) = g_n$ is the labor-supply (and employment) balanced path growth rate.

Under appropriate conditions a model may have a balanced growth path, a state of equilibrium in which all the model variables that tend to growth over time (trending variables) grow at the same rate. As all trending variables grow at the same rate, it is possible to extract the common trend between them and transform a non-stationary model into a stationary one, i.e. a model that has a steady-state. To detrend these variables we follow Pfeifer (2015) denoting with lowercase letters non-trending variables and writing every generic variable D_t as $(D_t = A_t H_t^{1-\alpha} d_t)$.[2] We will now proceed with the above equations detrending.

The utility function (3.1) can then be rewritten as

$$\begin{aligned} A_t H_t^{1-\alpha} u_t &= \left\{ \frac{(A_t H_t^{1-\alpha})^{1-\sigma} (c_t^{\#})^{1-\sigma}}{1-\sigma} - \frac{(A_t H_t^{1-\alpha} x_t)^{1-\sigma} \theta_t}{(z_t^l)^{\varphi} H_t^{1+\varphi}} \int_0^1 \frac{[H_t n_t(i)]^{1+\varphi}}{1+\varphi} \right\} \\ u_t &= (A_t H_t^{1-\alpha})^{-\sigma} \left[\frac{(c_t^{\#})^{1-\sigma}}{1-\sigma} - \frac{x_t^{1-\sigma} \theta_t}{(z_t^l)^{\varphi}} \int_0^1 \frac{n_t(i)^{1+\varphi}}{1+\varphi} \right] \end{aligned}$$

remembering that $A_t = A_{t-1} e^{g_{a,t}}$ with $g_{a,t} = (1-\rho_a) g_a + \rho_a g_{a,t-1} + \varepsilon_t^a$, and $H_t = (N_{t-1})^{d_l} (H_{t-1})^{(1-d_l)} e^{g_{n,t}}$ with $g_{n,t} = (1-\rho_b) g_n + \rho_n g_{n,t-1} + \varepsilon_t^n$; if we make $\varepsilon_t^a = \varepsilon_t^n = 0 \; \forall \; t$ and $d_l = 0$,[3] we have $A_t = A_{t-1} e^{g_a}$ and $H_t = H_{t-1} e^{g_n}$ iterating backward these equations one obtains $A_t = A_0 e^{g_a t}$ and $H_t = H_0 e^{g_n t}$, normalizing $A_0 = H_0 = 1$ results in $A_t = e^{g_a t}$ and $H_t = e^{g_n t}$. Besides that, we set $x_t = 1 \; \forall \; t$ or otherwise $X_t = A_t H_t^{1-\alpha}$. This is a condition sufficient to guarantee a steady-state for the utility function (3.1). To see this note that in the steady-state $c^{\#}$ and n are constants, $z_t^l = \theta_t = 1$ and, therefore, x should be constant as well. As x_t is a detrended variable its possible shocks would only have transitory effects and so it would be indistinguishable of z_t^l and thus redundant. Substituting these equations $A_t = e^{at}$, $H_t = e^{g_n t}$ and $x_t = 1$ in the utility function above, we

[2] In this text we are using the following convention for a generic variable like X_t: a uppercase letter X_t means a variable with trend; a lowercase letter x_t indicates a detrended variable; a lowercase letter x without the subscript t indicates a detrended variable at their steady-state value (except if the letter is representing a parameter)

[3] Making $d_l = 0$ is not strictly necessary because with $\varepsilon_t^a = \varepsilon_t^n = 0 \; \forall \; t$ the cointegrated and non-cointegrated models for the labor-supply and employment growth both coincide.

have

$$
\begin{aligned}
u_t &= \left[e^{g_a t} e^{(1-\alpha) g_n t}\right]^{-\sigma} \left[\frac{(c_t^{\#})^{1-\sigma}}{1-\sigma} - \frac{\theta_t}{(z_t^l)^{\varphi}} \int_0^1 \frac{n_t(i)^{1+\varphi}}{1+\varphi}\right] \\
&= \exp(-\sigma g_y t) \left[\frac{(c_t^{\#})^{1-\sigma}}{1-\sigma} - \frac{\theta_t}{(z_t^l)^{\varphi}} \int_0^1 \frac{n_t(i)^{1+\varphi}}{1+\varphi}\right] \qquad (3.7)
\end{aligned}
$$

where $c_t^{\#} = c_t - \tilde{h} c_{t-1}$ with $\tilde{h} = h \exp(g_y)$, and $g_y = g_a + (1-\alpha) g_n$ is the output (Y_t) balanced growth rate.

Looking at the budget constraint (3.3) now, it can be detrended in this way

$$
\begin{aligned}
A_t H_t^{1-\alpha} c_t + \frac{P_t^b}{z_t^c} A_t H_t^{1-\alpha} b_t^r &\leq \frac{1}{(1+\pi_t)} A_t H_t^{1-\alpha} b_{t-1}^r + A_t H_t^{1-\alpha} w_t^r(i) n_t(i) + A_t H_t^{1-\alpha} t_t^r \\
c_t + \frac{P_t^b}{z_t^c} b_t^r &\leq \frac{1}{(1+\pi_t)} b_{t-1}^r + w_t^r(i) n_t(i) + t_t^r \qquad (3.8)
\end{aligned}
$$

the production function (3.4) can be detrended in the same way

$$
A_t H_t^{1-\alpha} y_t(v) = z_t^a A_t [H_t n_t(z)]^{1-\alpha} + \frac{\Phi - 1}{\Phi} y \implies y_t(v) = z_t^a n_t(v)^{1-\alpha} + \frac{\Phi - 1}{\Phi} y \qquad (3.9)
$$

the labor demand (3.5) detrending is done like this

$$
\frac{N_t(i)}{H_t} = \left[\frac{W_t^n(i)}{W_t^n}\right]^{-\epsilon_{w,t}} \frac{N_t}{H_t} \implies n_t(i) = \left[\frac{W_t^n(i)}{W_t^n}\right]^{-\epsilon_{w,t}} n_t \qquad (3.10)
$$

and the output demand (3.6) detrending is

$$
\frac{Y_t(v)}{A_t H_t^{1-\alpha}} = \left[\frac{W_t^n(i)}{W_t^n}\right]^{-\epsilon_{p,t}} \frac{Y_t}{A_t H_t^{1-\alpha}} \implies y_t(v) = \left[\frac{P_t(v)}{P_t}\right]^{-\epsilon_{p,t}} y_t \qquad (3.11)
$$

Now all the models variables as a well defined steady-state and we can write down the model and calculate its first order conditions.

3.3 CONSUMPTION, LABOR-SUPPLY, WAGE-SETTING AND UNEMPLOYMENT

Following Erceg et al. (2000), it is assumed that in each period only a randomly selected fraction $(1-\theta_w)$ of occupations has permission to adjust optimally the nominal wage rate

(W_t^{n*}). The fraction $(1-\theta_w)$ is constant over time, which means that the occupations selection to optimally adjust the wage rate is independent of the time elapsed since the last optimal adjust. The remaining fraction θ_w of the occupations that is not selected to optimally adjust the wage rate, indexes their nominal wage to past and long-run (steady) inflation and to productivity growth as we will see ahead. For the $(1-\theta_w)$ family fraction who can set their wage in period t, they know they face a $(\theta_w)^k$ probability that the wage they choose in t, (W_t^{n*}), will still be prevailing (k) periods ahead. When the occupation (i) representatives set their nominal wage (W^{n*}), they take this into account.

Again, it is assumed the representative household seeks to maximize the present value of utility,

$$\max E_0 \sum_{k=0}^{\infty} \tilde{\beta}^k \left[\frac{(c_t^{\#})^{1-\sigma}}{1-\sigma} - \frac{\theta_t}{(z_t^l)^{\varphi}} \int_0^1 \frac{n_t(i)^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$c_t + \frac{P_t^b}{z_t^c} b_t^r = \frac{1}{(1+\pi_t)} b_{t-1}^r + w_t^r(i) n_t(i) + t_t^r(i) \quad \text{and} \quad n_t(i) = \left(\frac{W_t^{n*}}{W_t^n} \right)^{-\epsilon_{w,t}} n_t$$

where $\hat{\beta} = \beta \exp(-\sigma g_y)$.

The Lagrangian for this problem is:

$$\mathcal{L} = E_t \sum_{k=0}^{\infty} \hat{\beta}^k \Bigg\{ \left[\frac{(c_{t+k}^{\#})^{1-\sigma}}{1-\sigma} - \frac{\theta_{t+k}}{(z_{t+k}^l)^{\varphi}} \int_0^1 \frac{n_t^{1+\varphi}(i)}{1+\varphi} \right] - \Lambda_{t+k} \Big[(c_{t+k}^{\#} + \tilde{h} c_{t+k-1}) + \frac{P_{t+k}^b}{z_{t+k}^c} b_{t+k}^r - \frac{1}{(1+\pi_{t+k})} b_{t+k-1}^r - w_{t+k}^r n_t(i) - t_{t+k}^r \Big] \Bigg\} \tag{3.12}$$

or

$$\mathcal{L} = E_t \sum_{k=0}^{\infty} \hat{\beta}^k \Bigg\{ \left[\frac{(c_{t+k}^{\#})^{1-\sigma}}{1-\sigma} - \frac{\theta_w^k \theta_{t+k}}{(z_{t+k}^l)^{\varphi}} \int_0^1 \frac{(W_t^{n*}/W_{t+k}^n)^{[-(1+\varphi)\epsilon_{w,t+k}]} n_t^{1+\varphi}}{1+\varphi} \right] - \Lambda_{t+k} \Big[(c_{t+k}^{\#} + \tilde{h} c_{t+k-1}) + \frac{P_{t+k}^b}{z_{t+k}^c} b_{t+k}^r - \frac{1}{(1+\pi_{t+k})} b_{t+k-1}^r - \theta_w^k \frac{1}{P_{t+k}} \frac{W_t^{n*(1-\epsilon_{w,t+k})}}{W_{t+k}^{n\;(-\epsilon_{w,t+k})}} n_{t+k} - t_{t+k}^r \Big] \Bigg\} \tag{3.13}$$

3.3.1 Consumption

The first-order condition of the problem (3.12) in relation to consumption is

$$\frac{\partial \mathcal{L}}{\partial c_t^{\#}} = (c_t^{\#})^{-\sigma} - \Lambda_t = 0 \tag{3.14}$$

and the bonds demand first order condition in problem (3.12) is:

$$\frac{\partial \mathcal{L}}{\partial b_t^r} = -\hat{\beta}^t \Lambda_t \frac{P_t^b}{z_t^c} b_t^r + \hat{\beta}^{t+1} E_t \Lambda_{t+1} \frac{1}{(1+\pi_{t+1})} b_t^r = 0 \tag{3.15}$$

Combining equation (3.14) with (3.15), we obtain the consumption Euler equation:

$$P_t^b = \frac{1}{(1+r_t)} = \hat{\beta} E_t \left[\left(\frac{c_t^{\#}}{c_{t+1}^{\#}} \right)^{\sigma} \frac{1}{(1+\pi_{t+1})} \right] z_t^c$$

where $\log(z_t^c) = \tilde{z}_t^c = \rho_c \tilde{z}_{t-1}^c + \varepsilon_t^c$, $\rho_c \in [0,1]$, and $\varepsilon_t^c \sim (0, \sigma_c^2)$. Log linearizing the above equation around the steady-state yields the New Keynesian IS curve in log deviations:

$$\hat{c}_t = \frac{\tilde{h}}{1+\tilde{h}} \hat{c}_{t-1} + \frac{1}{1+\tilde{h}} E_t \hat{c}_{t+1} - \frac{1-\tilde{h}}{(1+\tilde{h})\sigma} (\hat{r}_t - E_t \hat{\pi}_{t+1}) + \hat{z}_t^c \tag{3.16}$$

where we used the fact that $\hat{c}_t^{\#} = (\hat{c}_t - \tilde{h}\hat{c}_{t-1})/(1-\tilde{h})$; $\hat{h} = h \exp(g_y)$, where is the consumption external habit parameter and g_y is the output (Y) balanced growth path.[4]

3.3.2 Desired Labor Supply

The first order condition of the problem (3.12) in relation to labor supply is

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial l_t(i)} &= -\frac{\theta_t}{(z_t^l)^{\varphi}} n_t(i)^{\varphi} + \Lambda_t w_t^r(i) = 0 \\ &= -\frac{\theta_t}{(z_t^l)^{\varphi}} \left(\int_0^{l_t(i)} j dj \right)^{\varphi} + \Lambda_t w_t^r(i) = 0 \end{aligned} \tag{3.17}$$

where l_t is the “desired” labor supply in substitution to “effective labor supply” n_t, and

[4]In this text we are using a additional variables representation convention: a lowercase letter with a tilde $\tilde{x}_t = \log(x_t)$ indicates the log of a detrended variable (except if the letter is representing a parameter, e.g., $\tilde{\beta}$); a lowercase letter with a bar $\bar{x}_t = \log(x)$ indicates the log of a detrended variable at their steady-state value; and finally a lowercase letter with a hat ($\hat{x}_t = \log(x_t) - \log(x) = \tilde{x} - \bar{x}$) indicates the log-deviation of a detrended variable from its log steady-state value.

the second equality used the fact given in (3.1) that $n_t(i) = \int_0^{n_t(i)} j dj$ for the effective labor supply or $l_t(i) = \int_0^{l_t(i)} j dj$ for the desired labor supply.

Now consider an individual specialized in labor type (i) and with marginal rate of substitution between labor and consumption given by $mrs_t(i)$, taking the real wage prevailing $w_t^r(i)$ as given, this individual will be willing to work in period t if and only if $w_t^r \geq mrs_t(i)$. Combining equation (3.17) with (3.14), we obtain the desired labor supply $l_t(i)$:

$$
\begin{aligned}
w_t^r(i) &\geq mrs_t(i) = \theta_t (c_t^{\#})^{\sigma} \left(\int_0^{l_t(i)} j dj \right)^{\varphi} (z_t^l)^{-\varphi} \\
w_t^r(i) &= mrs_t(i) = (z_t^s)^{\sigma} l_t(i)^{\varphi} (z_t^l)^{-\varphi} \qquad (3.18)
\end{aligned}
$$

where mrs_t is the marginal rate of substitution between labor and leisure, and the second equality used the fact that $\theta_t = (z_t^s / c_t^{\#})^{\sigma}$.

