UNIVERSIDADE FEDERAL DO PARANÁ DEPARTAMENTO DE ECONOMIA

Doctoral Thesis

ESSAYS ON FISCAL POLICY

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Curitiba, June 02, 2014

UNIVERSIDADE FEDERAL DO PARANÁ DEPARTAMENTO DE ECONOMIA

THESIS OF DOCTORATE

ESSAYS ON FISCAL POLICY

A thesis submitted
in fulfilment of the partial requirement
for the degree of Doctor in Economic
Development in the Economics
Department - Programa de Pós-Graduação em
Desenvolvimento Econômico - PPGDE

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Curitiba, June 02, 2014

TERMO DE APROVAÇÃO

Celso José Costa Júnior

"Essays on fiscal Policy"

TESE APROVADA COMO REQUISITO PARCIAL PARA OBTENÇÃO DO GRAU DE DOUTOR NO PROGRAMA DE PÓS-GRADUAÇÃO EM DESENVOLVIMENTO ECONÔMICO DA UNIVERSIDADE FEDERAL DO PARANÁ, PELA SEGUINTE BANCA EXAMINADORA:

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02 de junho de 2014

"Um político pensa nas próximas eleições.
Um estadista pensa na próxima geração."

James Clarke
Governador do Estado
do Arkansas, 1854-1916

Resumo

Há uma sensação generalizada na sociedade brasileira sobre a vulnerabilidade fiscal da economia do Brasil. Em vista disso, o objetivo deste trabalho é contribuir para a discussão sobre o papel da política fiscal como ferramenta de estímulo econômico. Para tanto, analisou-se algumas propostas de política fiscal para a economia brasileira, tanto do lado do gasto público, quanto do lado da desoneração tributária. Para tanto, foi desenvolvido um modelo DSGE Novo-keynesiano para uma economia pequena e fechada com seis choques fiscais: gasto corrente do governo; investimento público; transferência de renda às famílias; desoneração do imposto sobre o consumo; desoneração do imposto sobre a renda do trabalho; e desoneração do imposto sobre a renda do capital. Os parâmetros fiscais foram estimados usando a metodologia Bayesiana, enquanto os outros parâmetros foram calibrados. As políticas de desoneração tributária apresentaram melhores resultados do que as políticas de gasto público. Principalmente, a desoneração do tributo sobre a renda do trabalho. Entre as políticas de gasto, o investimento público foi o único choque que apresentou resultado positivo em relação ao PIB. Resumidamente, o resultado obtido é que políticas voltadas para o aumento da oferta agregada possuíram um resultado mais expressivo e sem pressionar a taxa de inflação.

Abstract

There is a widespread feeling in Brazilian society about the fiscal vulnerability of Brazil's economy. In view of this, the objective this work is to contribute to the discussion about the role of fiscal policy as economic stimulus tool. We discuss some proposals of fiscal policy for the Brazilian economy, both on the side of public spending and on the side of tax reduction. Thus, a New-Keynesian DSGE model for a small closed economy with six fiscal shocks (current government spending, public investment, income transfers to households, tax reduction on consumption, tax reduction on labor income and tax reduction on capital income) was developed. The fiscal parameters were estimated using the Bayesian methodology, while the other parameters were calibrated. The policies of tax reduction showed better results than the policies of public spending. Mostly, the tax reduction on labor income. Among policies spending, public investment was the only shock that showed positive results in relation to GDP. Briefly, the result is that policies aimed at increasing aggregate supply had a more significant result, without to press the inflation rate.

Acknowledgements

The fact that my academic experience during my Ph.D. program at PPGDE-UFPR has been great is largely due to my adviser Armando Sampaio. He has been the source of numerous suggestions and comments, teaching me how to become a better researcher. His guidance and support have been invaluable.

I would like to also thank to Fermando Motta Correia, José Basílio Pereima Neto, Flávio de Oliveira Gonçalves, Marcelo Luiz Curado and Maurício Vaz Lobo Bittencourt that contributed to my academic formation.

I could hot have completed the Ph.D. program without the continuous love and support from Priscilla and Manuela, my wife and daughter, respectively. I cannot imagine my life without them.

I woudl also like to thank to my mother Dair. Since I was very young they explained to me the importance of studying and having a strong professional formation.

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Chapter 1

Introduction

In the 1980s and 1990s, the Brazilian economic literature was concerned to diagnose and propose solutions to the inflationary process. With the implementation of the Plano Real, the country experienced a panorama of low inflation. So combat this problem has stopped occupy the center of attention of researchers, giving place to the study of fiscal vulnerability, considered the largest and most direct threat to the development of the Brazilian economy.

To collaborate with the discussion of the Brazilian fiscal vulnerability, this thesis studies the role of fiscal policy as economic stimulus tool. So, this study will analyze possibilities of fiscal policy for the Brazilian economy, both on the side of public spending and on the side of tax reduction. For this purpose six stochastic shocks will be performed in a DSGE model: current government spending; public investment; transfer income to households; tax reduction on consumption; tax reduction on labor income; and tax reduction on capital income.

The chapters are structured following the procedures of building the DSGE model of this study, solving it, estimating it and analyzing it. Figure 1.1 gives a visual overview of all steps.

Chapter 1 has the objective to present a small closed economy model. The basic framework contains equations that describe the behavior of the private sector for consumption, labor, production, the pricing decisions, the setting of monetary

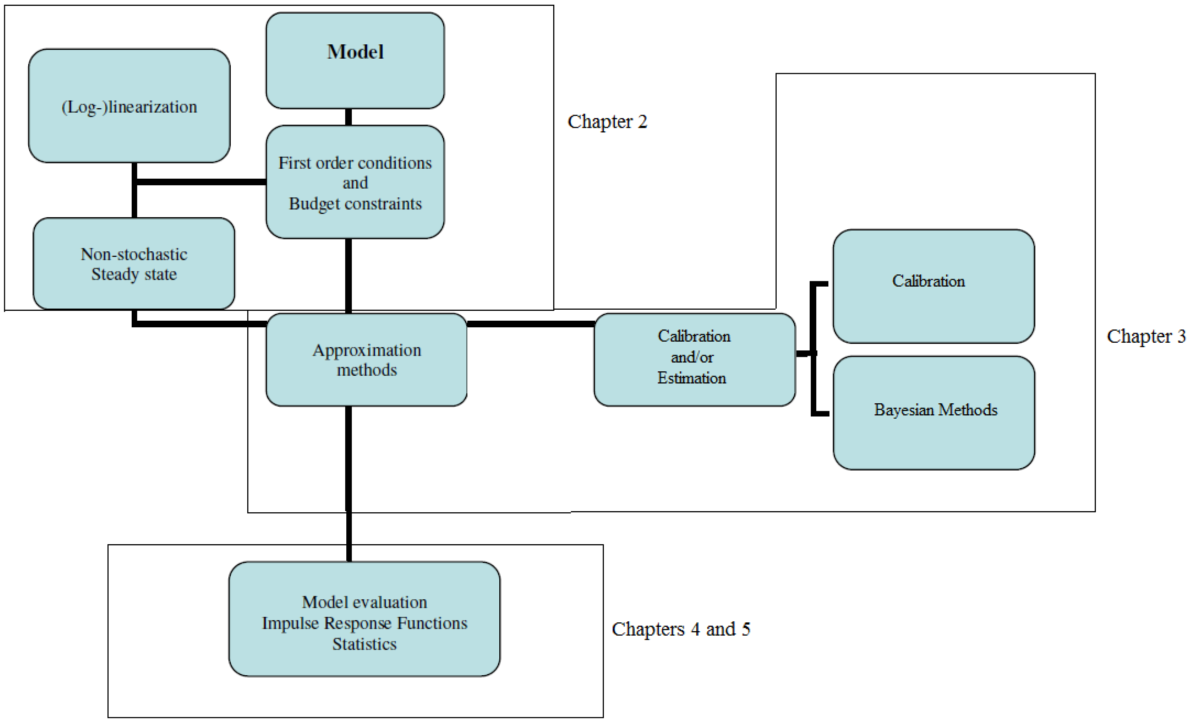

FIGURE 1.1: Overview of Solution Method to DSGE Model of This Study.
Source: Prepared by the author.

and fiscal policies. The model is very simple, it has two frictions: monopolistic competition and staggered pricing *a la Calvo*.

To solve it, we set up the economic model; derived the first-order equilibrium conditions, together with the structural equations, these build a system of non-linear stochastic difference equations; as this system usually does not have a closed analytical solution, the solution is approximated in the neighborhood of a given point, in most cases the non-stochastic steady state which is determined in next step; and either log-linear approximation around the steady-state leading to a system of linear difference equations in state-space form and solution of this system with the help of the usual procedures or second (or higher) order approximation around the steady-state.

In the chapter 2, the model of this work has been solved, to proceed further we need to parametrize the model economy, so this is the objective of chapter 3. The parameters already known and well accepted in DSGE Brazilian literature were calibrated, the main calibration procedure adopted here is to obtain the values of parameters from other relevant DSGE work in the literature.

The discussions on shocks that represent the proposed fiscal policies will be discussed in chapters 4 (shocks to public spending) and 5 (shocks to tax reduction). The analysis is based using the variance decomposition and the impulse response functions for each of the shocks[1].

[1]In the body of the thesis only fiscal shocks will be presented, other shocks are presented in Appendix A

Chapter 2

DSGE Model: Basic Structure

Introduction

The 1980s has witnessed a major breakthrough in the field of macroeconomic modeling. The first examples of this new methodology emerged from the models of real business cycles (RBC), primarily through the groundbreaking work of Kydland and Prescott (1982). Its builders were criticized for focusing the analysis on only one type of shock and a kind of economic structure and for not recognizing any active role for monetary policy. Therefore from the perspective of a central bank, it was difficult to see how these models could bring any positive contribution to the discussion of monetary policy.

Twenty years later, this controversy was completely dissipated. The main reason was that the methodological innovation overlying the RBC models brought the introduction of frictions that allowed the incorporation of Keynesian principles and new shocks to the initial modeling. The success of this new model made it possible for the main economic institutions to develop their own DSGE models as did Central Bank of Brazil (SAMBA), European Central Bank (NAWM), Bank of Canada (Totem), Bank of England (BEQM), Bank of Japan (JEM), Bank of Chile (MAS), European Community (QUESTIII) and the International Monetary Fund (GEM)). Nowadays, DSGE models are used to answer almost any behavior of an economic phenomenon, including issues related to fiscal policy.

The objective of this chapter is to present a small closed economy model. The basic framework contains equations that describe the behavior of the private sector for consumption, labor, production, the pricing decisions, the setting of monetary and fiscal policies. The model is very simple, it has two frictions: monopolistic competition and staggered pricing *a la Calvo*. However, this model is enough to the objective this work: to contribute to the discussion about fiscal policy on Brazilian Economy. It will be analyzed the behavior of the macroeconomics variables in relation to three alternatives of public expenditure: current spending; income transfers to households; and investment in public capital. About government revenue, it will be studied tax reduction in the taxes on consumption, on labor income and on capital income.

One of the central assumption of the neoclassical approach is the existence of perfect competition in goods and input markets. This resulted in market prices that are equal the marginal cost of production, zero profits for firms and a price for the productive factors equal to the marginal productivity. In this context, there is no market power affecting the determinations of prices. However, empirical evidence shows the existence of markups in the goods and services markets, which means that the prices of these goods are higher than their marginal cost.

Another neoclassical approach's principle is the fully flexible price model, however a more realistic model has need to incorporate sticky price setting. The new neoclassical synthesis models represent a response to the rational expectations revolution. In this latest paradigm, economic agents are modeled as forward-looking, and information is gleaned from the structure of the macroeconomic. However, this approach also has that agents can be backward-looking, meaning there is inertia in price setting due to institutional constraints such as overlapping contracts. Here we consider the existence of imperfect competition in the production sector and staggered pricing *a la Calvo*. In particular, the model this work can be called New Keynesian Model. Because following much of the literature, a model of staggered price setting due to Calvo (1993) and characterized by random price durations is referred to this denomination.

Despite the importance of the government as being responsible for public investment in infrastructure to support the private sector, there are few references about this subject. Some theoretical work of the 1970s incorporated public capital in the aggregate production function, with examples by Arrow and Kurz (1970), Weitzman (1970) and Pestieau (1974). However, it is from the work of Barro (1990) that these initial ideas are recovered. Another sequence of works following this line of thought as in Barro and Sala-i-Martin (1992), Finn (1993), Glomm and Ravikumar (1994), Cashin (1995) and Bajo (2000), among others. Among the empirical studies that use this same approach, one can highlight Mera (1973), Ratner (1983) and Aschauer (1989). The main assumption in relation to public investment in this study follows this form of treatment.

This chapter begins with this introduction and section one presenting the economic model, with section two defines the values of the steady state. The work continues with the log-linearization of the model.

2.1 The Model

The economic model of this work is a small and closed economy with sectors for households, firms, and government (Fiscal Authority and Monetary Authority). This model has two frictions: monopolistic competition and staggered pricing *a la Calvo*. The latter friction aims to avoid the model to have a very fast adjustment in relation to shocks, a factor noticed in empirical evidence.

2.1.1 Households

We assume that households are able to maximize its intertemporal utility by choosing consumption, savings, investment and leisure. For saving, the household can choose between two different savings instruments - physical capital and government bonds, indexed by j. Briefly, with the disposable income after payment of taxes, the household can purchase consumer goods, capital goods, and/or government bonds.

Relying on the behavior described about the households, this agent chooses how much to consume, how much to work and how much to acquire financial assets and physical capital to maximize the discounted stream of the expected utility[1][2],

$$\max E_t \sum_{t=0}^{\infty} \beta^t S_t^C \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - S_t^L \frac{L_{j,t}^{1+\psi}}{1+\psi} \right] \tag{2.1}$$

subject to their budget constraint,

$$P_t \left(1 + \frac{\tau_c}{\phi_t^c}\right)(C_{j,t} + I_{j,p,t}) + \frac{B_{j,t+1}}{RIS_t R_t^B} = W_t L_{j,t} \left(1 - \frac{\tau_l}{\phi_t^l}\right) + R_t K_{j,p,t} \left(1 - \frac{\tau_k}{\phi_t^k}\right) + B_{j,t} + TRANS_{j,t} \tag{2.2}$$

and in relation to the following law of motion of capital,

$$K_{j,p,t+1} = (1 - \delta_p) K_{j,p,t} + I_{j,p,t} \tag{2.3}$$

where E_t is the expectations operator, $\beta \in (0, 1)$ is the intertemporal discount factor, C is the consumption of household, L is the labor, S^C is the intertemporal shock, S^L is the shock on labor supply, ψ is the marginal disutility of labor and σ is the coefficient of relative risk aversion.

In the budget constraint, P is the general price level, I_p is the investment of private sector, B is the government bond maturing in one period, RIS is the risk premium, R^B is the rate of return on government bond (basic interest rate), W is the wage, R is the return to capital, K_p is the private stock of capital, $TRANS$ is

[1]The most common utility function to represent the choices of Household Representative is the utility function of constant relative risk aversion (CRRA) (Gali, 2008; Lim and McNelis, 2008; Clarida et al, 2002; Gali and Monacelli, 2005; Christoffel and Kuester, 2008; Christoffel et al, 2009; Ravenna and Walsh, 2006). There are other common parameterizations for the utility function in the literature, examples: logarithmic utility function, $U(C_t, L_t) = \ln C_t + \frac{L_t}{L_0} A \ln(1 - L_0)$ (Hansen, 1985); and utility function that would be a combination of the logarithmic and of the CRRA, $U(C_t, L_t) = \ln(C_t) - \frac{\upsilon}{1+\chi} L_t^{1+\chi}$ (Gertler and Karadi, 2011).

[2]A utility function must has certain characteristics: $U_C > 0$ and $U_L < 0$, this means that consumption and labor have a positive and a negative effects, respectively, over the happiness of the households. On the other hand, $U_{CC} < 0$ and $U_{LL} < 0$, indicating that the utility function is concave. This represents that if the consumption increases the utility level also increases, but in a smaller and smaller proportion. Another assumption regarding the utility function says that this function is additionally separable in time. This assumption allows to speak of an instantaneous utility function, wherein the agent receives utility solely from consumption that performs at a given moment in time.

the income transfers to households by government, ϕ^c, ϕ^l and ϕ^k are the stochastic components of tax reduction of the consumption tax, income tax on labor and income tax on capital, respectively. While τ_c, τ_l, τ_k represent the static components of the tax on consumption, income tax on labor, income tax on capital, respectively. In this work, is being adopted the convention that B_t is the nominal bond issued in (t-1) and matured in t[3,4]. Then, B_{t+1} and K_{t+1} are decided in t.

The household purchases of consumer goods (C) and investment goods (I_p) at the price level (P), also buys or sells government bonds(B) maturing in one period. These bonds pay a rate (R^B) with risk premium (RIS) (see Smets and Wouters (2007) and Christoffel et al. (2008)), which interest is controlled by the monetary authority.

This kind of agents pays three types of taxes (consumption tax, income tax on labor and income tax on capital). Their income comes from four sources: labor income, which depends on the level of nominal wages (W); return on capital rental to firms, which is a function of the rate of return to capital (R); income transfers ($TRANS$) received of the government; and income from government bonds acquired in the previous period (B).

To solve the problem of the household, a Lagrangian function is used:

$$\begin{aligned}\mathscr{L} = E_t \sum_{t=0}^{\infty} \beta^t \Bigg\{ S_t^C \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - S_t^L \frac{L_{j,t}^{1+\psi}}{1+\psi} \right] \\ -\lambda_t \Bigg[P_t \left(1 + \frac{\tau_c}{\phi_t^c}\right) (C_{j,t} + K_{j,p,t+1} - (1-\delta_p) K_{j,p,t}) + \frac{B_{j,t+1}}{RIS_t R_t^B} \\ -W_t L_{j,t} \left(1 - \frac{\tau_l}{\phi_t^l}\right) - R_t K_{j,p,t} \left(1 - \frac{\tau_k}{\phi_t^k}\right) - B_{j,t} - TRANS_{j,t} \Bigg] \Bigg\}\end{aligned} \tag{2.4}$$

The first order conditions associated with the choices of C_t, L_t, $K_{p,t+1}$ and B_{t+1} are respectively:

[3]In practice, each period a government usually issues bonds that mature at different times in the future. For convenience, we assume that all bonds are issued for one period.

[4]The price of bonds can be written as:

$$P_t^B = \frac{1}{RIS_t R_t^B}$$

$$\frac{\partial \mathscr{L}}{\partial C_{j,t}} = S_t^C C_{j,t}^{-\sigma} - \lambda_t P_t \left(1 + \frac{\tau_c}{\phi_t^c}\right) = 0 \tag{2.5}$$

$$\frac{\partial \mathscr{L}}{\partial L_{j,t}} = -S_t^C S_t^L L_{j,t}^{\psi} + \lambda_t W_t \left(1 - \frac{\tau_l}{\phi_t^l}\right) = 0 \tag{2.6}$$

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial K_{j,p,t+1}} = -\lambda_t P_t \left(1 + \frac{\tau_c}{\phi_t^c}\right) + \beta E_t \lambda_{t+1} \Bigg[(1-\delta_p) P_{t+1} \left(1 + \frac{\tau_c}{\phi_{t+1}^c}\right) \\ + R_{t+1} \left(1 - \frac{\tau_k}{\phi_t^k}\right) = 0\end{aligned} \tag{2.7}$$

$$\frac{\partial \mathscr{L}}{\partial B_{j,t+1}} = -\frac{\lambda_t}{RIS_t R_t^B} + \beta E_t \lambda_{t+1} = 0 \tag{2.8}$$

From equation (2.5),

$$\lambda_t = \frac{S_t^C C_{j,t}^{-\sigma}}{P_t \left(1 + \frac{\tau_c}{\phi_t^c}\right)} \tag{2.9}$$

Substituting the equation (2.9) into (2.6), it results in the equation of labor supply of the households:

$$S_t^L L_{j,t}^{\psi} C_{j,t}^{\sigma} \left[\frac{\left(1 + \frac{\tau_c}{\phi_t^c}\right)}{\left(1 - \frac{\tau_l}{\phi_t^l}\right)}\right] = \frac{W_t}{P_t} \tag{2.10}$$

An increase in the real wage rate motivates households to increase labor supply and consumption demand. The same result is found in tax reduction in the tax on consumption and tax on labor income[5]. Wealth effects can arise from shifts in production function (positive shock on productivity) (discussion in section 2.1.2). These effects show up first on firms' profits. But it is important to remember that the profits go to the households that own the firms. One new consideration is the wealth effect from a change in the real wage rate, given the position of the

[5] We can summarize the results by writing:
$L^s = L^s \left[\left(\underset{(+)}{\frac{W}{P}}\right), \left(\underset{(+)}{\phi_t^c}\right), \left(\underset{(+)}{\phi_t^l}\right)\right]$
and
$C^d = C^d \left[\left(\underset{(+)}{\frac{W}{P}}\right), \left(\underset{(+)}{\phi_t^c}\right), \left(\underset{(+)}{\phi_t^l}\right)\right]$
where L^s is labor supply and C^d consumption demand.

production function. An increase in W/P benefits the households that sell labor services. But this benefit is matched by an extra cost for the firms, which buy labor services. Since the firms are owned by households, the overall wealth effect on households from a change in W/P is nil (Barro, 1997).

Substituting equation (2.9) in equations (2.7) and (2.8), we obtain the Euler equations:

$$S_t^C C_{j,t}^{-\sigma} = \beta E_t \left\{ \frac{S_{t+1}^C C_{j,t+1}^{-\sigma}}{P_{t+1}\left(1+\frac{\tau_c}{\phi_{t+1}^c}\right)} \left[(1-\delta_p)P_{t+1}\left(1+\frac{\tau_c}{\phi_{t+1}^c}\right) + R_{t+1}\left(1-\frac{\tau_k}{\phi_{t+1}^k}\right)\right]\right\} \tag{2.11}$$

$$\frac{S_t^C C_{j,t}^{-\sigma}}{P_t\left(1+\frac{\tau_c}{\phi_t^c}\right)} = RIS_t R_t^B \beta E_t \left[\frac{S_{t+1}^C C_{j,t+1}^{-\sigma}}{P_{t+1}\left(1+\frac{\tau_c}{\phi_{t+1}^c}\right)}\right] \tag{2.12}$$

It is possible to give an intuitive explanation for Euler equation (see Wickens, 2011). Consider the following problem: if we reduce $C_{j,t}$ by a small amount $dC_{j,t}$, how much larger must $C_{j,t+1}$ be to fully compensate for this while leaving the value of current and future utility ($V_{j,t}$) unchanged? We suppose that consumption beyond period t+1 remains unaffected. Thus we let $V_{j,t} = U(C_{j,t}) + \beta E_t U(C_{j,t+1})$.

Taking the total differential of $V_{j,t}$, $0 = dV_{j,t} = DU_{j,t} + \beta E_t dU_{j,t+1} = U'(C_{j,t})dC_{j,t} + \beta E_t U'(C_{j,t+1})dC_{j,t+1}$. Hence we need to increase $C_{j,t+1}$ by $E_t dC_{j,t+1} = -\frac{U'(C_{j,t})}{\beta E_t U'(Cj,t+1)} dC_{j,t}$.

Using the equations (2.11) or (2.12),

$$E_t dC_{j,t+1} = -E_t \left[\frac{S_t^C C_{j,t}^{-\sigma}}{\beta E_t S_{j,t+1}^C C_{j,t+1}^{-\sigma}}\right] dC_{j,t}$$

As the resource constraint (equation (2.2)) must be satisfied in every period, in periods t and t+1 we require that,

$$P_t\left(1+\frac{\tau_c}{\phi_t^c}\right)(dC_{j,t} + E_t dK_{j,p,t+1} - (1-\delta_p)dK_{j,p,t}) = R_t dK_{j,p,t}\left(1-\frac{\tau_k}{\phi_t^k}\right)$$

$$E_t\left[P_{t+1}\left(1+\frac{\tau_c}{\phi_{t+1}^c}\right)\right][E_t dC_{j,t+1} + E_{t+1} dK_{j,p,t+2} - (1-\delta_p)E_t dK_{j,p,t+1}]$$

$$= E_t \left[R_{t+1} dK_{j,p,t+1} \left(1 - \frac{\tau_k}{\phi^k_{t+1}} \right) \right]$$

As $K_{j,t}$ is given and beyond period t+1 we are constraining the capital stock to be unchanged, only the capital stock in period t+1 can be different from before. Thus $dK_{j,p,t} = dK_{j,p,t+2} = 0$. The resource constraints for periods t and t+1 can therefore be rewritten as,

$$E_t dK_{j,p,t+1} = -dC_{j,t}$$

$$E_t \left\{ P_{t+1} \left(1 + \frac{\tau_c}{\phi^c_{t+1}} \right) [dC_{j,t+1} - (1-\delta_p) dK_{j,p,t+1}] \right\} = E_t \left[R_{j,t+1} dK_{j,p,t+1} \left(1 - \frac{\tau_k}{\phi^k_{t+1}} \right) \right]$$

getting,

$$E_t dC_{j,t+1} = -E_t \left\{ \left[(1-\delta_p) + \frac{R_{t+1}\left(1 - \frac{\tau_k}{\phi^k_{t+1}}\right)}{P_{t+1}\left(1 + \frac{\tau_c}{\phi^c_{t+1}}\right)} \right] \right\} dC_{j,t}$$

This can interpreted that the output no longer consumed in period t is invested and increases output in period t+1 by $-E_t \left[\frac{R_{t+1}\left(1 - \frac{\tau_k}{\phi^k_{t+1}}\right)}{P_{t+1}\left(1 + \frac{\tau_c}{\phi^c_{t+1}}\right)} \right] dC_{j,t}$. All of this can be consumed in period t+1. And as we do not wish to wish to increase the capital stock beyond period t+1, the undepreciated increase in the capital stock, $-(1-\delta_p)dC_{j,t}$, can also be consumed in period t+1. This gives the total increase in consumption in period t+1. The discounted utility of this extra consumption as measured in period t is,

$$\beta E_t [U'(C_{j,t+1}) dC_{j,t+1}] = -\beta E_t \left\{ U'(C_{j,t+1}) \left[(1-\delta_p) + \frac{R_{t+1}\left(1 - \frac{\tau_k}{\phi^k_{t+1}}\right)}{P_{t+1}\left(1 + \frac{\tau_c}{\phi^c_{t+1}}\right)} \right] \right\} dC_{j,t}$$

To keep $V_{j,t}$ constant, this must be equal to the loss of utility in period t. Thus,

$$U'(C_{j,t}) dC_{j,t} = \beta E_t \left\{ U'(C_{j,t+1}) \left[(1-\delta_p) + \frac{R_{t+1}\left(1 - \frac{\tau_k}{\phi^k_{t+1}}\right)}{P_{t+1}\left(1 + \frac{\tau_c}{\phi^c_{t+1}}\right)} \right] \right\} dC_{j,t}$$

Canceling dC_t from both sides gives the Euler equation (equation (2.11)).

2.1.1.1 Aggregation

The aggregate of this work follows the functional form ($X = (1 - \omega)X_p + \omega X_g$)[6] very common in this type of literature (Boscá et al, 2010; Galí et al, 2007; Itawa, 2009; Coenen and Straub, 2004; Furlanetto, 2007; Dallari, 2012; Mayer et al, 2010; Stahler and Thomas, 2011; Swarbrick, 2012; Motta and Tirelli, 2010; Díaz, 2012; Colciago, 2011; Mayer and Stahler, 2009; and Forni et al, 2009).

Thus, aggregate investment is given by:

$$I_t = (1 - \omega_G)I_{p,t} + \omega_G I_{g,t} \tag{2.13}$$

where ω_G is the participation of the public investment in the aggregate investment.

2.1.1.2 Shocks Related to Households

There are two shocks related to households' behavior: the shock in intertemporal preferences (S^C) and the shock on labor supply (S^L). While the first affects the choice of intertemporal consumption, the second affects labor supply and determination of nominal wages. The shock S^C was included to capture changes in valuation between the present and the future which the literature on intertemporal behavior suggested as a key to the understanding of aggregate fluctuations (Primiceri et al. 2006). Additionally the shock S^L was added to model changes in labor supply that Hall (1997) and Chari et al. (2007) identified as responsible for major changes in employment over the business cycle. There are three other shocks in the stochastic components of the taxes on consumption (ϕ^c), on labor income (ϕ^l) and on capital income (ϕ^k). These shocks were included to characterize the stochastic component related to these three types of taxes. And the shock in the risk premium. In fact, a positive shock to RIS reduces both consumption and investment (because the households demand more public bonds), generating a positive co-movement between those two aggregate demand components (Castro et al, 2011).

[6] $X_t = \int_0^1 X_{h,t} dh = (1 - \omega)X_{R,t} + \omega X_{NR,t}$.

Thus, the movement rules of such shocks are presented below:

$$\log S_t^C = (1-\rho_{sc}) \log S_{ss}^C + \rho_{sc} \log S_{t-1}^C + \epsilon_{sc,t} \tag{2.14}$$

$$\log S_t^L = (1-\rho_{sl}) \log S_{ss}^L + \rho_{sl} \log S_{t-1}^L + \epsilon_{sl,t} \tag{2.15}$$

$$\log \phi_t^c = (1-\rho_c) \log \phi_{ss}^c + \rho_c \log \phi_{t-1}^c + \epsilon_{c,t} \tag{2.16}$$

$$\log \phi_t^l = (1-\rho_l) \log \phi_{ss}^l + \rho_l \log \phi_{t-1}^l + \epsilon_{l,t} \tag{2.17}$$

$$\log \phi_t^k = (1-\rho_k) \log \phi_{ss}^k + \rho_k \log \phi_{t-1}^k + \epsilon_{k,t} \tag{2.18}$$

$$\log RIS_t = (1-\rho_R) \log RIS_{ss} + \rho_R \log RIS_{t-1} + \epsilon_{R,t} \tag{2.19}$$

where $\epsilon_{sc,t}$, $\epsilon_{sl,t}$, $\epsilon_{c,t}$, $\epsilon_{l,t}$, $\epsilon_{k,t}$, $\epsilon_{R,t}$ are exogenous shocks, and ρ_{sc}, ρ_{sl}, ρ_c, ρ_l, ρ_k, ρ_R are autoregressive components, of the intertemporal shock, of the shock on labor supply, of the shock of the taxes on consumption, on labor income, on capital income and risk premium, respectively.

2.1.2 Firms

Following the comments of the introduction about the reasons for using a New Keynesian model, the procedure to include monopolistic competition and sticky pricing is to divide the productive sector of the economy into two subsectors: firm producers of finished goods (retail); and firm producers of intermediate goods (wholesale). The wholesale sector is formed by a great number of firms, each producing a different good according to the structure of monopolistic competition. In the retail industry, there is a single firm that aggregates intermediate goods in a single good (Y) that will be consumed by economic agents. Besides these features, it should be mentioned that the markets for productive factors follow a structure of perfect competition. We consider a variant of this that is close related to work by Galí, Gertler and López-Salido (2001).

2.1.2.1 Firm Producers of Finished Goods (Retail)

First, it is necessary to define the aggregator behavior of the production function. The finished good is produced by a single firm that operates in perfect competition. For this purpose, the firm combines a continuum of intermediate goods and aggregates them into a single finished good using the following technology:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varphi-1}{\varphi}} dj \right)^{\frac{\varphi}{\varphi-1}} \tag{2.20}$$

where Y is aggregate output, Y_j is the intermediate product j, φ is the elasticity of substitution between intermediate goods. The form adopted to aggregate the assets is called an Dixit-Stiglitz aggregator (Dixit e Stiglitz, 1977).

As mentioned, the finished goods producer is in perfect competition and maximizes its profit by using the technology of equation (2.20), whereas the prices of intermediate goods are given. Therefore, the problem of the retail firm is:

$$\max_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{2.21}$$

substituting (2.20) into (2.21),

$$\max_{Y_{j,t}} P_t \left(\int_0^1 Y_{j,t}^{\frac{\varphi-1}{\varphi}} dj \right)^{\frac{\varphi}{\varphi-1}} - \int_0^1 P_{j,t} Y_{j,t} dj$$

The first order condition for each intermediate good j is:

$$P_t \left(\int_0^1 Y_{j,t}^{\frac{\varphi-1}{\varphi}} dj \right)^{\frac{\varphi}{\varphi-1}-1} Y_{j,t}^{\frac{\varphi-1}{\varphi}-1} - P_{j,t} = 0$$

$$Y_{j,t} = Y_t \left(\frac{P_t}{P_{j,t}} \right)^{\varphi} \tag{2.22}$$

Equation (2.22) demonstrates that the demand for intermediate good j is a decreasing function of its relative price and increasing in relation to the aggregate output of the economy.

