UNIVERSIDADE FEDERAL DO PARANÁ

LUCAS PIOLI REHBEIN KÜRTEN IHLENFELD

DISSERTAÇÃO DE MESTRADO

POWER TRANSFORMER PASSIVITY ENFORCEMENT: PRE- AND POST-PROCESSING APPROACHES

CURITIBA

2015

LUCAS PIOLI REHBEIN KÜRTEN IHLENFELD

POWER TRANSFORMER PASSIVITY ENFORCEMENT: PRE- AND POST-PROCESSING APPROACHES

Dissertação apresentada como requisito parcial à obtenção do grau de Mestre. Programa de Pós-Graduação em Engenharia Elétrica, Setor de Tecnologia, Universidade Federal do Paraná.
Orientador: Prof. Dr. Gustavo Henrique da Costa Oliveira

CURITIBA

2015

I25p Ihlenfeld, Lucas Pioli Rehbein Kürten
Power transformer passivity enforcement : pre- and post-processing approaches/ Lucas Pioli Rehbein Kürten Ihlenfeld. – Curitiba, 2015.
94 f. : il. color. ; 30 cm.

Dissertação - Universidade Federal do Paraná, Setor de Tecnologia, Programa de Pós-graduação em Engenharia Elétrica, 2015.

Orientador: Gustavo Henrique da Costa Oliveira .
Bibliografia: p. 86-94.

1. Transformadores elétricos. 2. Perturbação (Matemática). 3. Teoria da aproximação. 4. Modelagem de dados. 5. Transitórios (Eletricidade). I. Universidade Federal do Paraná. II.Oliveira, Gustavo Henrique da Costa. III. Título.

CDD: 621.31042

TERMO DE APROVAÇÃO

LUCAS PIOLI REHBEIN KÜRTEN IHLENFELD

POWER TRANSFORMER PASSIVITY ENFORCEMENT: PRE- AND POST-PROCESSING APPROACHES

Dissertação aprovada como requisito parcial à obtenção do grau de Mestre no Programa de Pós-Graduação em Engenharia Elétrica da Universidade Federal do Paraná, pela Comissão formada pelos professores:

Prof. Dr. Gustavo Henrique da Costa Oliveira (Orientador)
PPGEE, UFPR

Profa. Dra. Elizete Maria Lourenço
PPGEE, UFPR

Prof. Dr. Eduardo Gonçalves de Lima
PPGEE, UFPR

Prof. Dr. Antonio Carlos Siqueira de Lima
COPPE, UFRJ

Curitiba, 14 de Setembro de 2015.

LUCAS PIOLI REHBEIN KÜRTEN IHLENFELD

POWER TRANSFORMER PASSIVITY ENFORCEMENT: PRE- AND POST-PROCESSING APPROACHES

Dissertação aprovada como requisito parcial à obtenção do grau de Mestre no Programa de Pós-Graduação em Engenharia Elétrica da Universidade Federal do Paraná, pela Comissão formada pelos professores:

Orientador: Prof. Dr. Gustavo Henrique da Costa Oliveira
Departamento de Engenharia Elétrica, UFPR

Profa. Dra. Elizete Maria Lourenço
Departamento de Engenharia Elétrica, UFPR

Prof. Dr. Eduardo Gonçalves de Lima
Departamento de Engenharia Elétrica, UFPR

Prof. Dr. Antonio Carlos Siqueira de Lima
COPPE, UFRJ

Curitiba, 14 de Setembro de 2015

RESUMO

Esta dissertação trata, em bases matemáticas, do estudo das técnicas de aferição e imposição da passividade, uma propriedade qualitativa, geral e fundamental de transformadores. Para esse propósito, são propostas duas novas abordagens: uma de perturbação de dados no domínio da frequência, chamada pré-processamento, bem como um novo procedimento de perturbação de parâmetros no domínio do tempo, denominado pós-processamento. Inicialmente, métodos de aferição da passividade são empregados para distinguir sistemas passivos dos não-passivos bem como caracterizar as violações. Verificadas violações de passividade nos dados, usualmente devidas ao processo de medição, estes mesmos dados são perturbados, configurando o pré-processamento, de modo que todas as violações sejam suprimidas. Tal procedimento envolve encontrar matrizes de perturbação que, em cada frequência, atinjam esse objetivo causando, ao mesmo tempo, e em certo sentido, a menor perturbação possível. Os dados já passivos podem ser identificados e um modelo então obtido. Como dados passivos não garantem a obtenção de um modelo passivo, faz-se mister a imposição da passividade ao modelo obtido. Apesar de conduzir a resultados mais precisos, o pré-processamento de dados não é condição *sine qua non* para obtenção de modelos passivos. O procedimento de pós-processamento é que, *per se*, assegura a passividade, permitindo que este seja empregado de forma independente daquele. Por meio de resultados obtidos com dados experimentais, demonstra-se, de forma individual e conjunta, a validade das técnicas ora propostas.