The endogenous preferences shifter (θ_t) was introduced by Galí et al. (2012) based on Jaimovich and Rebelo (2009) in order to control the wealth (income) effect strength on labor supply. Such modification was necessary to overcome a counterfactual prediction of the standard New-Keynesian model. In the standard model, without the endogenous preferences shifter θ_t, the labor supply relation (3.18) - in logs and disregarding the habit formation in consumption $(c_t^{\#} = c_t)$ and shocks in labor supply $(z_t^l = 0 \; \forall \; t)$ - is given by:

$$
\tilde{w}_t^r = \sigma \tilde{c}_t + \varphi \tilde{l}_t
$$

where $\tilde{w}^r$ is the log of real wage, $\tilde{c}$ is the log of consumption and $\tilde{l}$ is the log of labor supply.

According to Christiano et al. (2010), this specification leads to counterfactual predictions about the labor force evolution. Evidence suggests labor force decreases in response to an negative monetary shock, while consumption decreases more than real wages. This does not occur in the standard model without an endogenous preference shifter. Incorporating the preference shifter θ_t into the model, labor supply now is given by (again disregarding consumption habits and labor supply shocks):

$$\tilde{w}_t^r = \sigma \tilde{z}_t^s + \varphi \tilde{l}_t$$
$$= \sigma[\gamma_c \tilde{c}_t + (1-\gamma_c)\tilde{z}_{t-1}^s] + \varphi \tilde{l}_t$$

where $\gamma_c \in [0,1]$, controls the wealth effect on labor supply. For sufficiently low values of γ_c, the counterfactual predictions of the standard model can be overturned.

Now let's go back to the equation (3.18). Recalling that $\log z_t^s = \tilde{z}_t^s = \gamma_c \tilde{c}_t^{\#} + (1-\gamma_c)\tilde{z}_{t-1}^s$ and iterating backwards, this relation can be written as $\tilde{z}_t^s = \gamma_c \sum_{k=0}^{\infty}(1-\gamma_c)^k \tilde{c}_{t-k}^{\#}$, then the labor supply can be written (in logs) as $\tilde{w}_t^r(i) = \sigma\gamma_c \sum_{k=0}^{\infty}(1-\gamma_c)^k \tilde{c}_{t-k}^{\#} + \varphi \tilde{l}_t(i) - \tilde{z}_t^l$. The infinite sum allows us to apply the Koyck (1954) transformation to the relation of labor supply (in logs) what results in

$$\tilde{w}_t^r(i) = (1-\gamma_c)\tilde{w}_{t-1}^r(i) + \sigma\gamma_c \tilde{c}_t^{\#} + \varphi \tilde{l}_t(i) - (1-\gamma_c)\varphi \tilde{l}_{t-1}(i) - \varphi\left[\tilde{z}_t^l - (1-\gamma_c)\tilde{z}_{t-1}^l\right]$$

aggregating the above equation in i and writing it in log deviations from the steady-state, we have the following aggregate labor supply relation

$$\hat{w}_t^r = (1-\gamma_c)\hat{w}_{t-1}^r + \frac{\sigma\gamma_c}{1-\tilde{h}}(\hat{c}_t - \tilde{h}\hat{c}_{t-1}) + \varphi\hat{l}_t - (1-\gamma_c)\varphi\hat{l}_{t-1} - \varphi\left[\hat{z}_t^l - (1-\gamma_c)\hat{z}_{t-1}^l\right] \quad (3.19)$$

where we used the fact that $\hat{c}_t^{\#} = (\hat{c}_t - \tilde{h}\hat{c}_{t-1})/(1-\tilde{h})$; $\hat{w}_t^r \approx \int_0^1 \hat{w}_t^r(i)di$, and $\hat{l}_t \approx \int_0^1 \hat{l}_t(i)di$.

3.3.3 Wage-Setting

The first order condition of the problem (3.13) in relation to nominal wage w_t^* is:

$$\frac{\partial \mathcal{L}}{\partial w_t^*} = \sum_{k=0}^{\infty} \hat{\beta}^k \Bigg\{ -\frac{\theta_w^k \theta_{t+k}}{(z_{t+k}^l)^{\varphi}}[-(1+\varphi)\epsilon_{w,t+k}](w_t^*)^{-1}\left(\frac{w_t^{n*}}{w_{t+k}^n}\right)^{-\epsilon_{w,t+k}} n_{t+k}^{1+\varphi}$$
$$+\Lambda_{t+k}\theta_w^k \frac{1}{p_{t+k}}(1-\epsilon_{w,t+k})\left(\frac{w_t^*}{w_{t+k}^n}\right)^{-\epsilon_{w,t+k}} n_{t+k} \Bigg\} = 0$$

$$
\begin{aligned}
&= \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k \left\{ \frac{\theta_{t+k}\epsilon_{w,t+k}}{(z^l_{t+k})^{\varphi}} (w^*_t)^{-1} \left(\frac{w^*_t}{w^n_{t+k}} \right)^{-\epsilon_{w,t+k}(1+\varphi)} n^{1+\varphi}_{t+k} \right. \\
&\left. -\Lambda_{t+k} \frac{1}{p_{t+k}} (\epsilon_{w,t+k} - 1) \left(\frac{w^*_t}{w^n_{t+k}} \right)^{-\epsilon_{w,t+k}} n_{t+k} \right\} = 0 \\
&= \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k \left\{ \frac{\theta_{t+k}\epsilon_{w,t+k}}{(z^l_{t+k})^{1+\varphi}} n^{1+\varphi}_{t+k}(i) - (c^{\#}_{t+k})^{-\sigma} \frac{w^*_t}{p_{t+k}} (\epsilon_{w,t+k} - 1) n_{t+k}(i) \right\} = 0 \\
&= \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k \left\{ \frac{w^*_t}{p_{t+k}} - \frac{\epsilon_{w,t+k}}{(\epsilon_{w,t+k} - 1)} \frac{\theta_{t+k}}{(z^l_{t+k})^{1+\varphi}} (c^{\#}_{t+k})^{\sigma} n^{\varphi}_{t+k}(i) \right\} = 0 \\
&= \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k \left\{ \frac{w^*_t}{p_{t+k}} - \mu^n_{w,t+k} mrs_{t+k} \right\} = 0
\end{aligned} \tag{3.20}
$$

Isolating w^*_t in the above equation, we obtain the nominal wage w^*_t chosen by the workers

$$
w^*_t = (1 - \hat{\beta}\theta_w) \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k \left[p_{t+k} \mu^n_{w,t+k} mrs_{t+k} \right] \tag{3.21}
$$

where $\mu^n_{w,t+k} = \epsilon_{w,t+k}/(\epsilon_{w,t+k} - 1)$ is the natural gross wage markup.

Log-linearizing the above equation around the steady-state results in an equation for the optimal wage w^*_t in log deviations

$$
\hat{w}^*_t = (1 - \hat{\beta}\theta_w) E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k [\hat{mrs}_{t+k}(i) + \hat{p}_{t+k} + \hat{\mu}^n_{w,t}]
$$

this equation can be written as

$$
\hat{w}^*_t = (1 - \hat{\beta}\theta_w) E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k [\hat{mrs}_{t+k}(i) - \hat{mrs}_{t+k} + \hat{mrs}_{t+k} + \hat{p}_{t+k} + \hat{\mu}^n_{w,t}]
$$

considering that $\hat{\mu}_{w,t+k} = \hat{w}^n_{t+k} - \hat{mrs}_{t+k} - \hat{p}_{t+k}$, the above equation can be rewritten as

$$
\hat{w}^*_t = (1 - \hat{\beta}\theta_w) E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k [\hat{mrs}_{t+k}(i) - \hat{mrs}_{t+k} + \hat{w}^n_{t+k} - (\hat{\mu}_{w,t+k} - \hat{\mu}^n_{w,t+k})] \tag{3.22}
$$

From equation (3.18), the average marginal rate of substitution for "employed" workers is $mrs_t = (z^l_t)^{-\varphi} (z^s_t)^{\sigma} n_t(i)^{\varphi}$, where $n_t = \int_0^1 n_t(i) di$ is the aggregate employment.

Multiplying and dividing the equation (3.18) by n_t^{φ}, we have:

$$mrs_t(i) = (z_t^l)^{-\varphi}(z_t^s)^{\sigma} n_t^{\varphi} \frac{n_t(i)^{\varphi}}{n_t{}^{\varphi}}$$
$$= mrs_t \left[\frac{n_t(i)}{n_t}\right]^{\varphi}$$

where the last equality used the fact that $mrs_t = (z_t^l)^{-\varphi}(z_t^s)^{\sigma} n_t(i)^{\varphi}$.

The log-linearization of equation above around the steady-state results in

$$m\hat{r}s_t(i) = m\hat{r}s_t + \varphi[\hat{n}_t(i) - \hat{n}_t]$$

On the other hand, log-linearizing the equation (3.10) around the steady-state $[\hat{n}_t(i) = -\epsilon_w(\hat{w}_t^n - \hat{w}_t)]$ and substituting this result in equation above results in

$$m\hat{r}s_t(i) = m\hat{r}s_t - \varphi\epsilon_w(\hat{w}_t^* - \hat{w}_t^n) \tag{3.23}$$

Substituting equation (3.23) into (3.22) results in a new equation for the optimal wage $\hat{w}_t^*$ log deviation

$$\hat{w}_t^* = \frac{(1-\hat{\beta}\theta_w)}{(1+\varphi\epsilon_w)} E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_w)^k \left[(1+\varphi\epsilon_w) w_{t+k}^n - (\hat{\mu}_{w,t+k} - \hat{\mu}_{w,t+k}^n)\right]$$
$$= \hat{\beta}\theta_w \hat{w}_{t+1}^* + (1-\hat{\beta}\theta_w)\hat{w}_t^n - \frac{(1-\hat{\beta}\theta_w)}{(1+\varphi\epsilon_w)}(\hat{\mu}_{w,t} - \hat{\mu}_{w,t}^n) \tag{3.24}$$

Families that can not optimally adjust wages choose a wage $w_t^{\#}$ according to a rule-of-thumb that indexes wages to past prices inflation π_{t-1}^p (Galí et al., 2012), and the balanced growth path productivity growth rate g_a,

$$w_t^{\#} = w_{t-1}^n (1+g_a)(1+\pi_{t-1}^p)^{\gamma_w}(1+\bar{\pi}^p)^{(1-\gamma_w)} \tag{3.25}$$

log-linearizing the above equation around the steady-state

$$\hat{w}_t^{\#} = \hat{w}_{t-1}^n + \gamma_w \hat{\pi}_{t-1}^p \tag{3.26}$$

Since a fraction $(1-\theta_w)$ can choose its wage rate w_t^* optimally, and the remaining fraction θ_w fixes its wage $w_t^{\#}$ according to the rule given in (3.25), the average nominal wage is given by

$$w_t^n = \left[\theta_w (w_t^{\#})^{1-\epsilon_{w,t}} + (1-\theta_w)(w_t^*)^{1-\epsilon_{w,t}}\right]^{\frac{1}{1-\epsilon_{w,t}}}$$

Log-linearizing the above equation around the steady-state results in

$$\hat{w}_t^n = \theta_w \hat{w}_t^{\#} + (1-\theta_w)\hat{w}_t^* \tag{3.27}$$

Substituting equations (3.24) and (3.26) into (3.27), we obtain the following equation for wage inflation

$$\hat{\pi}_t^w = \tilde{\beta} E_t \left(\hat{\pi}_{t+1}^w - \gamma_w \hat{\pi}_t^p\right) + \gamma_w \hat{\pi}_{t-1}^p - \lambda_w \left(\hat{\mu}_t^w - \hat{\mu}_{w,t}^n\right) \tag{3.28}$$

where $\lambda_w = \frac{(1-\hat{\beta}\theta_w)(1-\theta_w)}{(1+\varphi\epsilon_w)\theta_w}$; $\tilde{\mu}_{w,t}^n = (1-\rho_w)\bar{\mu}^w + \rho_w \tilde{\mu}_{w,t-1}^n + \varepsilon_{w,t}$ and $\varepsilon_{w,t} \sim i.i.d.(0, \sigma_w^2)$; and $\hat{\mu}_t^w = (\hat{w}_t - \hat{p}_t) - \hat{mrs}_t$. Without indexation to past inflation ($\gamma_w = 0$), the wage inflation equation becomes

$$\hat{\pi}_t^w = \hat{\beta} E_t \hat{\pi}_{t+1}^w - \lambda_w \left(\hat{\mu}_t^w - \hat{\mu}_{w,t}^n\right)$$

the inflation of wages depends positively on the expected wages inflation rate and negatively on the difference between effective and natural markup.

3.3.4 Unemployment Rate and the New Keynesian Wage Phillips Curve

We will now introduce the unemployment rate into the model and insert it into the wage inflation equation (3.28). Making ($k = 0$) in equation (3.21), we have

$$\begin{aligned}\frac{w_t^n}{p_t} = w_t^r &= \mu_{w,t} mrs_t \\ &= \mu_{w,t} (z_t^l)^{-\varphi} (z_t)^{\sigma} n_t^{\varphi}\end{aligned}$$

the second equality used the equation (3.18) multiplied by the average gross markup $\mu_{w,t}$, and mrs_t is the marginal rate of substitution between consumption and labor for the employed workers and n_t is the employment. Log-linearization of the above equation around the steady-state yields

$$\begin{aligned}\hat{\mu}_{w,t} &= \hat{w}_t^r - m\hat{r}s_t \\ &= \hat{w}_t^r - \sigma\hat{z}_t^s - \varphi\hat{n}_t + \varphi\hat{z}_t^l\end{aligned} \quad (3.29)$$

Equation (3.29) says that the log mean markup is given by the difference (in logs) between the average real wage and the employed workers marginal rate of substitution (mrs). Looking back at the equation (3.18) ($w_t^r(i) = mrs_t(i) = (z_t^s)^\sigma l_t(i)^\varphi (z_t^l)^{-\varphi}$), the reasoning behind this equation is that a worker (i) decides to offer work if $w_t^r \geq mrs_t$. When $mrs_t \neq z_t^l(z_t^s)^\sigma l_t^\varphi$, the worker will adjust his labor supply l_t to the point where $w_t^r = mrs_t$. Log-linearizing the equation (3.18) around the balanced growth path yields

$$\begin{aligned}\hat{w}_t^r &= m\hat{r}s_t \\ &= \sigma\hat{z}_t^s + \varphi\hat{l} - \varphi\hat{z}_t^l\end{aligned} \quad (3.30)$$

Recalling that the observed mean real wage w_t^r is the same for employed and unemployed workers, we can insert equation (3.29) in equation (3.30) and obtain the following relation between the wage markup $\hat{\mu}_{w,t}$ and the labor-force l_t and employment n_t log deviations

$$\hat{\mu}_{w,t} = \varphi(\hat{l}_t - \hat{n}_t) \quad (3.31)$$

What $(\hat{l}_t - \hat{n}_t)$ will be depends on whether the labor supply L is cointegrated or non-cointegrated with the employment N. We saw earlier that if the labor supply is cointegrated with employment, then ($d_l = 0$) in the utility function (3.1). On the other hand, if the labor supply is non-cointegrated with employment, then ($d_l = 1$) in the utility function (3.1). If the labor supply is cointegrated with employment, the unemployment rate will be stationary and will converge to a long-run mean as predicted by the natural rate hypothesis of unemployment rate. To see this consider the detrended labor supply ($l_t = L_t/H_t$) and detrended employment ($n_t = N_t/H_t$), where $H_t = (N_{t-1})^{d_l}(H_{t-1})^{(1-d_l)}e^{g_{n,t}}$ is the labor-supply and employment common trend. If $d_l = 0$, $H_t = H_{t-1}e^{g_{n,t}}$ and we have

$$l_t = \frac{L_t}{H_t} = \frac{L_t}{H_{t-1}e^{g_{n,t}}}$$

or in logs deviations from steady-state

$$\hat{l}_t = \hat{L}_t - \hat{H}_{t-1} - \hat{g}_{n,t} \tag{3.32}$$

and

$$n_t = \frac{N_t}{H_t} = \frac{N_t}{H_{t-1}e^{g_{n,t}}}$$

or in logs deviations from steady-state

$$\hat{n}_t = \hat{N}_t - \hat{H}_{t-1} - \hat{g}_{n,t} \tag{3.33}$$

Subtracting (3.33) from (3.33), we have

$$\hat{u}_t = \hat{l}_t - \hat{n}_t = \hat{L}_t - \hat{N}_t \tag{3.34}$$

where $\hat{u}_t = u_t - u^n$ is the unemployment rate u_t deviation from the long-run natural rate of unemployment (u^n) that coincides with the steady-state unemployment rate . In equation (3.34), we used the fact that $\log(1-u_t) = \log(N_t/L_t) \approx -u_t$ and therefore $\hat{u}_t \approx \hat{L}_t - \hat{N}_t$.