The assumption that there is perfect competition in the final goods market, allows us to derive the price of the final goods. Substituting equation (2.22) in (2.20):

$$Y_t = \left\{ \int_0^1 \left[Y_t \left(\frac{P_t}{P_{j,t}} \right)^{\varphi} \right]^{\frac{\varphi-1}{\varphi}} dj \right\}^{\frac{\varphi}{\varphi-1}}$$

$$P_t = \left(\int_0^1 P_{j,t}^{\frac{\varphi-1}{\varphi}} dj \right)^{\frac{\varphi}{\varphi-1}} \tag{2.23}$$

2.1.2.2 Firm Producers of intermediate goods (Wholesalers)

The wholesaler firms solve the problem in two steps. In the first step, firms take as given the prices of production factors: wages (W) and return to capital (R). They determine the quantities of those inputs that will minimize their costs. In the second stage, firms determine the optimal price of good j and they determine the quantity that will be produced in accordance with this price.

First Step

The objective of the first step is to minimize the cost of production,

$$\min_{L_{j,t}, K_{p,j,t}} W_t L_{j,t} + R_t K_{p,j,t} \tag{2.24}$$

subject to the following technology[7],

$$Y_{j,t} = A_t K_{p,j,t}^{\alpha_1} L_{j,t}^{\alpha_2} K_{g,j,t}^{\alpha_3} \tag{2.25}$$

This technology follows the works of Cassou and Lansing (1998), Lansing (1998), Baxter and King (1993) and Ambler and Paquet (1996), where they introduced

[7] As in the case of the utility function of the households, the production function must have some properties: to be strictly increasing ($F_K > 0$ and $F_L > 0$); to be strictly concave ($F_{KK} < 0$ and $F_{LL} < 0$); and to be twice differentiable. It is also assumed that the production function has constant returns to scale, $F(zK_t, zL_t) = zY_t$. Still, this function must fulfill the calls Inada conditions: $\lim_{K\to 0} = \infty$; $\lim_{K\to\infty} = 0$; $\lim_{L\to 0} = \infty$; and $\lim_{L\to\infty} = 0$.

the stock of public capital in the production function. So, $K_{g,t}$ is the stock of public capital, α_1, α_2, α_3 are participation of private capital, of labor and of public capital in the production of good j, respectively [8], e A is the productivity, whose law of motion is:

$$\log A_t = (1 - \rho_A) \log Ass + \rho_A \log A_{t-1} + \epsilon_{A,t} \tag{2.26}$$

where $\epsilon_{A,t}$ is exogenous shocks and ρ_A is autoregressive components of the productivity shock.

Using the Lagrangian function to solve the previous problem of wholesaler firm:

$$\mathscr{L} = W_t L_{j,t} + R_t K_{p,j,t} - \mu_t (A_t K_{p,j,t}^{\alpha_1} L_{j,t}^{\alpha_2} K_{g,j,t}^{\alpha_3} - Y_{j,t}) \tag{2.27}$$

The first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial L_{j,t}} = W_t - \alpha_2 \mu_t A_t K_{p,j,t}^{\alpha_1} L_{j,t}^{\alpha_2 - 1} K_{g,j,t}^{\alpha_3} = 0 \tag{2.28}$$

$$\frac{\partial \mathscr{L}}{\partial K_{p,j,t}} = R_t - \alpha_1 \mu_t A_t K_{p,j,t}^{\alpha_1 - 1} L_{j,t}^{\alpha_2} K_{g,j,t}^{\alpha_3} = 0 \tag{2.29}$$

From equations (2.28) and (2.29), we arrive at:

$$W_t = \mu_t \alpha_2 \frac{Y_{j,t}}{L_{j,t}} \tag{2.30}$$

$$R_t = \mu_t \alpha_1 \frac{Y_{j,t}}{K_{p,j,t}} \tag{2.31}$$

and from equations (2.30) and (2.31),

$$\frac{W_t}{R_t} = \left(\frac{\alpha_2}{\alpha_1}\right) \frac{K_{p,j,t}}{L_{j,t}} \tag{2.32}$$

[8]It is assumed that firms have constant returns to scale, $\alpha_1 + \alpha_2 + \alpha_3 = 1$, following Cassou and Lanzing (1998), corroborated by empirical studies from Aschauer (1989) and Ai and Cassou (1995).

Second Step

In the second step, the wholesale firm maximizes its profit by choosing the price of its good j,

$$\max_{P_{j,t}} P_{j,t}Y_{j,t} - W_tL_{j,t} - R_tK_{p,j,t} \tag{2.33}$$

substituting (2.22), (2.30) and (2.31) in (2.33):

$$\max_{P_{j,t}} P_{j,t}Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi} - \mu_t\alpha_2\frac{Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi}}{L_{j,t}}L_{j,t} - \mu_t\alpha_1\frac{Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi}}{K_{p,j,t}}K_{p,j,t}$$

$$\max_{P_{j,t}} P_{j,t}Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi} - \mu_t\alpha_2Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi} - \mu_t\alpha_1Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi}$$

$$\max_{P_{j,t}} P_{j,t}Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi} - \mu_t(\alpha_1+\alpha_2)Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi}$$

It lies in the following first order condition,

$$(1-\varphi)Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi} + \varphi\mu_t(\alpha_1+\alpha_2)Y_t\left(\frac{P_t}{P_{j,t}}\right)^{\varphi}P_{j,t}^{-1} = 0$$

$$\mu_t = \left(\frac{\varphi-1}{\varphi}\right)\left(\frac{1}{\alpha_1+\alpha_2}\right)P_{j,t} \tag{2.34}$$

substituting (2.34) into (2.30) and (2.31), and knowing that these firms have the same technology - $P_{j,t} = P_t$ e $Y_{j,t} = Y_t$ - the results for demand of the factors of production are:

$$\frac{W_t}{P_t} = \left(\frac{\varphi-1}{\varphi}\right)\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)\frac{Y_t}{L_t} \tag{2.35}$$

$$\frac{R_t}{P_t} = \left(\frac{\varphi-1}{\varphi}\right)\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)\frac{Y_t}{K_{p,t}} \tag{2.36}$$

We can use the equation (2.35) to see how various changes affects the demand for labor[9]. It follows at once that a decrease in the real wage rate, W/P, means a

[9] The same reasoning can be used for private capital.

higher quantity of labor demanded. When the real cost of hiring workers decrease, firms expand employment until labor's marginal product falls by as much as the decrease in W/P (Barro, 1997).

An upward shift in the schedule for labor's marginal product leads to a greater quantity of labor demanded at any given real wage rate. Employment expands until the marginal product again equals W/P[10].

2.1.2.3 Pricing *a la Calvo*

The wholesale firm chooses how much to produce in each period, but following a rule *a la Calvo* (Calvo, 1983), this is perhaps the most popular pricing model, that says they fail to choose the price of their good in all periods. At each period t, a fraction $0 < 1 - \theta < 1$ of firms are randomly selected and allowed to choose the price of their good for period t, $P^*_{j,t}$. The remaining firms (the ratio θ of firms) keeps the price of the previous period ($P_{j,t} = P_{j,t-1}$) for the product.

Thus, solving equation (2.32) to $L_{j,t}$:

$$L_{j,t} = \left(\frac{\alpha_2}{\alpha_1}\right)\frac{R_t K_{p,j,t}}{W_t}$$

and substituting this result in the production function (equation (2.25)):

$$Y_{j,t} = A_t K_{p,j,t}^{\alpha_1}\left[\left(\frac{\alpha_2}{\alpha_1}\right)\frac{R_t K_{p,j,t}}{W_t}\right]^{\alpha_2} K_{g,j,t}^{\alpha_3}$$

getting,

$$K_{p,j,t} = \left(\frac{Y_{j,t}}{A_t K_{g,j,t}^{\alpha_3}}\right)^{\frac{1}{\alpha_1+\alpha_2}}\left[\left(\frac{\alpha_1}{\alpha_2}\right)\frac{W_t}{R_t}\right]^{\frac{\alpha_2}{\alpha_1+\alpha_2}} \tag{2.37}$$

[10]We can summarize the results by writing down:
$L^d = L^d\left[\left(\underset{(-)}{\frac{W}{P}}\right)\right]$
where L^d is labor demand.

and,

$$L_{j,t} = \left(\frac{Y_{j,t}}{A_t K_{g,j,t}^{\alpha_3}} \right)^{\frac{1}{\alpha_1+\alpha_2}} \left[\left(\frac{\alpha_1}{\alpha_2} \right) \frac{W_t}{R_t} \right]^{\frac{-\alpha_1}{\alpha_1+\alpha_2}} \tag{2.38}$$

Since in any period, all firms face the same wages, rentals, and technology, the cost are the same for all firms. So, substituting these factor demands into the cost equation (CT) gives,

$$CT_t = \left(\frac{Y_{j,t}}{A_t K_{g,j,t}^{\alpha_3}} \right)^{\frac{1}{\alpha_1+\alpha_2}} \left\{ W_t^{\frac{\alpha_2}{\alpha 1+\alpha_2}} R_t^{\frac{\alpha_1}{\alpha 1+\alpha_2}} \left[\left(\frac{\alpha_1}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} + \left(\frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} \right] \right\}$$

and nominal marginal cost,

$$\begin{aligned} MC_t = & \left(\frac{1}{\alpha_1+\alpha_2} \right) Y_{j,t}^{\frac{\alpha_3}{\alpha_1+\alpha_2}} \left(\frac{1}{A_t K_{g,j,t}^{\alpha_3}} \right)^{\frac{1}{\alpha_1+\alpha_2}} \\ & W_t^{\frac{\alpha_2}{\alpha 1+\alpha_2}} R_t^{\frac{\alpha_1}{\alpha 1+\alpha_2}} \left[\left(\frac{\alpha_1}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} + \left(\frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} \right] \end{aligned} \tag{2.39}$$

The marginal cost represents the cost, relative to each production factor, of additional unit of the intermediate goods. Notice, specially, an increase in the productivity (A) or/and in the public capital stock (K_g) falls the marginal cost. The improvement of the marginal cost raises the supply of goods $(Y_{j,t})$.

The wholesale firm has a probability θ to keep the price of the previous period for the good and the probability $(1-\theta)$ to choose the price optimally. Once fixing the price in period t, there is the probability θ that this price will remain fixed in period t+1, a probability θ^2 that this price will remain fixed in period t+2, and so on. This firm should take into account these probabilities when choosing the price of its own good in it capacity to perform this adjustment.

Thus, the problem of the firm able to adjust the price of the good is:

$$\max_{P_{j,t}^*} E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[P_{j,t}^* Y_{j,t+i} - R_{t+i} K_{j,t+i} - W_{t+i} L_{j,t+i} \right] \tag{2.40}$$

where θ is the factor of rigidity in the adjustment of prices and $P^*_{j,t}$ is the optimal price set by the firm with the ability to adjust the price of your product. Equation (2.40) is the discounted profit of the firm during the period which the price $P^*_{j,t}$ is in progress.

substituting (2.22), (2.37) and (2.38) in (2.40):

$$\max_{P^*_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left\{ P^*_{j,t} Y_{t+i} \left(\frac{P_{t+i}}{P^*_{j,t}} \right)^{\varphi} - \left[Y_{t+i} \left(\frac{P_{t+i}}{P^*_{j,t}} \right)^{\varphi} \frac{1}{A_t K^{\alpha_3}_{g,j,t}} \right]^{\frac{1}{\alpha_1+\alpha_2}} W_t^{\frac{\alpha_2}{\alpha_1+\alpha_2}} R_t^{\frac{\alpha_1}{\alpha_1+\alpha_2}} \left[\left(\frac{\alpha_1}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} + \left(\frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} \right] \right\}$$

Arriving at the following first order condition:

$$E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left(1 - \varphi + \frac{\varphi}{P^*_{j,t}} MC_{t+i} \right) = 0$$

$$P^*_{j,t} = \frac{\varphi}{\varphi - 1} E_t \sum_{i=0}^{\infty} (\beta\theta)^i MC_{t+i} \tag{2.41}$$

Combining the pricing rule of equation (2.23), and the assumption that all firms with the ability to adjust define equal value and that firms without this ability retains the same price, the overall price level is obtained by the equation:

$$P_t = \left[\theta P_{t-1}^{1-\varphi} + (1-\theta) P_t^{*1-\varphi} \right]^{\frac{1}{1-\varphi}} \tag{2.42}$$

Dividing both sides by P_{t-1},

$$\pi_t = \left[\theta + (1-\theta) \left(\frac{P^*_t}{P_{t-1}} \right)^{1-\varphi} \right]^{\frac{1}{1-\varphi}} \tag{2.43}$$

where $\pi = \frac{P_t}{P_{t-1}}$ is the gross inflation rate between t-1 and t. Notice that, as shown below, all firms will choose the same price because they face an identical problem. It follows from (2.43) that in a steady state with zero inflation ($\pi = 1$), $P^*_t = P_{t-1} = P_t$ for all t.

The combination of the equations (2.41) and (2.43) result in the New Keynesian Phillips curve (see Roberts 1995 and 1997; Clarida et al., 1999; McCallum and Nelson, 1999; Svensson and Woodford, 2003 and 2004; Woodford, 2003; Giannoni and Woodford, 2005). Briefly, inflation will increase if marginal cost rise.

2.1.3 Government

In this section, we introduce government into our model. The principal role of governments is to provide public goods and services. These expenditures must be paid for. This can be achieved through taxation or by borrowing[11]. Other objective of the government is to keep price stability. It is reached using monetary policy. Hence, the government sector in this work is divided into two subsectors: Fiscal Authority and Monetary Authority.

2.1.3.1 Fiscal Authority

The government collects taxes and issues bonds to finance its spending on goods and services. Therefore, the change in public debt is given by the following rule:

$$\frac{B_{t+1}}{RIS_t R_t^B} - B_t = TS_t - TAX_t \tag{2.44}$$

where TS and TAX are total spending and tax revenue of the government, respectively.

As could not be otherwise, the total spending of the government is sensitive to the size of the public debt (current debt (B_t) relative to its steady-state level, B_{ss}). In other words, government total spending has an automatic stabilizing property (Lim and McNelis, 2008):

$$TS_t - TS_{ss} = \chi(B_t - B_{ss}) \tag{2.45}$$

[11] In reality, printing money is possible, but this work doesn't has money.

where χ is the sensitivity of government total spending relative to the size of the public debt.

The government total spending is obtained by:

$$TS_t = G_t P_t + I_{g,t} P_t + TRANS_t \tag{2.46}$$

where G, $I_{g,t}$ and $TRANS$ are current expenditures, public investments and transfers of income, respectively.

These spending are obtained by the rules:

$$G_t = \xi_G S_t^G Y_t \tag{2.47}$$

$$\log S_t^G = (1 - \rho_G) \log S_{ss}^G + \rho_G \log S_{t-1}^G + \epsilon_{G,t} \tag{2.48}$$

$$I_{g,t} = \xi_{I_g} S_t^{I_g} Y_t \tag{2.49}$$

$$\log S_t^{I_g} = (1 - \rho_{I_g}) \log S_{ss}^{I_g} + \rho_{I_g} \log S_{t-1}^{I_g} + \epsilon_{I_g,t} \tag{2.50}$$

$$\frac{TRANS_t}{P_t} = \xi_{Tr} S_t^{Tr} Y_t \tag{2.51}$$

$$\log S_t^{Tr} = (1 - \rho_{Tr}) \log S_{ss}^{Tr} + \rho_{Tr} \log S_{t-1}^{Tr} + \epsilon_{Tr,t} \tag{2.52}$$

where $\epsilon_{G,t}$, $\epsilon_{I_g,t}$ and $\epsilon_{Tr,t}$ are exogenous shocks, and ρ_G, ρ_{I_g} and ρ_{Tr} are autoregressive components of the current expenditures, of the public investments and of the transfers of income, respectively.

The stock of public capital follows the following law of motion:

$$K_{g,t+1} = (1 - \delta_g)K_{g,t} + I_{g,t} \tag{2.53}$$

We now consider alternative ways of taxation. Hence, tax revenue is obtained by the following equation:

$$TAX_t = \frac{\tau_c}{\phi_t^c}P_t(C_t + I_{p,t}) + \frac{\tau_l}{\phi_t^l}W_t L_t + \frac{\tau_k}{\phi_t^k}R_t K_{p,t} \tag{2.54}$$

2.1.3.2 Monetary Authority

The Central Bank of Brazil appears in this work following a simple Taylor rule (1993) with the dual goal of output growth and maintenance of price stability. Here, we are considering a variant of the work made by Lim and McNelis (2008).

$$R_t^B = \eta R_{t-1}^B + (1 - \eta)\left[a(Y_t - Y_{ss}) + b(\pi_t - \pi_{ss}) + R_{ss}^B S_t^M\right] \tag{2.55}$$

where S_t^M is monetary shock, η is smoothing parameter, and it allows lagged interest rates play a significant role in the determination of the current interest rate, (a) and (b) are the sensitivities of the basic interest rate in relation to the product and to the inflation rate, respectively.

$$\log S_t^M = (1 - \rho_M)\log S_{ss}^M + \rho_M S_{t-1}^M + \epsilon_{M,t} \tag{2.56}$$

where $\epsilon_{M,t}$ and ρ_M are exogenous shocks and autoregressive components of the monetary authority.

2.1.4 Equilibrium Condition of Goods Market

To complete the model it is necessary to use the equilibrium condition in the goods market. Wherein aggregate production Y_t is demanded by households (C_t and I_t)

and Government (G_t):

$$Y_t = C_t + I_t + G_t \tag{2.57}$$

2.2 Steady State

Once obtained the economic equilibrium, the next step is to define the values of the steady state. In fact, the model is stationary, in the sense that there is a value for the variables sustained intertemporally. Thus, an endogenous variable x_t will be in steady state for all t, if $E_t x_{t+1} = x_t = x_{t-1} = x_{ss}$.

Some endogenous variables have their values previously determined at steady state. That is, the variables involved in exogenous shocks. The next step to calculate the steady state is to remove the time indicators of the variables. Therefore, the structural model becomes:

From (2.3),

$$I_{p,ss} = \delta_p K_{p,ss} \tag{2.58}$$

From (2.10),

$$L_{ss}^{\psi} C_{ss}^{\sigma} \left(\frac{1+\tau_c}{1-\tau_l} \right) = \frac{W_{ss}}{P_{ss}} \tag{2.59}$$

From (2.11),

$$R_{ss}^{B} = \frac{1}{\beta} \tag{2.60}$$

From (2.12),

$$\frac{R_{ss}}{P_{ss}} = \left(\frac{1+\tau_c}{1-\tau_k} \right) \left[\frac{1}{\beta} - (1-\delta_p) \right] \tag{2.61}$$

From (2.13),

$$I_{ss} = (1-\omega_G) I_{p,ss} + \omega_G I_{g,ss} \tag{2.62}$$

From (2.25)

$$Y_{ss} = K_{p,ss}^{\alpha_1} L_{ss}^{\alpha_2} K_{g,ss}^{\alpha_3} \tag{2.63}$$

From (2.35),

$$\frac{W_{ss}}{P_{ss}} = \left(\frac{\varphi - 1}{\varphi}\right)\left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right)\frac{Y_{ss}}{L_{ss}} \tag{2.64}$$

From (2.36),

$$\frac{R_{ss}}{P_{ss}} = \left(\frac{\varphi - 1}{\varphi}\right)\left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right)\frac{Y_{ss}}{K_{p,ss}} \tag{2.65}$$

From (2.39),

$$\begin{aligned} MC_{ss} = \left(\frac{1}{\alpha_1 + \alpha_2}\right)\left(\frac{Y_{ss}}{K_{g,ss}}\right)^{\frac{\alpha_3}{\alpha_1+\alpha_2}} \\ W_{ss}^{\frac{\alpha_2}{\alpha 1+\alpha_2}} R_{ss}^{\frac{\alpha_1}{\alpha 1+\alpha_2}} \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} + \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha_1+\alpha_2}}\right] \end{aligned} \tag{2.66}$$

From (2.41),

$$P_{ss} = \frac{\varphi}{\varphi - 1}\left(\frac{1}{1 - \beta\theta}\right) MC_{ss} \tag{2.67}$$

From (2.43),

$$\pi_{ss} = 1 \tag{2.68}$$

From (2.44),

$$B_{ss}\left(\frac{1}{R_{ss}^B} - 1\right) = TS_{ss} - TAX_{ss} \tag{2.69}$$

From (2.46),

$$TS_{ss} = G_{ss}P_{ss} + I_{g,ss}P_{ss} + TRANS_{ss} \tag{2.70}$$

From (2.47),

$$G_{ss} = \xi_G Y_{ss} \tag{2.71}$$

From (2.49),

$$I_{g,ss} = \xi_{I_g} Y_{ss} \tag{2.72}$$

From (2.51),

$$TRANS_{ss} = \xi_{Tr} Y_{ss} P_{ss} \tag{2.73}$$

From (2.53),

$$I_{g,ss} = \delta_g K_{g,ss} \tag{2.74}$$

From (2.54),

$$TAX_{ss} = \tau_c P_{ss}(C_{ss} + I_{p,ss}) + \tau_l W_{ss} L_{ss} + \tau_k R_{ss} K_{p,ss} \tag{2.75}$$

From (2.57),

$$Y_{ss} = C_{ss} + I_{ss} + G_{ss} \tag{2.76}$$

Combining (2.72) and (2.74),

$$K_{g,ss} = \left(\frac{\xi_{I_g}}{\delta_g}\right) Y_{ss} \tag{2.77}$$

From (2.65),

$$K_{p,ss} = \left(\frac{\varphi - 1}{\varphi}\right)\left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right)\frac{Y_{ss}}{\frac{R_{ss}}{P_{ss}}}$$

with,

$$A_1 = \left(\frac{\varphi - 1}{\varphi}\right)\left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right)\frac{1}{\frac{R_{ss}}{P_{ss}}}$$

$$K_{p,ss} = A_1 Y_{ss} \tag{2.78}$$

Substituting (2.77) and (2.78) into (2.63),

$$Y_{ss} = (A_1 Y_{ss})^{\alpha_1} L_{ss}^{\alpha_2} \left[\frac{\xi_{I_g}}{\delta_g} Y_{ss}\right]^{\alpha_3}$$

$$\frac{Y_{ss}}{L_{ss}} = \left[A_1^{\alpha_1}\left(\frac{\xi_{I_g}}{\delta_g}\right)^{\alpha_3}\right]^{\frac{1}{\alpha_2}} \tag{2.79}$$

with,

$$A_2 = \left[\left(\frac{1}{A_1^{\alpha_1}}\right)\left(\frac{\delta_g}{\xi_{Ig}}\right)^{\alpha_3}\right]^{\frac{1}{\alpha_2}}$$

and,

$$L_{ss} = A_2 Y_{ss} \tag{2.80}$$

Substituting (2.79) into (2.64),

$$\frac{W_{ss}}{P_{ss}} = \left(\frac{\varphi - 1}{\varphi}\right)\left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right)\left[A_1^{\alpha_1}\left(\frac{\xi_{I_g}}{\delta_g}\right)^{\alpha_3}\right]^{\frac{1}{\alpha_2}} \tag{2.81}$$

Substituting (2.78) into (2.58),

$$I_{p,ss} = \delta_p A_1 Y_{ss} \tag{2.82}$$

Combining (2.72), (2.82) and (2.62),

$$I_{ss} = \left[(1 - \omega_G)\delta_p A_1 + \omega_G \xi_{I_g}\right] Y_{ss} \tag{2.83}$$

Substituting (2.71) and (2.83) into (2.63),

$$Y_{ss} = C_{ss} + \left[(1 - \omega_G)\delta_p A_1 + \omega_G \xi_{I_g}\right] Y_{ss} + \xi_G Y_{ss}$$

$$C_{ss} = \left\{1 - \left[(1 - \omega_G)\delta_p A_1 + \omega_G \xi_{I_g} + \xi_G\right]\right\} Y_{ss}$$

with,

$$A_3 = 1 - \left[(1 - \omega_G)\delta_p A_1 + \omega_G \xi_{I_g} + \xi_G\right]$$

$$C_{ss} = A_3 Y_{ss} \tag{2.84}$$

Substituting (2.80), (2.84), into (2.59),

$$Y_{ss} = \left[\frac{W_{ss}}{P_{ss}}\left(\frac{1 - \tau_l}{1 + \tau_c}\right)\left(\frac{1}{A_3^{\sigma} A_2^{\psi}}\right)\right]^{\frac{1}{\sigma + \psi}} \tag{2.85}$$

Substituting (2.71), (2.72), (2.73) into (2.70),

$$TS_{ss} = (\xi_G + \xi_{I_g} + \xi_{Tr}) Y_{ss} P_{ss} \tag{2.86}$$

Substituting (2.64), (2.65), (2.80) and (2.84) into (2.75),

$$TAX_{ss} = \left[\tau_c(A_3 + \delta_p A_1) + (\tau_l \alpha_2 + \tau_k \alpha_1)\left(\frac{\varphi - 1}{\varphi}\right)\left(\frac{1}{\alpha_1 + \alpha_2}\right)\right] P_{ss} Y_{ss}$$

$$A_4 = \tau_c(A_3 + \delta_p A_1) + (\tau_l \alpha_2 + \tau_k \alpha_1)\left(\frac{\varphi - 1}{\varphi}\right)\left(\frac{1}{\alpha_1 + \alpha_2}\right)$$

$$TAX_{ss} = A_4 P_{ss} Y_{ss} \tag{2.87}$$

Substituting (2.86), (2.87) into (2.69), and assuming $\frac{B_{ss}}{Y_{ss}} = 1$[12],

$$(\beta - 1) = \left[(\xi_G + \xi_{I_g} + \xi_{Tr}) - A_4\right] P_{ss} Y_{ss}$$

$$P_{ss} = \frac{(\beta - 1)}{(\xi_G + \xi_{I_g} + \xi_{Tr}) - A_4} \tag{2.88}$$

2.3 Log-linearization - (Uhlig's Method)

The conditions for optimizing the model forms a system of nonlinear difference equations. This system has not a closed analytic solution, generally. In most cases, it is easier and more convenient to use approximations to characterize the solution of the dynamic model (Fernandez-Villaverde, 2009). This section summarizes the method of log-linear approximation customarily used in the literature.

Uhlig (1999) recommends a simple method of log-linearization of functions that do not require differentiation. Thus, consider a set of variables X_t and setting $\widetilde{X}_t = \ln X_t - \ln X_{ss}$. Then we can write the original variables as: $X_t = X_{ss} e^{\widetilde{X}_t}$. Uhlig also proposes the following block of aid for the log-linearization:

$$e^{(\widetilde{X}_t + a\widetilde{Y}_t)} \approx 1 + \widetilde{X}_t + a\widetilde{Y}_t$$

[12] Castro et al, (2011) assuming $B^y = 2$ (Net government debt-to-GDP ratio (quarterly GDP))

$$\widetilde{X}_t\widetilde{Y}_t \approx 0$$

$$E_t\left[ae^{\widetilde{X}_{t+1}}\right] \approx a + aE_t\left[\widetilde{X}_{t+1}\right]$$

From (2.3),

$$K_{p,ss}\widetilde{K}_{p,t+1} = (1-\delta_p)K_{p,ss}\widetilde{K}_{p,t} + I_{p,ss}\widetilde{I}_{p,t} \tag{2.89}$$

From (2.10),

$$C_{ss}^{\sigma}L_{ss}^{\psi}\left[(\widetilde{S}_t^L + \sigma\widetilde{C}_t + \psi\widetilde{L}_t)(1+\tau_c) - \tau_c\widetilde{\phi}_t^c\right] = \frac{W_{ss}}{P_{ss}}\left[(\widetilde{W_t} - \widetilde{P}_t)(1-\tau_l) + \tau_l\widetilde{\phi}_t^l\right] \tag{2.90}$$

From (2.11),

$$\left[\sigma(\widetilde{C_{t+1}} - \widetilde{C}_t) + \widetilde{\pi_{t+1}}\right](1+\tau_c) = (\widetilde{RIS}_t + \widetilde{S_{t+1}}^c - \widetilde{S}_t{}^c + \widetilde{R^B}_t)(1+\tau_c) + \tau_c(\widetilde{\phi}_{t+1}^c - \widetilde{\phi}_t^c) \tag{2.91}$$

From (2.12),

$$\frac{P_{ss}}{\beta}\left[(\widetilde{P}_{t+1} + \widetilde{S}_t^c - \sigma\widetilde{C}_t)(1+\tau_c) - \tau_c\widetilde{\phi}_{t+1}^c\right] = P_{ss}(1-\delta_p)\left[(\widetilde{S}_{t+1}^c + \widetilde{P}_{t+1} - \sigma\widetilde{C}_{t+1}^c)(1+\tau_c)\right.$$
$$\left. - \tau_c\widetilde{\phi}_{t+1}^c\right] + R_{ss}\left[(\widetilde{S}_{t+1}^c + \widetilde{R}_{t+1} - \sigma\widetilde{C}_{t+1})(1-\tau_k) + \tau_k\widetilde{\phi}_{t+1}^k\right] \tag{2.92}$$

From (2.13),

$$I_{ss}\widetilde{I}_t = (1-\omega_G)I_{p,ss}\widetilde{I}_{p,t} + \omega_G I_{g,ss}\widetilde{I}_{g,t} \tag{2.93}$$

From (2.14),

$$\widetilde{S}_t^C = \rho_{sc}\widetilde{S}_{t-1}^C + \epsilon_{sc,t} \tag{2.94}$$

From (2.15),

$$\widetilde{S}_t^L = \rho_{sl}\widetilde{S}_{t-1}^L + \epsilon_{sl,t} \tag{2.95}$$

From (2.16),

$$\widetilde{\phi}_t^c = \rho_c \widetilde{\phi}_{t-1}^c + \epsilon_{c,t} \tag{2.96}$$

From (2.17),

$$\widetilde{\phi}_t^l = \rho_l \widetilde{\phi}_{t-1}^l + \epsilon_{l,t} \tag{2.97}$$

From (2.18),

$$\widetilde{\phi}_t^k = \rho_k \widetilde{\phi}_{t-1}^k + \epsilon_{k,t} \tag{2.98}$$

From (2.19),

$$\widetilde{RIS}_t = \rho_R \widetilde{RIS}_{t-1} + \epsilon_{R,t} \tag{2.99}$$

From (2.25),

$$\widetilde{Y}_t = \widetilde{A}_t + \alpha_1 \widetilde{K}_{p,t} + \alpha_2 \widetilde{L}_t + \alpha_3 \widetilde{K}_{g,t} \tag{2.100}$$

From (2.26),

$$\widetilde{A}_t = \rho_A \widetilde{A}_{t-1} + \epsilon_{A,t} \tag{2.101}$$

From (2.35),

$$\widetilde{W}_t - \widetilde{P}_t = \widetilde{Y}_t - \widetilde{L}_t \tag{2.102}$$

From (2.36),

$$\widetilde{R}_t - \widetilde{P}_t = \widetilde{Y}_t - \widetilde{K}_{p,t} \tag{2.103}$$

From (2.39),

$$(\alpha_1 + \alpha_2)\widetilde{MC}_t = \alpha_2 \widetilde{W}_t + \alpha_1 \widetilde{R}_t + \alpha_3 \widetilde{Y}_t - \widetilde{A}_t - \alpha_3 \widetilde{K}_{g,t} \tag{2.104}$$

From (2.41) e (2.42),

Log-linearizing the equation(2.41),

$$P^*_{ss}(1+\widetilde{P}^*_t) = \left(\frac{\varphi}{\varphi-1}\right)\left(\frac{MC_{ss}}{1-\beta\theta}\right)(1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^i(1+\widetilde{MC}_{t+i})$$

$$\widetilde{P}^*_t = (1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^i\widetilde{MC}_{t+i} \tag{2.105}$$

Log-linearizing the equation(2.42),

$$\widetilde{P}_t = \theta\widetilde{P}_{t-1} + (1-\theta)\widetilde{P}^*_t \tag{2.106}$$

Substituting (2.105) into (2.106),

$$\widetilde{P}_t = \theta\widetilde{P}_{t-1} + (1-\theta)(1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^i\widetilde{MC}_{t+i} \tag{2.107}$$

Multiplying both sides of equation (2.107) by $(1-\beta\theta L^{-1})$, knowing that $LX_t = X_{t-1}$ and $X_{t+1} = L^{-1}X_t$,

$$\widetilde{P}_t - \beta\theta E_t\widetilde{P}_{t+1} = \theta\widetilde{P}_{t-1} + (1-\theta)(1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^i\widetilde{MC}_{t+i}$$

$$\beta\theta\theta\widetilde{P}_t - \beta\theta(1-\theta)(1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^i\widetilde{MC}_{t+1+i}$$

$$\widetilde{P}_t - \beta\theta E_t\widetilde{P}_{t+1} = \theta\widetilde{P}_{t-1} - \beta\theta\theta\widetilde{P}_t + (1-\theta)(1-\beta\theta)\widetilde{MC}_t$$

$$\widetilde{P}_t - \widetilde{P}_{t-1} = \beta(E_t\widetilde{P}_{t+1} - \widetilde{P}_t) + \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widetilde{MC}_t - \widetilde{P}_t)$$

$$\widetilde{\pi}_t = \beta E_t\widetilde{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}(\widetilde{MC}_t - \widetilde{P}_t) \tag{2.108}$$

From (2.44),

$$\left(\frac{B_{ss}}{R^B_{ss}}\right)(\widetilde{B}_{t+1} - \widetilde{RIS}_t - \widetilde{R}^B_t) - B_{ss}\widetilde{B}_t = TS_{ss}\widetilde{TS}_t - TAX_{ss}\widetilde{TAX}_t \tag{2.109}$$

From (2.45),

$$TS_{ss}\widetilde{TS}_t = \chi B_{ss}\widetilde{B}_t \tag{2.110}$$

From (2.46),

$$TS_{ss}\widetilde{TS}_t = G_{ss}P_{ss}(\widetilde{G}_t + \widetilde{P}_t) + I_{g,ss}P_{ss}(\widetilde{I}_{g,t} + \widetilde{P}_t) + TRANS_{ss}\widetilde{TRANS}_t \tag{2.111}$$

From (2.47),

$$\widetilde{G}_t = \widetilde{S}^G_t + \widetilde{Y}_t \tag{2.112}$$

From (2.48),

$$\widetilde{S}^G_t = \rho_G \widetilde{S}^G_{t-1} + \epsilon_{G,t} \tag{2.113}$$

From (2.49),

$$\widetilde{I}_{g,t} = \widetilde{S}^{I_g}_t + \widetilde{Y}_t \tag{2.114}$$

From (2.50),

$$\widetilde{S}^{I_g}_t = \rho_{I_g} \widetilde{S}^{I_g}_{t-1} + \epsilon_{I_g,t} \tag{2.115}$$

From (2.51),

$$\widetilde{TRANS}_t = \widetilde{S}^{Tr}_t + \widetilde{Y}_t + \widetilde{P}_t \tag{2.116}$$

From (2.52),

$$\widetilde{S}^{Tr}_t = \rho_G \widetilde{S}^G_{t-1} + \epsilon_{G,t} \tag{2.117}$$

From (2.53),

$$K_{g,ss}\widetilde{K}_{g,t+1} = (1-\delta_g)K_{p,ss}\widetilde{K}_{g,t} + I_{g,ss}\widetilde{I}_{g,t} \tag{2.118}$$

From (2.54),

$$TAX_{ss}\widetilde{TAX}_t = \tau_c P_{ss}\left[C_{ss}(\widetilde{P}_t + \widetilde{C}_t - \widetilde{\phi}^c_t) + I_{ss}(\widetilde{P}_t + \widetilde{I}_t\widetilde{\phi}^c_t)\right]$$
$$+ \tau_l W_{ss}L_{ss}(\widetilde{W}_t + \widetilde{L}_t - \widetilde{\phi}^l_t) + \tau_k R_{ss}K_{ss}(\widetilde{R}_t + \widetilde{K}_t - \widetilde{\phi}^k_t) \tag{2.119}$$

From (2.55),

$$R^B_{ss}\widetilde{R}^B_t = \eta R^B_{ss}\widetilde{R}^B_{t-1}(1-\eta)(aY_{ss}\widetilde{Y}_t + b\pi_{ss}\widetilde{\pi}_t + R^B_{ss}\widetilde{S}^M_t) \tag{2.120}$$

From (2.56),

$$\widetilde{S}^M_t = \rho_M\widetilde{S}^M_{t-1} + \epsilon_{M,t} \tag{2.121}$$

From (2.57),

$$Y_{ss}\widetilde{Y}_t = C_{ss}\widetilde{C}_t + G_{ss}\widetilde{G}_t + I_{ss}\widetilde{I}_t \tag{2.122}$$

Chapter 3

Empirical Implementation

Introduction

In this chapter, we will be to parametrize the model economy, i.e., numerical values must be assigned to the parameters. Often, we have two alternatives: to estimate the parameters using some econometric technique or to calibrate the parameters. Calibration involves to calculate the value of the parameters in some way. On the other hand, the estimation approach, via maximum likelihood or Bayesian methods, is rather more complex and consists in adjusting the model to the observed data in order to determine the parameter values. The latter method has recently been gaining preference among macroeconomists.