Palavras-chave: Transformadores de Potência, Perturbação de dados, Perturbação de Parâmetros , *Passivity-Enforcement*, Modelagem, Análise de Transitórios.

ABSTRACT

This dissertation addresses the problem concerning the mathematical assessment and enforcement of passivity, a qualitative, general and fundamental property of power transformers. For serving that purpose, two novel approaches are introduced: a pre-processing approach consisting of frequency-domain data perturbation as well as a post-processing one comprising a time-domain parameter perturbation. Initially, passivity assessment methods can be used to distinguish passive systems from non-passive ones and characterize passivity violations. As data can reveal passivity violations owing to the data acquisition process, it is pre-processed so that violations be suppressed. This procedure entails finding a data perturbation matrix that achieves such objective and causes a least possible perturbation, in some sense. Passive data can be identified and a model then extracted. Since passive data does not ensure the extraction of a passive model whatsoever, the employment of passivity enforcement is an indispensable resource for fully guaranteed model passivity. Despite leading to more accurate results, pre-processing is not a *sine qua non* for obtaining passive models. It is passivity enforcement that *per se* ensures model passivity, thus allowing post-processing to be used regardless of pre-processing. Underpinned by results achieved upon experimental data, the effectiveness of the methods herein proposed are individually and jointly confirmed.

Key-words: Power Transformers, Data Perturbation, Model Parameter Perturbation, Passivity-Enforcement, Modelling, Transient Analysis.

To Amanda,

fide et amore

ACKNOWLEDGEMENTS

First and foremost I would like to thank my diligent advisor Professor Gustavo Henrique da Costa Oliveira for being generous with his time, expertise, guidance and encouragement. There are other faculty members whom I am most grateful to, I particularly thank Professors Eduardo Gonçalves de Lima, Elizete Maria Lourenço and Antonio Carlos Siqueira de Lima. Their valuable suggestions and judicious comments removed some deficiencies, corrected some errors of interpretation and improved some of the ideas contained in this document. Of course, none of them are to blame for any remaining errors.

I am deeply indebted to my family for supporting me along this journey, specially my wife, mother and grandmother: Amanda, Viviane and Lindamir. It is my greatest pleasure to acknowledge the constant care, assistance and encouragement of my love and wonderful wife, Amanda. Special thanks to my uncle, Mauro Pioli Rehbein, for both support and encouragement.

Finally, it is also a pleasure to publicly acknowledge the financial support provided by CAPES, the Brazilian Federal Agency for Support and Evaluation of Graduate Education within the Ministry of Education of Brazil.

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

$\{w_i\}_{i=1}^{K}$ - set of measurement frequencies

ω_i - ith frequency in the set of measurement frequencies

$m \times n$ - m and n are a matrix's input/output dimensions

il - matrix entry indexes associated with the ith output and lth input

$\mathbf{Y}(jw)$ - Admittance Matrix at frequency w

$Y_{il}(jw)$ - Admittance Matrix il entry at frequency w

$\mathbf{Z}(jw)$ - Impedance Matrix at frequency w

$Z_{il}(jw)$ - Impedance Matrix il entry at frequency w

$\mathbf{S}(jw)$ - Scattering Matrix at frequency w

$S_{il}(jw)$ - Scattering Matrix il entry at frequency w

$\mathbf{H}(jw)$ - Transfer/Hybrid Matrix at frequency w

$H_{il}(jw)$ - Transfer/Hybrid Matrix il entry at frequency w

$\mathbf{G}(jw)$ - Conductance Matrix at frequency w

$G_{il}(jw)$ - Conductance Matrix il entry at frequency w

$\mathbf{B}(jw)$ - Susceptance Matrix at frequency w

$B_{il}(jw)$ - Susceptance Matrix il entry at frequency w

$\mathbf{i}(jw)$ - Current Vector at frequency w

$I_i(jw)$ - ith component of a current vector

$\mathbf{I}$ - Identity Matrix

$\mathbf{v}(jw)$ - Voltage Vector at frequency w

$\mathbf{v}^{+}(jw)$ - Incident scattering variable vector at frequency w

$\mathbf{v}^{-}(jw)$ - Reflected scattering variable vector at frequency w

$V_i(jw)$ - ith component of a voltage vector

Superscript * - complex conjugate

Superscript ^{T} - Matrix transpose

Superscript ^{H} - Matrix conjugate (Hermitian) transpose

$\lambda(.)$ - operator extracting the set of eigenvalues of a given matrix

λ_{min} - minimum eigenvalue in the set of eigenvalues of a given matrix

$\sigma(.)$ - operator extracting the set of singular-values of a given matrix

σ_{max} - maximum singular-value in the set of singular-values of a given matrix

$(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ - is a state-space realization of an LTI system