Substituting (3.34) in (3.31), one obtains

$$\hat{\mu}_{w,t} = \varphi(u_t - u^n), \quad \hat{\mu}^n_{w,t} = \varphi(u^n_t - u^n) \quad \text{and} \quad \bar{\mu}^n_w = \varphi u^n \tag{3.35}$$

where $\bar{\mu}^n = \log[\epsilon_w/(\epsilon_w - 1)]$.

If the labor supply is no cointegrated with employment, the unemployment rate will be difference-stationary and its level will be a $I(1)$ process as predicted by the total (full) hysteresis hypothesis for the unemployment rate. Again consider the detrended labor supply ($l_t = L_t/H_t$) and detrended employment ($n_t = N_t/H_t$), where $H_t = (N_{t-1})^{d_l}(H_{t-1})^{(1-d_l)}e^{g_{n,t}}$ is the labor-supply and employment common trend. If

$d_l = 1$, $H_t = N_{t-1}e^{g_{n,t}}$ and we have

$$l_t = \frac{L_t}{H_t} = \frac{L_t}{H_{t-1}e^{g_{n,t}}}$$

or in logs deviations from steady-state

$$\hat{l}_t = \hat{L}_t - \hat{L}_{t-1} - \hat{g}_{n,t} \tag{3.36}$$

and

$$n_t = \frac{N_t}{H_t} = \frac{N_t}{N_{t-1}e^{g_{n,t}}}$$

or in logs deviations from steady-state

$$\hat{n}_t = \hat{N}_t - \hat{N}_{t-1} - \hat{g}_{n,t} \tag{3.37}$$

Subtracting (3.36) from (3.37), we have

$$\Delta\hat{u}_t = \Delta\hat{l}_t - \Delta\hat{n}_t = \Delta\hat{L}_t - \Delta\hat{N}_t \tag{3.38}$$

where $\Delta\hat{u}_t = u_t - u_{t-1}$ is the unemployment rate first differences. In equation (3.38), we used the fact that $\log(1-u_t) = \log(N_t/L_t) \approx -u_t \approx \hat{N}_t - \hat{L}_t$ and therefore $\Delta\hat{u}_t \approx \Delta\hat{L}_t - \Delta\hat{N}_t \approx \Delta\hat{l}_t - \Delta\hat{n}_t$.

In this case substituting equation (3.38) in (3.31) one obtains

$$\hat{\mu}_{w,t} = \varphi\Delta u_t \tag{3.39}$$

where $\Delta u_t = u_t - u_{t-1}$ and $\hat{\mu}_{w,t} = \mu_{w,t} - \bar{\mu}^n_w$.

Substituting equation (3.35) into equation (3.28), we obtain the New Keynesian Wage Phillips Curve in terms unemployment rate deviations from its natural rate, or the No-hysteresis Wage Phillips Curve:

$$\hat{\pi}^w_t = \hat{\beta}E_t\left(\hat{\pi}^w_{t+1} - \gamma_w\hat{\pi}^p_t\right) + \gamma_w\hat{\pi}^p_{t-1} - \lambda_w\varphi(u_t - u^n_t) \tag{3.40}$$

where $u^n_t = (1/\varphi)\hat{\mu}^n_{w,t}$.

Regarding the hysteretic model, substituting equation (3.39) into equation (3.28), we obtain the New Keynesian Wage Phillips Curve in terms of the unemployment first differences, or the With Hysteresis Wage Phillips Curve (WHWPC):

$$\hat{\pi}_t^w = \tilde{\beta} E_t \left(\hat{\pi}_{t+1}^w - \gamma_w \hat{\pi}_t^p \right) + \gamma_w \hat{\pi}_{t-1}^p - \lambda_w \varphi \Delta u_t + \lambda_w \hat{\mu}_{w,t}^n \tag{3.41}$$

3.4 FIRMS BEHAVIOR

We assume there is a continuum $v \in [0,1]$ of monopolistic competitive firms, each of them producing a differenced good $v \in [0,1]$ with the following production technology:

$$y_t(j) = z_t^a \left[n_t(v) \right]^{(1-\alpha)} + \frac{\Phi - 1}{\Phi} y \tag{3.42}$$

where, $n_t(v) = \left[\int_0^1 n_t(i,v)^{(\epsilon_{w,t}-1)/\epsilon_{w,t}} di \right]^{\epsilon_{w,t}/(\epsilon_{w,t}-1)}$, is a CES index of the different types of labor i employed by the firm v; and $z_t^a = (z^a)^{(1-\rho_{za})} (z_{t-1}^a)^{\rho_{za}} e_t^{\varepsilon_t^{za}}$ is an exogenous temporary productivity shifter with $\rho_{za} \in [0,1]$, and $\varepsilon_t^{za} \sim i.i.d.(0, \sigma_{za}^2)$.

Following Calvo (1983), it is assumed that in each period only a randomly selected fraction $(1-\theta_p)$ of firms has permission to adjust optimally their prices. The fraction $(1-\theta_p)$ is constant over time, which means that firms selection is independent of the time elapsed since the last optimal adjust. The remaining fraction θ_p of firms that is not selected to optimally adjust their prices, indexes their prices $p_t^{\#}$ to last inflation rate and to trend inflation $p_t^{\#} = (1+\pi_t^p)^{\gamma_p} (1+\bar{\pi}^p)^{(1-\gamma_p)}$, where $\bar{\pi}^p$ is the trend inflation rate or the long-run inflation rate. The firm that can optimally adjust its price in period t, set p_t^* to maximize:

$$\max E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_p)^k \left[p_t^* y_{t+k}(v) - p_{t+k} y_{t+k}(v) mc_{t+k}(v) \right] \tag{3.43}$$

subject to

$$y_{t+k}(v) = \left(\frac{p_t^*}{p_{t+k}} \right)^{-\epsilon_{p,t+k}} y_{t+k} \tag{3.44}$$

where $mc_t(v)$ is the real marginal cost of the activity v, and $\epsilon_{p,t}$ is the substitution elasticity between the goods $y_t(v)$. The first order condition to problem above is:

$$E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_p)^k \left[\frac{\epsilon_{p,t+k}}{\epsilon_{p,t+k} - 1} p_{t+k} mc_{t+k}(v) - p_t^* \right] y_{t+k}(v) = 0$$

Solving the equation above to p_t^*, we have

$$p_t^* = \frac{E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_p)^k \mu_{p,t+k} p_{t+k} mc_{t+k}(v) y_{t+k}(v)}{E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_p)^k y_{t+k}(v)}$$

where $\mu_{p,t} = \epsilon_{p,t}/(\epsilon_{p,t} - 1)$ is the gross price markup (markup over marginal cost) with $\log \mu_{p,t} = \tilde{\mu}_{p,t} = (1 - \rho_p)\tilde{\mu}_p + \rho_p \tilde{\mu}_{p,t-1} + \varepsilon_t^p$, $\rho_p \in [0, 1]$, $\varepsilon^p \sim$ i.i.d.$(0, \sigma_p^2)$, and $\tilde{\mu}_p$ is the steady-state markup in log.

Log-linearizing the above equation around the long-run inflation, we obtain an equation for the optimal log price choice (p_t^*) in terms of long run inflation deviations (in logs):

$$\hat{p}_t^* = (1 - \hat{\beta}\theta_p) E_t \sum_{k=0}^{\infty} (\hat{\beta}\theta_p)^k \left[\hat{p}_{t+k} + \hat{mc}_{t+k}(v) + \hat{\mu}_{t+k}^p \right] \tag{3.45}$$

Let's now derive a relationship between the (log deviation form steady-state) marginal cost of a firm v $[\hat{mc}_t(v)]$ and the economy's average marginal cost (in log deviation from steady-state) $(\hat{mc}_t)$. The latter is defined by $\hat{mc}_t = \hat{w}_t^r - \hat{mpn}_t$, where $\hat{w}_t^r$ is the (log deviation form steady-state) real wage $(\tilde{w}_t^r = \tilde{w}_t^n - \tilde{p}_t)$, $\hat{mpn}_t$ is the economy average marginal product of labor (in log deviations from steady-state) given by $\hat{mpn}_t = \hat{z}_t^a - \alpha \hat{n}_t = \{1/[\Phi(1 - \alpha)]\} (\Phi \hat{z}_t^a - \alpha \hat{y}_t)$, where the last equality made use of the production function (3.42) log-linearized around the steady-state. So the economy's average marginal cost (in log deviations from steady-state) is

$$\hat{mc}_t = \hat{w}_t^r + \frac{1}{\Phi(1 - \alpha)} [\alpha \hat{y}_t - \Phi \hat{z}_t^a] \tag{3.46}$$

and a similar relationship holds for the marginal cost (in log deviations from steady-state) of a firm v: $\hat{mc}_t(v) = \hat{w}_t^r + \{1/[\Phi(1 - \alpha)]\} [\alpha \tilde{y}_t(v) - \Phi \tilde{z}_t^a]$. Combining these two marginal cost relations we obtain a equation relating individual and average marginal costs: $\hat{mc}_t(v) = \hat{mc}_t + \{\alpha/[\Phi(1 - \alpha)]\} [\hat{y}_t(v) - \hat{y}_t]$, log-linearizing the demand function (3.11) around the steady-state $[\hat{y}_t(v) = -\epsilon_p[\hat{p}_t^* - \hat{p}_t] + \hat{y}_t]$ and combining with $\hat{mc}_t(v) = \hat{mc}_t + \{\alpha/[\Phi(1 - \alpha)]\} [\hat{y}_t(v) - \hat{y}_t]$, we obtain a new relation between individual and average marginal costs:

$$\hat{mc}_t(v) = \hat{mc}_t - \{\alpha\epsilon_p/[\Phi(1-\alpha)]\}\,(\hat{p}_t^* - \hat{p}_t) \tag{3.47}$$

Combining the equation (3.47) with (3.45), one obtains another relation to the new price set by firms allowed to adjust prices:

$$\begin{aligned}\hat{p}_t^* &= (1-\hat{\beta}\theta_p)E_t\sum_{k=0}^{\infty}(\hat{\beta}\theta_p)^k\left[\hat{p}_{t+k} + \frac{\Phi(1-\alpha)}{\Phi(1-\alpha)+\alpha\epsilon_p}(\hat{mc}_{t+k} + \hat{\mu}_{t+k}^p)\right] \\ &= \hat{\beta}\theta_p E_t\hat{p}_{t+1}^* + (1-\hat{\beta}\theta_p)\left[\hat{p}_t + \frac{\Phi(1-\alpha)}{\Phi(1-\alpha)+\alpha\epsilon_p}(\hat{mc}_t + \hat{\mu}_t^p)\right]\end{aligned} \tag{3.48}$$

Following Christiano et al. (2010), firms that can not optimally adjust price choose a new price $p_t^{\#}$ according to a rule-of-thumb that indexes prices to past inflation π_t^p and to trend inflation $\bar{\pi}^p$,

$$p_t^{\#} = p_{t-1}(1+\pi_{t-1}^p)^{\gamma_p}(1+\bar{\pi}^p)^{1-\gamma_p} \tag{3.49}$$

log-linearization of the above equation around the trend inflation results in

$$\hat{p}_t^{\#} = \hat{p}_{t-1} + \gamma_p\hat{\pi}_{t-1}^p \tag{3.50}$$

Since a fraction $(1-\theta_p)$ of firms can choose P_t^* optimally, and the remaining fraction θ_p adjusts prices according to the rule in (3.50), the average price index of the economy is given by

$$p_t = \left[\theta_p(p_t^{\#})^{1-\epsilon_{p,t}} + (1-\theta_p)(p_t^*)^{1-\epsilon_{p,t}}\right]^{\frac{1}{1-\epsilon_{p,t}}}$$

log-linearization of the above equation around the trend inflation yields

$$\hat{p}_t = \theta_p\hat{p}_t^{\#} + (1-\theta_p)\,\hat{p}_t^* \tag{3.51}$$

Substituting the equations (3.48) and (3.50) in (3.51), we finally obtain the New Keynesian Phillips Curve:

$$\hat{\pi}_t = \frac{\gamma_p}{1+\hat{\beta}\gamma_p}\hat{\pi}_{t-1} + \frac{\hat{\beta}}{1+\hat{\beta}\gamma_p}E_t\hat{\pi}_{t+1} + \frac{\lambda_p}{1+\hat{\beta}\gamma_p}(\hat{mc}_t + \hat{\mu}_t^p) \tag{3.52}$$

where $\lambda_p = [(1-\alpha)(1-\theta_p)(1-\hat{\beta}\theta_p)]/[\theta_p(1-\alpha+\alpha\epsilon_p)]$, and $\hat{\mu}_t^p = \rho_p\hat{\mu}_{t-1}^p + \varepsilon_t^p$ is the firms desired markup log deviation from steady-state with $\varepsilon_t^p \sim i.i.d.(0,\sigma_p^2)$.