Bayesian analysis differs substantially from the classical one. In classical analysis the probability of an event is the limit of the relative frequency of that event. Moreover, the parameters of a model are treated as fixed, unknown quantities. In this framework, unbiased estimators are important because the average value of the sample estimator converges to the true value via some Law of Large Numbers. Also, minimum variance estimators are preferable because they yield values closer to the true parameter. Finally, estimators and tests are evaluated in repeated samples since this insures that they give correct results with high probability.

Bayesian analysis takes a different point of view on all these issues. Probabilities measure the level of beliefs that a researcher has in an event. Parameters are random variables with a probability distribution. Properties of estimators and tests in repeated samples are uninteresting since beliefs are not necessarily related to the relative frequency of an event in large number of hypothetical experiments. Finally, estimators are chosen to minimize expected loss function (with expectations taken with respect to the posterior distribution), conditional on the data.

In general, regarding the use of classical econometric approach to macroeconomic general equilibrium models, Bayesian econometrics has some important advantages, such as: i) it is a flexible way to introduce a prior information about the structure of the data; ii) because that its procedure is based on the likelihood function, it is possible to take advantage of all the constraints resulting from a general equilibrium model.

In addition to this introductory section, this chapter is structured as follows. It begins with a description of the data used in the estimation of the parameters of the standard deviations of the stochastic shocks. Then it shows the calibrated parameters, *prior* and *posterior* of the parameters to Bayesian estimation. Finishing with estimation, which it was run on the Dynare platform [1]

3.1 Data

We estimated the model using quarterly data spanning from 2003Q1 to 2013Q4 (44 data points). We use 11 model variables as observables (P, $TRANS$, Ig, RTL, RTKp, RTC[2], R^B, I, G, C and L) which they are described in the Table 3.1. We have chosen this set of observables taking into account data availability and their

[1]Dynare is a software platform for the treatment of a wide class of macroeconomic models, in particular models of Dynamic Stochastic General Equilibrium (DSGE) and Overlapping Generations (OLG). The models solved by Dynare include the rational expectations hypothesis, but Dynare is also able to handle models where expectations are formed differently: on one extreme, models where agents perfectly anticipate the future; at the other extreme, the models where the agents have limited rationality or imperfect knowledge and thus form their expectations through a learning process. In terms of types of agents, it is possible to incorporate in Dynare: consumers, productive enterprises, government, monetary authorities, investors and financial intermediaries. Some level of heterogeneity can be achieved by the inclusion of several different classes of agents in each of the categories of the listed agents (Adjemian et al., 2011).

[2]$RTL = \tau_l WL$, $RTKp = \tau_k RKp$ and $RTC = \tau_c(C + Ip)$.
where RTL, RTKp and RTC are revenues from the taxes on labor income, on private capital income and on consumption.

relevance to the objective of this work. Furthermore, a large set of observables reduces the problem of identification.

DSGE models are designed to characterize a stationary economy: the stochastic behavior of the variables is in the form of temporary departures from steady state values. Corresponding data are represented analogously. DeJong and Dave (2008) explain the need seasonal adjustment and removing trend by means of an example of a business cycle model which is proposed to characterize patterns of fluctuations in the data that recur at business cycle frequencies: between approximately 6 and 40 quarters. It not intended to characterize seasonal fluctuations. Yet unless additional steps are taken, the removal of the trend will leave such fluctuations intact, and their presence can have a detrimental impact on inferences involving business cycle behavior.

Consider the observations $Z_t, t = 1, ..., N$ of a time series. Notice that the decomposition of a series consists to write Z_t as a sum of three unobservable components,

$$Z_t = T_t + S_t + a_t \tag{3.1}$$

where T_t and S_t represent the trend and seasonality, respectively, while a_t is a random component with zero mean and constant variance σ_a^2. So we can think a_t as a stationary process. It follows that Z_t is in general a non-stationary series.

To seasonality adjustment, the main interest in considering a model of type (3.1), it will estimate S_t and build a free series of seasonality. That is, if $\hat{S}_t$ for an estimated S_t,

$$Z_t^{SA} = Z_t - \hat{S}_t \tag{3.2}$$

it will be a seasonally adjusted.

There is no one right way to detrend a time series. The question is not one of right or wrong, but of useful or not useful (Hoover, 2012). Constant trends correspond to an equation with fixed coefficients, and include linear an exponential trends. If we believe that, despite cyclical fluctuations, the average growth rate of a series does not change much over a long period, then it reasonable to assume that the trend has a constant rate of growth and can be described by an equation $T = a(1+b)^t$ or $T = a\exp(bt)$ where t is time, and a and b are constants. If the time series grows at a steady proportionate rate, the trend is then described by a linear function $T = a+bt$. Other detrend way is to use moving-average trends.

These methods truly decompose the trend and the cycle into separate parts. Sometimes we may not really care about the trend but just want to focus on fluctuations. This is easily done by taking the first difference of the data $\Delta X_t = X_t - X_{t-1}$ or $\hat{X}_t = \frac{\Delta X_t}{X_{t-1}}$. Estimating T_t and S_t and subtracting them from Z_t we obtain an estimate of the random component a_t.

So, to prepare the data for the model estimation, we deflated using the IPCA, detrented and seasonally adjusted non stationary series using the software X12-ARIMA[3] and applied first log-difference[4]. Figure 3.1 provides the graphs of the series after being transformed.

Table 3.1: Observables variables of the model.

Variable	Series	Source
P	Series constructed using the IPCA (%a.m.)	IBGE/SNIPC
TRANS	Benefícios assistenciais (LOAS e RMV) R$ (milhões)	Min. Fazenda/STN
Ig	Custeio e investimento - R$ (milhões)	Min. Fazenda/STN
RTL	IR - pessoas físicas R$ (milhões)	Min. Fazenda/SRF
RTKp	IR - pessoas jurídicas R$ (milhões)	Min. Fazenda/SRF
RTC	ICMS and IPI R$ (milhões)	Min. Fazenda/SRF
R^B	Selic Over (% a.m.)	BCB Boletim/M. Finan.
I	Capital - formação bruta - R$ (milhões)	IBGE/SCN 2000 Trim.
G	Consumo final - adm. pública - R$ (milhões)	IBGE/SCN 2000 Trim.
C	Consumo final - famílias - R$ (milhões)	IBGE/SCN 2000 Trim.
L	Horas pagas - indústria - índice (média 2006 = 100)-SP	Fiesp

[3]X12-ARIMA was developed by US Census Bureau as an extended and improved version of the X11- ARIMA method of Statistics Canada. The program runs through the following steps. First the series is modified by any user-defined prior adjustments. Then the program fits a regARIMA model to the series in order to detect and adjust for outliers and other distorting effects for improving forecasts and seasonal adjustment. The program then uses a series of moving averages to decompose a time series into three components. In the last step a wider range of diagnostic statistics are produced, describing the final seasonal adjustment, and giving pointers to possible improvements which could be made.

[4]first log-difference$(X_t) = \ln\left(\frac{X_t}{X_{t-1}}\right)$

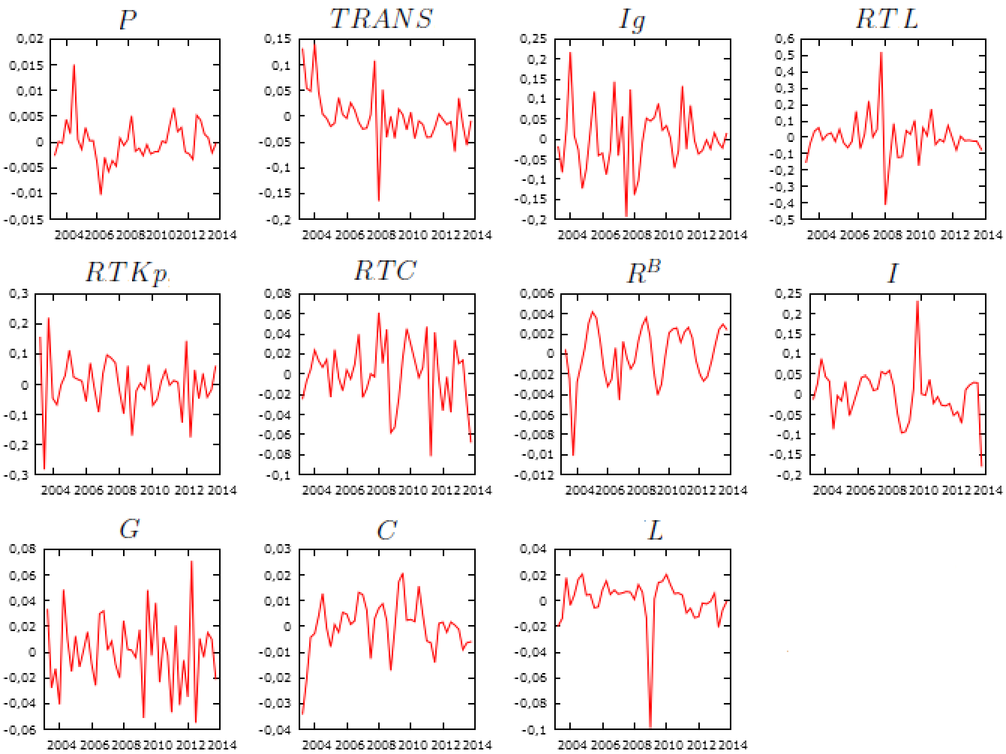

FIGURE 3.1: Data series (after transformation).
Source: Prepared by the author.

3.2 Calibrated Parameters and Prior Values

The parameters that are not directly related to the objective of this work were calibrated, while those relevant parameters in the analysis of the propagation of shocks were estimated using the Bayesian methodology. The main calibration procedure adopted here is to obtain the values of parameters from other relevant DSGE work in the literature.

Cavalcanti and Vereda (2010) analyzed the dynamic properties of a DSGE model for Brazil under alternative parameterizations. Therefore, they identified 'allowable ranges' of values for some of the key parameters in the literature. Using the results of these authors, it was decided to use the parameters in common between the two studies, which were the discount factor (β); the rate of capital depreciation (δ_p)(using the same value to rate of public depreciation (δ_g)); the coefficient of relative risk aversion (σ); and the marginal disutility of labor (ψ).

For parameters related to the monetary side, the sensitivities of the basic interest rate on the product (a) and on the inflation rate (b) plus the interest smoothig parameter (η) were obtained from Castro et al (2011). Finally, the parameters related to the structure of the firms were calibrated from two studies. The shares of private capital (α_1), of labor (α_2) and of public capital (α_3) in output were obtained from Mussolini (2011) while the index of price stickiness (θ) and the elasticity of substitution between intermediate goods (φ) was obtained from Lim and McNelis (2008). Table 3.2 summarizes the calibration of the parameters.

TABLE 3.2: Calibration of the Parameters.

Parameters	Value	Source
β	0,985	Cavalcanti and Vereda (2010)
δ	0,025	Cavalcanti and Vereda (2010)
σ	2	Cavalcanti and Vereda (2010)
ψ	1,5	Cavalcanti and Vereda (2010)
a	0,16	Castro et al (2011)
b	2,43	Castro et al (2011)
η	0,79	Castro et al (2011)
α_1	0,32	Mussolini (2011)
α_2	0,6	Mussolini (2011)
α_3	0,08	Mussolini (2011)
θ	0,85	Lim and McNelis (2008)
φ	6	Lim and McNelis (2008)

The prior values were obtained follows. The parameters related to taxation were obtained from Araújo and Ferreira (1999). The procedure adopted by these authors was to find the share of each type of tax to GDP and the ratio of the related variable with the tax to GDP. For the tax rate on consumption (τ_c), $\tau_c C = 0,1282Y$, with $C/Y = 0,8045$, resulting in $\tau_c = 0,1594$. For the tax rate on capital income (τ_k), $\tau_k RK = 0,0399Y$, with $R = 0,1647$ and $K/Y = 2,98$, obtaining $\tau_k = 0,0813$. For the tax rate on labor income (τ_l), $\tau_l WL = 0,0881$, with $WL/Y = 0,5092$, finding $\tau_l = 0,1730$. And the sensitivity of government spending relative to public debt (χ) was obtained from Lim e McNelis (2008).

We used the series *Benefícios Assistenciais (LOAS e RMV), Custeio e Investimento, Consumo Final - Adm. Pública and PIB - Preços de Mercado* to calculate the parameters values of ω_G, ξ_g, ξ_{Ig} and ξ_{Tr} using the relationship $I_g/I = 0,42$,

$G/Y = 0,2$, $I_g/Y = 0,05$ and $TRANS/Y = 0,007$, respectively. The autoregressive components (ρ_R, ρ_A, ρ_{sc}, ρ_{sl}, ρ_c, ρ_l, ρ_k, ρ_M, ρ_G, ρ_{I_g} and ρ_{Tr}) and exogenous shocks (ϵ_R, ϵ_A, ϵ_{sc}, ϵ_{sl}, ϵ_c, ϵ_l, ϵ_k, ϵ_M, ϵ_G, ϵ_{I_g} and ϵ_{Tr}) were got of the procedure from Castro et al (2011)[5], but for reason of identification, the values of ϵ_l and ϵ_k were changed. Table 3.3 summarizes the prior distribution of the model.

TABLE 3.3: Prior distribution of the model.

Parameters	Distribution	Average	Standard Deviation	Source
τ_c	gamma	0,1594	0,05	Araújo and Ferreira (1999)
τ_k	gamma	0,0813	0,04	Araújo and Ferreira (1999)
τ_l	gamma	0,1730	0,05	Araújo and Ferreira (1999)
χ	beta	0,1	0,05	Lim and McNelis (2008)
ω_G	beta	0,42	0,04	Author
ξ_g	beta	0,2	0,015	Author
ξ_{Ig}	beta	0,05	0,005	Author
ξ_{Tr}	beta	0,007	0,002	Author
ρ_A	beta	0,5	0,1	Castro et al (2011)
ρ_{sc}	beta	0,5	0,1	Castro et al (2011)
ρ_{sl}	beta	0,5	0,1	Castro et al (2011)
ρ_c	beta	0,5	0,1	Castro et al (2011)
ρ_l	beta	0,5	0,1	Castro et al (2011)
ρ_k	beta	0,5	0,1	Castro et al (2011)
ρ_G	beta	0,5	0,1	Castro et al (2011)
ρ_{I_g}	beta	0,5	0,1	Castro et al (2011)
ρ_{Tr}	beta	0,5	0,1	Castro et al (2011)
ρ_R	beta	0,5	0,1	Castro et al (2011)
ρ_M	beta	0,5	0,1	Castro et al (2011)
ϵ_A	invgamma	1	inf	Castro et al (2011)
ϵ_{sc}	invgamma	1	inf	Castro et al (2011)
ϵ_{sl}	invgamma	1	inf	Castro et al (2011)
ϵ_c	invgamma	1	inf	Castro et al (2011)
ϵ_l	invgamma	5*	inf	Author
ϵ_k	invgamma	5*	inf	Author
ϵ_G	invgamma	1	inf	Castro et al (2011)
ϵ_{I_g}	invgamma	1	inf	Castro et al (2011)
ϵ_{Tr}	invgamma	1	inf	Castro et al (2011)
ϵ_R	invgamma	1	inf	Castro et al (2011)
ϵ_M	invgamma	1	inf	Castro et al (2011)

Due to criterion of identification.

3.2.1 Identification and Sensitivity

DSGE models consist of a system of non-linear equations involving a vector **z** of endogenous variables, a **u** vector of random structural shocks and a $\theta \subset \Theta$ k-dimensional vector of deep parameters. Most applications use a linear approximation of the original model, i.e. expressing the model in terms of stationary

[5] Castro et al (2011) chose the same mean value and standard deviation for the autoregressive component and for the stochastic shocks.

variables linearized around their steady-states. The unique solution (if it exists), can be expressed as

$$z_t = \mathbf{A}(\theta)z_{t-1} + \mathbf{B}(\theta)u_t \tag{3.3}$$

where $\mathbf{A}$ and $\mathbf{B}$ are functions of θ. For later use, and using Iskrev (2010) notation, denote τ as a vector collecting all the reduced-form coefficients from the DSGE model, i.e. the elements in $\mathbf{A}$, $\Omega = \mathbf{BB'}$ and the steady-state of z_t that depend on θ, respectively $\tau = [\tau_z, \tau_A, \tau_\Omega]$.

A condition for identification is that distinct values of θ imply distinct values of the probability density function of the data, as the latter contains all available sample information about the value of the parameter vector of interest θ. Usually, the distribution of the data $\mathbf{X}$ is unknown or assumed to be Gaussian and estimation of θ is based on the first moments of the data. If $\mathbf{X}$ is not normally distributed, higher-order moments may provide additional information about θ, not contained in the first two moments. Define $\mathbf{mT}$:= $[\mu', \sigma'_T]$ as the vector collecting the first and second order moments of the observable variables. Identification based on the mean and the variance of $\mathbf{X}$ is only sufficient but not necessary for identification with the complete distribution, so that the mapping from the population moments of the sample, $\mathbf{mT}$, to θ is unique.

Global identification cannot, in general, be established for unique solutions of systems of non-linear equations, but local identification can be verified by means of a rank condition of the Jacobian matrix $J_T = \frac{\partial m_T}{\partial \theta'}$. Indeed, θ_0 is said to be locally identified if J_T has full column rank when evaluated at θ_0, although this does not guarantee that the model is locally identified everywhere in the parameter space. However, studying the rank of J_T is helpful, as local identifiability ensures consistent estimation of the parameters of interest, and, moreover, it will help to pinpoint the parameters that cause identification problems. For example, the column of J_T corresponding to an unidentified parameter θ_j will be a vector of zeros and the rank condition will fail. Another possibility may occur when the columns of J_T are not linearly independent, due to parameters entering the solution in a way that makes them indistinguishable (partial identification).

While numerical differentiation could be used to obtain J_T, this is unreliable when the model is highly non-linear. Instead, Iskrev (2010) proposes using analytical derivatives, employing implicit derivation, breaking down the mapping from θ to **mT** in two steps: i) a transformation of θ to τ, ii) a transformation from τ to **mT**. The Jacobian can then be express as

$$J_T = \frac{\partial m_T}{\partial \tau'} \frac{\partial \tau'}{\partial \theta'} \tag{3.4}$$

The first term $J_1 = \frac{\partial m_T}{\partial \tau'}$ may be obtained by direct differentiation. Regarding the second term, Iskrev (2010) establishes a necessary condition for identification: the point θ_0 is locally identifiable only if the rank of $H_T = \frac{\partial \tau'}{\partial \theta}$ evaluated at θ_0 is equal to k. The second term can be split into the three components of τ. Note that H_T does not depend on the data, thus implying that it is possible to detect lack of identification, inherent to the structure of the DSGE model, before taking the model to the data. On the other hand, the rank of J_T will provide information on the identification θ given the set of observable variables and the sample size.

The above suggests a procedure based on Monte Carlo exploration of the parameter space Θ of model parameters. One starts by constructing a sample of draw from Θ (discarding values that do not ensure stability and determinacy). This step can be guided by use of priors, specifying a theoretically admissible range and/or a particular distribution for θ. The identifiability of each draw for θ_j is then established by studying the rank of J_T and H_T, resorting to the necessary and sufficient conditions enumerated in Iskrev (2010):

- if H_T is rank-deficient at θ_j, this particular point is unidentifiable (full rank of H_T is necessary for identification)
- if H_T full rank but J_T does not, the θ_j cannot be identified for the particular set of observables and contemporaneous and lagged moments under consideration, i.e. given **X** and **T**
- weak identification issues when the columns of J_T and H_T are nearly linearly dependent (multicollinearity analysis of the re-scalet Jacobians)

Thus, a given parameter θ_j may be poorly identified because it has little impact on the reduced-from coefficients of the model ($\frac{\partial \tau}{\partial \theta_j} \approx 0$ or because its impact on the reduced-form coefficients is approximately to a linear combination of $\frac{\partial \tau}{\partial \theta_i}$ of other parameter).

The 'point estimate' mode is the default option in Dynare, in which local identification checks are done for the whole set, or only a subset, of the parameters in the model at a chosen central tendency measure, either at the defined prior or the estimated posterior, or at the calibrated value if no prior is declared. If priors have been defined, a Monte Carlo exploration is also available, in which the identification checks are based on samples from the prior distribution.

The result of identification procedure then reveals whether or not there are identification problems, stemming from J_T and/or H_T, as illustrated below for the model of this work (Box 3.1):

==== Identification analysis ====
Testing prior mean
All parameters are identified in the model (rank of H).
All parameters are identified by J moments (rank of J)
==== Identification analysis completed ====
BOX 3.1: Result of the Identification analysis

If the rank condition fails, the procedure indicates which parameters are responsible for identification problems. The result of identification analysis this work (Box 3.1) indicates that all parameters are identified in the model.

A further output of the *identification* routine of the Dynare is the analysis of identifications 'strengh', i.e., focusing on weak identification, summarized in two additional plots. The procedures are based on either the asymptotic or moments information matrix. The first can be obtained given a sample of size T, whereas the second can be computed based on Monte Carlo simulations for samples of size T, from which sample moments of the observed variables are computed, forming a sample of N replicas of simulated moments. The corresponding information matrix is then obtained as $I_T(\theta|m_T) = H_T \sum_{m_T} H_T$, where $\sum_{m_T}$ is the covariance matrix of simulated moments.

The 'strength' of identification for θ_i is computed as

$$s_i = \sqrt{\frac{\theta_i^2}{I_T(\theta)^{-1}_{(i,i)}}} \tag{3.5}$$

which works like a t-test for θ_i. This can be decomposed into a 'sensitivity' and 'correlation' parts, the first referring to the case when weak identification arises when the moments do not change with θ, and the second when colinearity dampens the effect of θ_i. The latter is defined as

$$s_i = \sqrt{\theta_i^2 I_T(\theta)_{(i,i)}} \tag{3.6}$$

Thus, the identification in the Dynare plots the measures described above (also normalized relative to the prior standard deviation for θ_i), where large bars imply strong identification, while low bars signal potential weak identification for the respective θ_i. The Figure 3.2 shows the identification in the moments for the this work model. The result was quite good, but the parameters ρ_k, ϵ_k, ρ_c and ϵ_c have weak identification. The main result of the identification analysis was to demonstrate that all fiscal variables (which is the object of study of this thesis: τ_c, τ_k, τ_l, χ , ω_G, ξ_g, ξ_{Ig} and ξ_{Tr}) have strong identification.

3.3 Bayesian Methodology

The information that we have about a quantity of interest θ is fundamental in the statistic. The true value of θ is unknown and the idea is to try to reduce this issue. Furthermore, the intensity of uncertainty about θ can assume different levels. Bayesian point of view, these different levels of uncertainty is represented using probabilistic Models. In this context, it is natural that different researchers may have different levels of uncertainty (specifying different models). Thus, there is not distinction between observable quantities and the parameters of a statistical model, all are considered random quantities.

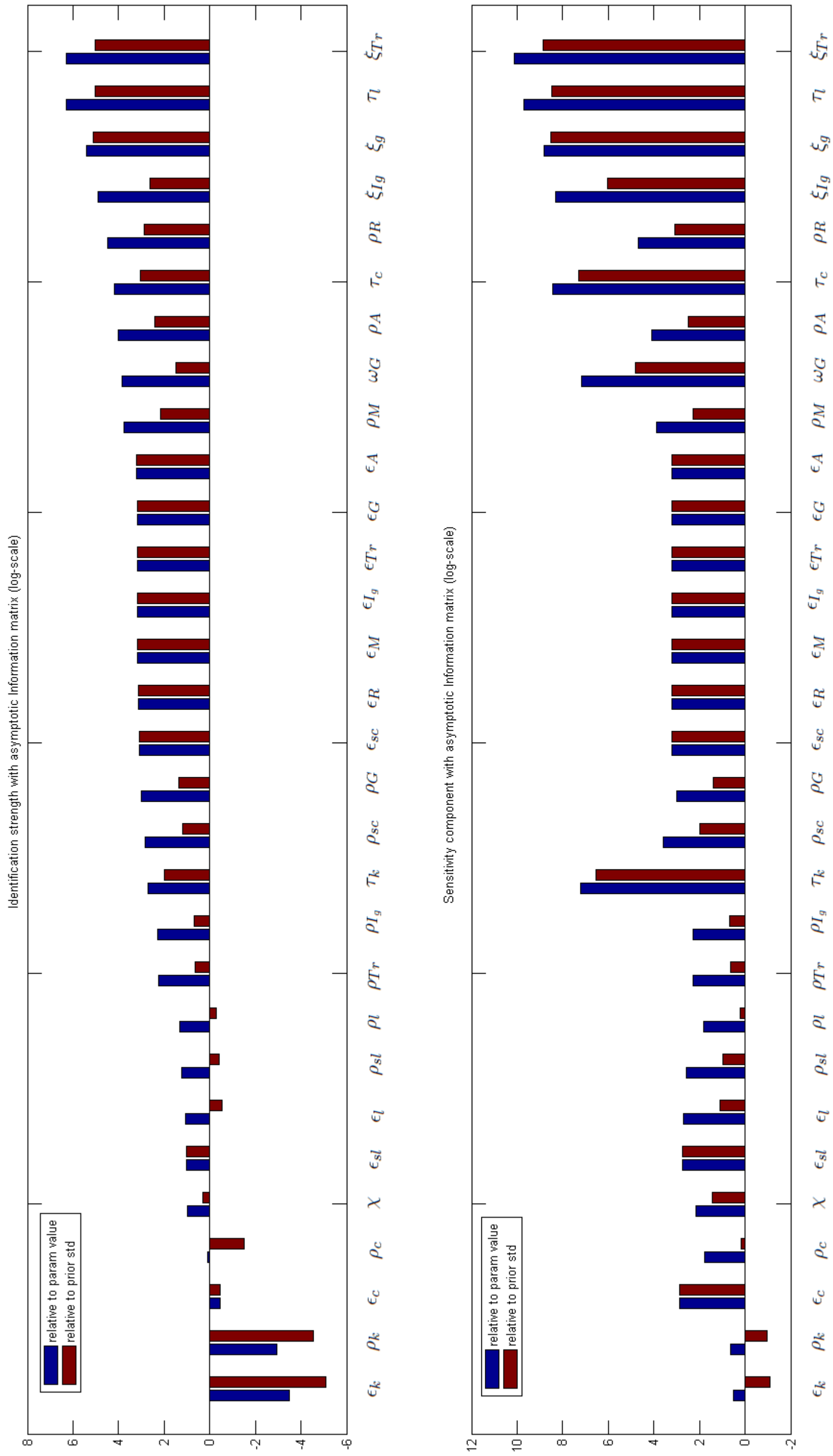

FIGURE 3.2: Identification strength.
Source: Prepared by the author.

3.3.1 Bayes Theorem

Consider an unknown amount of interest θ (typically unobservable). The information we have about θ, summarized probabilistically by $p(\theta)$, it can be enhanced by observing a random amount X related with θ. The sampling distribution $p(x|\theta)$ defines this relationship. The idea that after observing $X = x$ the amount of information about θ increases is quite intuitive and Bayes' theorem is the update rule used to quantify this increase in information,

$$p(\theta|x) = \frac{p(x,\theta)}{p(x)} = \frac{p(x|\theta)p(\theta)}{p(x)} = \frac{p(x|\theta)p(\theta)}{\int p(\theta,x)d\theta} \tag{3.7}$$

Note that $1/p(x)$ does not depend of θ, works as a normalizing constant $p(\theta|x)$.

For a fixed value of x, the function $l(\theta;x) = p(x|\theta)$ provides the likelihood of each of the possible values of θ while $p(\theta)$ is called as the a prior distribution of θ. These two sources of information, prior and likelihood are combined leading to the posterior distribution of θ, $p(\theta|x)$. Thus, the usual form of Bayes' theorem is

$$p(\theta|x) \propto l(\theta;x)p(\theta) \tag{3.8}$$

Note that by omitting the term $p(x)$, the equality in (3.7) is replaced by a proportionality. This simplified form of Bayes' theorem is useful in problems involving parameter estimation since the denominator is just a normalizing constant.

3.3.2 Monte Carlo Markov Chain

Methods of Monte Carlo Markov Chain (MCMC) are an alternative to non-iterative methods for complex problems. The idea is to obtain a sample from the posterior distribution and calculate sample estimates of characteristics of this distribution. The difference is that here we use an iterative simulation techniques, based on Markov chains, and thus the generated values will no longer be independent.

3.3.2.1 Metropolis-Hastings Algorithm

The Metropolis-Hastings algorithms use the same idea of the methods of rejection, i.e. a value is generated from an auxiliary distribution and accepted with a given probability. This correction mechanism ensures the convergence of the chain to the equilibrium distribution, which in this case is the posterior distribution.

Suppose that the chain is in the state θ and a value of θ' is generated from a proposal distribution $q(\bullet|\theta)$. Note that the proposal distribution can depend on the current state of the chain, for example $q(\bullet|\theta)$ could be a normal distribution distribution centered in θ. The new value θ' is accepted with probability

$$\alpha(\theta, \theta') = \min\left(1, \frac{\pi(\theta')q(\theta|\theta')}{\pi(\theta)q(\theta'|\theta)}\right) \tag{3.9}$$

where π is the distribution of interest.

An important feature is that you only need to know π partially. This is critical in Bayesian applications where we do not know fully the posterior. Note also that the chain can remain in the same state for many iterations and in practice it is customary to monitor this by calculating the average percentage of iterations for which new values are accepted.