$\mathbf{x}(t)$ - state vector $\in \mathbf{R}^n$

$\dot{\mathbf{x}}(t)$ - state-vector derivative $\in \mathbf{R}^n$

$\mathbf{y}(t)$ - output vector $\in \mathbf{R}^m$

$\mathbf{M}_\alpha$ - Hamiltonian Matrix for hybrid parameters

α - Hybrid Hamiltonian Matrix's parameter for adjusting threshold

$\mathbf{M}_\gamma$ - Hamiltonian Matrix for scattering parameters

γ - Scattering Hamiltonian Matrix's parameter for adjusting threshold

Ω - Set of purely imaginary eigenvalues of the Hamiltonian matrix in ascending order

$\mathbf{T}$ - Half-size test matrix

$\otimes$ - Kronecker product

$\mathbf{\Theta}$ - vector of parameters

$\mathbf{\Delta}$ - Perturbation matrix

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Introduction

Dynamical models are mathematical constructs chiefly devised to yield the finest possible predictions of every imaginable aspect of the surrounding world. In the early days of dynamics the astounding accuracy achieved in celestial mechanics to describe planetary motion prompted an obsession with prediction making at the highest level that has persisted unabated throughout time. Along this intellectual journey many fruitful ideas have flourished leaving a rich legacy of conceptual frameworks that once resided in the arcane realms of pure science and eventually spread out to a vast breadth of application fields to solve a myriad of problems. Progress however comes in fits and starts, the real pinnacles or revolutionary ideas come very infrequently. In electrical engineering, dynamic systems abound. A formidable example of intricacy is provided by electrical networks and their constituent parts, each part a system in its own right. Among others, a ubiquitous, prominent and essential component of such networks are power transformers. These are not only indispensable to a reliable steady-state system operation but also highly sensitive to the seemingly inexorable occurrence of transient events which may ultimately lead to their own failure and possibly widespread power disruption. Today's ferocious energy-dependent economies cannot afford accidental power outage due to isolated incidents, not to mention the prohibitive costs impinging on the budgets of electricity utilities whenever power transformers have to be replaced prematurely. These critical needs have prompted significant research from both manufacturers and researchers over the causes of transformer failure. There is a wide diversity of attributable causes related to transformer malfunction and failure. These claims are substantiated by statistics drawn from JWG-A2/C4.39 (2014), Hori et al. (2007) and Shipp et al. (2011) that attribute equipment malfunction and eventual breakdown to electromagnetic transients generated by dynam-

ical interactions between transformers and other neighbouring network components. An investigation conducted in Brazil Bechara (2010) reported that 6 out of 20 step-up transformers under analysis most likely owed their failures to very fast transient over-voltages. In many transient studies, e.g. lightning or switching overvoltage studies, power transformers have been successfully modeled as linear time-invariant dynamical systems since non-linearities are of lesser importance in the frequency range of interest. The use of linearity would have been precluded in case core non-linear effects to simulate inrush currents and ferroresonance must be considered.

General-purpose time domain simulation tools for studying transient behavior of electrical networks have been constantly evolving in both scope and capabilities, illustrative instances of such computer-aided tools include the Electromagnetic Transients Program (EMTP), Alternative Transients Program (ATP), PSCAD and so forth. These software packages have long underpinned multiple transformer engineering decisions ranging from the early product-design phase to everyday operational occurrences thus further subsidizing improved design choices (chiefly those concerning insulation design and coordination, as in Transformers-Switchgear-Committee (2010) and JWG-A2/C4.39 (2014)), as well as permitting more accurate security measures, contingency plans and locating potential sources of problem. Indeed, time domain electromagnetic transient simulations can provide much understanding of and fascinating insights into transformer dynamics, with the caveat that the simulator be correctly parameterized. Most engineers endeavouring to generate models from measured data or running network-level time domain simulations are almost certainly no experts in dynamic systems, system identification, rational approximation, passive networks or any other seemingly arcane research area. However, a holistic approach to the problem requires some system-level knowledge so that meaningful results be achieved, precluding the unintentionally cavalier attitude when one sometimes hopes to accidentally type in a valid parameter set, thus corroborating the GIGO paradigm (Garbage in, garbage out). Consequently, transient simulations can only be effective if well underpinned by carefully devised models.