3.5 MODEL SUMMARY IN LOG-LINEAR FORM

Since our model is a closed economy with no capital and no government, the condition of equilibrium in the goods market is $y_t = c_t$, so Euler's equation (3.16) (New Keynesian IS) can be rewritten for both models as:

$$\hat{y}_t = \frac{\tilde{h}}{1+\tilde{h}}\hat{y}_{t-1} + \frac{1}{1+\tilde{h}}E_t\hat{y}_{t+1} - \frac{1-\tilde{h}}{(1+\tilde{h})\sigma}(\hat{r}_t - E_t\hat{\pi}_{t+1}^p) + \hat{z}_t^y \tag{3.53}$$

where $\tilde{h} = \exp(g_y)h$, g_y is the balanced growth path of the output, and $\hat{z}_t^y = \rho_y\hat{z}_{t-1}^y + \varepsilon_t^y$ with $\rho_y \in [0,1]$ and $\varepsilon_t^y \sim i.i.d.(0,\sigma_y^2)$.

The equation (3.19) reproduced below represents the labor supply or the labor force in both models, i.e., people willing to work at the current average wage:

$$\hat{w}_t^r = (1-\gamma_c)\hat{w}_{t-1}^r + \frac{\sigma\gamma_c}{1-\tilde{h}}(\hat{c}_t - \tilde{h}\hat{c}_{t-1}) + \varphi\hat{l}_t - (1-\gamma_c)\varphi\hat{l}_{t-1} - \hat{z}_t^l + (1-\gamma_c)\hat{z}_{t-1}^l \tag{3.54}$$

where $\tilde{h} = \exp(g_y)h$, $\hat{z}_t^l = \rho_l\hat{z}_{t-1}^l + \varepsilon_t^l$ with $\rho_l \in [0,1]$ and $\varepsilon_t^l \sim i.i.d.(0,\sigma_l^2)$.

Given the equilibrium output, log-linearizing (3.42) and summing up over v, employment in log deviation from the steady-state is for the model "without hysteresis":

$$\hat{n}_t = \frac{(\hat{y}_t - z_t^a)}{\Phi(1-\alpha)} \tag{3.55}$$

For the model without hysteresis, the unemployment rate can be defined has:

$$\hat{u}_t = \hat{l}_t - \hat{n}_t \tag{3.56}$$

While for the model with hysteresis, the unemployment rate can be defined has:

$$\Delta\hat{u}_t = \hat{l}_t - \hat{n}_t \tag{3.57}$$

The inflation rate for both models is given by equation (3.52) repeated below

$$\hat{\pi}_t = \frac{\gamma_p}{1+\tilde{\beta}\gamma_p}\hat{\pi}_{t-1} + \frac{\tilde{\beta}}{1+\tilde{\beta}\gamma_p}E_t\hat{\pi}_{t+1} + \frac{\lambda_p}{1+\tilde{\beta}\gamma_p}(\hat{mc}_t + \hat{\mu}_t^p) \tag{3.58}$$

where $\lambda_p = [(1-\alpha)(1-\theta_p)(1-\tilde{\beta}\theta_p)]/[\theta_p(1-\alpha+\alpha\epsilon_p)]$, and $\hat{\mu}_t^p = \rho_p\hat{\mu}_{t-1}^p + \varepsilon_t^p$, with $\rho_p \in [0,1]$ and $\varepsilon_t^p \sim i.i.d.(0,\sigma_p^2)$.

The marginal cost represented in equation (3.46) is reproduced below

$$\hat{mc}_t = \hat{w}_t^r + \frac{1}{\Phi(1-\alpha)}[\alpha\hat{y}_t - \Phi\hat{z}_t^a] \tag{3.59}$$

where $\hat{z}_t^a = \rho_{za}\hat{z}_{t-1}^a + \epsilon_t^{za}$, with $\rho_{za} \in [0,1]$, and $\epsilon_t^{za} \sim i.i.d.(0,\sigma_{za}^2)$.

The real wage log deviations from steady-state $\hat{w}_t^r$ can be written has

$$\hat{w}_t^r = \hat{w}_{t-1}^r + \hat{\pi}_t^w - \hat{\pi}_t^p - \hat{g}_{a,t} + \alpha\hat{g}_{n,t} \tag{3.60}$$

Wage inflation in the model "without" hysteresis given in equation (3.40) is reproduced below

$$\hat{\pi}_t^w = \tilde{\beta}E_t\left(\hat{\pi}_{t+1}^w - \gamma_w\hat{\pi}_t^p\right) + \gamma_w\hat{\pi}_{t-1}^p - \lambda_w\varphi u_t + \lambda_w\hat{\mu}_{w,t}^n \tag{3.61}$$

where $w_1 = \tilde{\beta} = \beta\exp(g_y)$, $\lambda_w = \frac{[(1-\tilde{\beta}\theta_w)(1-\theta_w)]}{[(1+\varphi\epsilon_w)\theta_w]}$, and $\hat{\mu}_{w,t}^w = \rho_w\hat{\mu}_{w,t-1}^w + \epsilon_t^w$, with $\rho_w \in [0,1]$, and $\epsilon_t^w \sim i.i.d.(0,\sigma_w^2)$.

Wage inflation in the model "with" hysteresis given in equation (3.41) is reproduced below

$$\hat{\pi}_t^w = \tilde{\beta}E_t\left(\hat{\pi}_{t+1}^w - \gamma_w\hat{\pi}_t^p\right) + \gamma_w\hat{\pi}_{t-1}^p - \lambda_w\varphi\Delta u_t + \lambda_w\hat{\mu}_{w,t}^n \tag{3.62}$$

where $\Delta u_t = u_t - u_{t-1}$, $w_1 = \tilde{\beta} = \beta\exp(g_y)$, $\lambda_w = \frac{[(1-\tilde{\beta}\theta_w)(1-\theta_w)]}{[(1+\varphi\epsilon_w)\theta_w]}$, and $\hat{\mu}_{w,t}^w = \rho_w\hat{\mu}_{w,t-1}^w + \epsilon_t^w$, with $\rho_w \in [0,1]$, and $\epsilon_t^w \sim i.i.d.(0,\sigma_w^2)$.

To close the model, we assume the following Taylor rule:

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1-\phi_r)\left(\phi_\pi \hat{\pi}_t^p + \phi_y \hat{y}_t\right) + z_t^r \tag{3.63}$$

where $\hat{z}_{w,t}^r = \rho_w \hat{z}_{w,t-1}^r + \epsilon_t^r$, with $\rho_r \in [0,1]$, and $\epsilon_t^r \sim i.i.d.(0, \sigma_r^2)$.

3.5.1 Equations of Observation

In this subsection we show the observation equations, i.e., equations that links the models theoretical variables to observed variables in the economy. The output observation equation is:

$$\Delta \text{LogGDP}_t = g_{yt} + y_t - y_{t-1} \tag{3.64}$$

where ΔdLogGDP$_t$ is the log(GDP) first difference, and g_{yt} is the trend growth of GDP given by:

$$g_{yt} = g_{at} + (1-\alpha) g_{nt} \tag{3.65}$$

where g_{at} is the trend growth of the productivity and g_{nt} is the trend growth of the labor-supply both given by

$$g_{at} = (1-\rho_a)\bar{g_a} + \rho_a g_{at} + \varepsilon_t^a \tag{3.66}$$

and

$$g_{nt} = (1-\rho_a)\bar{g_n} + \rho_a g_{nt} + \varepsilon_t^n \tag{3.67}$$

where $\bar{g_a}$ is the long-run growth rate of the productivity, and $\bar{g_n}$ is the long-run growth rate of the labor-supply.

The labor-supply observation equation in the model without hysteresis is given by:

$$\Delta \text{LogLaS}_t = g_{nt} + l_t - l_{t-1} \tag{3.68}$$

where ΔdLaS$_t$ is the first difference of the labor-supply (LaS) in logs, and g_{nt} is the trend growth of the labor-supply.

The labor-supply observation equation in the model with hysteresis is given by:

$$\Delta \mathrm{LogLaS}_t = g_{nt} + l_t \tag{3.69}$$

The wage inflation observation equation is:

$$\Delta \mathrm{LogW}_t^n = \hat{\pi}_t^w + \bar{\pi}^p + \bar{g_a} - \alpha \bar{g_n} \tag{3.70}$$

where $\Delta \mathrm{LogW}_t^n$ is the first difference in logs of the average nominal wage and $\bar{\pi}^p$ is the long-run mean inflation.

The price inflation observation equation is:

$$\Delta \mathrm{LogP}_t = \bar{\pi}^p + \hat{\pi}_t^p \tag{3.71}$$

where $\Delta \mathrm{LogP}_t$ is the first difference in logs of the consumer price index (CPI) and $\bar{\pi}^p$ is the long-run mean inflation.

The interest rate observation equation is:

$$\frac{R_t}{100} = \frac{(1 + \bar{\pi}^p)}{\beta \exp(-\sigma \bar{g_y})} - 1 + \hat{r}_t \tag{3.72}$$

where R_t is the central bank short-term interest rate and $\bar{g_y}$ is the steady-state growth rate of output given by

$$\bar{g_y} = \bar{g_a} + (1 - \alpha)\bar{g_n} \tag{3.73}$$

Finally, the unemployment rate observation equation in the model without hysteresis is:

$$\Delta \mathrm{UnR} = u_t - u_{t-1} \tag{3.74}$$

where UnR is the observed unemployment rate.

While the unemployment rate observation equation in the model with hysteresis is:

$$\Delta \mathrm{UnR} = \Delta u_t \tag{3.75}$$

3.6 ESTIMATION

The models were estimated by Bayesian techniques using the software Dynare 4.4.3 (Adjemian et al., 2014) from the 1999Q1 to 2015Q4. Subsequent distributions of the parameters were computed using the Metropolis-Hastings algorithm "that uses Monte Carlo Markov Chain (MCMC) procedures. The quarterly data used in the model estimation were the first differences in logs of the IBGE's quarterly GDP, the economically active population and the unemployment rate of the IBGE Monthly Employment Survey (PME in Portuguese), wage inflation is measured by the first differences in logs of average income from the principal labor of IBGE's (PME), the consumer's inflation rate measured by the first differences in logs of the IBGE's CPI (IPCA in Brazil), and the short-term nominal interest rate of the Central Bank of Brazil (Selic rate).

All series were seasonally adjusted except the nominal interest rate. In regard of economically active population, unemployment and wages the source of these data - the Monthly Employment Survey (PME) - was reformulated in 2002 and some methodological aspects were modified. To complete our data set, the new methodology series were chained to the old methodology series. *Priors* sources for estimates came mainly from Castro et al. (2011) who developed the SAMBA[5], a DSGE model for the Brazilian economy used by the Central Bank of Brazil. The *prior* of the parameter that controls the wealth effect on labor supply was obtained from Galí et al. (2012). Calibrated parameters are shown in (TABLE 3.1).

TABLE 3.1 – CALIBRATED PARAMETERS

	Parameters and Description	Value	Source
ϵ_p	Elasticity of Substitution among goods	11	Castro et al. (2011)
α	Labor Elasticity in Production Function	0 (zero)	-
β	Quarterly Time Discount Factor	0.99	Standard in Literature

In (TABLE 3.2) are shown the estimated parameters for both models: No-Hysteresis Model (NHM) and With-Hysteresis Model (WHM). Let's comment some of them. The intertemporal consumption elasticity of substitution (σ) and the consumption habits parameter (h), had similar estimated values in both models ($\sigma = 1.27$, $h = 0.59$ in the NHM

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and $\sigma = 1.26$, $h = 0.59$ in the WHM). The parameter that controls the wealth effect on labor supply (γ_c) was estimated at 0.48 in the NHM and 0.63 in the WHM. This difference can mean that the wealth effect on labor supply is higher in the NHM compared to the WHM. This parameter was estimated at 0.01 in the Galí et al. (2012) model, which may show that the wealth effect on labor supply is more important in the American economy.

TABLE 3.2 – ESTIMATED PARAMETERS IN BOTH MODELS

Parameters	Prior Distribution			No hysteresis			With hysteresis		
Preferences	Dist.	Mean	S.D.	Mean	90% interval		Mean	90% interval	
σ intertemporal EoS	N	1.30	0.05	1.27	1.18	1.33	1.26	1.15	1.32
h consumption habit	B	0.70	0.04	0.59	0.49	0.66	0.59	0.51	0.64
φ Inverse Frisch Elasticity	G	5.00	1.00	5.41	4.13	6.90	4.24	2.86	5.30
γ_c wealth effect control	B	0.50	0.20	0.48	0.18	0.79	0.63	0.43	0.87
Production	Dist.	Mean	S.D.	Mean	90% interval		Mean	90% interval	
Φ fixed cost	G	5.00	0.50	3.96	3.18	4.62	2.20	1.58	2.95
Wages and prices nominal rigity	Dist.	Mean	S.D.	Mean	90% interval		Mean	90% interval	
ϵ_w EoS among labor types	G	5.00	1.00	4.27	2.76	5.98	2.71	1.72	3.76
θ_w calvo wage rigity index	B	0.75	0.05	0.68	0.54	0.81	0.47	0.39	0.54
γ_w wage indexation	B	0.50	0.15	0.51	0.24	0.86	0.77	0.64	0.94
θ_p calvo price rigity index	B	0.08	0.02	0.0846	0.05	0.115	0.09	0.05	0.12
γ_p price indexation	B	0.500	0.15	0.51	0.32	0.73	0.54	0.32	0.74
Taylor Rule	Dist.	Mean	S.D.	Mean	90% interval		Mean	90% interval	
ϕ_r mon. policy smoothing	B	0.60	0.15	0.43	0.20	0.64	0.43	0.23	0.63
ϕ_π inflation coefficient	N	2.00	0.35	2.22	1.67	2.73	2.84	2.38	3.44
ϕ_y output coefficient	G	0.25	0.10	0.71	0.32	1.06	0.31	0.11	0.47
Shocks autocorrelation	Dist.	Mean	S.D.	Mean	90% interval		Mean	90% interval	
ρ_y consumption shock	B	0.50	0.20	0.05	0.01	0.10	0.05	0.03	0.11
ρ_l temporaty labor-supply	B	0.50	0.20	0.71	0.54	0.89	0.59	0.28	0.81
ρ_w wage markup shock	B	0.50	0.20	0.04	0.01	0.07	0.04	0.01	0.06
ρ_p price markup shock	B	0.50	0.20	0.50	0.18	0.80	0.55	0.24	0.76
ρ_{za} temporaty productivity	B	0.50	0.20	0.68	0.52	0.87	0.76	0.52	0.96
ρ_r interest rate shock	B	0.50	0.20	0.48	0.19	0.766	0.62	0.29	0.84
ρ_a permanent productivity	B	0.50	0.20	0.23	0.06	0.43	0.15	0.03	0.26
ρ_n permanent labor-supply	B	0.50	0.20	0.29	0.08	0.50	0.44	0.24	0.79
Shocks standard deviation	Dist.	Mean	S.D.	Mean	90% interval		Mean	90% interval	
σ_y consumption shock	IG	1.00	∞	0.67	0.31	1.10	0.62	0.25	1.05
σ_l temporaty labor-supply	IG	1.00	∞	0.69	0.25	1.20	1.05	0.38	1.92
σ_w wage markup shock	IG	1.00	∞	0.91	0.29	1.71	0.62	0.29	1.03
σ_{za} temporaty productivity	IG	1.00	∞	0.60	0.22	0.98	0.74	0.35	1.10
σ_p price markup shock	IG	1.00	∞	0.68	0.27	1.39	0.90	0.26	1.43
σ_r interest rate shock	IG	1.00	∞	0.58	0.23	0.94	1.13	0.34	1.76
σ_a permanent productivity	IG	1.00	∞	0.79	0.29	1.55	0.65	0.30	1.28
σ_n permanent labor-supply	IG	1.00	∞	0.60	0.26	0.94	0.65	0.28	0.96

The Calvo price rigidity index (θ_p) was estimated at 0.08 in the NHM and at 0.09 in the WHM. These values imply a price mean duration of 1 quarter in both models. The price indexation to inflation parameter (γ_p) was estimated in 0.51 in the NHM and

0.54 in the WHM. The Calvo wage rigidity index (θ_w) was estimated at 0.68 in the NHM and at 0.47 in the WHM. These values imply a wage rate mean duration of 3 quarters in the NHM and 2 quarters in the WHM. The wage indexation to inflation parameter (γ_w) was estimated in 0.51 in the NHM and 0.77 in the WHM. In regard to Taylor Rule parameters, monetary policy smoothing parameter estimates were similar between the two models, inflation coefficient was 0.71 in the NHM and 0.31 in the WHM and the output coefficient was 2.22 in the NHM and 2.84 in the WHM.