In practical terms, the Metropolis-Hastings algorithm can be specified by the following steps (Figure 3.3),

1. Initialize the iteration counter $t = 0$ and specify an initial value θ^0.
2. Generate a new value θ' of the distribution $q(\bullet|\theta)$.
3. Calculate the probability of acceptance $\alpha(\theta, \theta')$ and generate $u \sim U(0, 1)$.
4. Whether $u \leq \alpha$ then it accepts the new value and then $\theta^{(t+1)} = \theta'$, otherwise reject and do $\theta^{(t+1)} = \theta$.
5. Increment the counter of t to $t + 1$ and return to step 2.

Although the distribution proposal can be chosen arbitrarily in practice must take some care to ensure the efficiency of the algorithm. In Bayesian applications the distribution of interest is itself a posterior, i.e. $\pi = p(\theta|x)$ and the probability of acceptance takes a particular form,

$$\alpha(\theta, \theta') = \min\left(1, \frac{p(x|\theta')}{p(x|\theta)} \frac{p(\theta')}{p(\theta)} \frac{q(\theta|\theta')}{q(\theta'|\theta)}\right) \tag{3.10}$$

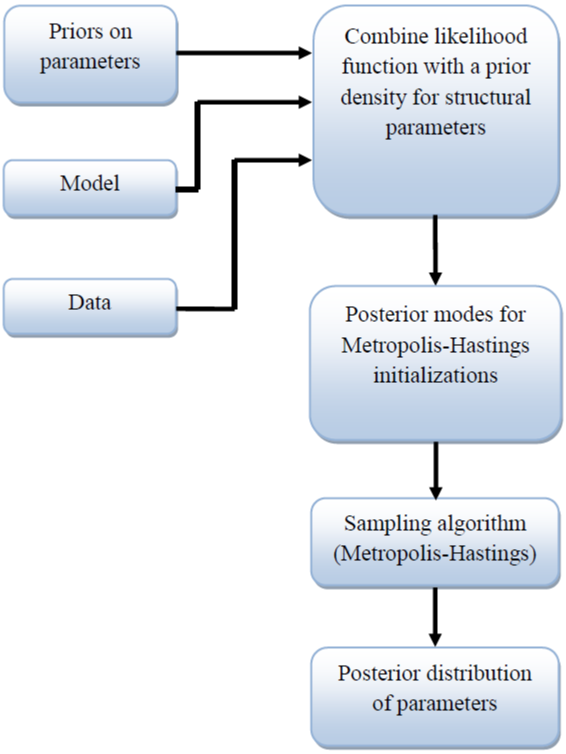

FIGURE 3.3: Bayesian Estimation Process.
Source: Prepared by the author.

3.4 Estimation Results

We used the Dynare program for estimation. Posterior distributions for the parameters were computed using the Metropolis-Hasting algorithm, which relies on the Monte Carlo Markov Chain (MCMC) procedure. This section shows the testing for convergence of MCMC and de posterior values.

3.4.1 Testing for Convergence of MCMC

Testing for convergence of the posterior distribution is notoriously difficult. Monte Carlo Markov Chain (MCMC) univariate diagnostic is the main source of feedback to gain confidence, or spot a problem, with results (Griffoli, 2011). For this test, Dynare utilizes some indicative statistics, summarized by diagrams, as recommended by Brooks and Gelman (1998). These diagrams are made up of:

- Three multivariate figures, representing convergence indicators for all parameters considered together
- Three figures for each parameter, representing univariate convergence indicators

The basic univariate test is motivated by standard Analysis of Variance Considerations. Suppose we generate m MCMC chains that are each run for 2n iterations, of which the first n are discarded to avoid the burn-in period. Consider a particular scalar summary φ of the draws φ_{jk}, $j = 1, ..., m$, $k = 1, ..., n$, where φ would typically be one the parameters θ_i. It is then a standard result that if the φ_{jk} are normally distributed with variance σ^2, then an unbiased estimator $\hat{\sigma}^2$ of σ^2 is given by

$$(mn-1)\hat{\sigma}^2 = \sum_{j=1}^{m}\sum_{k=1}^{n}(\varphi_{jk} - \varphi..)^2 \equiv \sum_{k=1}^{n}(\varphi_{jk} - \varphi.)^2 + n\sum_{j=1}^{m}(\varphi_{j}. - \varphi..)^2 \quad (3.11)$$

where $_j.$ represents the mean for the jth chain, and $\varphi..$ is the mean over all chains. One measure of convergence is that the . are all equal to $\varphi..$ i.e. that the initial value of the draw in each chain has not affected the mean. Another test is whether the variance is equal across all the chains. An obvious way of testing both of these together is to check whether the *Potential Scale Factor* $R_2 \equiv V/W$ is approaching 1, where,

$$V = \frac{1}{mn-1}\sum_{j=1}^{m}\sum_{k=1}^{n}(\varphi_{jk} - \varphi..)^2 \quad (3.12)$$

$$W = \frac{1}{m(n-1)}\sum_{j=1}^{m}\sum_{k=1}^{n}(\varphi_{jk} - \varphi_{j}.)^2 \quad (3.13)$$

Brooks and Gelman (1998) recommend that V and W are plotted sequentially for $k = 1, ..., n$; this means that one can check that as n increases, V and W tend individually to a limit, and that this is the same limit as k approaches n. If the posterior distribution is unimodal, this is essentially a check that both means and variances of all chains' estimates of φ are tending to the same limit. If the posterior distribution is not unimodal, then it makes sense to extend this to other moments, and Dynare does this for third moments as well:

$$R_3 = \frac{V_3}{W_3} = \frac{\frac{1}{mn-1}\sum_{j=1}^{m}\sum_{k=1}^{n}|\varphi_{jk} - \varphi_{..}|^3}{\frac{1}{m(n-1)}\sum_{j=1}^{m}\sum_{k=1}^{n}|\varphi_{jk} - \varphi_{j.}|^2} \tag{3.14}$$

One can extend this to the multivariate case, so that analogously to (3.11) we can write down an unbiased estimate $\hat{\Omega}$ of the covariance matrix of the vector of parameters θ as

$$(mn-1)\hat{\Omega} = \sum_{j=1}^{m}\sum_{k=1}^{n}(\varphi_{jk} - \varphi_{..})(\varphi_{jk} - \varphi_{..})^T \equiv \sum_{k=1}^{n}(\varphi_{jk} - \varphi_{j.})(\varphi_{jk} - \varphi_{j.})^T + n\sum_{j=1}^{m}(\varphi_{j.} - \varphi_{..})(\varphi_{j.} - \varphi_{..})^T \tag{3.15}$$

Matrices V and W are the defined analogously to their scalar versions above. One measure of how close these are is the maximum root statistic, which is the solution to $\max_a \frac{(a^T V a)}{a^T W a}$, which is given by the largest eigenvalue of $W^{-\frac{1}{2}} V W^{-\frac{1}{2}}$, which should tent to 1 if the chains are converging to the posterior distribution.

Another multivariate measure involves monitoring the determinants of V and W, and also the corresponding determinants for other moments e.g. the third moment, as in the univariate case.

Another measure recommended by Brooks and Gelman (1998) is an interval measure. This is based on the intuitive notion that R_2 also represents a squared ratio of the proportion of draws within a certain confidence interval. To perform this explicity, Brooks and Gelman (1998) suggest a measure $R_{interval}$ that uses, as before, the last n of the 2n draws of each chain, and then

1. From each chain find the empirical $100(1-\alpha)\%$ interval i.e. the number of draws within the empirical $100\frac{\alpha}{2}\%$ and $100\left(1-\frac{\alpha}{2}\%\right)$ points; Dynare sets $\alpha = 0,2$.

2. Do the same for all the mn draws from all the m chains.

3. Evaluate $R_{interval} \equiv \frac{V_{interval}}{W_{interval}}$ where $V_{interval}$=length of total-sequence interval, $W_{interval}$=mean length of within-sequence intervals. As before, it is insightful to plot both $V_{interval}$ and $W_{interval}$.

This can be extended to the multivariate case; here we count the number of draws for which each of the elements θ_i of the vector θ lie within their individual empirical $100(1-\alpha)\%$ intervals.

Reading this diagnostics, if the results from one chain are sensible, and the optimizer did not get stuck in an odd are or the parameter subspace, two things should happen. First, results within any of the however many iterations of Metropolis-Hastings simulation should similar. And second, results between the various chains should be close. More specifically, the red and blue lines on the charts represent specific measures of the parameter vector both within and between chains. For the results to be sensible, these should be relatively constant (although there will always be some variation) and they should converge (Griffoli, 2011).

Considering the result of this test for our model. The multivariate diagnostics (Figure 3.4) indicates that the chains have converged to similar means and distributions. However, examining the individual parameters (Figures 3.5, 3.6 and 3.7), the results is somewhat mixed.

In general, the parameters converge, especially in relation to the 'interval' (e.g. τ_c, τ_k and χ). On the other hand, the parameters of stochastic shocks and the autoregressive components presented the greatest variation during the process of convergence, especially ϵ_l, ϵ_k, ϵ_{sc}, ϵ_{sl}, ρ_{sl} and ρ_l. This less satisfactory performance for the estimation of these parameters is related to prior values of each. This feature was already noticed in Figure 3.2, which these components showed weak identification. Even realizing that lower performance in the estimation of

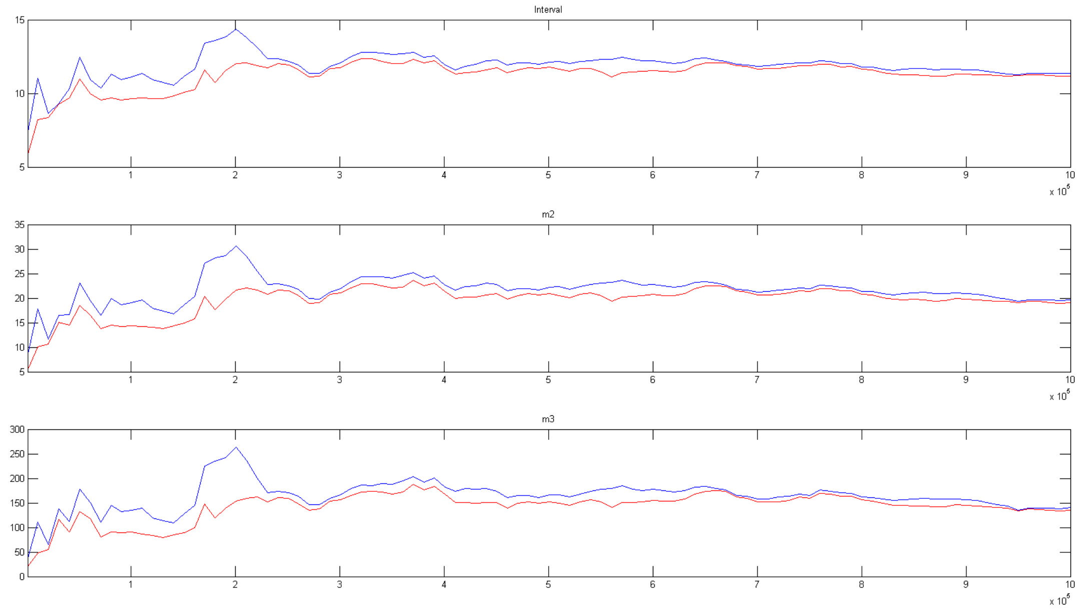

FIGURE 3.4: Multivariate MCMC Diagnostics for the model.
Source: Prepared by the author.

these parameters, we decided not to change their prior values. Because we consider the general result satisfactory, without any prejudice to the outcome of the work.

3.4.2 Posterior Values

Given the prior distributions of the parameters, we estimate the posteriors distributions using a Markov chain process via Metropolis-Hastings algorithm with 1.000.000 iterations, scale value 0,4 to be used for the jumping distribution and 5 parallel chains for Metropolis-Hastings algorithm. The results of the Bayesian estimation are shown in Table 3.4 and Figures 3.8 and 3.9.

These graphs are especially relevant and present key results, but they can also serve as tools to detect problems or build additional confidence in yours results. First, the prior and the posterior distribution should not be excessively different. Second, the posterior distributions should be close to normal, or at lest not display a shape that is clearly non-normal. Third, the green mode should not be too far

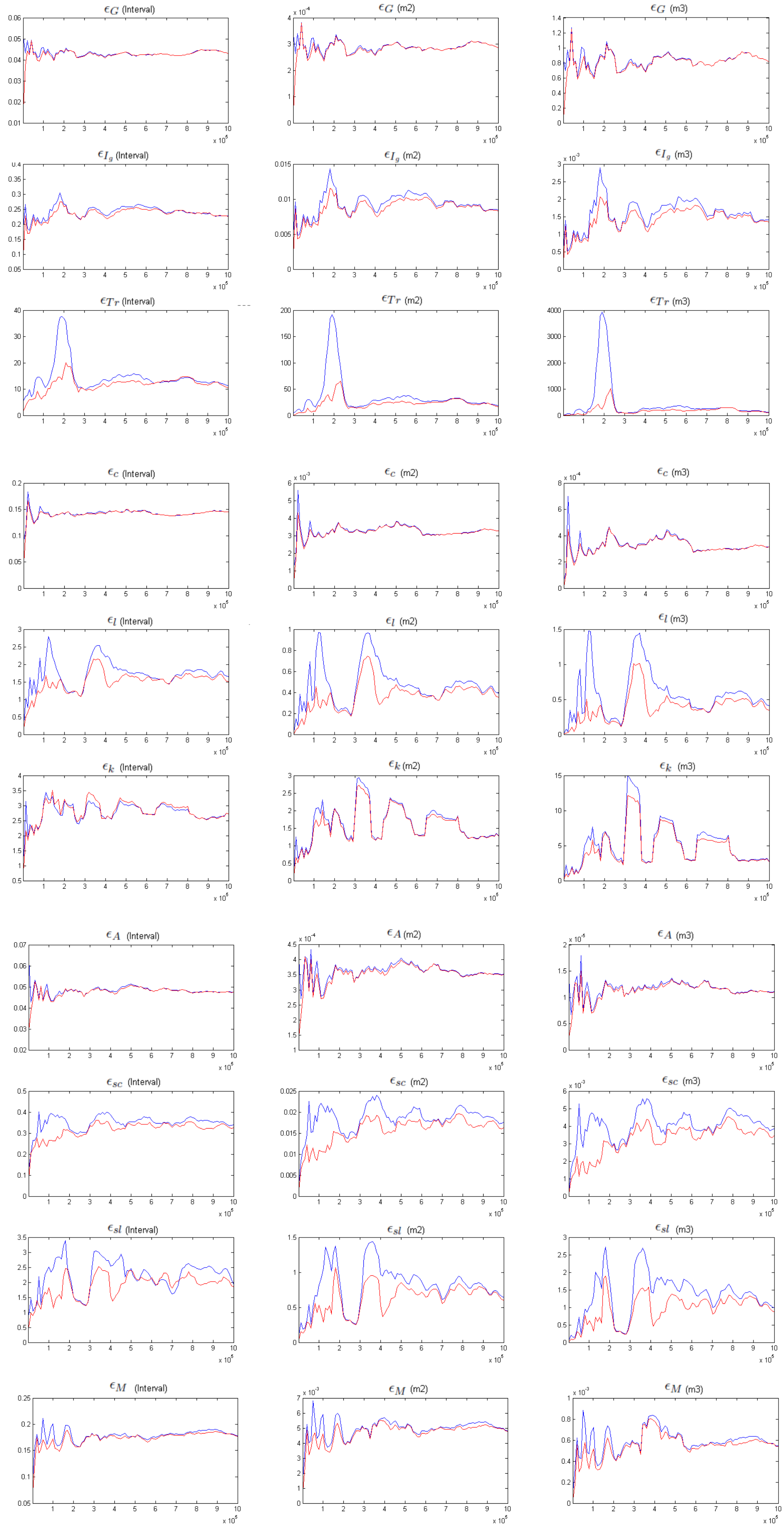

FIGURE 3.5: Univariate MCMC Diagnostics for the Model for ϵ_G, ϵ_{I_g}, ϵ_{Tr}, ϵ_c, ϵ_l, ϵ_k, ϵ_A, ϵ_{sc}, ϵ_{sl} and ϵ_M.
Source: Prepared by the author.

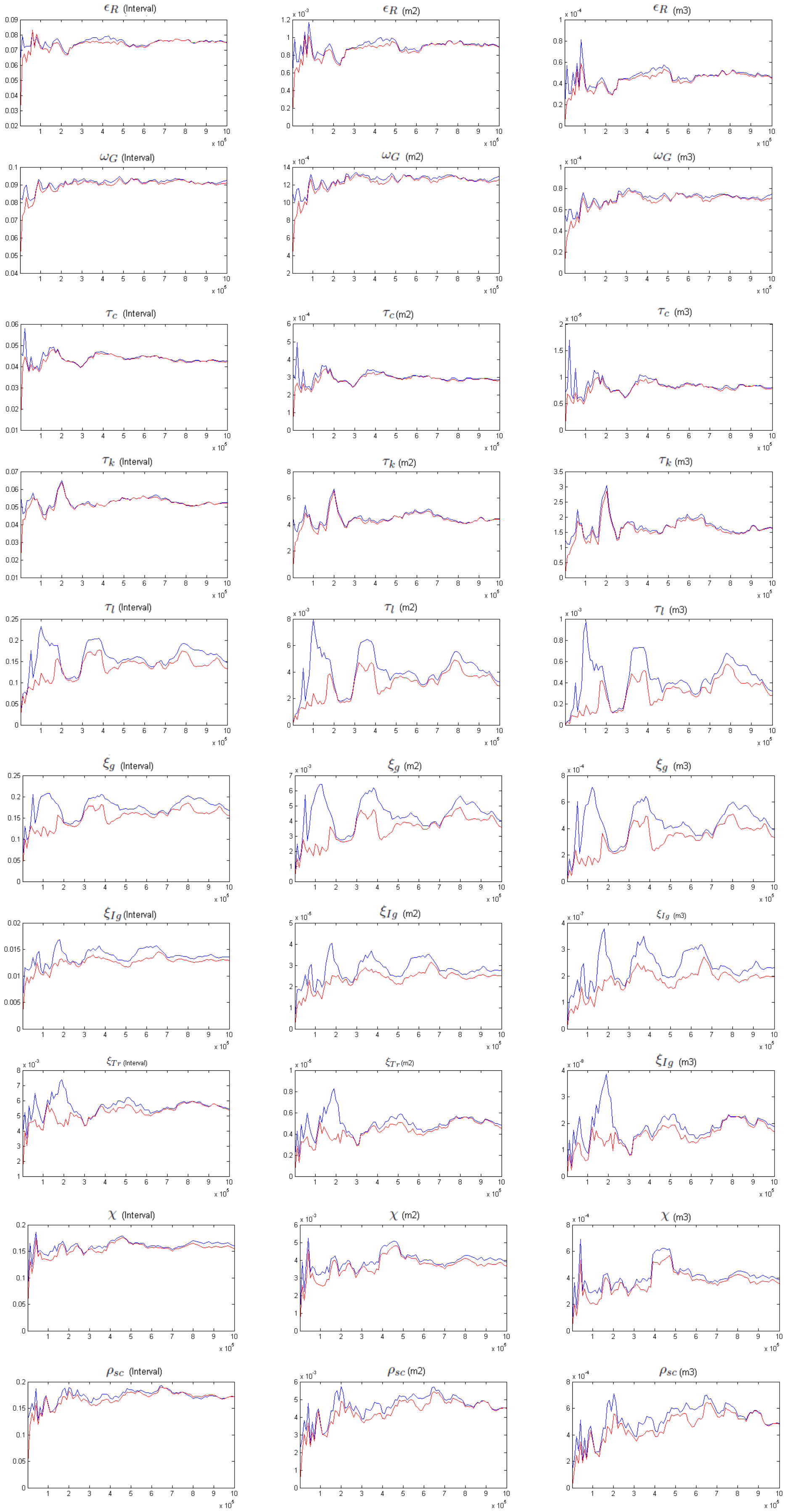

FIGURE 3.6: Univariate MCMC Diagnostics for the Model for ϵ_R, ω_G, τ_c, τ_k, τ_l, ξ_g, ξ_{Ig}, ξ_{Tr}, χ and ρ_{sc}.
Source: Prepared by the author.

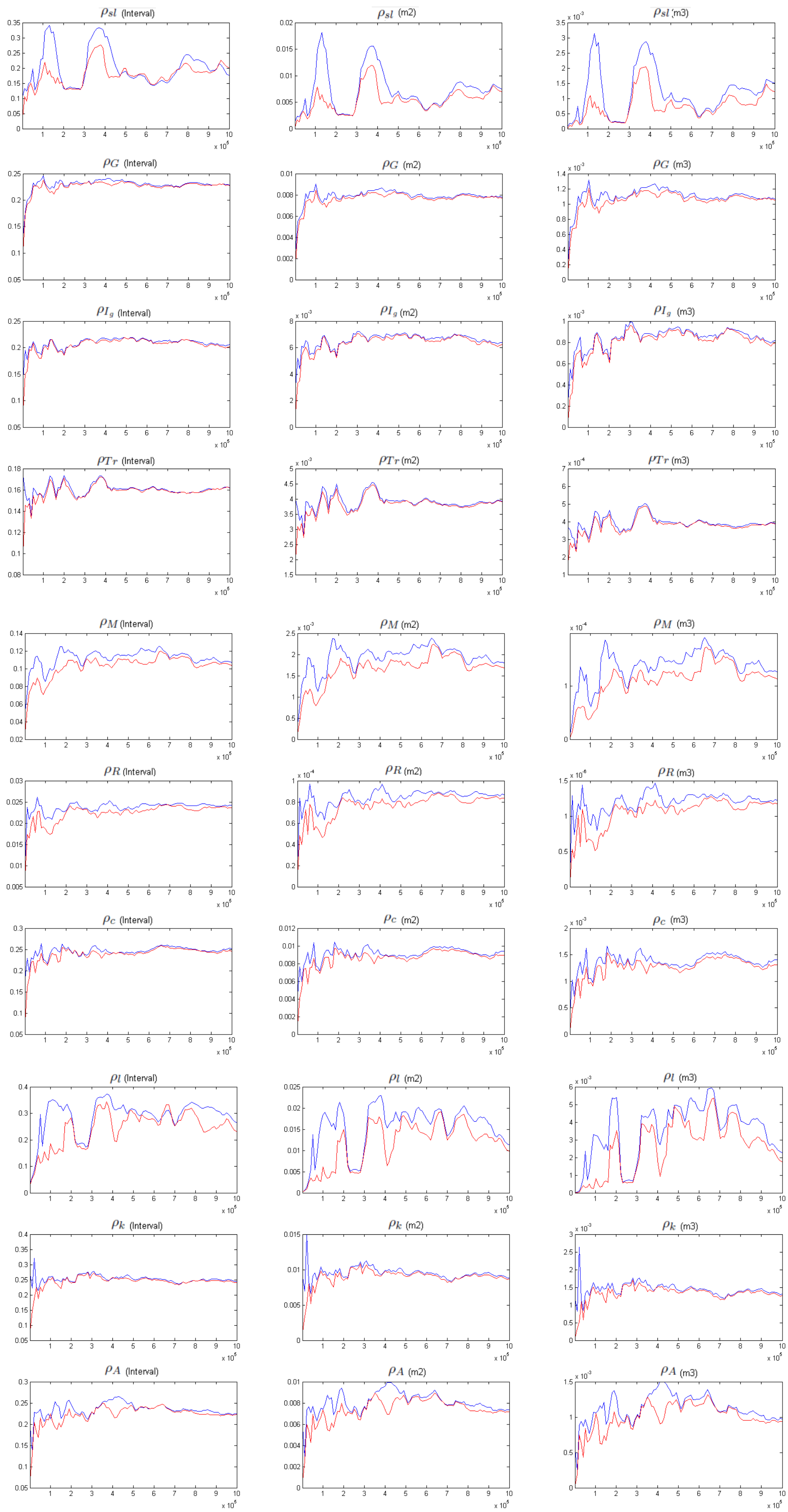

FIGURE 3.7: Univariate MCMC Diagnostics for the Model for ρ_{sl}, ρ_G, ρ_{I_g}, ρ_{Tr}, ρ_M, ρ_R, ρ_c, ρ_l, ρ_k and ρ_A.
Source: Prepared by the author.

TABLE 3.4: Posterior distribution of the model.

Parameter	prior mean	post. mean	90% HPD interval	prior	pstdev
Fiscal Parameters					
ω_G	0,42	0,379	0,318 0,4371	beta	0,04
τ_c	0,159	0,1495	0,1219 0,1767	gamma	0,05
τ_k	00,081	0,0452	0,0125 0,0767	gamma	0,04
τ_l	0,173	0,3489	0,2717 0,4535	gamma	0,05
ξ_g	0,2	0,5328	0,4364 0,6431	beta	0,15
ξ_{Ig}	0,05	0,0532	0,0445 0,0618	beta	0,005
ξ_{Tr}	0,007	0,0092	0,0058 0,0126	beta	0,002
χ	0,1	0,1599	0,0545 0,2568	beta	0,05
Autoregressive Components					
ρ_{sc}	0,5	0,3665	0,2558 0,476	beta	0,1
ρ_{sl}	0,5	0,2339	0,1088 0,3401	beta	0,1
ρ_G	0,5	0,4268	0,2787 0,569	beta	0,1
ρ_{I_g}	0,5	0,3523	0,2199 0,4805	beta	0,1
ρ_{Tr}	0,5	0,262	0,1596 0,3635	beta	0,1
ρ_M	0,5	0,6441	0,5758 0,7142	beta	0,1
ρ_R	0,5	0,8643	0,8487 0,879	beta	0,1
ρ_c	0,5	0,4664	0,3089 0,6258	beta	0,1
ρ_l	0,5	0,8066	0,6662 0,9486	beta	0,1
ρ_k	0,5	0,4825	0,3297 0,6346	beta	0,1
ρ_A	0,5	0,4791	0,3367 0,6133	beta	0,1
Exogenous Shocks					
ϵ_G	1	0,1504	0,1222 0,176	invg	Inf
ϵ_{I_g}	1	0,5766	0,4288 0,7195	invg	Inf
ϵ_{Tr}	1	15,7367	9,1234 23,1159	invg	Inf
ϵ_c	1	0,3044	0,213 0,395	invg	Inf
ϵ_l	5	2,2033	1,2851 3,3242	invg	Inf
ϵ_k	5	2,7941	1,2128 4,4605	invg	Inf
ϵ_A	1	0,1558	0,1248 0,1844	invg	Inf
ϵ_{sc}	1	0,5539	0,3549 0,7655	invg	Inf
ϵ_{sl}	1	2,1978	0,2811 3,0861	invg	Inf
ϵ_M	1	0,5123	0,398 0,6242	invg	Inf
ϵ_R	1	0,2462	0,1977 0,2932	invg	Inf

away from the mode of the posterior distribution.

Notice that the overall result was satisfactory. However, we should direct the comments to the most relevant results of this work, which they are related to fiscal parameters. Thus, note that: ω_G, τ_c, τ_k, τ_l, ξ_{Ig}, ξ_{Tr} and χ presented great results (they have normal distributions and their prior values do not differ considerably from the posteriors), while ξ_g showed acceptable result (because even with a normal distribution, the posterior value became distant from the prior). Briefly, none of the fiscal parameters showed unsatisfactory results.

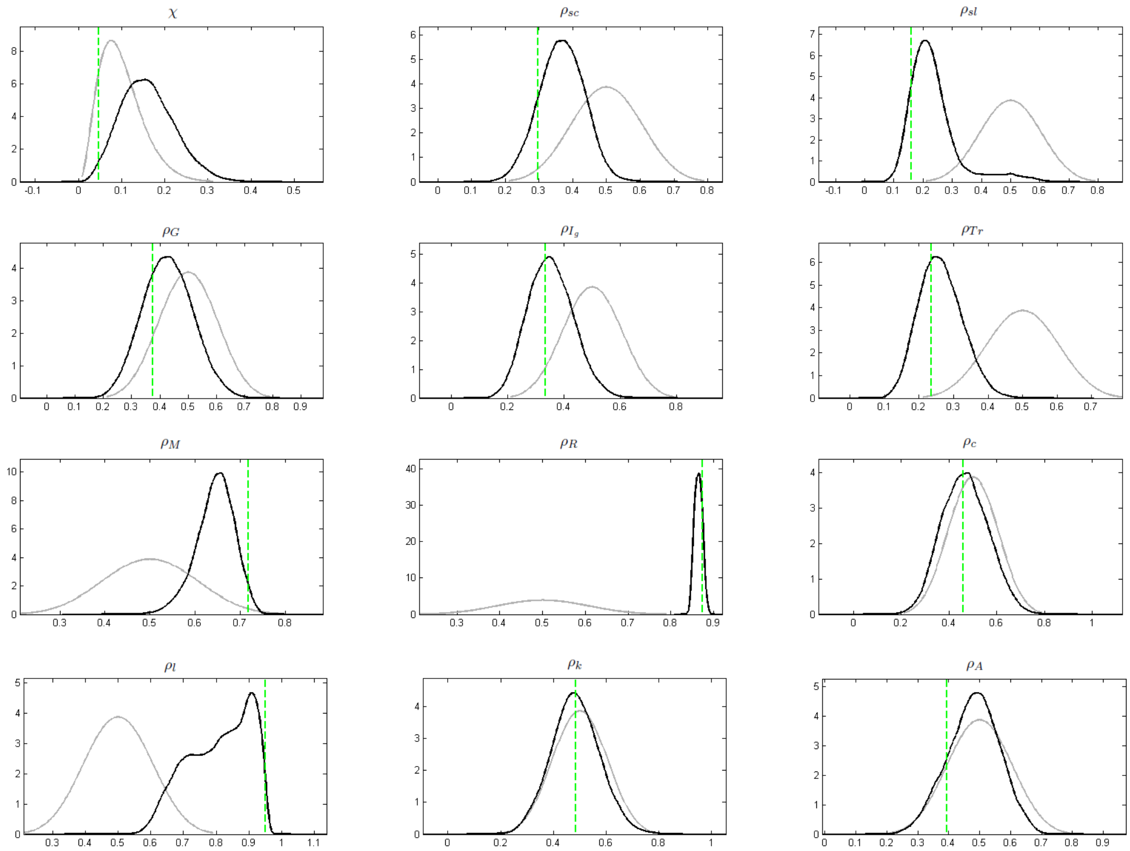

FIGURE 3.8: Priors and posteriors for χ, ρ_{sl}, ρ_G, ρ_{I_g}, ρ_{Tr}, ρ_M, ρ_R, ρ_c, ρ_l, ρ_k and ρ_A.
Source: Prepared by the author.

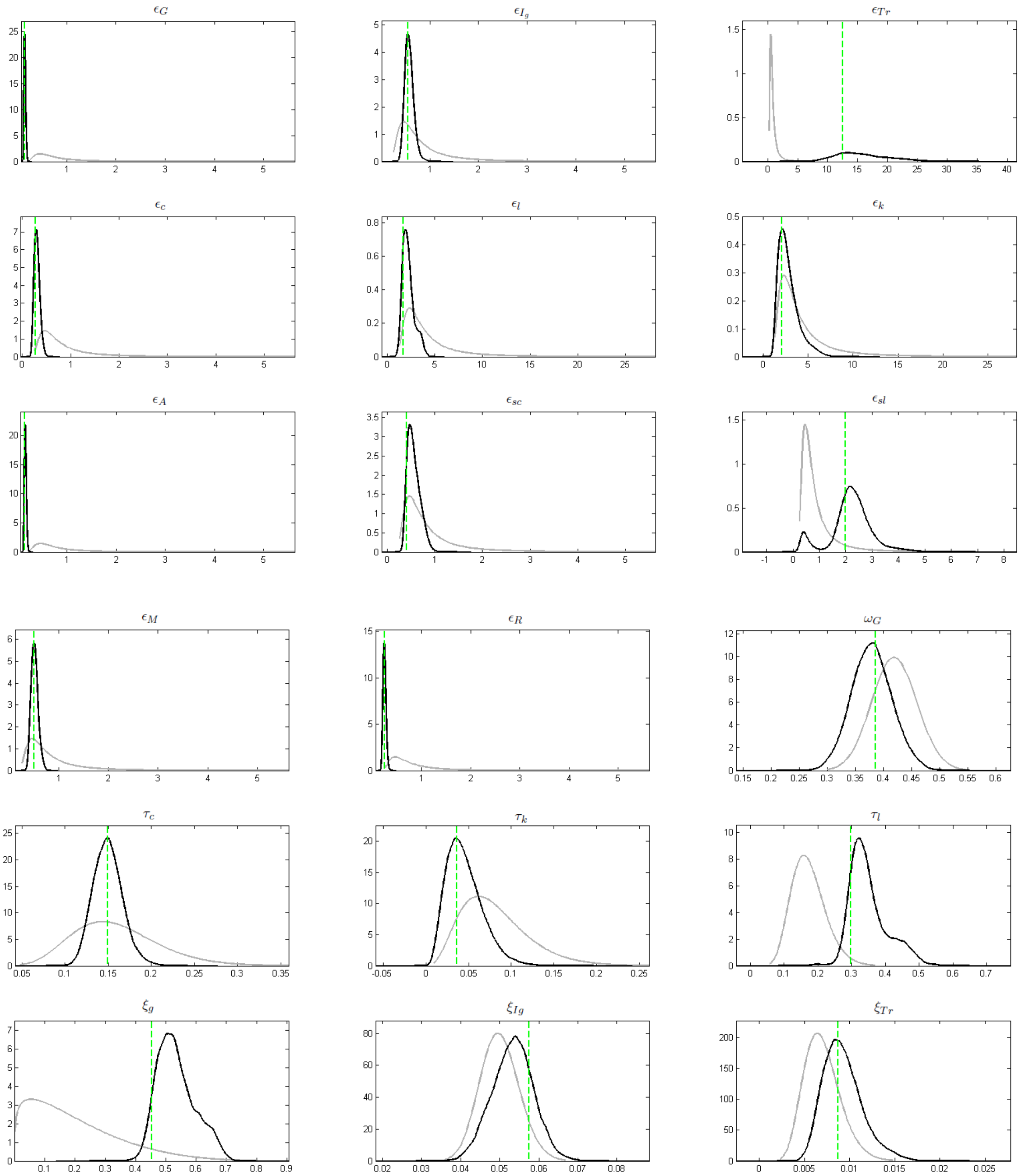

FIGURE 3.9: Priors and posteriors for ϵ_G, ϵ_{I_g}, ϵ_{Tr}, ϵ_c, ϵ_l, ϵ_k, ϵ_A, ϵ_{sc}, ϵ_{sl} and ϵ_M, ϵ_R, ω_G, τ_c, τ_k, τ_l, ξ_g, ξ_{Ig} and ξ_{Tr}.
Source: Prepared by the author.