In a more general setting, questions arise about what properties should models be

endowed with so that simulators achieve their best possible performance and thriving simulation results ensue. Ideally, a suitable model must accurately reflect every feature of the actual system to be emulated. In qualitative analysis of linear systems, the usual intrinsic properties pertaining to such systems can be enumerated as: causality, stability, passivity, controllability, observability etc. As described in Chen (2012), these fundamental properties comprise a necessary condition for obtaining a consistent description (model) of the physical system.

According to Aguirre (2007) and Ljung (2010), even though it is not unanimously standard in the literature, the model identification problem has been subsumed into three main categories, namely *white-box*, *black-box* and *grey-box* models, such that:

- *White-box* models comprise those based on first principles, i.e. the physical laws governing the system behavior hence requiring in-depth knowledge of all intrinsic properties of the actual system. By way of illustration, devising a *white-box* transformer model would require information usually regarded as strictly confidential, e.g., equipment geometry and properties of manufacturing materials, thus preventing this approach from being employed by most independent practitioners due to industrial and trade secrets.

- *Black-box* models constitute an experimental data-based technique which requires little or no prior knowledge of the system being modeled that has gained preference among many modelers for it only depends on exogenous input/output measurements, either in the time or frequency domains.

- *Grey-box* modeling is conceived as in-between *Black* and *White-box* models, usually relying on any auxiliary and additional information beyond the initial data, varying degrees of shade can be achieved depending on what and how information enters the problem, further details can be found in Ljung (2010).

Wide-band models have become the tool of the trade in transient simulations. Nonetheless, achieving representative simulation results is only possible should the model bear all the intrinsic properties of the actual system being emulated. For a broad perspective

and some illustrative case studies of black-box modeling of power transformers in the frequency-domain, refer to Akçay et al. (1998), Gustavsen & Semlyen (1998), Gustavsen (2010), Reginato & Oliveira (2007), Reginato & Oliveira (2008) , Reginato (2008),OLIVEIRA (2009), Maestrelli & Oliveira (2010), Maestrelli (2010) and Oliveira & Mitchell (2013).

1.2 Passive Systems

As pointed out by Coelho et al. (2004), there is a common misconception that passive data yields a passive model by simply estimating model parameters that result in a sufficiently faithful approximation, but it proves not to be a strong enough condition to rely on when devising a consistent model. However counter-intuitive this statement may sound, model accuracy implies neither model passivity nor consistency, not to mention the frequent occurrences of data already corrupted by passivity violations. According to Triverio et al. (2007), accuracy is a major concern but it is neither the only nor the most important feature of any model and, in order to be consistent, models of passive systems must bear three fundamental properties, namely stability, causality and passivity; such that the latter is the strongest for it implies both causality and stability. Network-level simulations require that different systems be interconnected to form an overall network. Intuitively, passive system are those unable to generate energy. This definition can be formalized in terms of Lyapunov functions as in reference Willems (1972) which also offers a proof that the interconnection of dissipative systems leads to an overall dissipative (passive) system, which mathematically means that passivity possesses a closure property. It was once thought stability was closed by compounding various systems but unlike passivity, stability and causality are not closed under system composition, thus also reinforcing the need to draw a clearer distinction between stable and passive systems, which is of major importance when devising models for transient simulations. Therefore, as mentioned in Coelho et al. (2004), merging stable models with other stable or even passive ones does not imply that the resulting system be stable. Moreover, network-level instability cannot be predicted at the moment each individual model is devised, for the ultimate response is strongly dependent upon the properties of each constituent model forming the

overall network, thereby leading to a complex interplay between the network and each of its constituent parts. For that reason, network-level compatibility can only be *a priori* guaranteed by enforcing model compliance with passive behavior. Otherwise, nonphysical anomalies may compromise any time-domain simulation, e.g., terminal quantities that grow arbitrarily with no bounds.

Consequently, passivity has been broadly explored in the literature as a golden model property and has been fully incorporated as a modeling task as well. Passivity is a power-transformer fundamental property which cannot remain an unmodeled effect. As a consequence, when deriving a mathematical model from transformer's own data, means to retain this characteristic feature are indispensable. Not only for the sake of transient simulation but also because power transformers are intrinsically passive systems. Nonetheless, the ubiquitous black-box modeling technique has strong reliance on data that has been invariably subjected to corrupting factors during its acquisition, since existing technology and instrumentation are so sensitive that isolation from external disturbances is often impossible, e.g., stray capacitances and inductances due to measurement cables, numerical noise, round-off error and so forth. As a result, the data thus produced is inherently noisy and systematic data collection constitutes a problem in its own right. Furthermore, even during model parameter estimation, there is no guarantee whatsoever that the passivity feature be preserved. As it cannot remain an unmodeled effect, models must undergo a passivity enforcement insofar as the system's original passive behavior be recovered. Since any model must retain the intrinsic principles and physical properties of the system it is supposed to emulate, irrespective of the technique employed in its derivation. From a passivity perspective, accuracy is somewhat relative.