In (FIGURE 3.1) we show the correlations and autocorrelations implicated by both models compared to the empirical counterparts. Both models replicate well the correlation between GDP growth with nominal wage inflation, GDP growth with CPI inflation and GDP growth with unemployment rate first differences. The autocorrelations of CPI inflation are well approximated by both models. GDP growth and unemployment rate first differences are poorly approximated by both models.

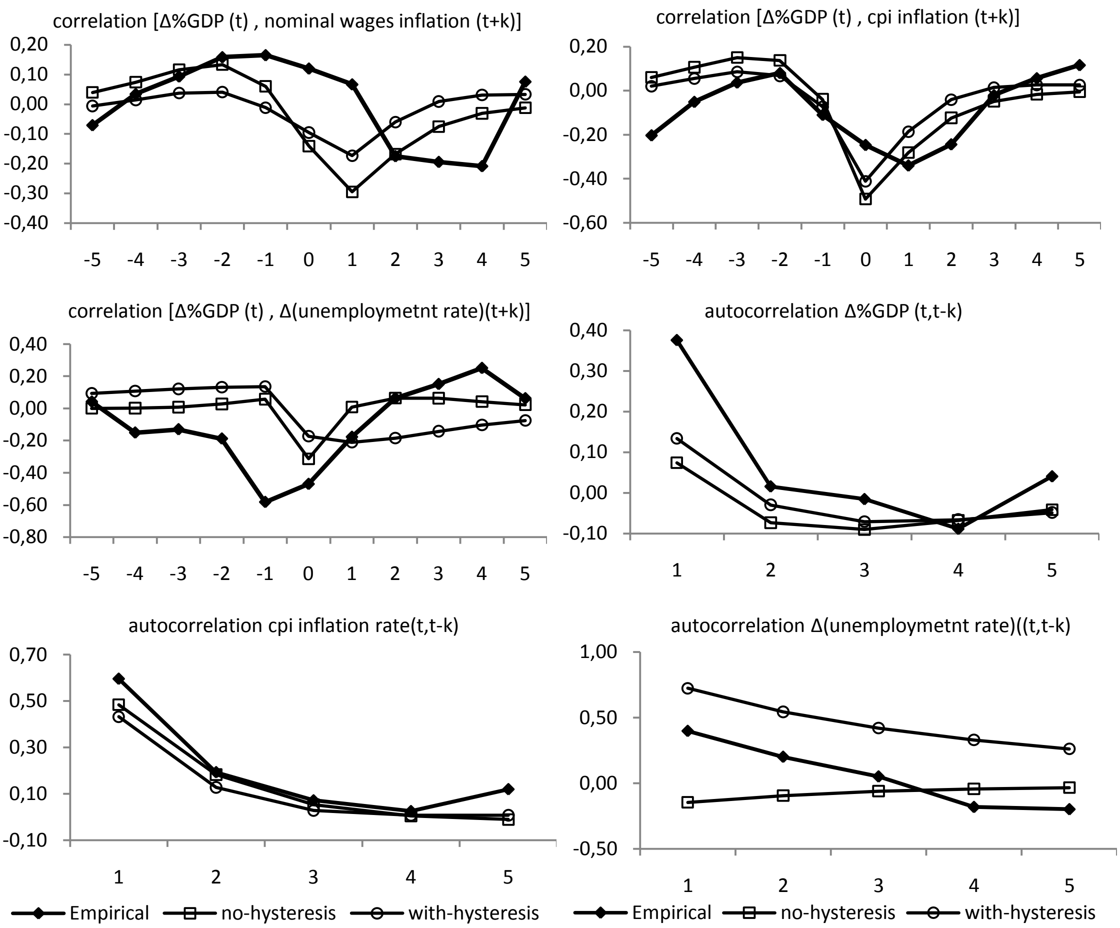

FIGURE 3.1 – DYNAMIC CORRELATION BETWEEN GDP GROWTH, CPI INFLATION, WAGE INFLATION AND UNEMPLOYMENT FIRST DIFFERENCES

3.6.1 Impulse Responses

In (FIGURE 3.2) the impulse response functions implied by the models in relation to a consumption-demand shock are presented. GDP, Wage inflation and CPI inflation responses are similar for both models. In the model with no hysteresis (WHM), the (log) real wage increases more than it does in the model with hysteresis (WHM), although the demand shock has no permanent effect on the real wage in both models. In relation to the unemployment rate, the demand shock produces a permanent fall in unemployment rate according to WHM, while in the NHM this fall is only temporary.

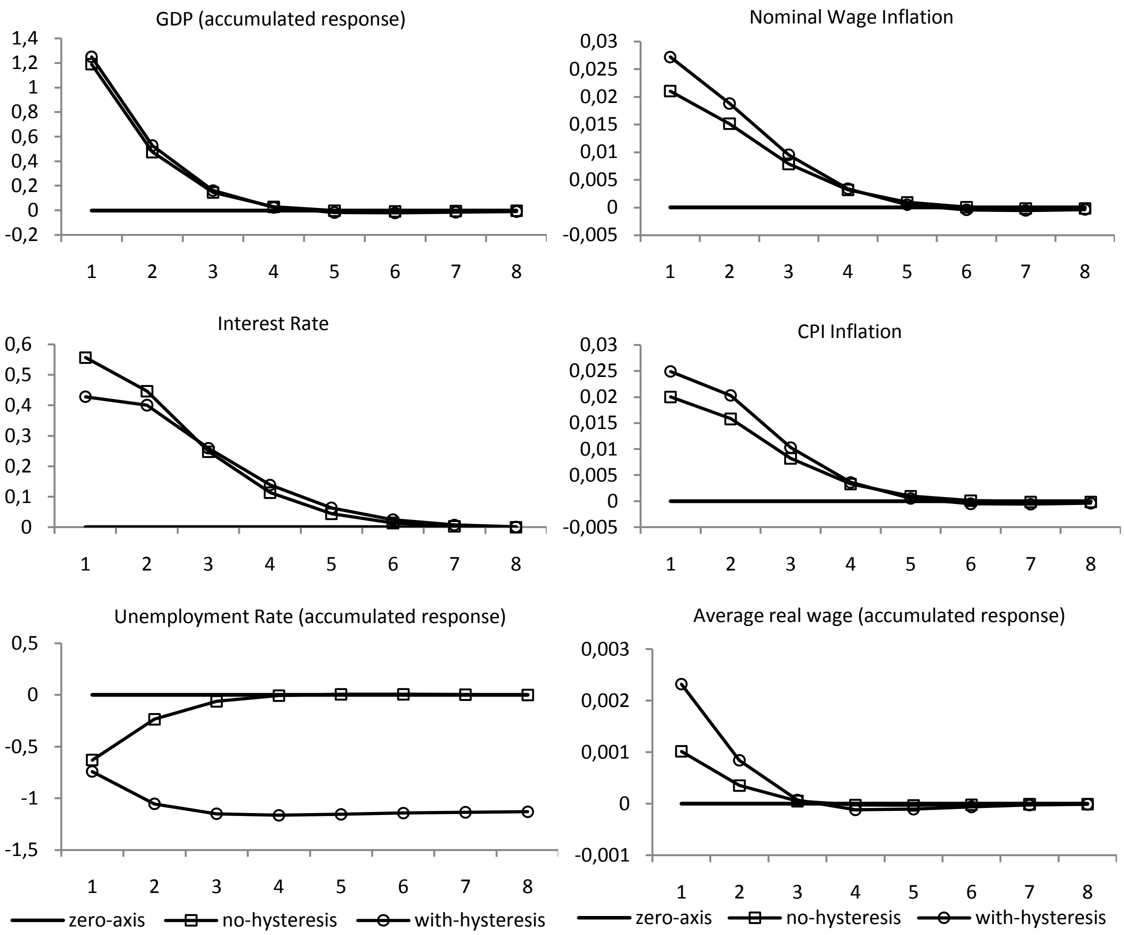

FIGURE 3.2 – IMPULSE RESPONSE FUNCTIONS IN (%) TO A 1% CONSUMPTION-DEMAND SHOCK - QUARTERS

In (FIGURE 3.3) the impulse response functions implied by the models in relation to a temporary productivity shock are presented. In the model with hysteresis a temporary positive productivity shock of 1% a permanent increase of 2.5% in the unemployment rate in the log-run. In the model without hysteresis, the temporary shock of productivity has a temporary effect on all variables. The other variables (GDP, wage inflation, CPI inflation and interest rate) responses is stronger in the model without hysteresis.

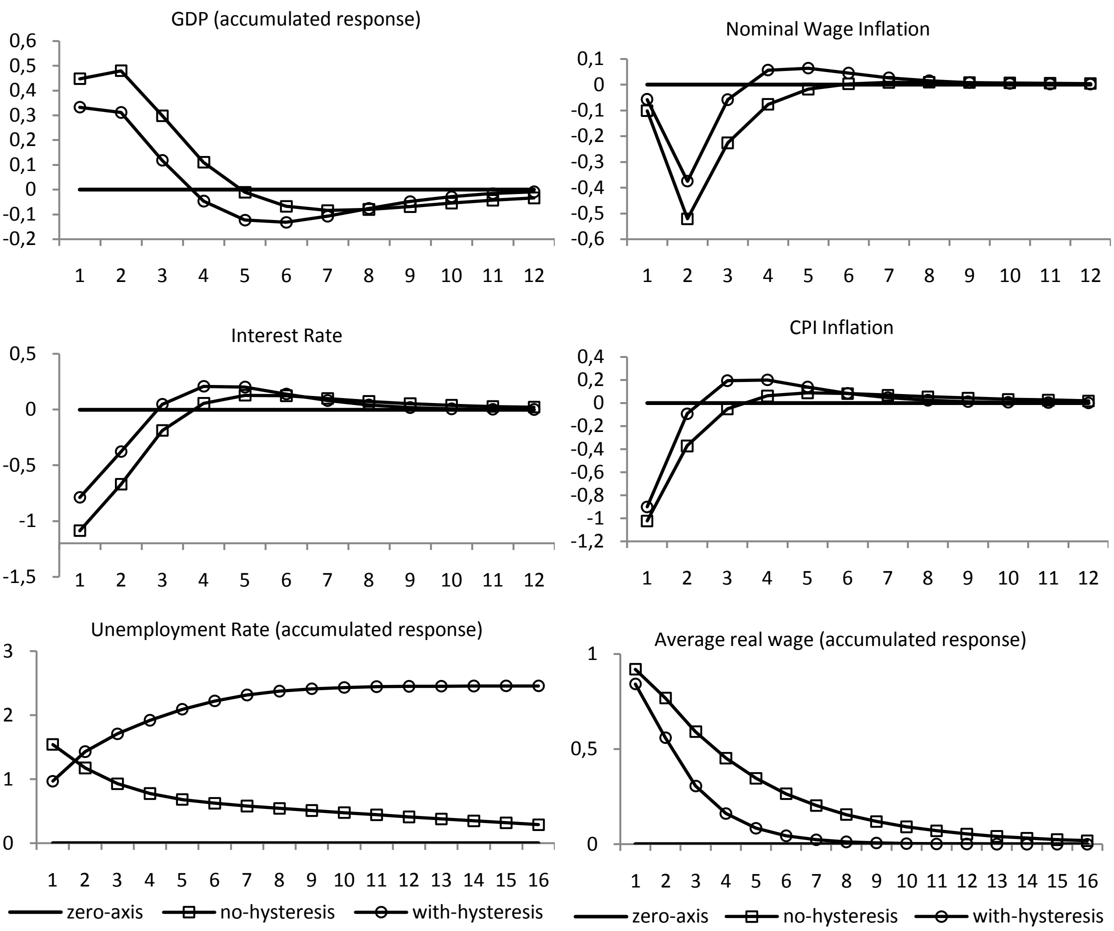

FIGURE 3.3 – IMPULSE RESPONSE FUNCTIONS IN (%) TO A 1% TEMPORARY PRODUCTIVITY SHOCK - QUARTERS

In (FIGURE 3.4) the impulse response functions implied by the models in relation to a permanent productivity shock are presented. Interest rate, CPI inflation rate, nominal wage inflation and real wage respond similarly to the permanent shock of productivity in both models. The GDP and real wage increases permanently in both models. In the model with hysteresis the unemployment rate decreases permanently by 1.25% in response to a 1% permanent productivity shock.