Chapter 4

Macroeconomics Effects Related to Composition of Brazilian Government Spending

Introduction

The public sector in Brazil has experienced considerable expansion over the past six decades. The available data indicate that between 1948 and 2012, total expenditures of the public sector rose from 17% of GDP in 1948 to 39% in 2013 (IMF, 2014). Some studies on the dynamic behavior of public expenditures have been conducted. Among them, some highlights are the contributions of Wagner (Bird, 1970) and Peacock and Wiseman (1970) who established hypotheses that explain the process of evolution in the public sector. Both studies emphasized the problem of identifying the determinants of the expansion of government expenditures based on empirical analysis. It is justified to look at the composition of public expenditures because the public administrator has to choose the types of expenditures that will probably leave some unsatisfied groups.

Table 4.1 shows the growth of public spending of some developed countries during the twentieth century. The first reason for the growth in public spending shown in this table is in the "war effort" that includes the reconstruction of the countries involved in the two great global conflicts. If the wars were the only cause of the increase in the relationship between public spending and GDP, that trend should have ceased after 1945. Yet, the overall share of the government in the economy grew significantly after 1960. This means that there were other causes that explain this phenomenon.

In this sense, the literature on public finance highlights two important causes. The first is represented by demographic factors associated with the aging of the population (that apparently explains the increase in public spending in the economies shown in Table 4.1). Given that a greater proportion of the population became older, the total expenditures on health as well as social security grew. The second cause is urbanization as an explanation for the increase in government spending in developing countries (however, this does not exactly correlate because for those countries, this phenomenon predates the period in Table 4.1). Between 1950 and 1980, the percentage of the Brazilian urban population increased from 36% to 68% of the total population. Just as had occurred in the advanced industrialized countries, this led to a major shift in demand for public spending (Giambiagi and Além, 2008).

Briefly, it is legitimate for a government to increase spending, but at the same time from the point of view of anti-inflationary policy, it is desirable that if this occurs, it is funded through taxes and/or with only a modest increase in public debt (Giambiagi and Além, 2008). However, Brazil has systematically raised the size of the government in the economy. Between 1991 and 2011, the ratio of its primary spending to GDP increased 57,66% (with an annual average of 2,37% shown in Figure 4.1), starting from 13,7% of GDP in 1991 to 21,6% of GDP in 2011. The excessive size of the Brazilian government is more evident when it is compared with other countries. Table 4.2 displays the government expenditures in various developed and developing countries. Among all the countries listed, Brazil has the thirteenth largest government expenditure in percentage of GDP, well ahead of developed countries like the United States, Japan and New Zealand.

For countries that make up the BRICS group, Brazil has the highest government spending at 39%, followed by Russia (35%), South Africa (30%), India (27%) and China (21%). It is worth noting that the two countries that have exhibited greater

Table 4.1: Growth of public expenditure in the world (% GDP). Source: Modified from Giambiagi and Além (2008)

Countries	Late nineteenth century		Pre-War		Post-War	
Germany	10		14,8		25	
Canada	-		-		16,7	
United States	7,3		7,5		12,1	
France	12,6		17		27,6	
Netherlands	9,1		9		13,5	
Italy	11,9		11,1		22,5	
Japan	8,8		8,3		14,8	
Norway	5,9		9,3		16	
United Kingdom	9,4		12,7		26,2	
Sweden	5,7		10,4		10,9	
Average	8,97		11,12		18,53	
Countries	Pre-Second World War	1960	1980	1990	2002	2012
Germany	34,1	32,4	47,9	45,1	47,95	44,67
Canada	25	28,6	38,8	46	43,39	44,84
United States	19,7	27	31,4	32,8	34,84	38,72
France	29	34,6	46,1	49,8	52,87	56,64
Netherlands	19	33,7	55,8	54,1	46,21	50,17
Italy	24,5	30,1	42,1	53,4	47,12	50,51
Japan	25,4	17,5	32	31,3	36,62	39,9
Norway	11,8	29,9	43,8	54,9	46,38	42,99
United Kingdom	30	32,2	43	39,9	38,08	44,77
Sweden	16,5	31	60,1	59,1	55,6	52,12
Average	23,5	29,7	44,1	46,64	44,906	46,533

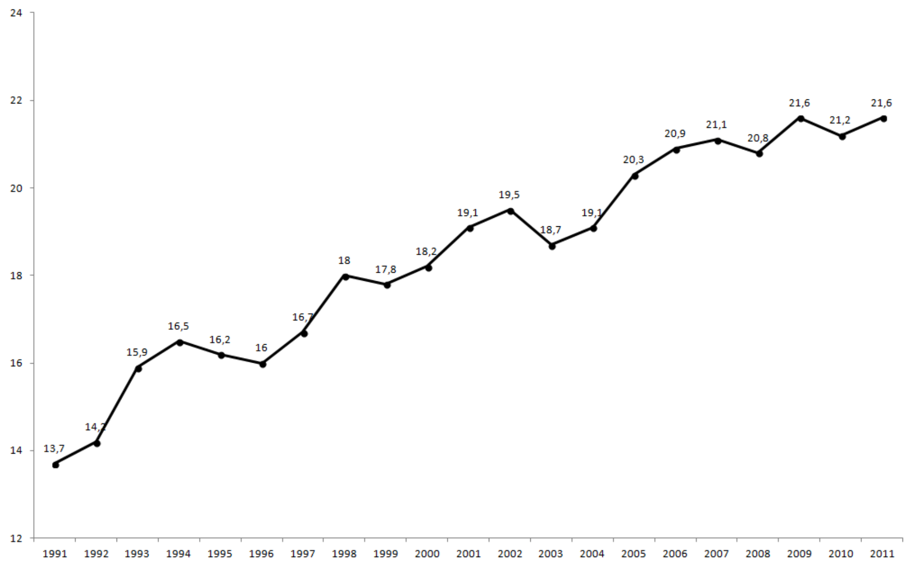

Figure 4.1: Primary Government Spending (%GDP).
Source: Secretaria de Política Econômica/Secretaria do Tesouro Nacional.

Table 4.2: Government Expenditures in Various Countries. Expressed as Ratios to Nominal GDP. Source: IMF(2014)

Country	Gover. Expen (Ratios to GDP)	Country	Gover. Expen (Ratios to GDP)
Paraguay	0,19	New Zealand	0,35
Peru	0,19	Japan	0,37
Chile	0,22	United States	0,38
Mexico	0,24	United Kingdom	0,42
Colombia	0,28	Norway	0,43
Ecuador	0,29	Canada	0,44
Uruguay	0,33	Israel	0,45
Egypt	0,35	Germany	0,46
Argentina	0,37	Portugal	0,47
Turkey	0,37	Netherlands	0,48
China	0,21	Greece	0,48
India	0,27	Italy	0,49
South Africa	0,30	Sweden	0,53
Russia	0,35	France	0,55
Brazil	0,39	Denmark	0,55

economic growth are those with the lowest public spending of the group (China and India). When comparing Brazil with developing countries, only two countries have a similar proportion of public spending: Argentina and Turkey, both at 37%. It is not surprising that the governments of the developed countries have higher spending as a share of GDP since they are the countries with the highest quality in the provision of public services (examples include Denmark and Sweden).

Figure 4.2 was designed with the aim of analyzing the behavior of the composition of public spending in Brazil: current spending, public investment and income transfers to households. Relationships were made between current spending and public investment (G/Ig); current spending and income transfers (G/TRANS); and public investment and income transfers (Ig/TRANS). Notice that the G/-TRANS and G/Ig relationships have decreased consistently during the entire period (2003-2013), with the fall of G/TRANS being most significant. On the other hand, the behavior of Ig/TRANS has an inflection point in 2008, in the beginning, it was decreasing and then reversed from that point forward.

In the remaining sections, this chapter presents the relevant literature about dynamic stochastic general equilibrium (DSGE) modeling of government expenditures. This is followed with the study analysis results that include the variance decomposition and impulse response functions.

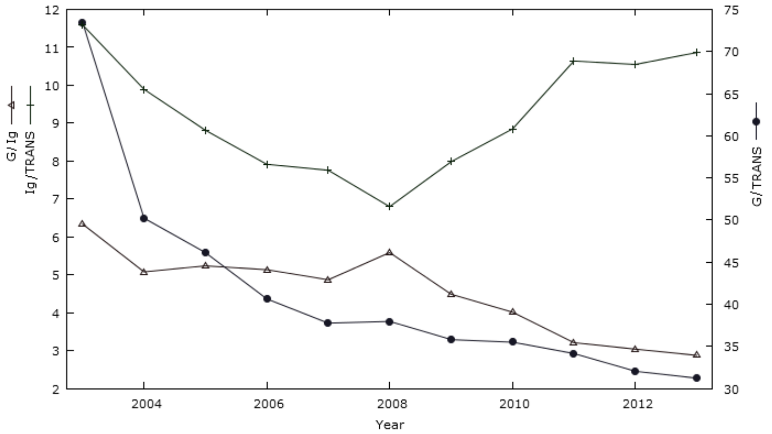

Figure 4.2: Composition of Brazilian Government Expenditure.
Series: Custeio e Investimento (Proxy to Ig); Benefícios Assistenciais (Proxy to TRANS); and Consumo final - adm. pública (Proxy to G).
Prepared by the author.

4.1 Literature Review

This section reviews the relevant economic literature on the use of DSGE models in the study of public spending. Among the models developed for the Brazilian economy, three "large models" have greater acceptance among Brazilian economists.

The first and most complete is the stochastic analytical model with a Bayesian approach (SAMBA). Castro et al. (2011) developed this model to be used as part of the macroeconomic modeling framework of the Central Bank of Brazil. The model embeds 1) the fiscal authority pursues an explicit target for the primary surplus-to-GDP ratio, in accordance with the fiscal regime in place since 1999; and 2) the significant part or consumer prices is regulated by the government, following contract rules.

The SAMBA model also includes two other features that are less usual in DSGE models but relevant in the case of the Brazilian economy. First, in Brazil and many other countries with relatively large manufacturing sectors, most imports are inputs used in the production function rather than being final consumption goods. So the model treats imports as inputs that are used to produce differentiated sectoral goods. Second, as supported by the data, the model assumes that a fraction of the imports must be financed abroad so that shocks from external financial conditions have an extra channel of transmission into the domestic economy. Other frictions are wage and sectoral price rigidities, habit persistence in consumption and non-Ricardian agent and adjustment costs in investments, exports and imports. Figure 4.3 exhibits the shocks to the government consumption in the SAMBA model of Castro et al. (2011).

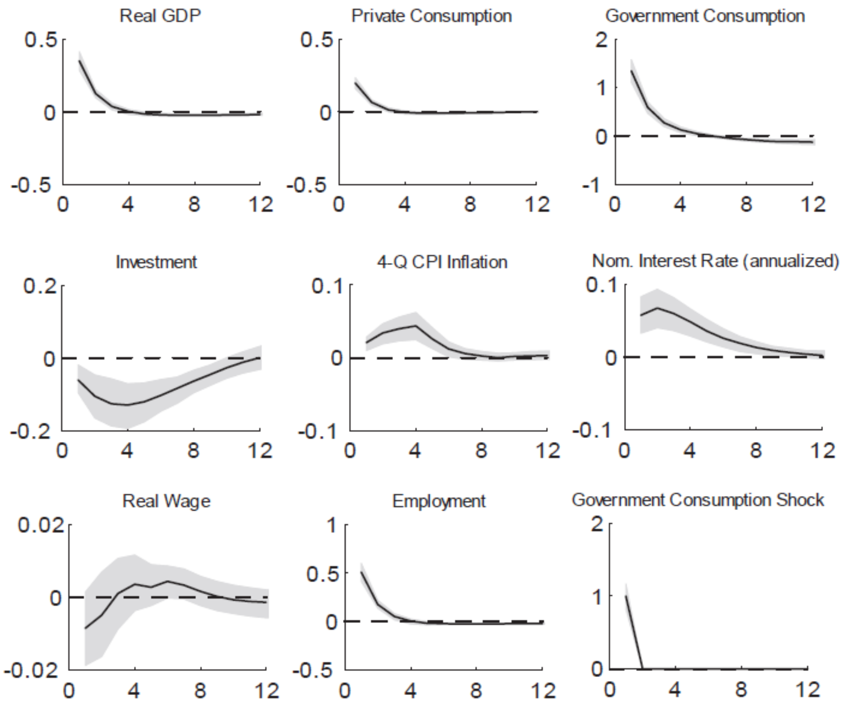

FIGURE 4.3: Impulse Responses to a Government Consumption Shock to the SAMBA.
Source: Modified from Castro et al (2011).

This study sought to answer: 1) how does the type of fiscal expenditure matter for the business cycle?; 2) to what extent can an expansionist shock to the primary surplus put accomplishment of the Central Bank's inflation target at risk?; and 3) has the conduct of fiscal policy in Brazil in recent years put pressure on inflation? Carvalho and Valli (2011) created another model with great acceptance among academics working with DSGE in Brazil. These authors introduced that governments intervene in the economy through the accumulation of public capital with an impact on factor productivity and in the overall demand for investment goods.

Different from this work where the firm does not choose the amount of public capital that one will use in its production process, Carvalho and Valli (2011) assumed that firms can selectively choose between public and private capital services. This modeling choice was intended to capture the significant presence of the Brazilian government in the productive sector of the economy. This model works with two types of households; the first has more specialized labor services[1] where the second does not. Wages and prices have rigidities but there are still consumption habits. Among the shocks proposed by the authors, there are shocks in the primary surplus/GDP (Figure 4.4), in the public investments (Figure 4.5) and in public transfers / GDP (Figure 4.6).

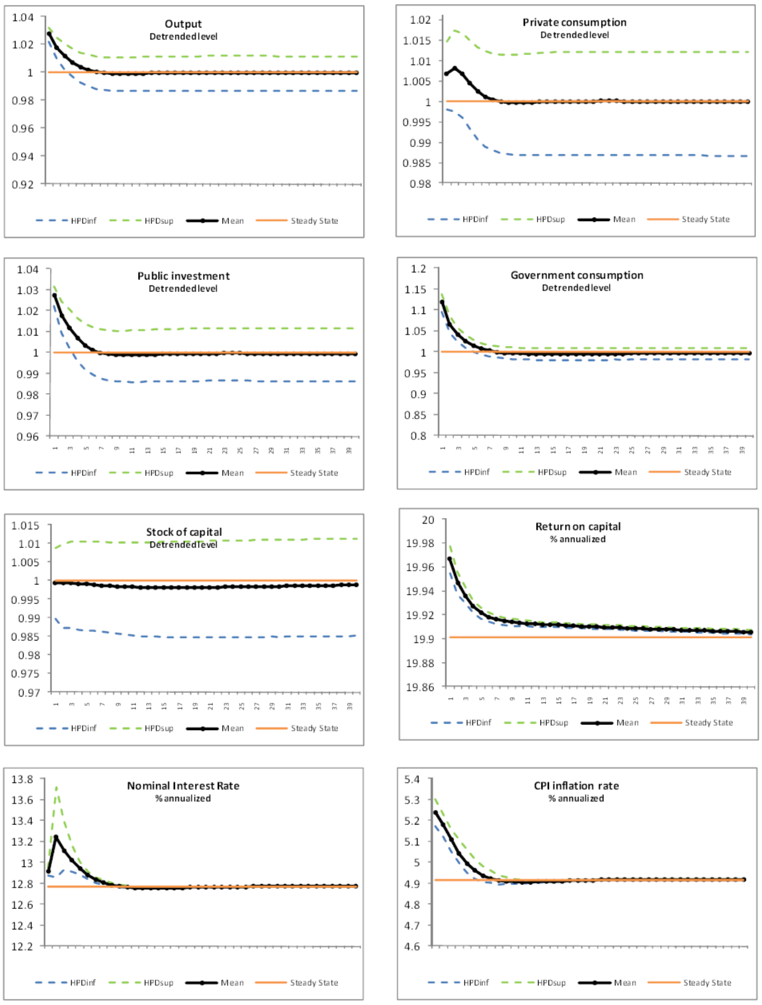

FIGURE 4.4: Impulse responses to a 1 s.d. shock to the primary surplus/GDP - Carvalho and Valli model.
Source: Modified from Carvalho and Valli (2011).

[1]Just a different way of denominating non-Ricardian agents.

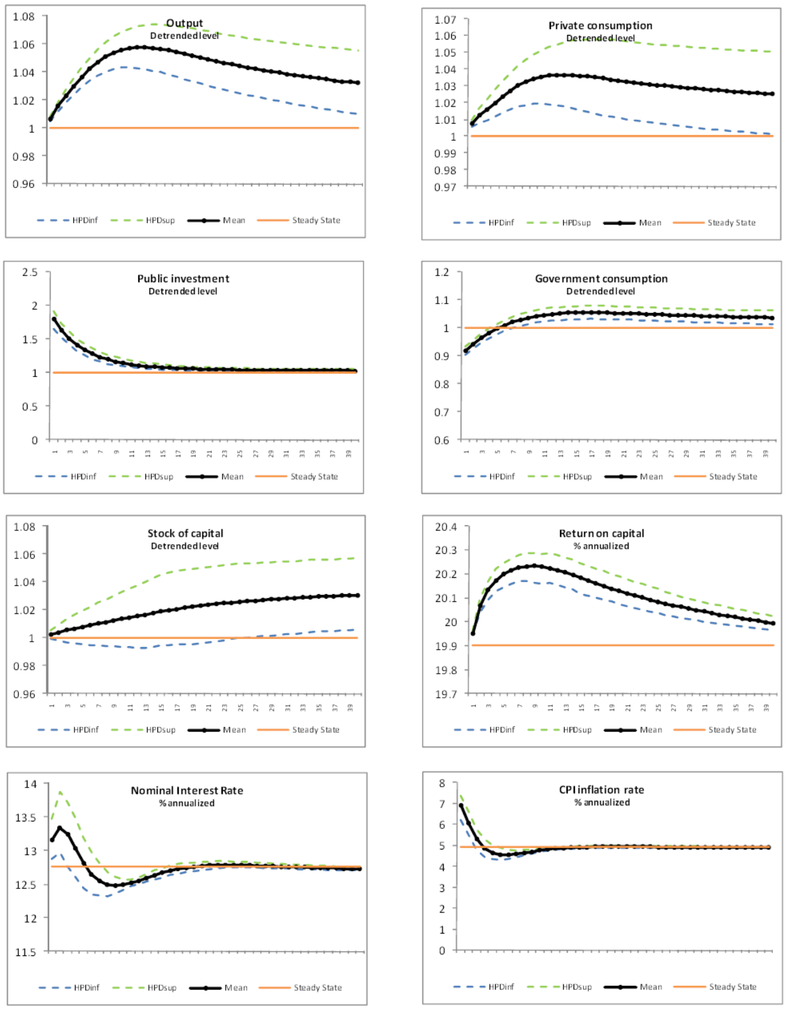

FIGURE 4.5: Impulse responses to a 1 s.d. shock to public investment - Carvalho and Valli model.
Source: Modified from Carvalho and Valli (2011).

Completing this set of three "large models", Vereda and Cavalcanti (2010) developed a model with the objective of identifying the impact of various shocks on the evolution of the main macroeconomic variables. It was especially to predict the effects of changes in the conduct of monetary and fiscal policies, productivity shocks, terms of trade shocks, foreign interest rate shocks and global growth shocks. Its structure is supported in the following characteristics: 1) firms with market power; 2) price and wages rigidities; 3) real frictions such as capital adjustment costs, variable capacity utilization and habit formation in consumption; and 4) non-Ricardian agents.

After reviewing these three large models, it is appropriate to highlight advances in

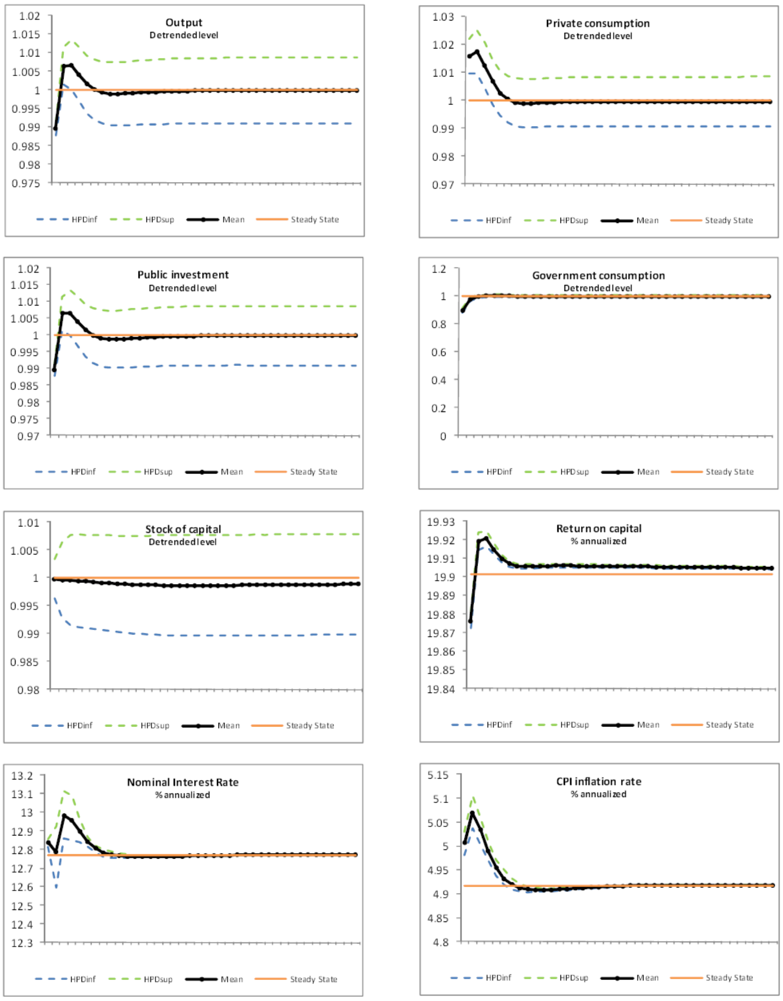

FIGURE 4.6: Impulse responses to a 1 s.d. shock to public transfer/GDP - Carvalho and Valli model.
Source: Modified from Carvalho and Valli (2011).

small DSGE models to fiscal policy. Mussolini and Teles (2012) developed a model of business cycles with government and public capital, with the aim of studying the effects of fiscal shocks on the economic cycle and representing the main stylized facts of fiscal policy in postwar Brazil. Their results indicated that a real business cycle (RBC) with public capital and fiscal policy shocks seems to show well these factors. However, it overstates the volatility of government consumption in the face of private consumption as it significantly underestimates the variation of the latter. The authors analyzed four types of shocks, of which three are of interest in this study: shocks in the income tax, in the government spending on consumption and in the public investment (Figure 4.7). Therefore, the models developed by Castro et al (2011), Carvalho and Valli (2011) and Mussolini and Teles (2012) were used as a kind of benchmark for this study, which justifies the display of some results from these works.

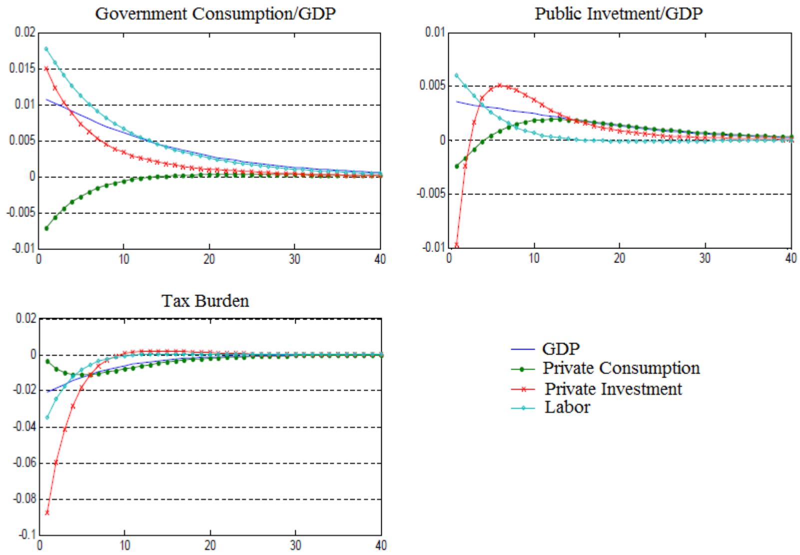

FIGURE 4.7: Impulse response function of the Mussolini and Teles model.
Source: Modified from Mussolini and Teles (2012).

The composition of government spending was studied by Straub and Tchakarov (2007) who analyzed the macroeconomic effects of the observed change in the composition of public spending in the European Union since the 1970s. The model of these authors includes habits of consumption, adjustment costs in the capital stock and wage and price frictions. The main result from their study is that both temporary and permanent increases in public investment generate larger fiscal multipliers than those from increases in public consumption.

4.2 Results

This chapter is motivated by recent debates about the relative effectiveness of public consumption vs. public investment and income transfer as demand management tools. In particular, the debate has focused on the veracity of the underlying hypothesis that public investment is a relatively more powerful instrument of fiscal policy than public consumption. Although there is substantial empirical literature that attempts to identify the sources of these developments, there is still a lack of understanding of the nature and the impact of a change in the composition of public spending.

With the purpose of helping to fill this gap, this study aimed to analyze how three types of government expenditures affect the main macroeconomic variables. Three spending shocks in a DSGE model were used. In the first, there is the possibility of the government stimulating the economy through current spending. In the second, the government increases spending on public investments [2]. In the third, the government provides income transfer to households [3].

4.2.1 Variance Decomposition

This section examines the contribution of each shock to the overall variance of each variable. It provides an initial analysis of the relationships between exogenous stochastic processes and endogenous variables. Table 4.3 and Figure 4.8 show the decomposition of the error variance for the variables analyzed in this work, considering only three shocks related to the composition of public spending (ϵ_g, ϵ_{Ig} and ϵ_{Tr}), which are all objects in the analysis.

In general, the shock in the current government spending (ϵ_g) has the greatest explanatory power in relation to the movements of most of the variables (blue columns of Figure 4.8). In some instances, the explanatory power exceeds 90%: product (Y), 90,37%; household consumption (C), 99,31%; private investment (I_p), 98,15%; total investment (I), 99,89%; current government spending (G), 99,86%; labor (L), 99,37%; private capital (K_p), 99%; interest rate (R^B), 97,88%; inflation rate (π), 92,93% ; and public debt (B), 96,31%.

The other six variables are also explained by this shock; however, this explanation is shared with the shock in nominal income transfer to households (ϵ_{Tr}): wages

[2] Investment in general refers to the use of capital in production methods to increase production capacity. Public investment refers to capital invested by the state to improve the quality of life of people. For example, there are expenditure on infrastructure such as roads, bridges, hospitals, ports, sanitation and schools that are essential to the economy and growth of a country, which depend almost exclusively on public investment. Other examples include public investments in social, urban, transport and productive factors.

[3] The income transfer programs aim to raise the income of the poor people until some minimum acceptable level is reached.

(W), 25,01%; return of private capital (R), 27,03%; general price level (P), 25,14%; total government spending (TS), 11,21%; and total government revenues (TAX), 24,29%. Which are explained by ϵ_{Tr} and the respective percentages.

Turning to analysis of the shock in public investment (ϵ_{Ig}), notice that four variables are influenced by it: gross domestic product - GDP (Y); stock of public capital (kg); inflation rate (π); and public debt (B). However, the stock of public capital is the only variable explained by this shock at 90%, and it shoukd not be otherwise.

Briefly, the analysis of variance decomposition showed that current government spending, among the compositional elements of public spending, had greater power to influence changes in the economy.

Table 4.3: Variance decomposition (in percent). Source: Prepared by the author.

	ϵ_g	ϵ_{Ig}	ϵ_{Tr}
Y	90,37	9,63	0
C	99,31	0,69	0
Ip	98,15	1,85	0
Ig	0,12	99,88	0
I	99,89	0,11	0
G	99,86	0,14	0
L	99,37	0,63	0
Kp	99	1	0
Kg	0,59	99,41	0
W	74,37	0,61	25,01
R	72,32	0,65	27,03
R_b	97,88	2,12	0
P	74,27	0,59	25,14
PI	92,93	7,07	0
Cmg	73,55	0,59	25,86
B	96,31	3,53	0,16
TS	88,5	0,28	11,21
TAX	75,13	0,58	24,29
TRANS	0,26	0	99,74

4.2.2 Analysis of Bayesian Impulse Response Functions (IRF)

This section presents the impulse response functions as a result of persistent shocks to the variables of the model. The dark lines show the response of the economy

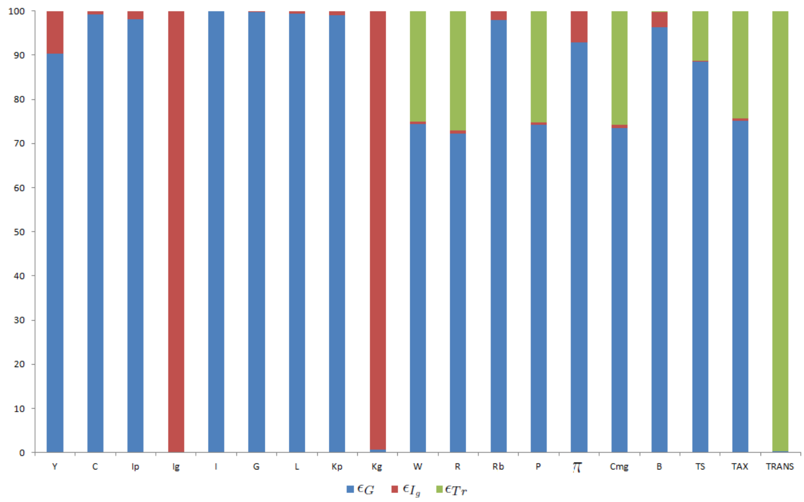

FIGURE 4.8: Variance decomposition (%).
Source: Prepared by the author.

calibrated with the average of the estimated parameter distributions. The gray area indicates the 10th and 90th percentiles. Thus, this section shows the importance of shocks to the endogenous variables of interests by analyzing the impulse response to the structural shocks in the models. The impulse response functions (IRFs) can be directly related to the reduced-form state space representation of the economic model. Consider the general state space form equations:

$$X_{t+1} = AX_t + B\epsilon_{t+1} \tag{4.1}$$

$$Y_t = CX_t \tag{4.2}$$

where X_t is the potentially unobservable state vector and Y_t is the vector of the observables, ϵ_t is the vector of economic shocks and measurement errors (where the latter are included among the state variables X_t) influencing the states and observables.

The impulse responses from the structural shocks ϵ_t to Y_t are given by the following moving average (MA) representation if the eigenvalues of A are inside the unit circle:

$$Y_t = d(L)\epsilon_t = \sum_{j=0}^{\infty} d_j L^j \epsilon_t \tag{4.3}$$

where L is the lag operator, $\sum_{j=0}^{\infty} trace(d_j d_j') < +\infty$, and $d_j = CA^{j-1}B$ for $j \geq 0$. Note that equation (4.3) transforms k shocks ϵ_t into n observables Y_{t+j}, $j \geq 0$.

IRFs are the expected future path of the endogenous variables, conditional on a shock in period 1 of one-standard-deviation. If a model is linearized up to a first order in the Dynare software, IRFs are simply the algebraic forward iteration of the model's computed decision rule from the state-space form (e.g. Hamilton, 1994). The Dynare software procedure runs an IRF (that starts from the exact steady state), by sampling shocks from the distribution (with 1 s.d.) to see how the system reacts for the given periods.

4.2.2.1 Current Government Spending Shock

Here, the study defined the shock in the current government spending, whose response functions are shown in Figure 4.9. The increase in current government spending pushes aggregate demand (equation 2.57) rising GDP (Y), which presses the marginal cost of firms (equation 2.39) generating inflation (π). The Central Bank, through Taylor's Rule (equation 2.55), responds by increasing the interest rate (R^B). The bond prices decrease, which increases their demand. Households alter their portfolios, changing investment assets (I_p) for government bonds (B_t), and consumption (C) follows the same trend of private investment due to the opportunity cost generated by the increase in interest rate (equation 2.2).

If one looks at the public budget, on one hand, the increase in the GDP (Y) raises the public investment (I_g) (equation 2.49) and the nominal income transfer ($TRANS$) (equation 2.51). On the other hand, the current government spending is a component of public spending, so if its value increases, other components of spending (Ig and $TRANS$) suffer a pressure to reduce, or as appropriate, the government can allow a rise in public debt (equations 2.46 and 2.44) since tax rates are unchanged. However, a higher interest rate makes it difficult to rollover debt, leaving this financing channel impaired. Inevitably, there is a need for spending containment, so there is a cut in nominal income transfer. This pressure for a smaller total government spend is also felt by the drop in revenues because the new composition of the household budget (higher demand for government bonds and lower acquisition of consumption and investment goods) decreases total government revenue (TAX) (equation 2.54).

Using Table 4.4 and Figures 4.3, 4.4 and 4.7, it is possible to analyze the correlation[4] among the results of this study and the three models chosen as study benchmark [5], comparing the results of this model with models CA and CV, only household consumption did not present a positive correlation. Moreover, model MT exhibited a positive correlation with respect to consumption; however, the result of the private investment presented inverted.

These divergent results are easy to explain. Starting from divergence with model MT, these authors work with a balanced budget that does not have public debt. If there was debt, the increase of the aggregate demand rises the pressure on prices and thus a reaction of the Central Bank to raise interest rates, an event that changes the portfolio composition of households (leaving acquire capital goods, I_p to purchase government bonds, B). However, in model MT, even with an increase in the interest rate, the households do not get to acquire capital goods.

The differences in consumption behavior between the models are in the assumptions about the rigidity of consumption (consumption habit formation and non-Ricardian agents). The study model, like model MT, does not have a restriction on consumption. Instead, consumption habit formation derives from when a behavior is repeated regularly, that behavior becomes automatic. In particular, consumption habit formation refers to consumer happiness not only being affected by current consumption but also by level of consumption in previous periods. The economic literature shows different elements that cause deviations from the theory of the permanent income life cycle. The main explanation for this result is that the capital market is not perfect and therefore there exists a liquidity constraint for

[4] This is not referring to the correction coefficient of statistical theory, but only to visual observation of the behavior of the same variable in different models.

[5] To facilitate reading from this point forward: the models of Castro et al (2011), Carvalho and Valli (2011) and Mussolini and Teles (2012) are called CA, CV and MT, respectively.

some individuals. Therefore, the inclusion of frictions in household consumption softens the fluctuations to a given shock. Then the results between the models would converge.

4.2.2.2 Public Investment Shock

Figure 4.10 shows the result of the shock in public investment. Two variables are initially more sensitive to this shock: GDP (Y) (equation 2.57) and the stock of public capital (K_g) (equation 2.53). These two variables are pushing the marginal cost (equation 2.39), one positively and the other negatively . While a higher aggregate demand forces an increase in marginal cost of the firm (since the demand for inputs increased), the increase in the stock of public capital facilitates the production process This relieves the pressure on this variable, as the latter result is greater than the first, resulting in a lower marginal cost.

This process generates a deflationary movement and the Central Bank, knowing that motion, decreases the basic interest rate (R^B) (equation 2.55). A lower interest rate improves government accounts because it facilitates the rollover of debt, which expands (B). This result leaves the fiscal policy more “loose”, without pressing the total government spending (TS) to decline, even with a drop in revenues (TAX).

One can make the same comparison with the benchmark using Table 4.4 and Figures 4.5 and 4.7. The result of the study model showed differences in relation to three variables when compared with model CV. This is the result of different assumptions on the behavior of firm producers of intermediate goods to minimize costs.

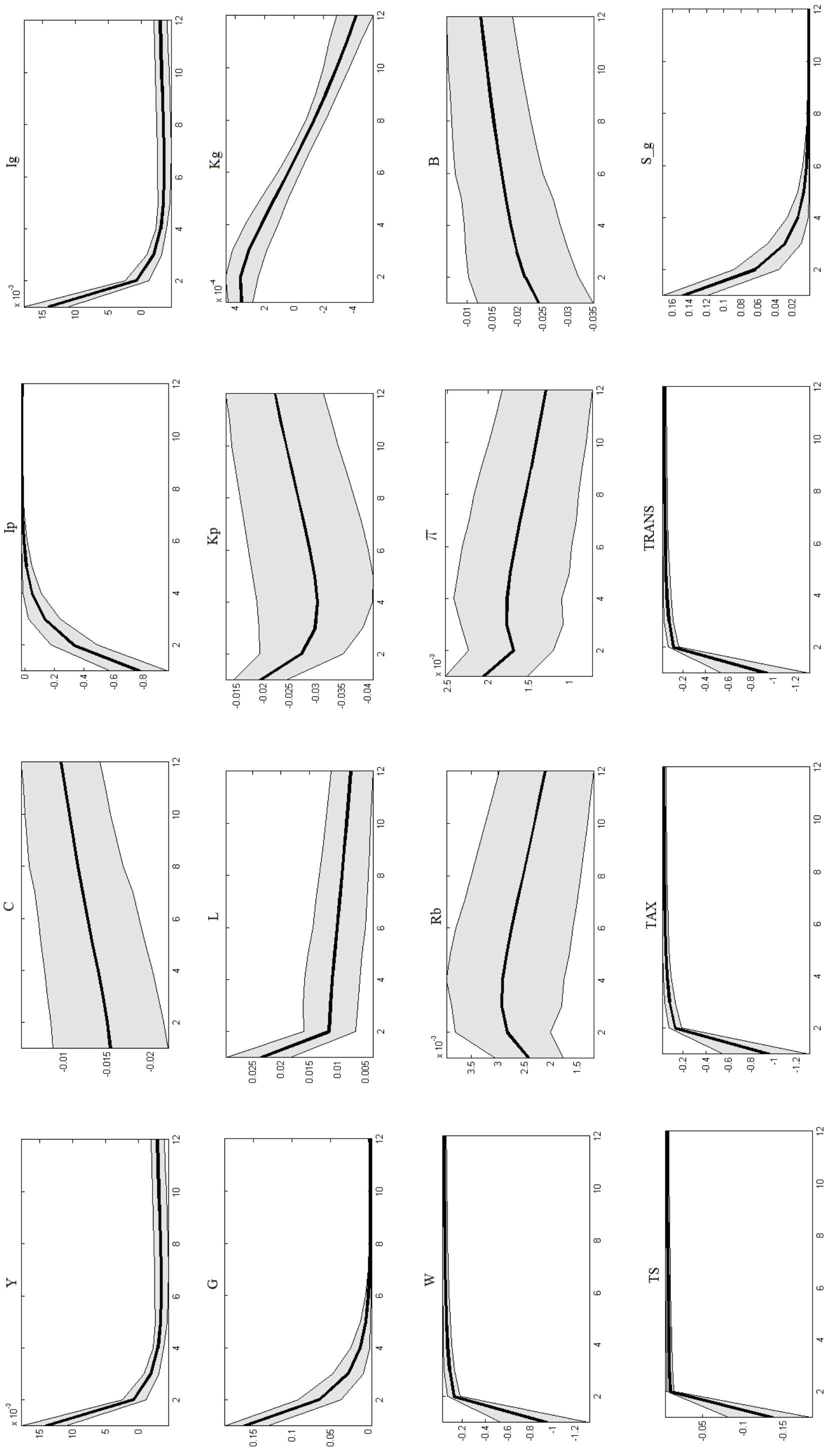

FIGURE 4.9: Current Government Spending shock.
Source: Prepared by the author.

TABLE 4.4: Correlation between the model of this paper with three 'benchmark' models. Source: Prepared by the author.

Current Government Spending shock								
MODEL	Y	C	IP	Ig	G	L	R^B	π
This work	↑	↓	↓	↑	↑	↑	↑	↑
Castro et al (2011)	↑	↑	↓	unused	↑	↑	↑	↑
Carvalho and Valli (2011)	↑	↑	unused	↑	↑	unused	↑	↑
Mussolini and Teles (2012)	↑	↓	↑	unused	unused	↑	unused	unused
Public Investment								
MODEL	Y	C	IP	Ig	G	L	R^B	π
This work	↑	↓	↓	↑	↑	↑	↓	↓
Castro et al (2011)	unused	unused	unused	unused	unused	unused	unused	unused
Carvalho and Valli (2011)	↑	↑	unused	↑	↑	unused	↓	↑
Mussolini and Teles (2012)	↑	↑	↑	unused	unused	↑	unused	unused
Income Transfer								
MODEL	Y	C	IP	Ig	G	L	R^B	π
This work	0	0	0	0	0	0	0	0
Castro et al (2011)	unused	unused	unused	unused	unused	unused	unused	unused
Carvalho and Valli (2011)	↑	↑	unused	↑	0	unused	↑	↑
Mussolini and Teles (2012)	unused	unused	unused	unused	unused	unused	unused	unused

Model CV has the assumption that firms can choose the composition of public and private capitals, given the cost of each input. On the other hand, the study model works on the assumption that a wholesale firm has a choice between private capital and labor inputs, given the values of W and R. These different assumptions mean that the marginal cost of the study model has more sensitivity to increases in the stock of public capital. In this model, an increase in public capital decreases the marginal cost, while for model CV, the opposite occurs. This event explains the inverted behavior of the price levels in both models.

The difference in the behavior of household consumption relative to models CV and MT, plus the private investment in the latter model, are similar to that previously discussed in the case of shock in the current government spending. To model CV, the explanation is on the friction of consumption, while in model MT it is on the assumption of balanced budget.

4.2.2.3 Income Transfer Shock

The third component of government spending is income transfer to households ($TRANS$) (Figure 4.11). The shock in this variable showed no significant changes in the real side of the economy (changes in GDP (Y), inputs (K, T) etc.). The declines in nominal wages (W) and return on capital (R) decrease the marginal costs of wholesale firms (equation 2.39), forcing a drop in the price level (P). This movement makes the real prices of these inputs ($\frac{W}{P}$ and $\frac{R}{P}$) stay stable, without changes in the demand of these variables.

The changes are perceptible in the variables of the government. The declines in nominal wages (W) and return on capital (R) decrease the total government revenue (TAX) (equation 2.54). To compensate for this drop, a spending restraint (equation 2.46) is not needed. This is facilitated instead by the fall in the price level, helping the public debt stay balanced.

Among these three models used to benchmark, just model CV has income transfer to households. In Table 4.4 and Figure 4.6, the shock in this variable is able

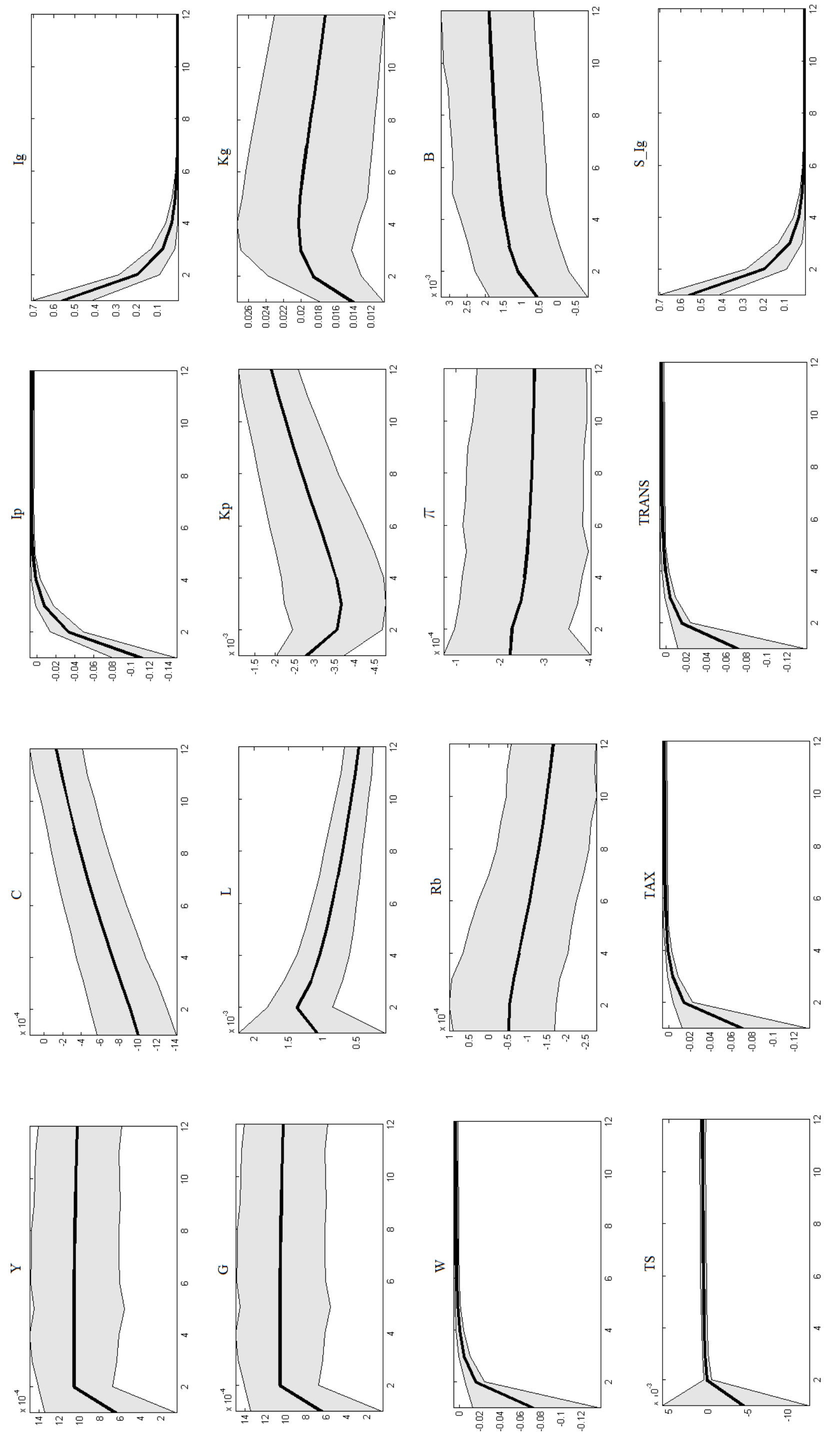

FIGURE 4.10: Public Investment shock.
Source: Prepared by the author.

to influence the economy, and in a similar proportion to the other two shocks. This shock influences the variables of model CV, but in the study model it does not. The explanation is that these authors worked with heterogeneous households. This difference between the models is what would be differentiating the results of the shock on income transfer to households.

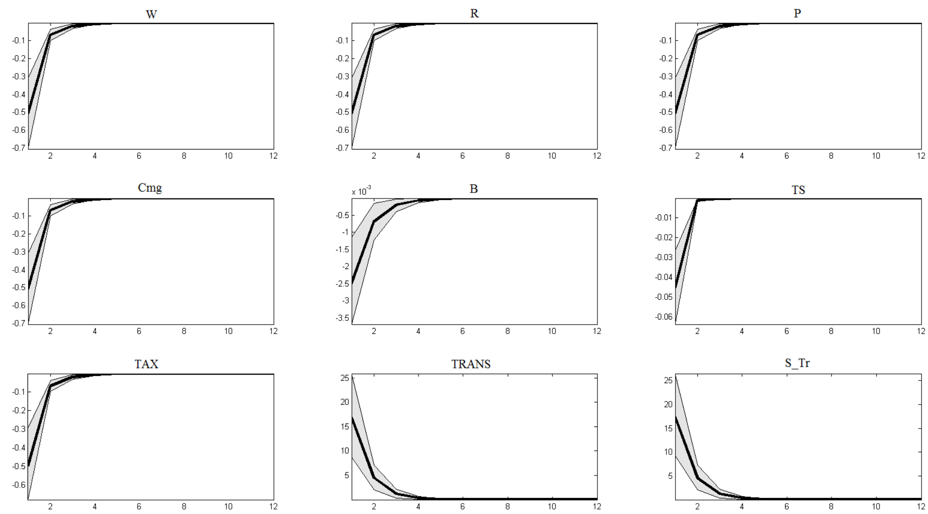

FIGURE 4.11: Income Transfer Shock.
Source: Prepared by the author.

4.2.2.4 Comparison Among the Shocks and Their Persistence

The study first analyzed the shocks alone. Then the goal was to compare each shock and its quantitative results between them. Figure 4.12 provides a comparison between the three shocks (ϵ_g, ϵ_{Ig} and ϵ_{Tr}) related to the variables that make up the composition of government spending. In the analysis of variance decomposition, the study was concerned about the explanatory power of each shock. In this section, the study is more concerned about the result of each shock.

The current government spending is the variable of composition in public spending that affects the economy more, as the variance decomposition has shown. However, should an increase in current government spending be used as an economic incentive? Looking at the GDP, one realizes that the current government spending affects GDP. This shock can stimulate positively the economy by two quarters.

After this period, GDP remains below its steady state for approximately 50 quarters. Then, it is noticed that the total result of this effect on GDP is negative.

Another negative consequence is the persistence of inflation. Even with the reaction of the Central Bank, it remains above its steady state level for 60 quarters, sustained by the level of labor that remains above the steady state during the same period. This features resistance in the rate of inflation. The effects on other variables last somewhere between 2 to 4 quarters.

On the other hand, even if the shock in public investment is showing a modest result in the short term, its effect is maintained in the long term. So the overall result is positive (Figure 4.13). Here, GDP growth is not enough to put pressure on inflation rate due to the decrease in the marginal cost of firms.

A final comment is on the stability of the model in relation to shocks. All variables go back to their steady states as seen in Figure 4.14.

4.3 Conclusions

In this chapter the objective was to study the relative effectiveness of public spending and to analyze the veracity of the underlying hypothesis that public investment is a relatively more powerful instrument of public spending.

The results showed that current government spending, among the compositional elements of public spending, had greater power to influence changes in the economy. Its effects in the GDP stimulated the economy by two quarters. After this period, GDP remained below its steady state for approximately 50 quarters, but its total result is negative. Another negative consequence was the persistence of inflation, sustained by the level of labor that remained above the steady state during the same period. Thus, we note the current government spending is not an economic policy that brings positive results for the economy.

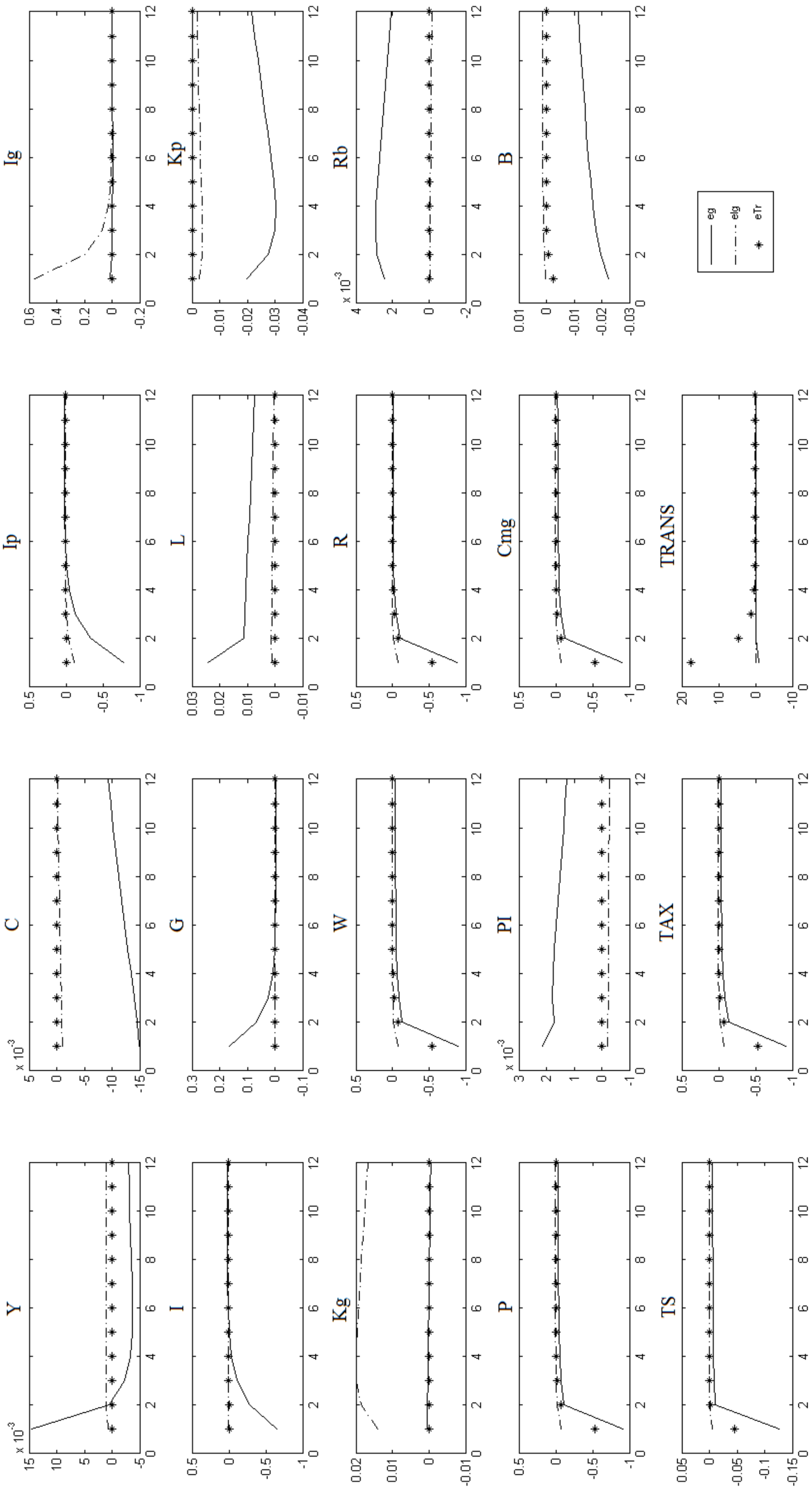

FIGURE 4.12: Comparison Among the Shocks.
Source: Prepared by the author.

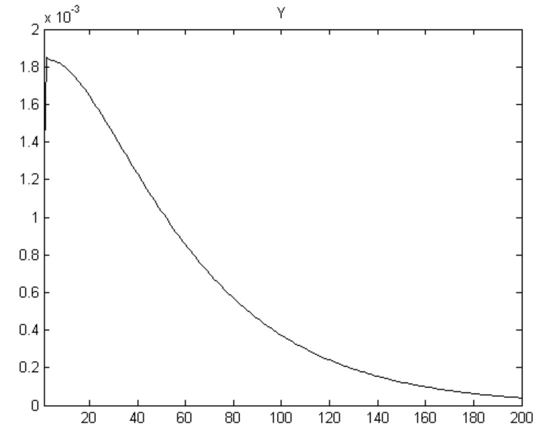

FIGURE 4.13: Long-term result on GDP of a shock in the Public Investment.
Source: Prepared by the author.

However, the shocks in public investment has a long term effect on GDP, a fact that confirms the earlier statement that o public investment is a relatively more powerful instrument of public spending, and this result is achieved without a pressure on the inflation rate. Finally, income transfers did not affect the real economy.

Briefly, public spending was not effective in stimulating the economy in the short term. But among these components, the public investment is more powerful instrument of fiscal policy.

The exercises of comparing the study model with the benchmark models were satisfactory, because they have many points in agreement with the benchmark models. However the inclusion of frictions in household consumption would improve the model results.

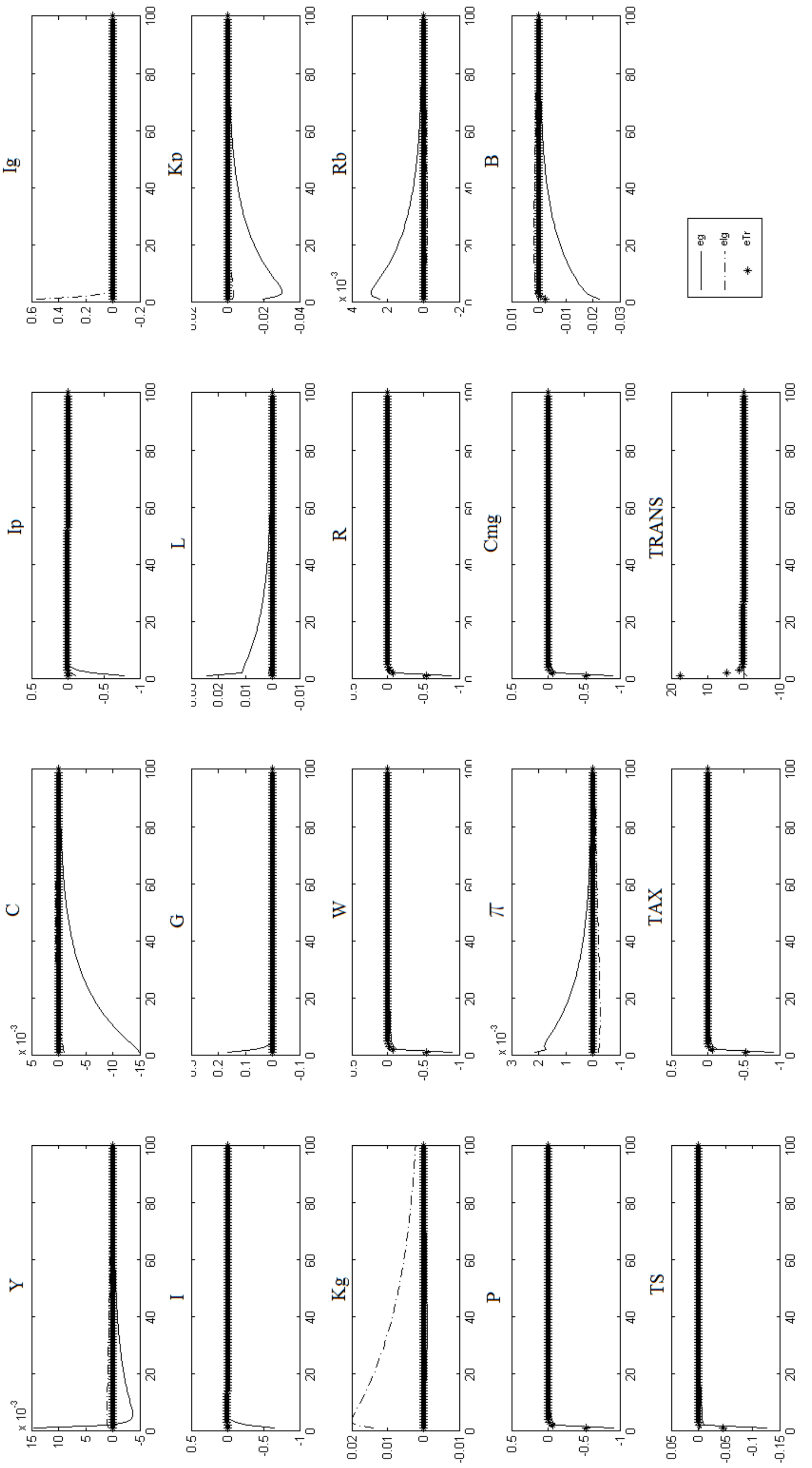

FIGURE 4.14: Shocks Persistence.
Source: Prepared by the author.

Chapter 5

Tax Reduction Policies and Their Impact on the Brazilian Economy

Introduction

One of the most hotly debated topics by Brazilian economists is that country's tax reforms. The general feeling is that the Brazilian Constitution of 1988 initially created a system of insufficient funding for the size of the state. Because of this, the government had to create a series of taxes to supplement state funding, without much concern for the economic rules of taxation. The main result of this policy, among other factors, was a tax system that adversely affected the competitiveness of the productive sector.

The tax literature shows significant differences among the tax reforms of many countries in recent decades. Sandford (1993) contributed a summary listing common elements in such reforms, including reducing the number of tax rates and their maximum marginal value in the income tax of individuals; reducing the aliquots of corporations; and increasing the share of consumption taxes, rather than income taxes.

This work aims to contribute to this discussion by analyzing tax reductions through a DSGE model[1]. To achieve this purpose, three stochastic shocks will be analyzed: the shock from the tax rate on consumption, the shock from the tax rate on labor income and the shock from the tax rate on capital income.

Extensive literature exist on the possible impact of tax reforms in Brazil. Cavalcanti and Silva (2010); Santana, Cavalcanti and Paes (2012); Paes and Bugarin (2006); Pereira and Ferreira (2010); Araújo and Ferreira (1999); Lledo (2005); and Salami and Fochezatto (2009) evaluated the impacts of proposed reforms in the national tax system. Menezes and Barreto (1999) and Teles and Andrade (2006) simulated the combined effects of tax and pension reforms. The literature cited was built using models of overlapping generations (OLG[2]). Instead, this work seeks to contribute to the discussion using a DSGE model.

The main critique of taxation in Brazil refers to the size of the tax burden (TB), which is excessive for a developing country. To verify this concern, Figure 5.1 provides the tax burden (General Government Revenue / GDP) and the growth rate of this variable over the period 2005 to 2013 for a set of 30 countries (developed and developing). We notice that inside this group, Brazil has the fourteenth-highest tax burden (37.22 %), outperforming all the BRICS countries (Russia, South Africa, China and India (35,80%, 28,90%, 22,94% and 20%, respectively)). It also outperforms such developed countries as Australia, Japan and United States (33,63%, 31,68% and 30,71%, respectively). In terms of empirical evidence, this critical comparison of Brazil's tax burden is quite pertinent.

It is not enough just to question the size of the tax burden. The evolution of this variable makes the situation more serious. Figure 5.1 shows the evolution of the tax burden and the public debt-to-GDP ratio in Brazil since 1996. It can be observed that the weight of taxes has been growing consistently since fiscal adjustments in the early 1990s. The positive side is the continuing decline in public debt, which could provide an increase in government savings. However, Brazilian

[1] *Dynamic Stochastic General Equilibrium.* The DSGE methodology attempts to explain aggregate economic phenomena, such as economic growth, business cycles, and the effects of monetary and fiscal policy, based on macroeconomic models with microfoundations.

[2] *Overlapping generations models* is a modeling type that uses representative agents who live a long enough finite period of time to overlap with at least one period of life of another agent.

TABLE 5.1: General Government Revenue em 2013. Source: IMF (2014).

Country	General Government Revenue	Revenue Average Growth(2005-2013)	Country	General Government Revenue	Revenue Average Growth(2005-2013)
Argentina	42,58	5,81	Australia	33,63	-0,4
Brazil	37,22	0,69	Canada	41,48	-0,41
Chile	22,99	0,72	China	22,94	3,41
Colombia	27,71	1,18	Denmark	57,28	0,37
France	52,87	0,59	Germany	44,69	0,13
Greece	44,76	1,03	India	20	1,23
Italy	47,78	0,77	Japan	31,68	0,89
Mexico	23,35	2,2	Netherlands	47,55	0,71
Norway	55,1	-0,06	Paraguay	20,94	1,97
Peru	21,47	2,08	Portugal	43,29	0,92
Russia	35,8	-0,13	South Africa	28,9	1,5
Spain	37,93	-0,07	Sweden	51,92	-0,37
Turkey	36,06	2,12	United Kingdom	37,74	0,46
United States	30,71	0,34	Uruguay	32,96	1,15

primary spending grew strongly over the period (see Figure 4.1 of the previous chapter).

This characteristic growth of the tax burden is not unique to Brazil. Most countries have experienced an increasing tax burden. Table 5.1 shows that only six countries out of 30 showed negative growth rates of the tax burden during the period (Norway, Russia, Spain, Australia, Canada and Sweden). These countries, with the exception of Russia, are developed. Brazil is the eighteenth-largest country in relation to growth of the tax burden. The relevance lies in the fact that no developing country has a TB growth rate higher than Brazil's. Briefly, the tax burden is not only overly large but has been increasing consistently over time.

FIGURE 5.1: Tax revenues and public debt as ratio to GDP.
Source: Prepared by the author.

Figure 5.2 aims to show the composition of direct and indirect taxes in Brazil. To represent these types of taxation, the following series was used: ICMS[3] and IPI[4] representing revenues on the acquisition of consumer and investment goods; individual tax on personal income [5] representing tax on labor income; and the tax on corporate income [6] representing tax on capital income.

Notice that consumption tax is more representative (average of 7,11 %), exceeding the value of the sum of the other two types of tax together (0,3 % and 2,5 % for taxes on labor income and on capital income, respectively). From 2009, consumption tax exceeded GDP growth; this was influenced by Brazilian Government incentives for household consumption (see Alvarenga et al. (2010)). On the other hand, the other two types of tax remained relatively stable throughout the period.

This chapter begins with this introduction, while Section Two presents the literature review. Section Three details the results and ends with the conclusions.

FIGURE 5.2: Relationship of Taxes as ratio to GDP. Tax on Labor Income (TLI), Tax on Capital Income (TCI) and Tax on Consumption (TC). Source: Prepared by the author.