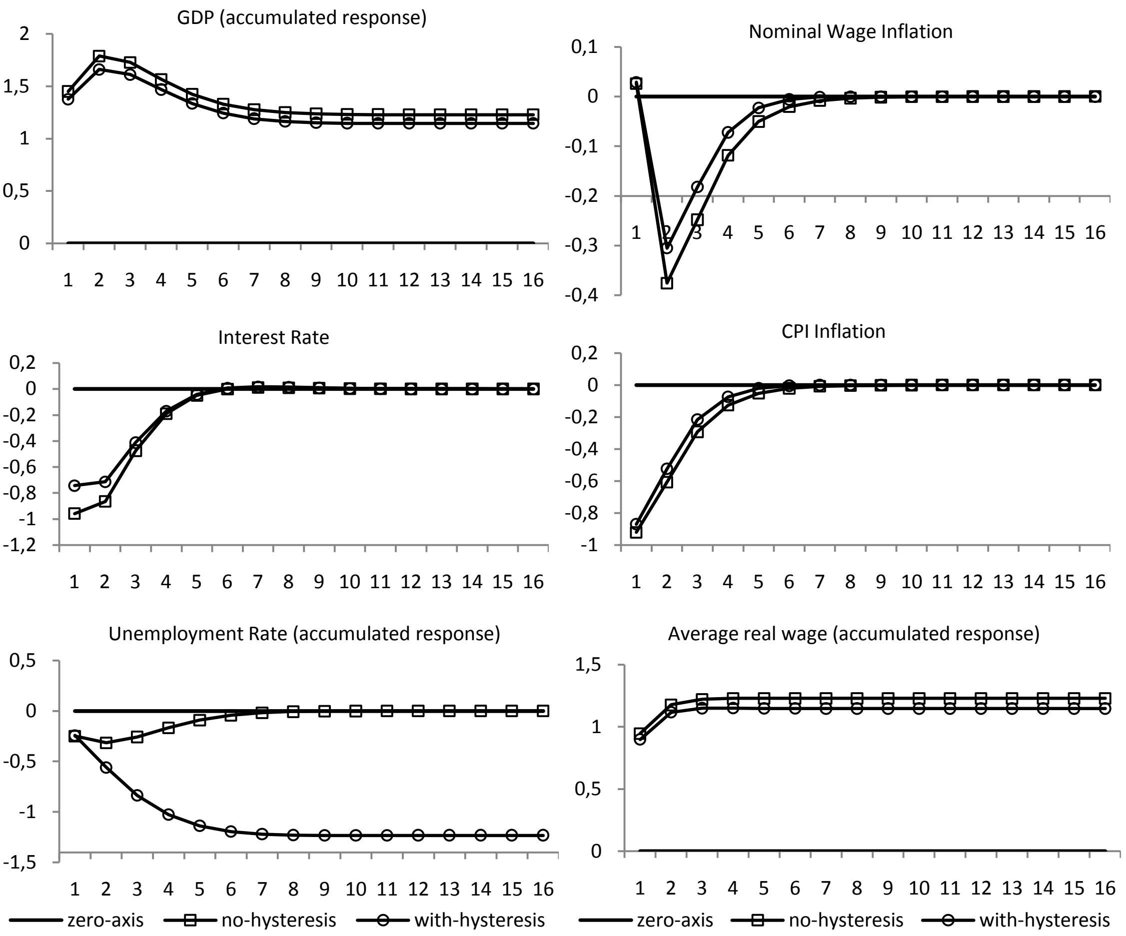

FIGURE 3.4 – IMPULSE RESPONSE FUNCTIONS IN (%) TO A 1% PERMANENT PRODUCTIVITY SHOCK - QUARTERS

In (FIGURE 3.5) the impulse response functions implied by the models in relation to a price markup shock are shown. Interest rate, CPI inflation rate, nominal wage inflation and real wage respond similarly to the price markup shock in both models. Unemployment rate decreases permanently in the WHM by 0.08% in response to a 1% price markup shock.

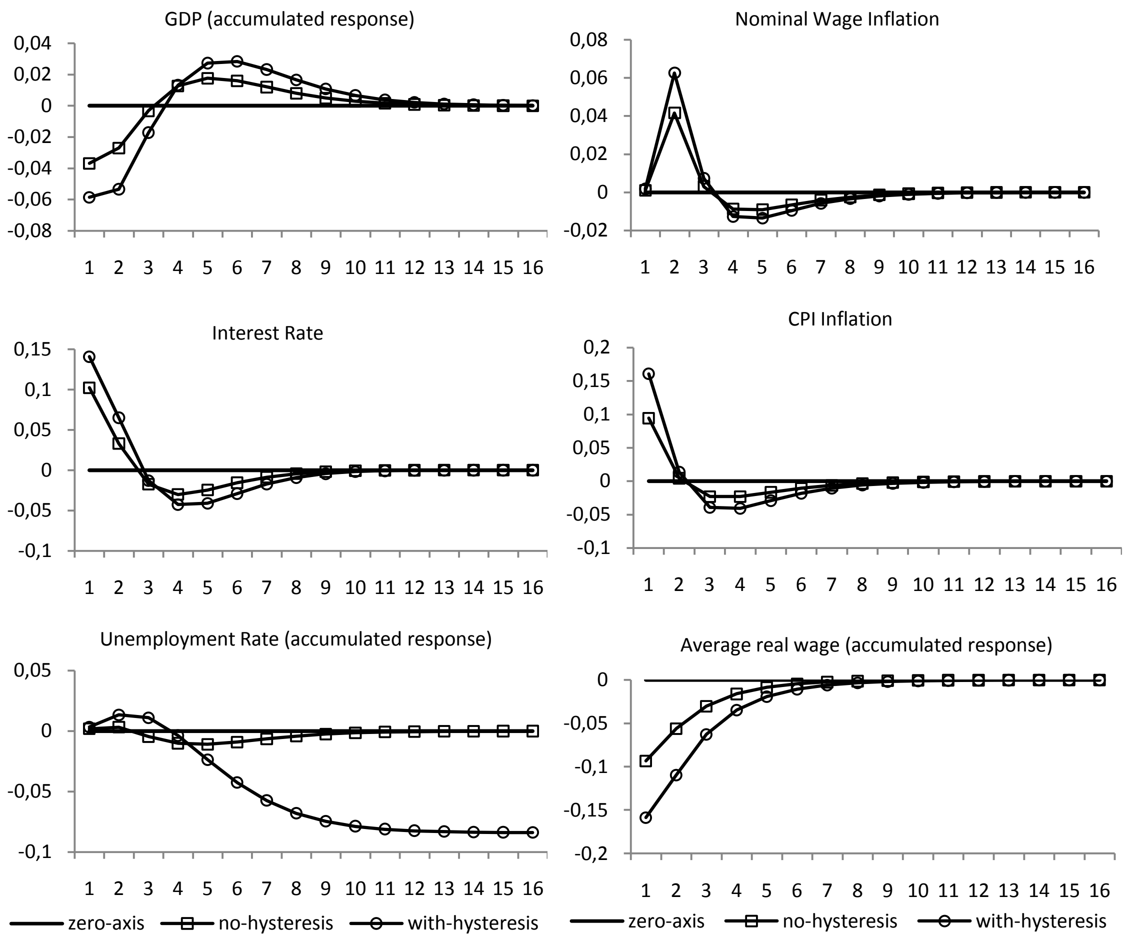

FIGURE 3.5 – IMPULSE RESPONSE FUNCTIONS IN (%) TO A 1% PRICE MARKUP SHOCK - QUARTERS

In (FIGURE 3.6) the impulse response functions implied by the models in relation to a wage markup shock are shown. The GDP, interest rate, real wage, CPI inflation and nominal wage inflation responses are similar in both models. The markup wage shock, as expected, produces a permanent increase in the unemployment rate of 1% in the model with hysteresis in response to a 1% wage markup shock, which does not occurs in the model without hysteresis.

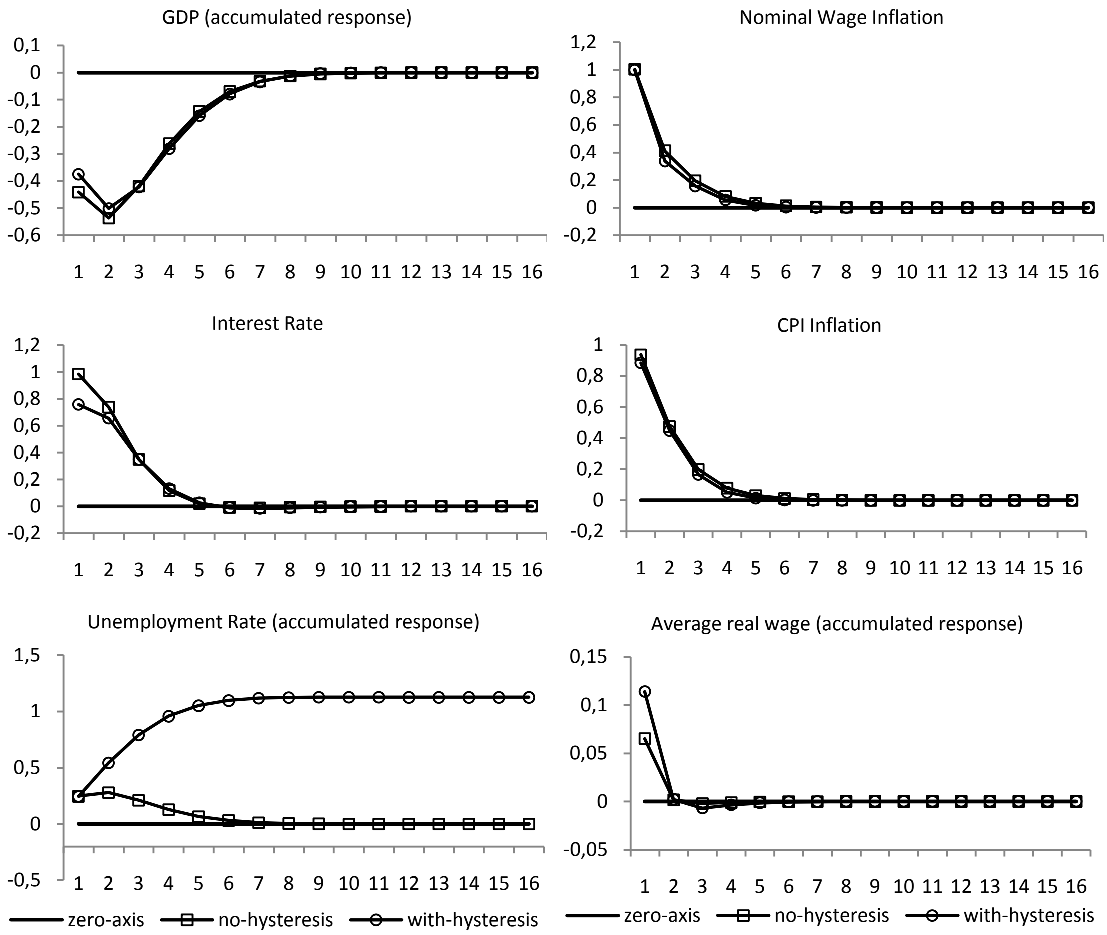

FIGURE 3.6 – IMPULSE RESPONSE FUNCTIONS IN (%) TO A 1% WAGE MARKUP SHOCK - QUARTERS

In (FIGURE 3.7), the GDP, interest rate, real wage, CPI inflation and nominal wage inflation responses are stronger in the model without hysteresis when compared with the no hysteresis model. Regarding unemployment rate, the temporary labor supply shock produce a permanent increase in the unemployment rate of 1.75% in the model with hysteresis in response to a 1% temporary labor-supply shock, what do not occur in the model without hysteresis.

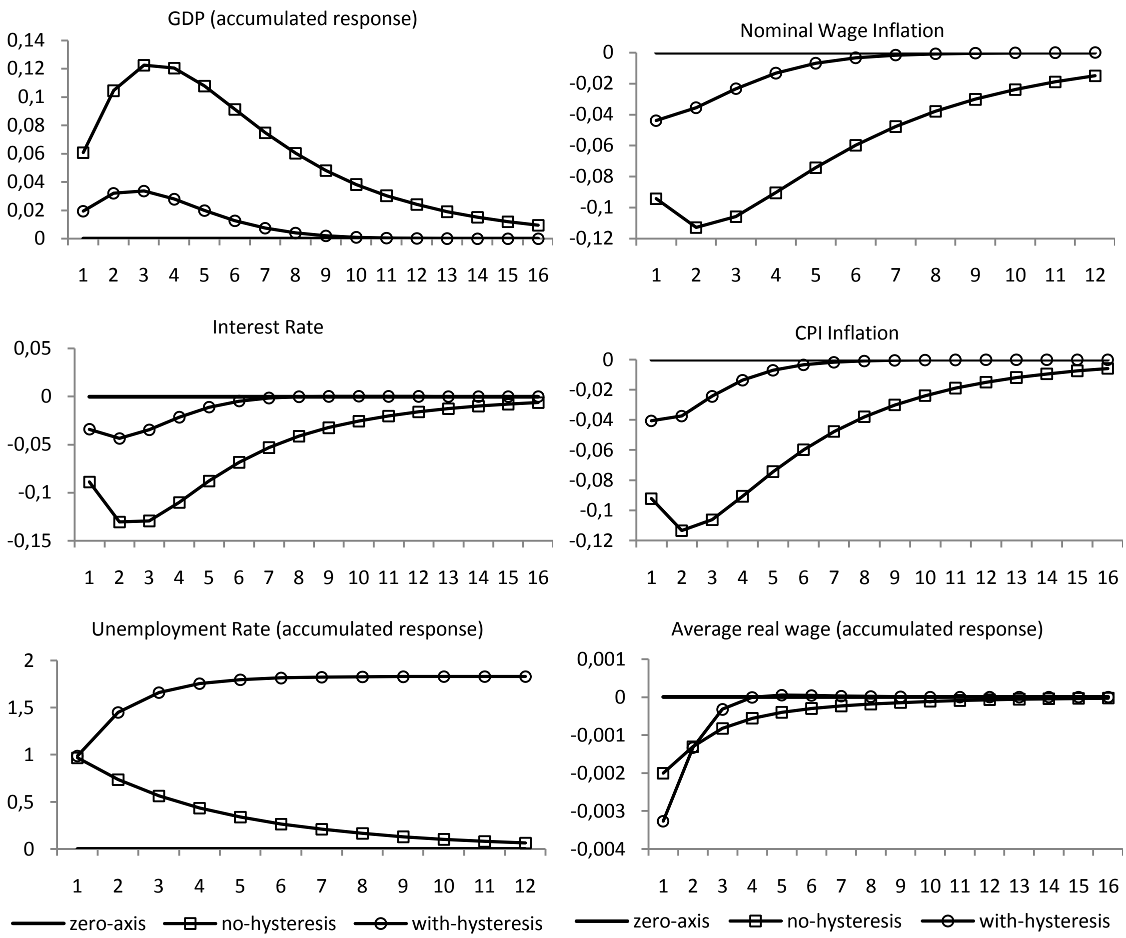

FIGURE 3.7 – IMPULSE RESPONSE FUNCTIONS IN (%) TO A 1% TEMPORARY LABOR-SUPPLY SHOCK - QUARTERS

In (FIGURE 3.8) the impulse response functions implied by the models in relation to a monetary policy shock are shown. The GDP, CPI inflation, nominal wage inflation and real wage are stronger in the with hysteresis model compared with the no hysteresis model. The unemployment rate rises permanently by 5% in the model with hysteresis in response to a 1 % positive monetary policy shock, while in the model without hysteresis unemployment increases only temporarily.