[3] O Imposto sobre Operações relativas à Circulação de Mercadorias e Prestação de Serviços de Transporte Interestadual e Intermunicipal e de Comunicação (ICMS) is a state tax whose primary taxable event is the movement of goods.

[4] O Imposto sobre Produtos Industrializados (IPI) is a federal tax that has as taxable events: customs clearance of imported products, and the exit of manufactured products from the plant.

[5] Individual tax on personal income is a federal tax levied on all persons who have obtained a gain above a certain minimum. The rate is variable and proportional to taxable income

[6] The tax on corporate income is a federal tax on all firms and individual companies. By choice or legal determination, firms and individual companies are taxed by the following forms: simple; deemed income; real profit; or arbitrated profit.

5.1 Literature Review

In this section, we describe the main results for tax reduction in Brazil and the DSGE models that have the same purpose. Importantly, the shortage of models of this methodology directed to studies on tax reduction in Brazil. Castro et al. (2011) and Mussolini and Teles (2012) have already been discussed in the previous chapter, and they will be the basis for comparisons in this chapter. Figure 5.3 is the result of a negative shock in net lump-sum tax in the model of Castro et al. (2011). Another important issue regarding the Brazilian literature on tax reduction (already mentioned in the introduction) is that the discussion focuses on models of overlapping generations. For this reason, this section will present some works that are references on tax reduction, but are not DSGE models.

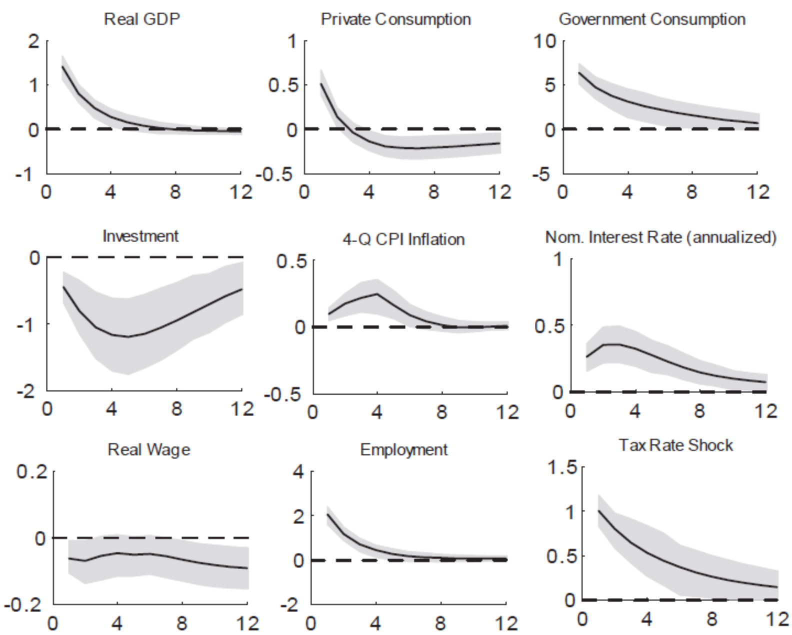

FIGURE 5.3: .
Source: Modified from Castro et al (2011).

Cavalcanti and Silva (2010) have the perception that there is no consensus on the optimal form of tax reduction for the production sector. To contribute to this discussion the authors propose a general equilibrium model with overlapping generations in which they seek to compare measures of tax reduction on labor and capital, both offset by increases in consumption taxation. Households pay taxes on labor income, on capital income, on consumption, and also contribute to the social

security system. The result of the authors' proposition is that the elimination of tax on capital can lead to substantial increases in capital and products relative to the tax reduction on labor.
on capital income, on consumption, and also contribute to the social security system. The result of the authors' proposition is that the elimination of tax on capital can lead to substantial increases in capital and products relative to the tax reduction on labor.

Salami and Fochezatto (2009) propose an analysis of the economic effects of different long-term tax options. To achieve this objective, the authors use a general equilibrium model with overlapping generations. The article presents a tax change with a decrease in direct taxes and an increase in indirect taxes, without changing the tax burden. Households pay consumption tax, income tax, and social contribution. The authors do not recommend replacing an indirect tax on consumption with a direct tax on income, while maintaining the same level of public revenue.

Araújo and Ferreira (1999) studied what impacts tax reforms in the Brazilian economy would have on welfare. They developed a dynamic general equilibrium model with households paying taxes on the purchase of consumption goods, on the acquisition of capital goods, on labor income and on property income. The purpose of their study was to show that tax reform remains unchanged with the participation of government revenue in the product. The result achieved is that the tax reform proposed would lead to positive results in the product, employment and capital stock.

Forni et al. (2009) used a DSGE model to estimate the effects of fiscal policy in the Euro area. The theoretical framework of this model has non-Ricardian agents; taxation is divided into consumption taxes, taxes on labor income and taxes on capital income. Government expenditures are divided into purchases of goods and services, compensation for public employees and transfers to households. The results indicate that reductions in tax rates on labor income and consumption have significant effects on output, while reductions in tax on capital income favor investment and products in the medium term (Figures 5.4, 5.5 and 5.6).

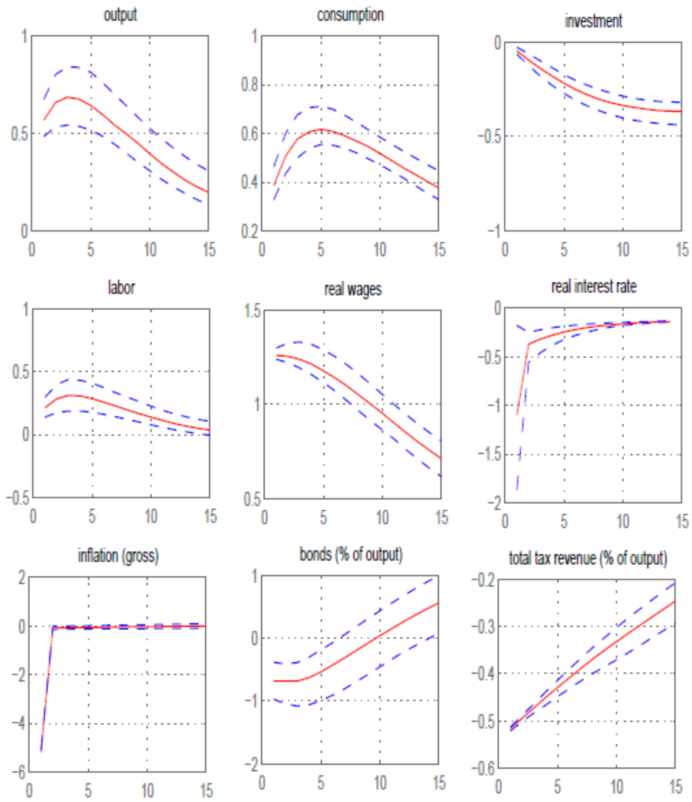

FIGURE 5.4: Tax on Consumption Shock of the Model's Forni et al (2009). Source: Modified from Forni et al (2011).

5.2 Results

As mentioned in the introduction, this chapter aims to study tax reduction for three types of taxes: tax on consumption; tax on labor income; and tax on capital income. In the literature, the best composition has several discrepancies (Cavalcanti and Silva, 2010). Thus, every study that attempts to distinguish the results of different tax reductions will certainly contribute to further reflections for designs of fiscal policy.

5.2.1 Variance Decomposition

The variance decomposition presented in this section seeks to demonstrate the explanatory power of the changes in each variable, given the proposed shocks in this study. Figure 5.7 and Table 5.2 show the decomposition of the error variance

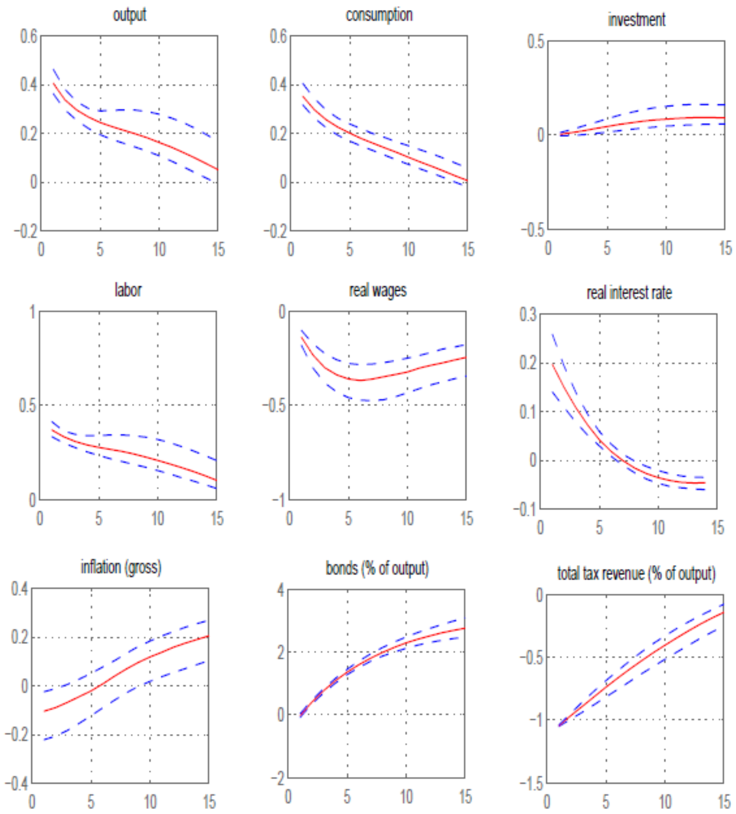

FIGURE 5.5: Tax on Labor Income Shock of the Model's Forni et al (2009). Source: Modified from Forni et al (2011).

for the three shocks of tax reduction (ϵ_c, ϵ_l and ϵ_k).

In a general way, the shock of tax reduction on labor income is that it has greater explanatory power in the variables. In all of these, the weight of explanation of this shock exceeds 80%: GDP (94,57%); household consumption (99,62%); private investment (91,83%); public investment (94,57%); current government spending (94,57%); labor (93,54%); stock of private capital (99,37%); stock of public capital (99,62%); wages (98,04%); return on capital (98,93%); interest rate (98,54%); inflation rate (99,74%); marginal cost of firms (99,74%); public debt (99,35%); total government spending (83,14%); total government revenue (99,1%); and income transfers to households (83,14%).

There are two striking features of these results. The first is the superiority of the tax reduction on labor income in relation to the other two tax reductions. Most impressive is the weak capacity of the tax reduction on capital income to stimulate

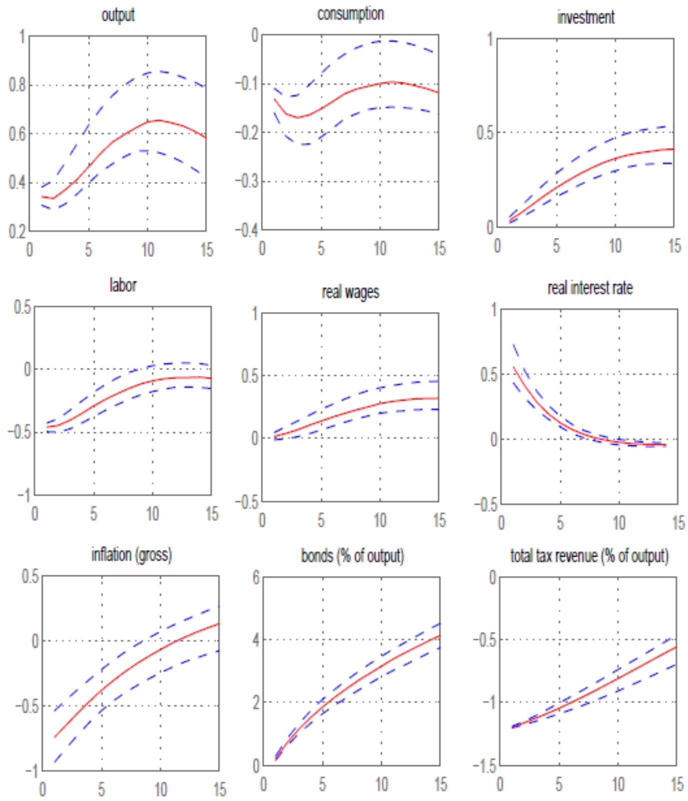

FIGURE 5.6: Tax on Capital Income Shock of the Model's Forni et al (2009). Source: Modified from Forni et al (2011).

increased private capital stock (among the three proposed shocks, this is the one that least encourages increased input).

5.2.2 Analysis of Bayesian Impulse Response Functions (IRF)

Given the estimation of the model, Bayesian impulse response functions are generated. These represent the dynamics of the main macroeconomic variables from an exogenous shock. In this section we discuss the implications of our estimates for the effects of three possible tax reductions.

TABLE 5.2: Variance decomposition (in percent). Source: Prepared by the author.

	ϵ_c	ϵ_l	ϵ_k
Y	5,34	94,57	0,08
C	0,36	99,62	0,01
Ip	8,01	91,83	0,16
Ig	5,34	94,57	0,08
I	7,88	91,96	0,16
G	5,34	94,57	0,08
L	6,36	93,54	0,1
Kp	0,61	99,37	0,02
Kg	0,37	99,62	0,01
W	1,33	98,04	0,63
R	0,51	98,93	0,57
R^B	1,44	98,54	0,02
π	0,25	99,74	0
Cmg	0,92	98,46	0,62
B	0,6	99,35	0,06
TS	13,09	83,14	3,77
TAX	0,38	99,1	0,52
TRANS	13,09	83,14	3,77

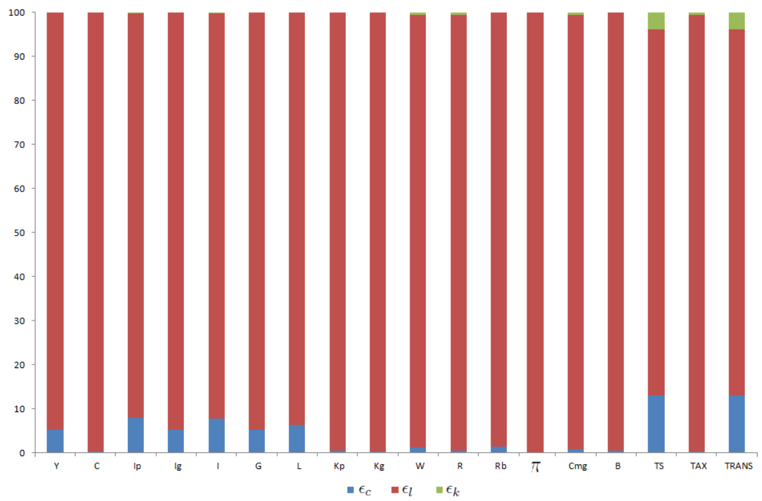

FIGURE 5.7: Variance decomposition (%). Source: Prepared by the author.

5.2.2.1 Tax on Consumption Shock

Figure 5.8 displays the result of the tax reduction on consumption. This shock initially affects the wage (W) (equation 2.10) and the return on capital (R) (equation 2.11) increasing demand for labor (L) (equation 2.35) and private capital (K_p) (equation 2.36) raising aggregate supply (Y) (equation 2.35). The nominal marginal cost of wholesale firms faces two effects, the first effect is positive and comes from a higher product (Y), and a second is negative, which originates from the falling prices of the inputs (W and R) (equation 2:39), and the second prevails over the first. With a higher output, current government spending (G) (2:47 equation) and public investment (I_g) (equation 2.49) receive a boost. This increases the stock of public capital, which also helps to decrease the nominal marginal cost. However, the price level decreases enough to increase the real marginal cost, and exert pressure on the inflation rate, causing the Central Bank to increase the interest rate (R^B) (equation 2.55).

The decrease in total government revenue, coupled with the increase in interest rates (which affects the rollover of debt) exerts pressure on the fiscal budget for a retrenchment. The government addresses this need by reducing income transfers to households ($TRANS$) (equation 2.44). The effect on the economy due to tax reduction on consumption is very short; it is held in just two quarters. The work of Forni et al (2009) (Figure 5.4) shows similar results for the Euro area. However, the effects of this type of tax is more persistent (15 quarters)[7].

5.2.2.2 Tax on Labor Income Shock

The tax reduction on labor income is presented in Figure 5.8. After the reduction of this tax, households accept a lower real wage (W/P) (equation 2.10) (since their disposable income increased). The wholesalers' firms demand more labor input (L) (equation 2.35), which increases aggregate supply (Y) (equation 2.25). This increase in output raises current government spending (G) (equation 2.47) and public investment (I_g) (equation 2.49). Households with greater disposable income increase their spending on consumption goods (C) and private investment (I_p), and demand a greater amount of government bonds (B) (equation 2.2).

After the maturation period of government investment, the stock of public capital is increased (K_g) (equation 2.53); this decreases the nominal marginal cost (equation 2.39), which creates a deflationary process (π), and the Central Bank sees space for an expansionary monetary policy (lower interest rates (R^B)) (equation 2.55). This decreases the demand for government bonds because of the increase in the price of the asset (if R^B decreases, $P^B = frac1R^B$ increases).

[7] There is a need for some caution in comparing the model of this work with the model of Forni et al. (2009) due to the fact that the first is directed at the Brazilian economy, while the second was developed for the Eurozone.

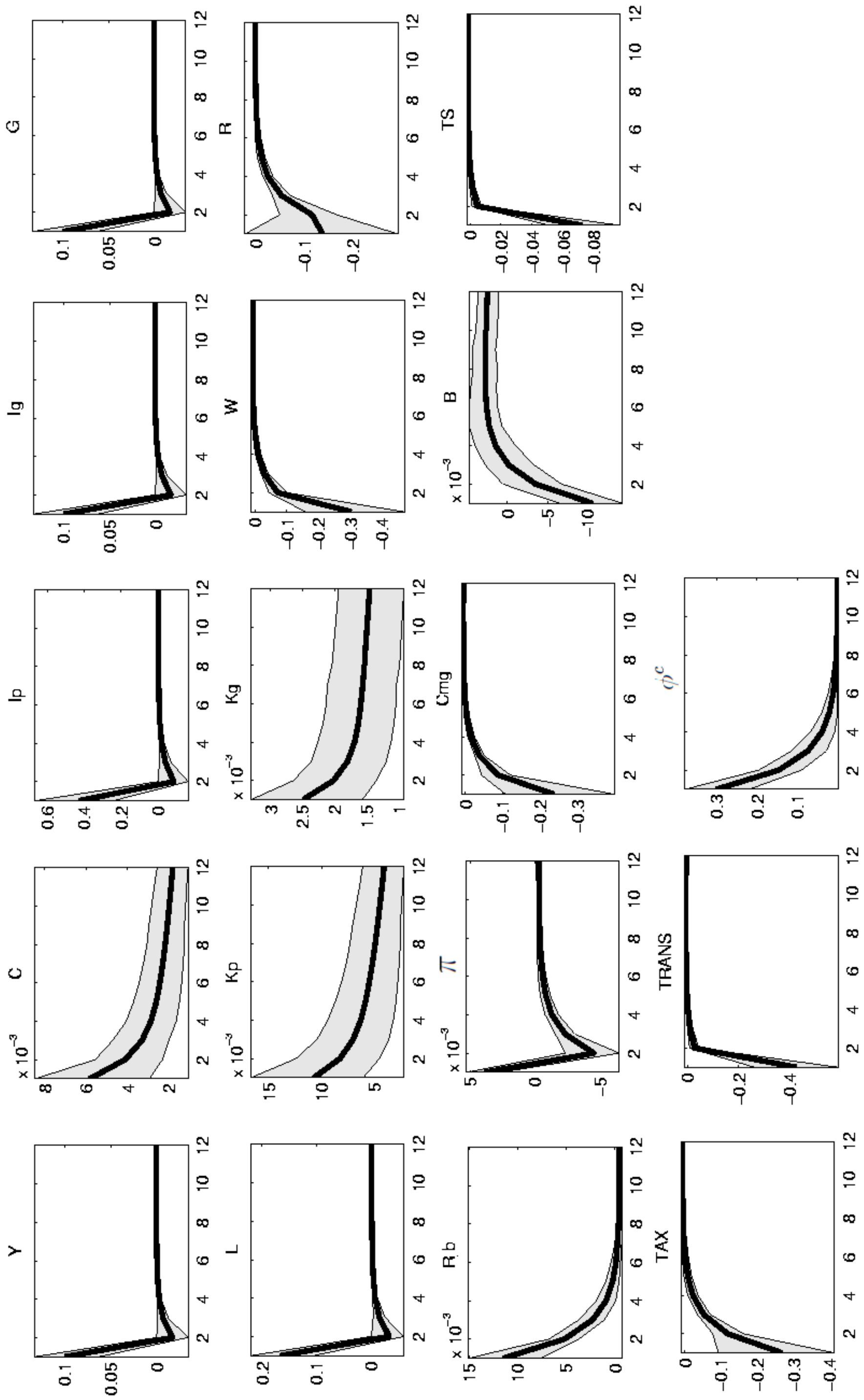

FIGURE 5.8: Tax on Consumption Shock.
Source: Prepared by the author.

In this process of falling interest rates, the government has space to work with a larger stock of government debt, a fact that helps maintain spending (equation 2.44), even with the drop in total revenue (equation 2.54). However, the government prefers to make cuts in income transfer to households ($TRANS$) (2.46 equation), because there is less need for this type of social policy, since the economy is expanding.

The shock in this tax also resembles the work of Forni et al. (2009) (Figure 5.5), with some differences in relation to the inflation rate and the interest rate. But here the tax reduction on income labor persists for more than 15 periods in both studies.

5.2.2.3 Tax on Capital Income Shock

The third tax reduction proposed in this chapter concerns the tax on capital income (Figure 5.9). This shock causes a decrease in real return on capital (R/P) (equation 2.11), thus firms require more private capital (K_p) (equation 2.36), raising the product (Y) (equation 2.25). Households perceive an opportunity to increase the purchase of investment goods (since tax is lower in this sector), which decreases aggregate consumption (C) (equation 2.2). Current government spending (G) (Equation 2.47) plus public investment (I_g) (Equation 2.49) grow due to higher product. The stock of public capital (K_g) increases by decreasing the nominal marginal cost. Inflation that initially rose now starts to sag due to the decrease in marginal cost.

With smaller total revenue, and with the difficulty of rolling government debt, the government decreases transfer income to households. Two other differences between the shocks deserve comment. Among the three shocks on tax reduction, household consumption decreases only in this shock. The reason is the increased demand for investment goods, given the opportunity cost due to lower tax on capital income. Furthermore, the increase in the stock of capital is the main result of this tax reduction; however, it is lower than the results of other shocks. Finally, the persistence of the effects is similar to the consumption tax, two quarters.

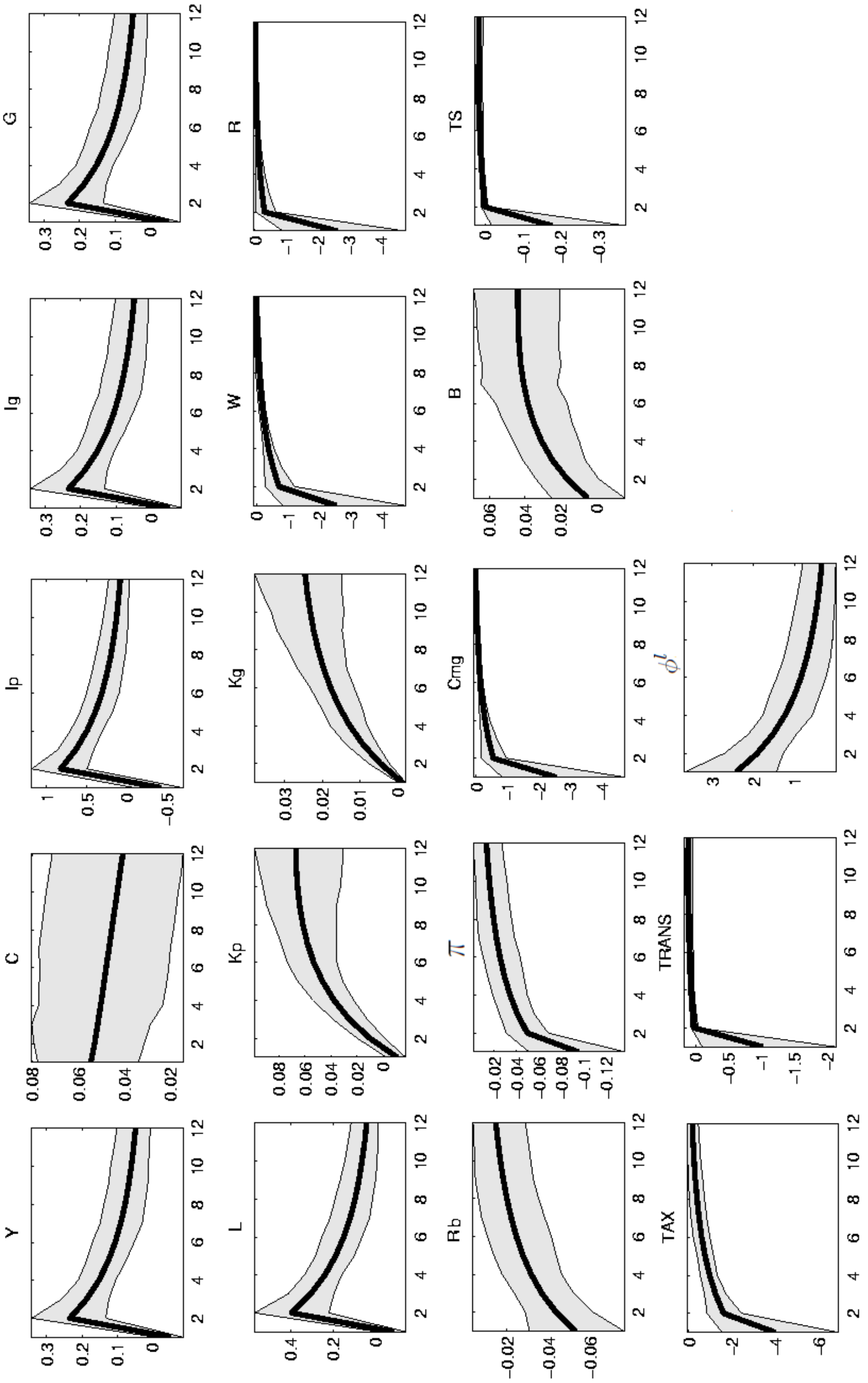

FIGURE 5.9: Tax on Labor Income Shock.
Source: Prepared by the author.

Comparing this shock with the same one of Forni et al. (2009), the results are similar. The effects on output, investment and interest rates are positive, while the effect on consumption is negative. Moreover, like the shock on consumption tax, the persistence of the effects diverges. While in this work the effects of the shock on capital income tax remain two quarters, in the work of Forni et al. (2009) they exceed 15 tax quarters.

5.2.2.4 Comparison between the Shocks and their Persistence

Analyzing Figure 5.11 the superiority of the result of the tax reduction on labor in relation to the two other alternatives is undeniable. Its effect on output persists for more than 20 quarters. In relation to household consumption and public and private capitals, the effects are greater than 60 quarters. The downside is the persistence of government debt. Regarding stability, all variables return to their steady state (Figure 5.12).

Given that the works of Castro et al. (2011) and Mussolini and Teles (2012) use a lump-sum tax, we are left to comment on the similarities among these results in this subsection. The results of Figure 5.11 and 5.3 are similar with regard to product, household consumption, government spending, real wage and amount of work. Mussolini and Teles (2012) obtained results similar to ours for the four variables presented by the authors, both in their shift directions as in the persistence of the shock (Figure 4.7) [8].

5.3 Conclusions

This chapter aimed to study possibilities for tax reduction through three stochastic shocks in the tax rates on consumption, on labor income and on capital income.

The results show that the tax reduction on labor income is the most effective in stimulating the economy. First, the size of the increases, and secondly by the

[8] This should reverse the reasoning for the comparison of this work with Mussolini and Teles (2012), as these authors worked with a shock of tax increase.

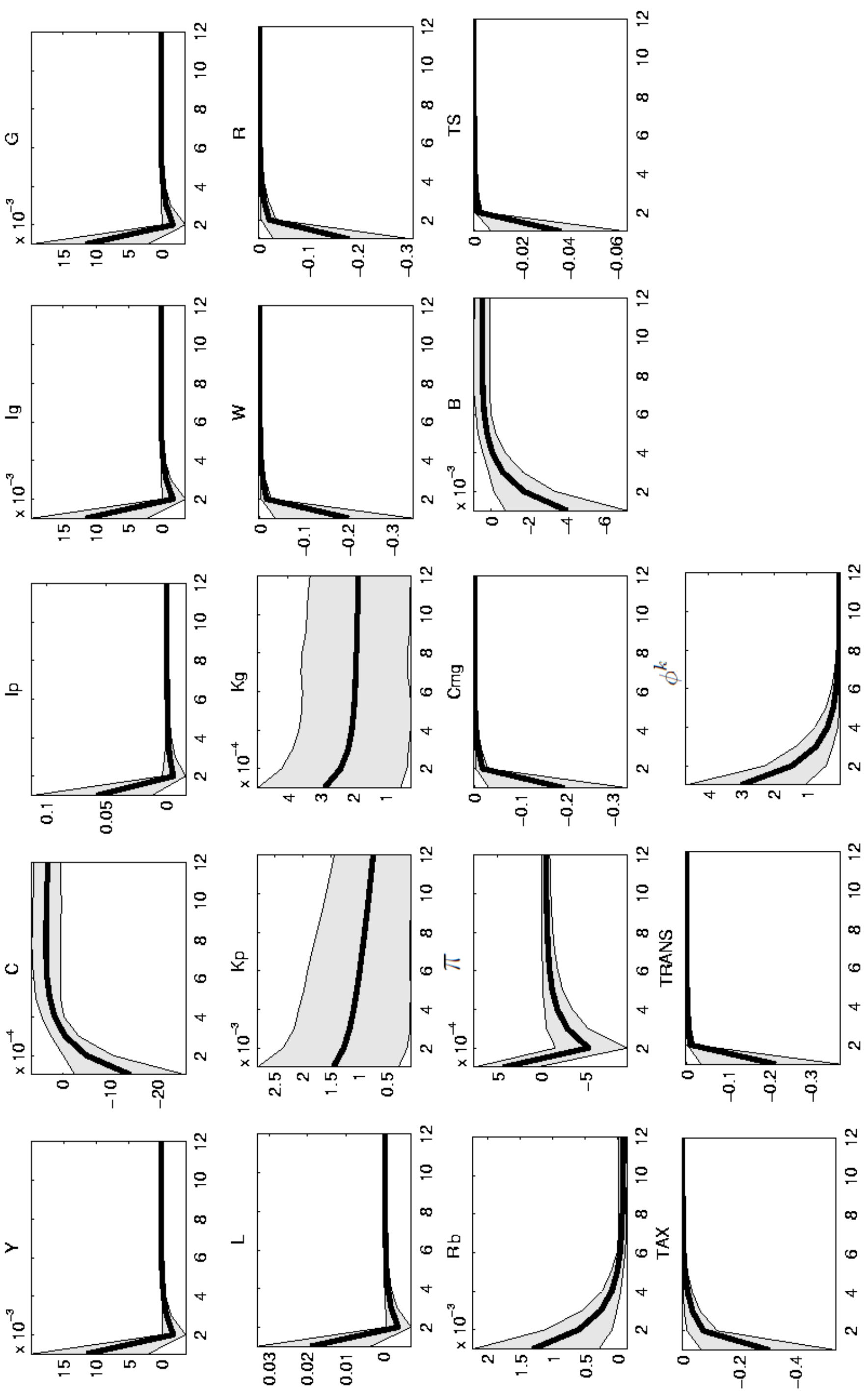

FIGURE 5.10: Tax on Capital Income Shock.
Source: Prepared by the author.

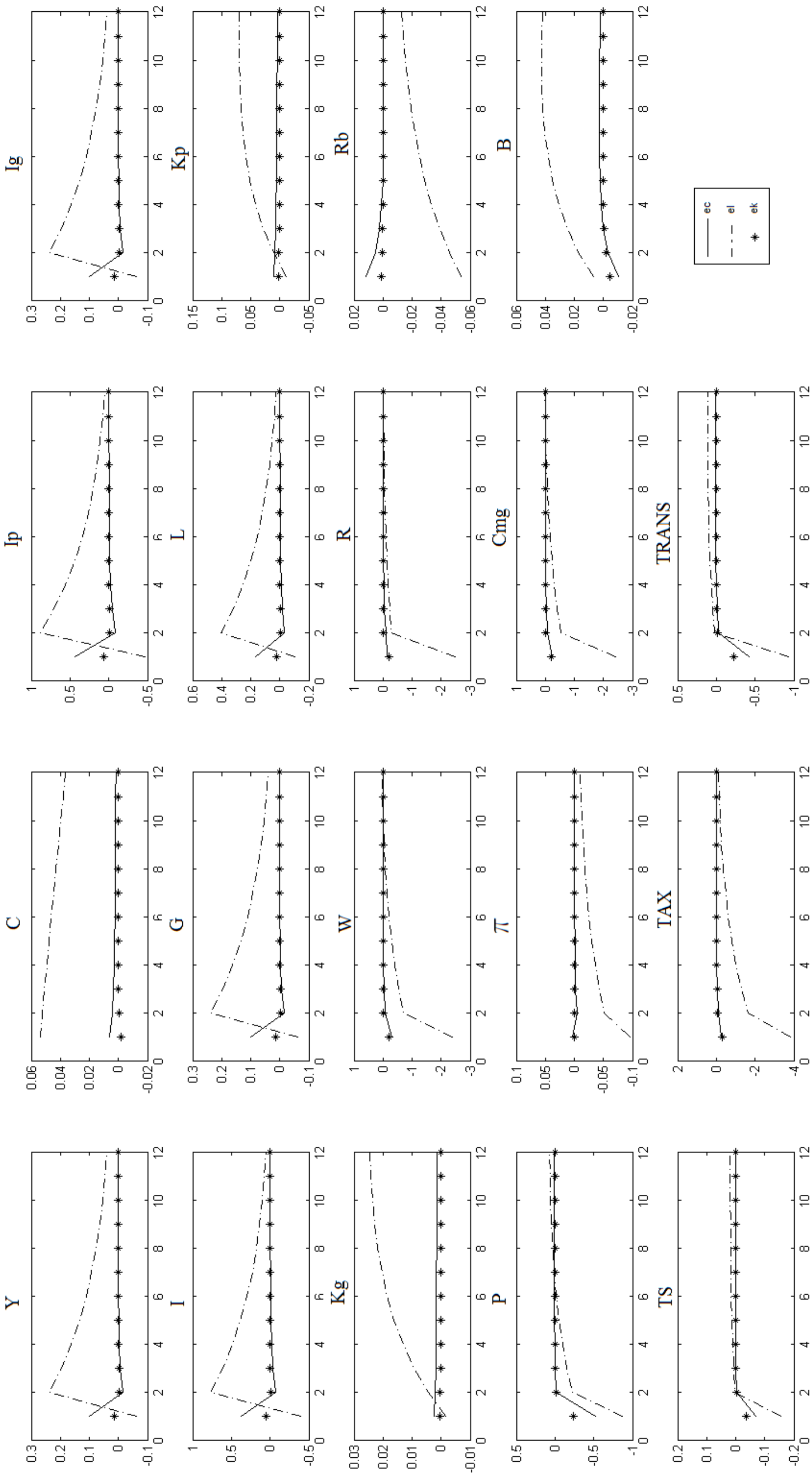

FIGURE 5.11: Comparison between the Shocks.
Source: Prepared by the author.

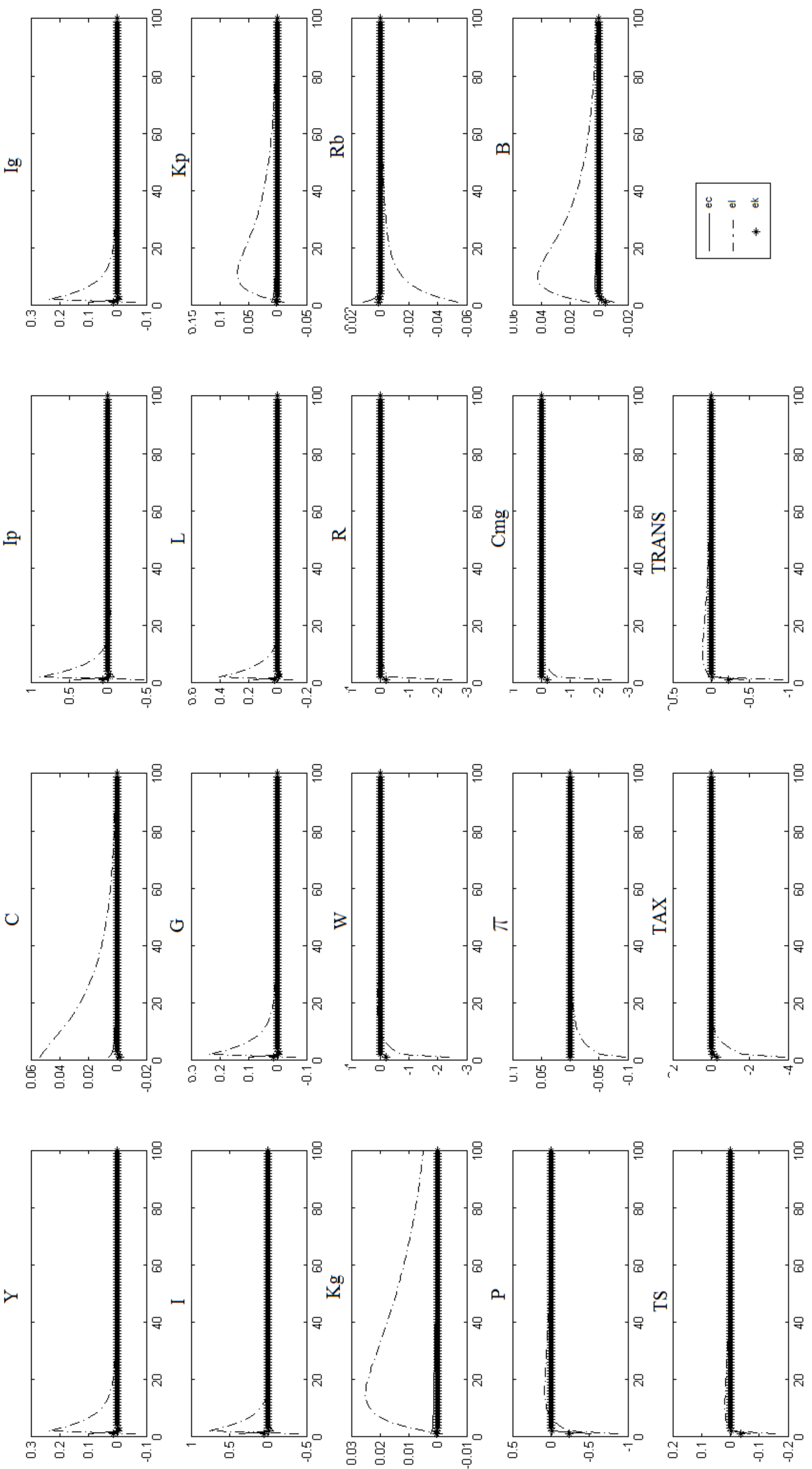

FIGURE 5.12: Shocks Persistence.
Source: Prepared by the author.

persistence of result. In this shock, the product is maintained for over 20 quarters, while for the other two shocks, incentives lose strength in two quarters. This result contradicts that obtained by Cavalcanti and Silva (2010) who found that the tax reduction on the capital factor has better results than on labor.

Looking at the public budget and tax reductions in consumption tax and capital income tax, there is a need for the government to reduce its share in the economy, since its revenue decreased, and the increase in interest rates undermined public debt management. Then, the channel for reduced spending was the income transfer to households. In the case of tax reduction on labor income, the decline in interest rates makes it easy to rolling the public debt, and even with a drop in revenues, the government does not need to introduce spending cuts, but decreases the income transfer to households by choice, unlike the other two cases.

Briefly, the best choice for tax reduction is on labor income, due to persistence in the GDP effects and the better overall state of the economy. There is an increase in the amount of input (L, K_p and K_g), which increases the product without pressure on inflation.

Chapter 6

Last Considerations

The main objective of this study was to analyze possibilities of fiscal policy for the Brazilian economy, both on the side of public spending and on the side of tax reduction. For this purpose six stochastic shocks were performed: current government spending; public investment; transfer income to households; tax reduction on consumption; tax reduction on labor income; and tax reduction on capital income.

The result of this study is that the shocks in government spending is lower than shocks in tax reduction (Figures 6.1 and 6.2). Of the three possibilities of policies spending, the only positive outcome is the public investment. Even so, it is less to the three policies of tax reduction. Importantly, the only fiscal policy that generated an inflationary process was the shock on current government spending. Even the other shock that stimulates household spending - tax reduction on consumption - the result was fall in inflation. The other three shocks negatively affect the marginal cost, event that drops the level of prices. Among the policies of tax reduction, the tax reduction on labor income showed more significant results with increased GDP and drop in the level of prices.

In order to complete this discussion on the better fiscal policy, we seek to identify the Keynesian multiplier for each of the six proposed this study. Thus, we tested a shock of the same proportion ($\epsilon = 1$) for each proposal. The multiplier is obtained in the same way of the "old Keynesian", $\alpha = \frac{\partial Y}{\partial public \quad spending}$ or $\alpha = \frac{\partial Y}{\partial tax \quad reduction}$, in simple terms, we are getting the gray area in Figure 6.1. Table 6.1 displays

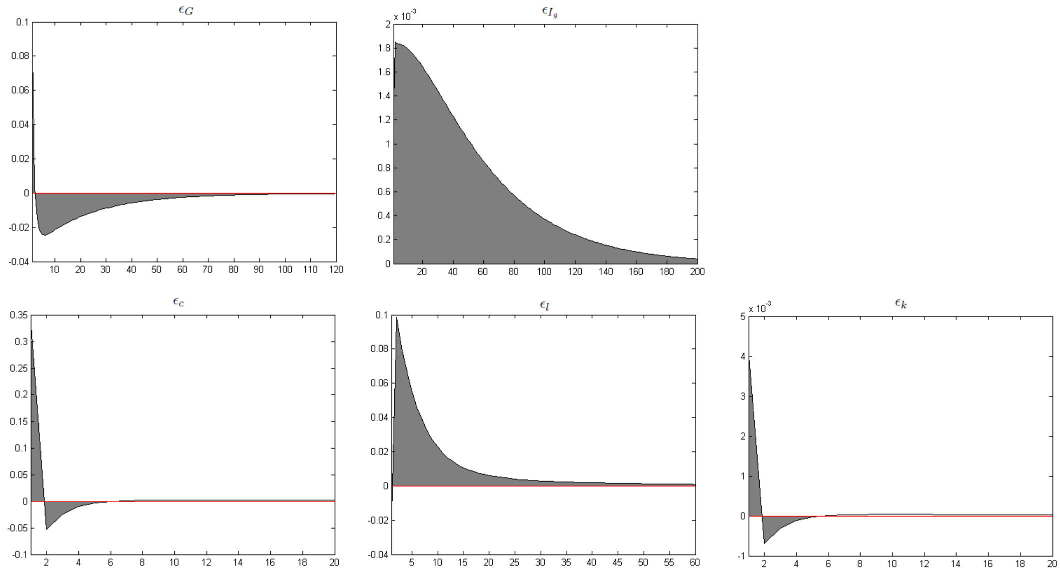

FIGURE 6.1: Keynesian multipliers. Here, all shocks are getting the same value, $\epsilon = 1$.
Source: Prepared by the author.

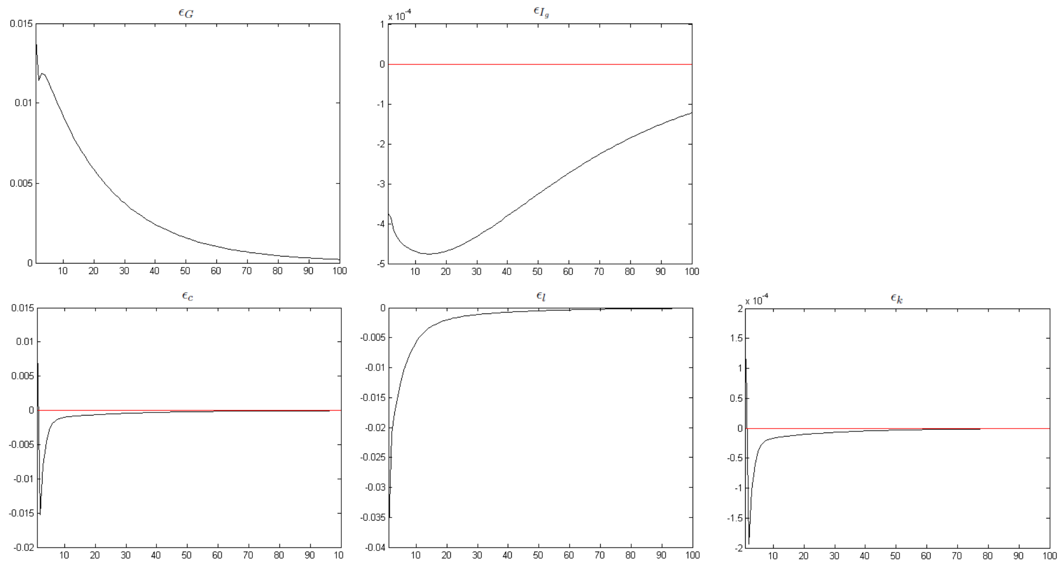

FIGURE 6.2: Inflationary Process.
Source: Prepared by the author.

these results.

The multiplier of current government spending had a negative result, meaning that for every R$ 1 billion increase in this fiscal policy the GDP falls in R$ 0,7 billion[1]. The other possibilities of fiscal policies had positive results, however the only fiscal incentive that has multiplier effect is the tax reduction on labor income ($\alpha = 1,2$). Indicating be the better proposal for fiscal stimulus.

TABLE 6.1: Keynesian multipliers (α). Source: Prepared by the author.

ϵ_G	ϵ_{Ig}	ϵ_{Tr}	ϵ_c	ϵ_l	ϵ_k
-0,7	0,15	0	0,23	1,2	0,23

Briefly, this study was able to answer the initial proposal (analyze possibilities of fiscal policy for the Brazilian economy). For future work, it would be interesting to consider some frictions not used here. Could be incorporated consumption habits, non-Ricardian agents, the cost of adjusting investment among others.

[1]This result is close to the value for the Euro area using the model Smets and Wouters (2003). Where results of the multipliers to possibilities of fiscal packages in 2011Q1 and 2013Q4 were -0,2 and -0,34, respectively (Cwik and Wieland, 2009).

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Appendix A

Other Shocks

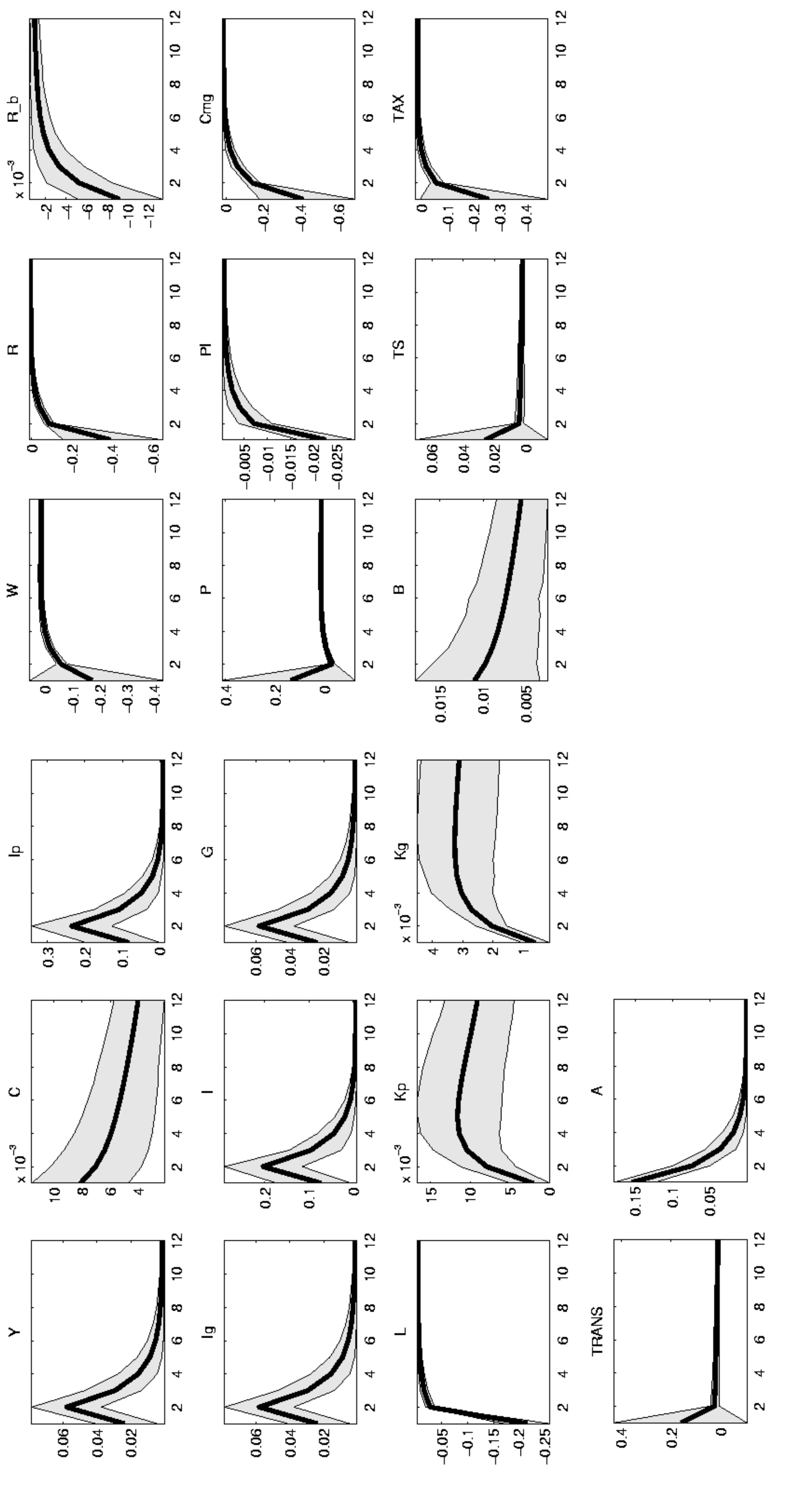

FIGURE A.1: Impulse Responses to a Transitory Technology Shock.
Source: Prepared by the author.

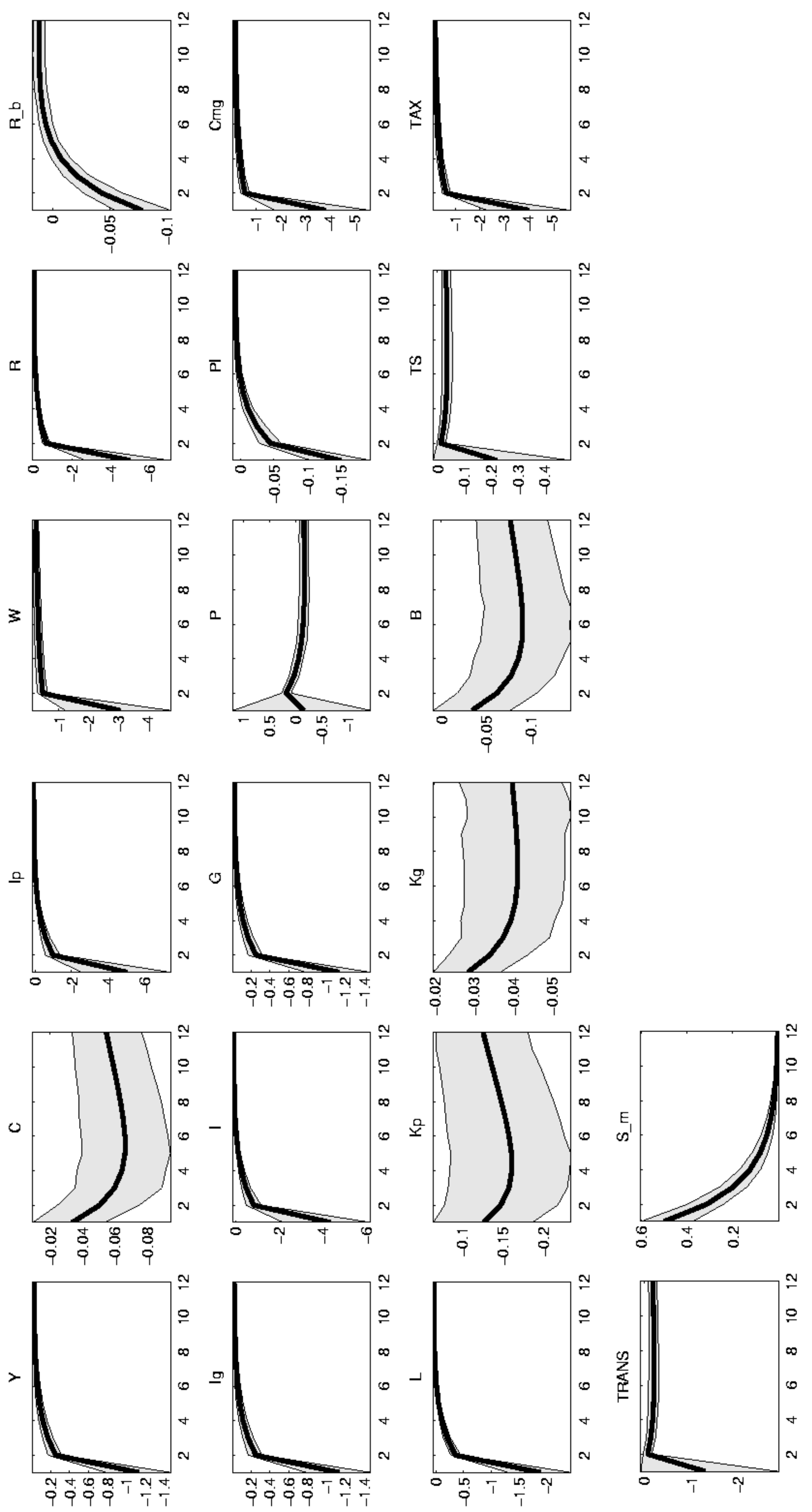

FIGURE A.2: Impulse Responses to a Monetary Policy Shock.
Source: Prepared by the author.

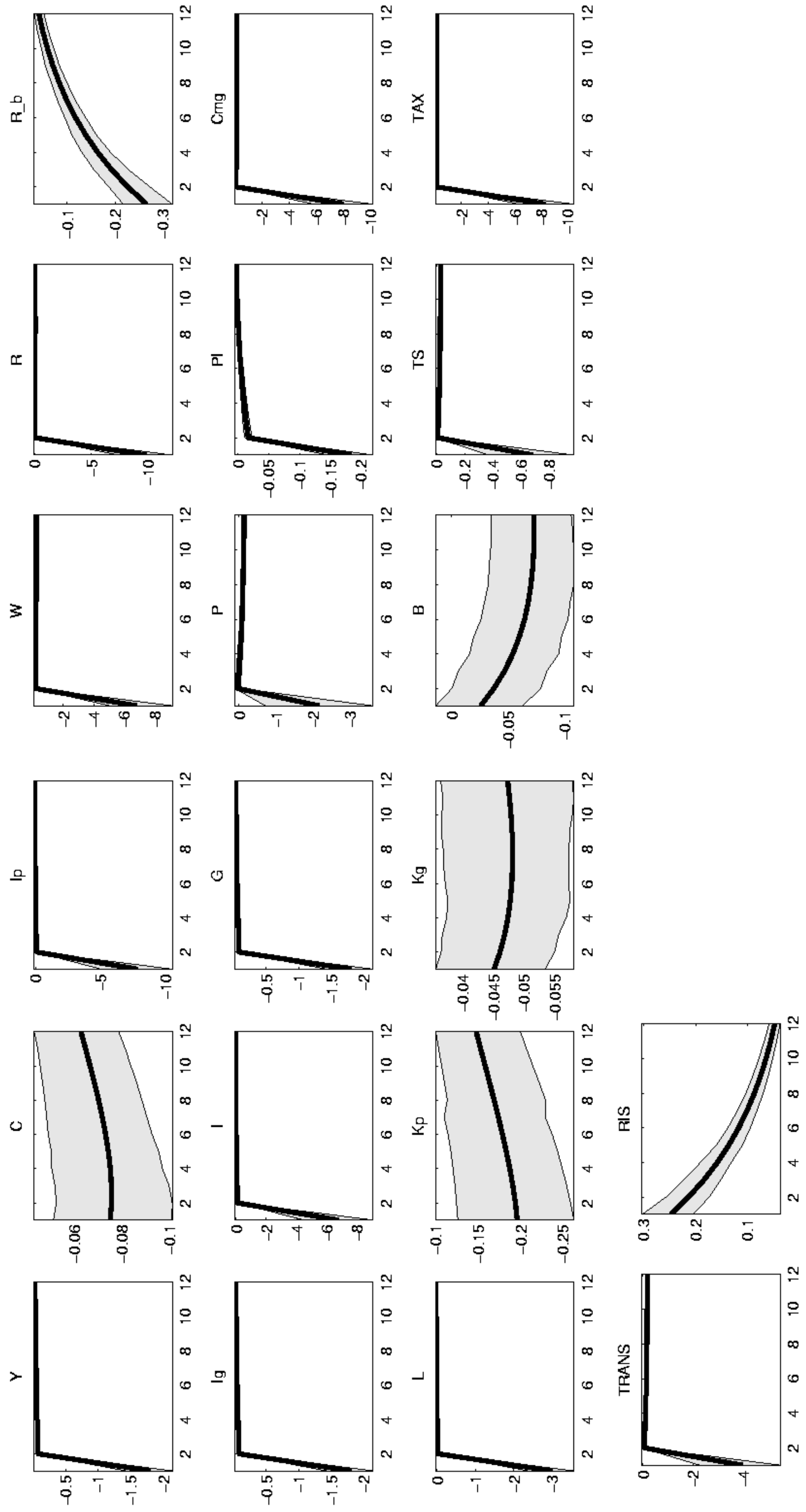

Figure A.3: Impulse Responses to a Risk Premium Shock.
Source: Prepared by the author.

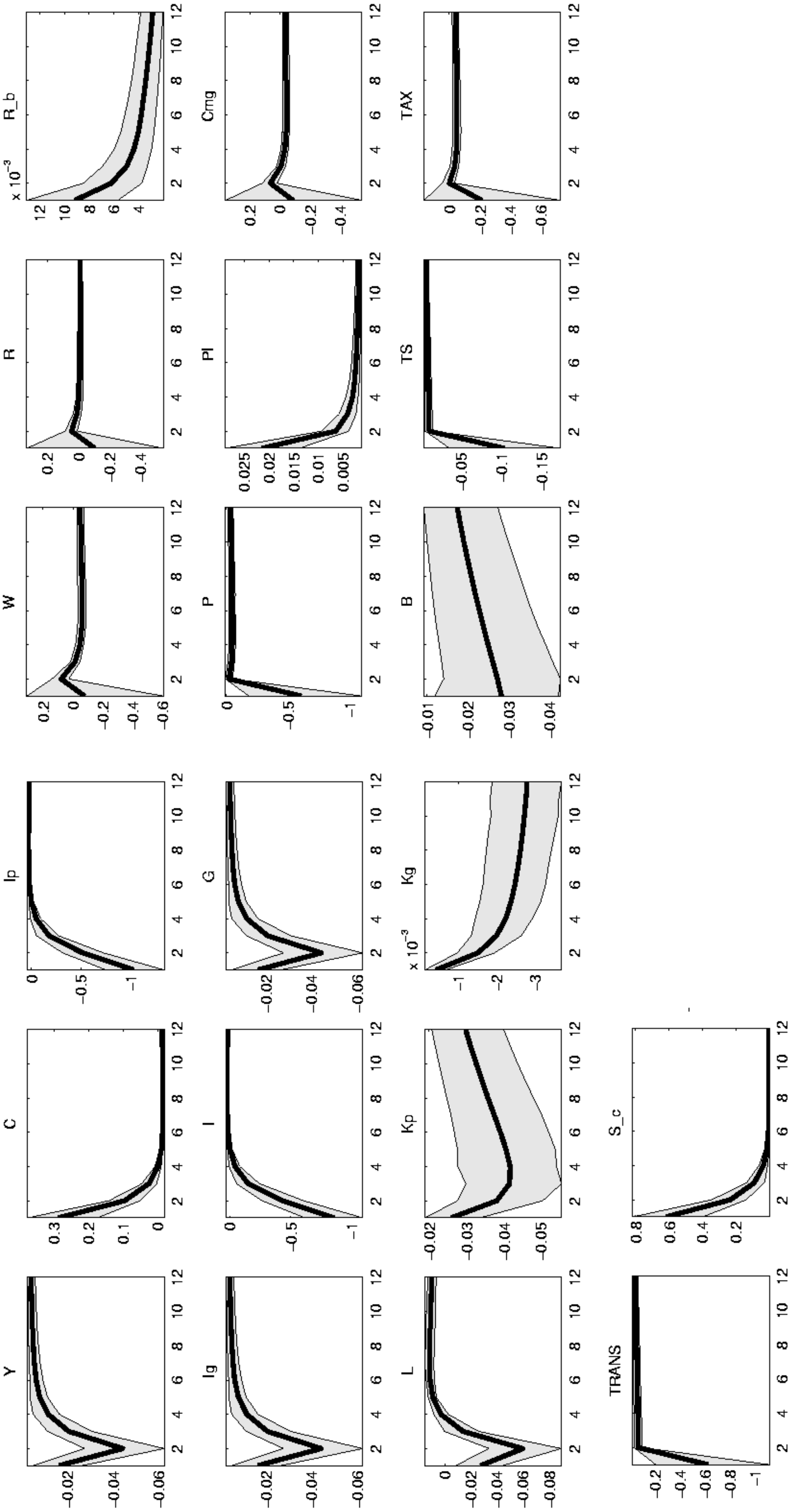

FIGURE A.4: Impulse Responses to a Household Preferences Shock.
Source: Prepared by the author.

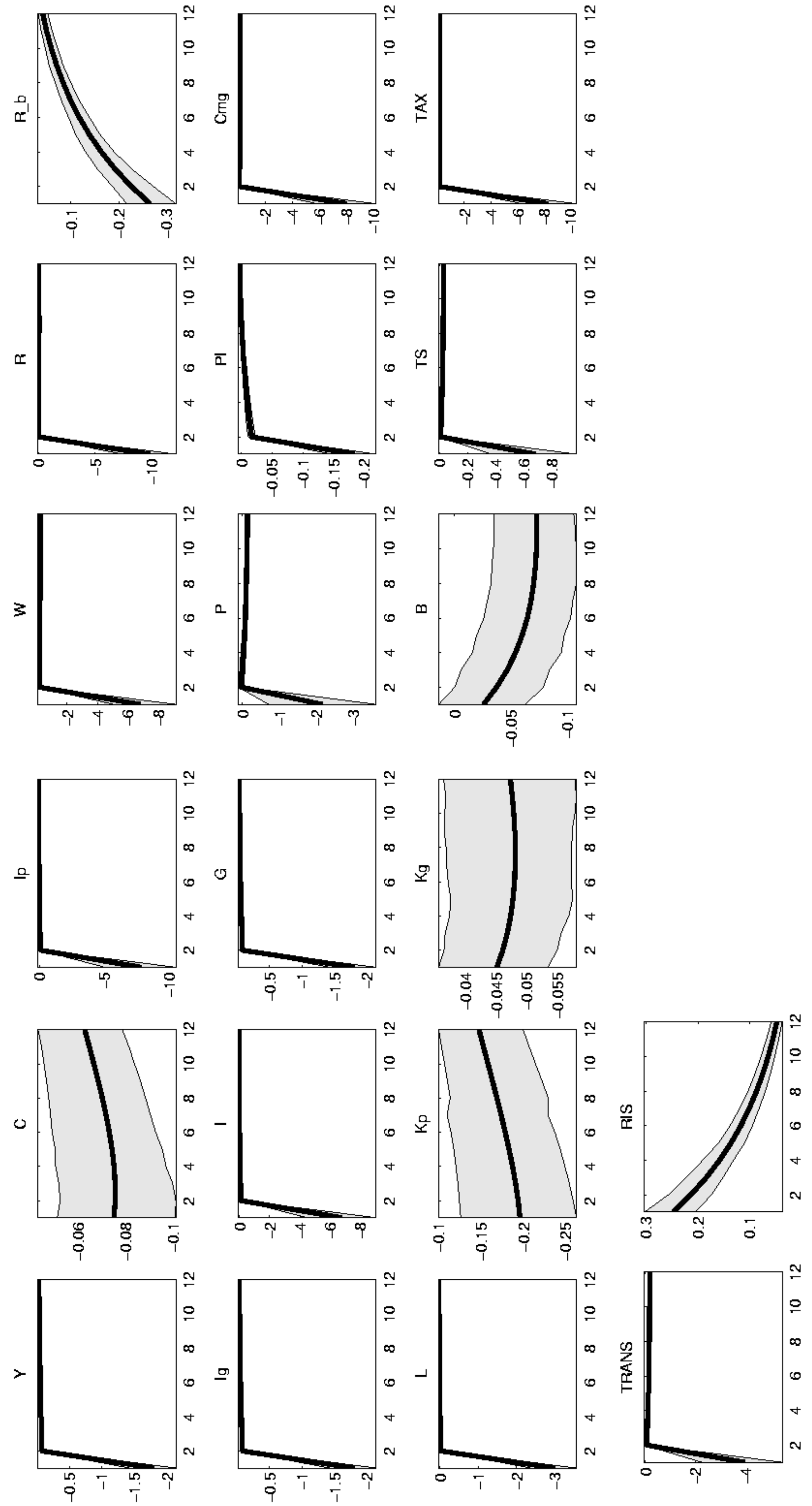

FIGURE A.5: Impulse Responses to a Supply Labor Shock.
Source: Prepared by the author.