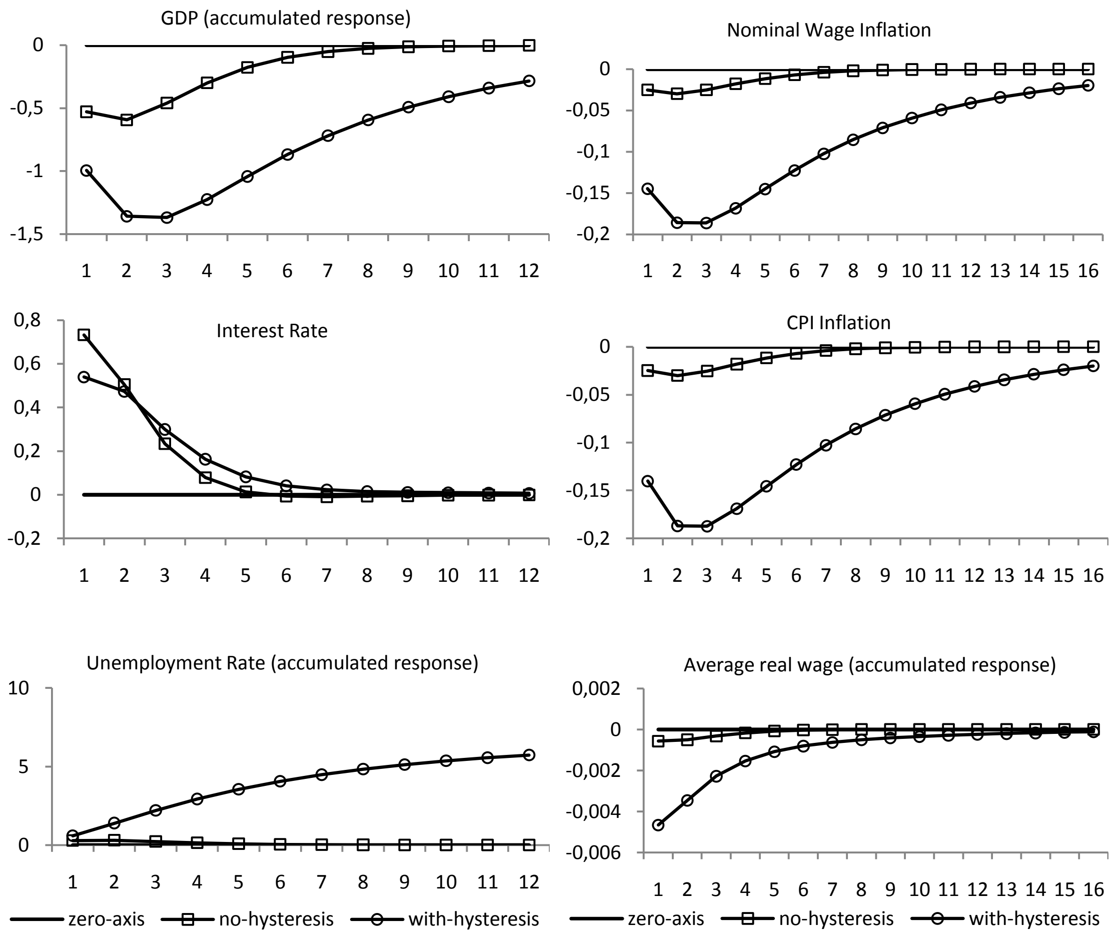

FIGURE 3.8 – IMPULSE RESPONSE FUNCTIONS IN (%) TO A 1% MONETARY POLICY SHOCK - QUARTERS

In comparing the two models impulse response functions, we can see that in the model with hysteresis demand, monetary policy and supply shocks can have permanent effects on the unemployment rate level. But the magnitude of these effects seem exaggerated for some shocks. We believe this is due to the simplified production function that we adopt in this our model. It is expected that a model with a production function that includes capital with variable utilization rate, costs of utilization and labor adjustment costs can produce more realistic impulse response functions for the unemployment rate.

3.6.2 Forecast Error Variance Decompositions

Looking at GDP growth in 1 and 4 quarter horizons, demand shocks explain about 33% (1st quarter) to 38% (4th quarter) of the forecast error variance of GDP growth (GDP FEV) in the no hysteresis model (NHM) and about 45% (1st quarter) to 49% (4th quarter) of the FEV in with hysteresis model (WHM). The permanent shocks of productivity explain about 41% (1st quarter) to 36% (4th quarter) of the GDP FEV in NHM and about 33% (1st quarter) to 29% (4th quarter) of the GDP FEV in WHM. The permanent shocks of labor supply, explain about 18% (1st quarter) to 17% (4th quarter) of GDP FEV in both models. The two models differ in the role of monetary policy shocks in the GDP FEV in horizons of 1 and 4 quarters . At 1 and 4 periods, the model without hysteresis about 5.4% and 5.3% of the GDP FEV is explained by monetary shocks, in the model with hysteresis these values is 17.3% and 16.5. In the long-run demand shocks play a similar role in the GDP FEV decomposition in both models, while monetary policy shocks are more important in the model with hysteresis (17.9% vs. 5.6%).

Analyzing the inflation FEV decomposition at the horizon 1, demand shocks explain about 0.01% of the inflation FEV in the NHM and 0.03% in the WHM. The monetary policy shock explains about 0.02% of the inflation FEV in the NHM and 0.82% in the WHM. The combined supply shocks explain 100% of the inflation FEV in the NHM at horizon 1 and 99% in the WHM at the same horizon. In the long-run, demand and monetary policy shocks combined explain about 0.09% of the FEV in the NHM and 5.9% in the WHM, while combined shocks of supply account for about 100% of the FEV in the NHM and 94% in the WHM.

Looking at the unemployment rate now, in regard to the model without hysteresis the three groups of shocks have similar weight in explaining the FEV decomposition of the unemployment rate first differences except the monetary policy shock. Demand and monetary policy shocks together account for about 20% to 23.5% of the unemployment rate FEV in the NHM at all horizons. In the hysteresis model (WHM), the combined demand and monetary policy shocks explain about 31% of the FEV at horizon 1 and explain

TABLE 3.3 – FORECAST ERROR VARIANCE DECOMPOSITION (%)

Shock	GDP		CPI Inflation		Δ(UR)	
Horizon 1	NHM	WHM	NHM	WHM	NHM	WHM
Demand	27.19	27.41	0.01	0.03	16.43	18.72
Monetary Policy	5.37	17.33	0.02	0.82	3.24	11.82
Productivity Temporary	3.86	1.94	37.44	33.83	36.38	32.08
Productivity Permanent	40.50	33.24	30.37	33.83	36.38	32.08
Price Markup	0.03	0.06	0.32	1.08	0.00	0.00
Wage Markup	3.74	2.47	31.52	32.64	2.51	2.04
Labor-supply Temporary	0.07	0.01	0.31	0.07	38.88	33.32
Labor-supply Permanent	19.25	17.55	0.00	0.00	0.00	0.00
Combined Supply Shocks	67.44	55.26	99.97	99.15	80.33	69.46
Horizon 4	NHM	WHM	NHM	WHM	NHM	WHM
Demand	32.91	32.25	0.02	0.04	20.64	11.19
Monetary Policy	5.26	16.46	0.07	3.76	3.24	37.10
Productivity Temporary	4.34	2.52	32.03	28.53	34.18	21.77
Productivity Permanent	36.21	28.81	35.50	34.40	2.75	4.65
Price Markup	0.04	0.08	0.27	0.93	0.00	0.01
Wage Markup	3.89	2.64	31.01	32.23	2.58	4.06
Labor-supply Temporary	0.10	0.01	1.10	0.12	36.60	21.23
Labor-supply Permanent	17.25	17.22	0.00	0.00	0.00	0.00
Combined Supply Shocks	61.83	51.29	99.91	96.30	76.12	51.71
Horizon ∞	NHM	WHM	NHM	WHM	NHM	WHM
Demand	32.41	31.37	0.02	0.04	20.17	9.15
Monetary Policy	5.56	17.88	0.07	5.82	3.38	47.33
Productivity Temporary	4.57	2.58	32.36	28.54	34.21	18.63
Productivity Permanent	36.16	28.43	35.13	33.32	2.98	4.03
Price Markup	0.04	0.08	0.28	0.94	0.00	0.02
Wage Markup	4.17	2.90	30.65	31.21	2.71	3.47
Labor-supply Temporary	0.12	0.01	1.50	0.12	36.55	17.37
Labor-supply Permanent	16.98	16.75	0.00	0.00	0.00	0.00
Combined Supply Shocks	62.03	50.75	99.91	94.14	76.45	43.52

NOTE: Δ(UR) is the unemployment rate first differences. Combined Supply Shocks refers to the sum of temporary and permanent productivity and labor-supply shocks with price and wage markup shocks.

about 56.5% in the infinite horizon (long-run). Thus, demand and monetary shocks have a larger role in explaining the short and long-run unemployment FEV decomposition in the model with hysteresis in comparison to the model without hysteresis, a result common in economic hysteresis literature.

3.7 CONCLUDING REMARKS

In this paper we expanded the Galí (2011a,b) unemployment model in order to consider hysteresis in unemployment rate. To introduce hysteresis in the Galí's model, we suppose that the labor supply path performs in a non-stationary way and is non-cointegrated with the employment. This combination produced a New-Keynesian model with a non-stationary unemployment rate, i.e., a unemployment rate with full (or strong) hysteresis. To demonstrate the implications of our hysteresis model, we estimated two small Bayesian DSGEs with closed economy and no government applied to the Brazilian economy. One of them adopts he standard model of Galí (2011a,b) and Galí et al. (2012) without hysteresis, while the other model adopts the hypothesis of unemployment rate hysteresis according to the assumption of non-stationary and no cointegrated with employment labor supply path that we adopt.

In the model developed in this paper we show that it is feasible to incorporate unemployment rate hysteresis within a New Keynesian model and that in this model demand shocks can have permanent effects on labor supply, employment and unemployment rate. To illustrate the applicability of our model we estimate two small Bayesian DSGE models for the Brazilian economy (closed economy and without government models). One of our main findings is that the monetary policy has greater weight in the forecast error variance decomposition in the model with hysteresis in comparison to the model without hysteresis.

In comparing the two models impulse response functions, we have seen that in the model with hysteresis demand, monetary policy and supply shocks have permanent effects on the unemployment rate level. But the magnitude of these effects seem exaggerated for some shocks. We believe this is due to the simplified production function that we adopt in this our model. It is expected that a model with a production function that includes capital with variable utilization rate, costs of utilization and maybe labor adjustment costs can produce more realistic impulse response functions for the unemployment rate. These improvements we leave to future research.

4 THESIS CONCLUSIONS

The aim of this thesis was to analyze the effects of possible hysteresis presence on the Brazilian unemployment rate. We pursued this objective through two essays or papers. In the first essay or paper, "Dynamic Effects of Hysteresis in Brazilian Unemployment", we tested the hypothesis of total hysteresis presence in the Brazilian unemployment rate through a cointegration model between real wage, real output per capita and unemployment rate proposed by Balmaseda et al. (2000). Our many findings were: first, the Brazilian unemployment rate measured by IBGE's PME can be considered a process with full hysteresis (unemployment rate is a I(1) variable) in all the sub-periods. Second, demand shocks play a similar role in explaining the forecast error variance (FEV) of the unemployment rate in all sub-periods. Third, productivity shocks was more important in explaining the (FEV) of the unemployment rate in the "before Real Plan" period, while labor-supply shocks were more important in the "after Real Plan" period. Third, real wages seemed be more flexible in the "before Real Plan" period compared with subsequent periods, mainly compared with the "Inflation Targeting" period.

In the second essay or paper, "Hysteresis in a New Keynesian DSGE", we expand the Galí (2011a,b) unemployment model to consider the hysteresis in unemployment rate hypothesis. With full hysteresis, the various shocks affecting the economy have a permanent effect on employment and unemployment rate. In an economy of this type the unemployment rate do not tend to a certain mean or to a "natural rate" of unemployment in the long-run. In the model developed in this paper we showed that it is feasible to incorporate unemployment rate hysteresis within a New Keynesian model and that in this model demand shocks can have permanent effects on employment and unemployment rate. To illustrate the applicability of our model we estimated two small Bayesian DSGE models for the Brazilian economy (closed economy and without government models). One of our main findings was that the monetary policy has greater weight in the forecast error variance decomposition in the model with hysteresis in comparison to the model without hysteresis.

In comparing the two models impulse response functions, we have seen that in the model with hysteresis demand, monetary policy and supply shocks have permanent effects on the unemployment rate level. But the magnitude of these effects seem exaggerated for some shocks. We believe this is due to the simplified production function that we adopt in this our model. It is expected that a model with a production function that includes capital with variable utilization rate and costs of utilization, fixed production costs, and labor adjustment costs can produce more realistic impulse response functions for the unemployment rate.

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A DYNARE CODE FOR THE MODEL WITHOUT HYSTERESIS

```
var y dLnGDP z_y l z_l pi_w dLnWn mu_w n dLnLaS z_a u dUnR pi_p
dLnIPCA mu_p mc w r Selic z_r gy ga gn;

varexo e_y e_l e_w e_za e_a e_n e_p e_r;

parameters h sigma rho_y gamma_c varphi rho_l gamma_w theta_w
ep_w rho_w beta alpha psi rho_za rho_a rho_n gamma_p theta_p
ep_p rho_p phi_r phi_pi phi_y rho_r sigma_ey sigma_el sigma_ew
sigma_eza sigma_ep sigma_er sigma_ea sigma_en  ga_bar gn_bar
pip_bar;

h = 0.7;
sigma = 1.3;
rho_y = 0.5;
gamma_c = 0.5;
varphi = 3;
rho_l = 0.5;
gamma_w = 0.65;
theta_w = 0.16;
ep_w = 26;
rho_w = 0.5;
beta = 0.99;
alpha = 0;
psi = 5;
rho_a = 0.5;
rho_za = 0.5;
rho_n = 0.5;
gamma_p = 0.65;
theta_p = 0.25;
ep_p = 5;
rho_p = 0.5;
phi_r = 0.79;
phi_pi = 2.43;
phi_y = 0.16;
rho_r = 0;
sigma_ey = 1;
sigma_el = 1;
sigma_ew = 1;
sigma_eza = 1;
sigma_ep = 1;
sigma_er = 1;
sigma_ea = 1;
```

```
sigma_en = 1;
ga_bar = 0.002;
gn_bar = 0.005;
pip_bar = 0.016;


model(linear);

// gy_bar (1)

#gy_bar = ga_bar + (1 - alpha)*gn_bar;

// h_tilde (2)

#h_tilde = h*exp(-gy_bar);

// beta_tilde (3)

#beta_tilde = beta*exp(-sigma*gy_bar);

// new keynesian IS curve (5)

y = (1/(1+h_tilde))*y(+1) + (h_tilde/(1+h_tilde))*y(-1)
- ((1-h_tilde)/(sigma*(1+h_tilde)))*(r - pi_p(+1)) + z_y;

z_y = rho_y*z_y(-1) + e_y;

// labor supply

w = (1-gamma_c)*w(-1) + sigma*(gamma_c/(1-h_tilde))*(y
- h_tilde*y(-1)) + varphi*l - (1-gamma_c)*varphi*l(-1)
- varphi*z_l + (1-gamma_c)*varphi*z_l(-1);

z_l = rho_l*z_l(-1) + e_l;

// labor demand

n = (1/(psi*(1 - alpha)))*(y - psi*z_a);

z_a = rho_za*z_a(-1) + e_za;

// unemployment rate

u =  l - n;

// cpi inflation (IPCA)
```

```
pi_p = (beta_tilde/(1+beta_tilde*gamma_p))*pi_p(+1)
+ (gamma_p/(1+beta_tilde*gamma_p))*pi_p(-1)
+ (((1 - theta_p)*(1 - beta_tilde*theta_p)
*(1 - alpha))/(theta_p*(1 - alpha + alpha*ep_p)))*mc
+ mu_p;

mu_p = rho_p*mu_p(-1) + e_p;

//marginal cost

mc = w + (1/(psi*(1 - alpha)))*(alpha*y - psi*z_a);

// real wage

w = w(-1) + pi_w - pi_p - (ga - ga_bar) + alpha*(gn - gn_bar);

//wage inflation

pi_w = gamma_w*pi_p(-1) + beta_tilde*(pi_w(+1) - gamma_w*pi_p)
- (((1-beta_tilde*theta_w)
*(1 - theta_w))/(theta_w*(1 + ep_w*varphi)))*varphi*u + mu_w;

mu_w = rho_w*mu_w(-1) + e_w;

// monetary policy (taylor-rule)

r = phi_r*r(-1) + (1 - phi_r)*(phi_pi*pi_p + phi_y*y) + z_r;

z_r = rho_r*z_r(-1) + e_r;

//observation equations

dLnGDP = gy + y - y(-1);
gy = ga + (1 - alpha)*gn;
ga = (1-rho_a)*ga_bar + rho_a*ga(-1) + e_a;
dLnLaS = gn + l - l(-1);
gn = (1 - rho_n)*gn_bar + rho_n*gn(-1) + e_n;
dLnWn = pip_bar + pi_w + ga_bar  - alpha*gn_bar;
dLnIPCA = pip_bar + pi_p;
Selic = ((1+pip_bar)/(beta*exp(-sigma*gy_bar))) - 1 + r;
dUnR = u - u(-1);

end;

initval;

y = 0;
```

```
z_y = 0;
l = 0;
z_l = 0;
pi_w = 0;
mu_w = 0;
n = 0;
z_a = 0;
u = 0;
pi_p = 0;
mu_p = 0;
mc = 0;
w = 0;
r = 0;
z_r = 0;
dLnGDP = ga_bar + (1 - alpha)*gn_bar;
gy = ga_bar + (1 - alpha)*gn_bar;
ga = ga_bar;
dLnLaS = gn;
dLnWn = pip_bar + ga_bar  - alpha*gn_bar;
gn = gn_bar;
dLnIPCA = pip_bar;
Selic = ((1+pip_bar)/(beta*exp(-sigma*(ga_bar
+ (1 - alpha)*gn_bar)))) - 1;
dUnR = 0;

end;

steady;

check;

shocks;

var e_y = 1;
var e_l = 1;
var e_w = 1;
var e_p = 1;
var e_r = 1;
var e_za = 1;
var e_a = 1;
var e_n = 1;


end;

estimated_params;
sigma, normal_pdf, 1.3, 0.05;
```

```
h, beta_pdf, 0.7, 0.04;
rho_y, beta_pdf, 0.5, 0.2;

varphi, gamma_pdf, 5, 1, 1;
gamma_c, beta_pdf, 0.5, 0.2;
rho_l, beta_pdf, 0.5, 0.2;

psi, gamma_pdf, 5, 0.5, 1;

ep_w, gamma_pdf, 5, 1, 1;
theta_w, beta_pdf, 0.75, 0.05;
gamma_w, beta_pdf, 0.5, 0.15;
rho_w, beta_pdf, 0.5, 0.2;

theta_p, beta_pdf, 0.08, 0.02;
gamma_p, beta_pdf, 0.5, 0.15;
rho_p, beta_pdf, 0.5, 0.2;

rho_za, beta_pdf, 0.5, 0.2;

phi_r, beta_pdf, 0.6, 0.15;
phi_pi, normal_pdf, 2, 0.35;
phi_y, gamma_pdf, 0.25, 0.1;
rho_r, beta_pdf, 0.5, 0.2;

ga_bar, normal_pdf, 0.002, 0.001;
rho_a, beta_pdf, 0.5, 0.2;
gn_bar, normal_pdf, 0.004, 0.002;
rho_n, beta_pdf, 0.5, 0.2;
pip_bar, normal_pdf, 0.0159, 0.008;

sigma_ey, inv_gamma_pdf, 1, Inf;
sigma_el, inv_gamma_pdf, 1, Inf;
sigma_ew, inv_gamma_pdf, 1, Inf;
sigma_eza, inv_gamma_pdf, 1, Inf;
sigma_ep, inv_gamma_pdf, 1, Inf;
sigma_er, inv_gamma_pdf, 1, Inf;
sigma_ea, inv_gamma_pdf, 1, Inf;
sigma_en, inv_gamma_pdf, 1, Inf;

end;


options_.relative_irf=1;

estimation(datafile=series1, mh_replic=10000, mh_nblocks=5,
mh_drop=0.5, mh_jscale=0.8, mode_compute=4, mode_check, order=1,
```

```
diffuse_filter, moments_varendo, irf =48, ar = 12,
conditional_variance_decomposition = [1:32]) dLnGDP dLnLaS dLnWn
dLnIPCA Selic u;

stoch_simul(periods=0, order = 1, ar = 8,
conditional_variance_decomposition = [1:32]);

varobs dLnGDP dLnLaS dLnWn dLnIPCA Selic dUnR;

disp('Initial period is $1999Q1')
```

B DYNARE CODE FOR THE MODEL WITH HYSTERESIS

```
var y dLnGDP z_y l z_l pi_w dLnWn mu_w n dLnLaS z_a du dUnR pi_p
dLnIPCA mu_p mc w r Selic z_r gy ga gn;

varexo e_y e_l e_w e_za e_a e_n e_p e_r;

parameters h sigma rho_y gamma_c varphi rho_l gamma_w theta_w ep_w
rho_w beta alpha psi rho_za rho_a rho_n gamma_p theta_p
ep_p rho_p phi_r phi_pi phi_y rho_r sigma_ey sigma_el sigma_ew
sigma_eza sigma_ep sigma_er sigma_ea sigma_en
ga_bar gn_bar pip_bar;

h = 0.85;
sigma = 1.3;
rho_y = 0.5;
gamma_c = 0.5;
varphi = 5;
rho_l = 0.5;
gamma_w = 0.5;
theta_w = 0.75;
ep_w = 5;
rho_w = 0.5;
beta = 0.99;
alpha = 0;
psi = 5;
rho_a = 0.5;
rho_za = 0.5;
rho_n = 0.5;
gamma_p = 0.5;
theta_p = 0.08;
ep_p = 5;
rho_p = 0.5;
phi_r = 0.5;
phi_pi = 2;
phi_y = 0.25;
rho_r = 0.5;
sigma_ey = 1;
sigma_el = 1;
sigma_ew = 1;
sigma_eza = 1;
sigma_ep = 1;
sigma_er = 1;
sigma_ea = 1;
```

```
sigma_en = 1;
ga_bar = 0.002;
gn_bar = 0.004;
pip_bar = 0.016;


model(linear);

// gy_bar (1)

#gy_bar = ga_bar + (1 - alpha)*gn_bar;

// h_tilde (2)

#h_tilde = h*exp(-gy_bar);

// beta_tilde (3)

#beta_tilde = beta*exp(-sigma*gy_bar);

// new keynesian IS curve (5)

y = (1/(1+h_tilde))*y(+1) + (h_tilde/(1+h_tilde))*y(-1)
- ((1-h_tilde)/(sigma*(1+h_tilde)))*(r - pi_p(+1)) + z_y;

z_y = rho_y*z_y(-1) + e_y;

// labor supply

w = (1-gamma_c)*w(-1) + sigma*(gamma_c/(1-h_tilde))
*(y - h_tilde*y(-1)) + varphi*l - (1-gamma_c)*varphi*l(-1)
- varphi*z_l + (1-gamma_c)*varphi*z_l(-1);

z_l = rho_l*z_l(-1) + e_l;

// labor demand

n = (1/(psi*(1 - alpha)))*(y - psi*z_a);

z_a = rho_za*z_a(-1) + e_za;

// unemployment rate and its first differences

du =  l - n;

// cpi inflation (IPCA)
```

```
pi_p = (beta_tilde/(1+beta_tilde*gamma_p))*pi_p(+1)
+ (gamma_p/(1+beta_tilde*gamma_p))*pi_p(-1)
+ (1 - theta_p)*(1 - beta_tilde*theta_p)
*(1/theta_p)*(1 - alpha)*(1/(1 - alpha + alpha*ep_p))
*(1/(1+beta_tilde*gamma_p))*mc + mu_p;

mu_p = rho_p*mu_p(-1) + e_p;

//marginal cost

mc = w + (1/(psi*(1 - alpha)))*(alpha*y - psi*z_a);

// real wage

w = w(-1) + pi_w - pi_p - (ga - ga_bar) + alpha*(gn - gn_bar);

//wage inflation

pi_w = gamma_w*pi_p(-1) + beta_tilde*(pi_w(+1) - gamma_w*pi_p)
- (((1-beta_tilde*theta_w)
*(1 - theta_w))/(theta_w*(1 + ep_w*varphi)))
*varphi*du + mu_w;

mu_w = rho_w*mu_w(-1) + e_w;

// monetary policy (taylor-rule)

r = phi_r*r(-1) + (1 - phi_r)*(phi_pi*pi_p + phi_y*y) + z_r;

z_r = rho_r*z_r(-1) + e_r;

//observation equations

dLnGDP = gy + y - y(-1);
gy = ga + (1 - alpha)*gn;
ga = (1-rho_a)*ga_bar + rho_a*ga(-1) + e_a;
dLnLaS = gn + l;
gn = (1 - rho_n)*gn_bar + rho_n*gn(-1) + e_n;
dLnWn = pip_bar + pi_w + ga_bar  - alpha*gn_bar;
dLnIPCA = pip_bar + pi_p;
Selic = log((1+pip_bar)/(beta*exp(-sigma*gy_bar))) + r;
dUnR = du;

end;

initval;
```

```
y = 0;
z_y = 0;
l = 0;
z_l = 0;
pi_w = 0;
mu_w = 0;
n = 0;
z_a = 0;
du = 0;
pi_p = 0;
mu_p = 0;
mc = 0;
w = 0;
r = 0;
z_r = 0;
dLnGDP = ga_bar + (1 - alpha)*gn_bar;
gy = ga_bar + (1 - alpha)*gn_bar;
ga = ga_bar;
dLnLaS = gn;
gn = gn_bar;
dLnWn = pip_bar + ga_bar  - alpha*gn_bar;
dLnIPCA = pip_bar;
Selic = log((1+pip_bar)/(beta*exp(-sigma*(ga_bar + (1 - alpha)*gn_bar
dUnR = 0;

end;

steady;

check;

shocks;

var e_y = 1;
var e_l = 1;
var e_w = 1;
var e_p = 1;
var e_r = 1;
var e_za = 1;
var e_a = 1;
var e_n = 1;


end;

estimated_params;
```

```
sigma, normal_pdf, 1.3, 0.05;
h, beta_pdf, 0.7, 0.04;
rho_y, beta_pdf, 0.5, 0.2;

varphi, gamma_pdf, 5, 1, 1;
gamma_c, beta_pdf, 0.5, 0.2;
rho_l, beta_pdf, 0.5, 0.2;

psi, gamma_pdf, 5, 1, 1;

ep_w, gamma_pdf, 5, 1, 1;
theta_w, beta_pdf, 0.75, 0.05;
gamma_w, beta_pdf, 0.5, 0.15;
rho_w, beta_pdf, 0.5, 0.2;

theta_p, beta_pdf, 0.08, 0.02;
gamma_p, beta_pdf, 0.5, 0.15;
rho_p, beta_pdf, 0.5, 0.2;

rho_za, beta_pdf, 0.5, 0.2;

phi_r, beta_pdf, 0.6, 0.15;
phi_pi, normal_pdf, 2, 0.35;
phi_y, gamma_pdf, 0.25, 0.1;
rho_r, beta_pdf, 0.5, 0.2;

ga_bar, normal_pdf, 0.002, 0.001;
rho_a, beta_pdf, 0.5, 0.2;
gn_bar, normal_pdf, 0.004, 0.002;
rho_n, beta_pdf, 0.5, 0.2;
pip_bar, normal_pdf, 0.0159, 0.008;

sigma_ey, inv_gamma_pdf, 1, Inf;
sigma_el, inv_gamma_pdf, 1, Inf;
sigma_ew, inv_gamma_pdf, 1, Inf;
sigma_eza, inv_gamma_pdf, 1, Inf;
sigma_ep, inv_gamma_pdf, 1, Inf;
sigma_er, inv_gamma_pdf, 1, Inf;
sigma_ea, inv_gamma_pdf, 1, Inf;
sigma_en, inv_gamma_pdf, 1, Inf;

end;

options_.relative_irf=1;

estimation(datafile=series2, mh_replic=10000, mh_nblocks=5,
mh_drop=0.5, mh_jscale=0.8, mode_compute=4, mode_check, order=1,
```

```
diffuse_filter, moments_varendo, irf =48, ar = 12,
conditional_variance_decomposition = [1:32]) dLnGDP dLnLaS dLnWn
dLnIPCA Selic l n;

stoch_simul(periods=0, order = 1, ar = 8,
conditional_variance_decomposition = [1:32]);

varobs dLnGDP dLnLaS dLnWn dLnIPCA Selic dUnR;

disp('Initial period is $1999Q1')
```