1.3 Systematic approaches to passivity

Many impressive strides have been made in the relentless quest to address the issue of passivity enforcement. Most endeavours have concentrated on model passivity as opposed to data passivity. The former is generally framed as a post-processing approach since it occurs in a stage subsequent to model identification whereas the latter comprises the pre-

processing approach which in turn is applied to raw data, i.e., preceding the model identification. Post-processing is a mature subject, pre-processing is still in its infancy. Model passivity enforcement, i.e., post-processing can be divided into two classes of enforcement strategies, namely perturbative and non-perturbative enforcement schemes. Perturbative enforcement relies on model parameter perturbation to recover passivity when a model reveals passivity violations. Perturbative enforcement algorithms use different strategies to achieve the final passive behaviour, some methods are based on discrete frequency methods such that common implementations include Linear Programming (LP) in Saraswat et al. (2004), Quadratic Programming (QP) in Gustavsen & Semlyen (2001) and Second-order Cone Programming (SOCP) in Grivet-Talocia & Ubolli (2008). Another perturbative method uses a Hamiltonian Perturbation as in Grivet-Talocia (2004). On the other hand, non-perturbative methods estimate parameters leading to passive behaviour. Instances of non-perturbative methods comprise those based on the positive-real lemma, namely Coelho et al. (2004) and the one originated from this research Oliveira et al. (2014), as well as the Positive Fraction Vector Fitting - PFVF in Tommasi et al. (2011).

As opposed to the traditional model passivity-enforcement, comparatively small amount of effort has been devoted to data pre-processing on the grounds that passive data does not ensure a passive model. In references Morched et al. (1993) and Gustavsen & Semlyen (1998) instances of preprocessing based on matrix diagonalization were used to obtain models and reduce computation times. A more recent example can be found in Gustavsen (2014), where the problem of accurately representing small eigenvalues is resolved by the introduction of a similarity transformation matrix, thus better revealing the eigenvalues of the admittance matrix; the transformation preserves passivity and symmetry. The herein proposed pre-processing ventured into a different direction: pre-processing is used to suppress passivity violations corrupting the data used in the further identification process.

1.4 Objectives

1.4.1 General Objective

This research's prime objective is to propose a systematic approach to the passivity problem pertaining to the modeling of power transformers derived from its own terminal data for the purposes of transient simulations. Passivity enforcement can be properly framed as a twofold task, since violations usually arise in both transformer data and model.

1.4.2 Specific Objectives

The primary research objective can be broken down into the following set of sub-objectives:

- Propose a pre-processing methodology based on data passivity compensation;
- Propose a post-processing methodology based on model passivity compensation;
- Unify the pre- and post-processing into a single passivity enforcement procedure;
- Apply and validate the proposed methodology on transformer actual data;

1.5 Research Contributions

The contributions of this research are stated as follows:

- A Pre-processing approach is herein formulated which can improve model accuracy by suppressing existing passivity violations corrupting transformer measured data. The Convex formulation employed is mathematically sound and preserves intrinsic properties pertaining to power transformers such as symmetry and passivity, thus judiciously obtaining perturbation matrices that are mathematically and physically consistent with the problem. This contribution has been published in the IEEE Transactions on Power Delivery, refer to Ihlenfeld et al. (2015).
- A Post-processing formulation is proposed which involves the building and formulation of a convex optimization methodology based on Linear Matrix Inequalities

(LMI-based), intended for obtaining an optimal, passive, black-box model for power transformers since there is no guarantee whatsoever of identifying passive black-box models without enforcement. This contribution has been published in the IEEE Transactions on Power Delivery, refer to Oliveira et al. (2014).

1.6 Document Outline

The outline of this document is as follows: in chapter one a basic exposure and contextualization of the problem in the realm of electrical engineering is made. Subsequently, essential background foundations as well as assessment methods are laid in chapter two. Next passivity enforcement schemes applicable to data and model form the theoretical substance of chapters three and four, respectively. Chapter five then introduces some case studies to corroborate and validate the previously expounded theory. Conclusions and prospects for further research constitute the closing chapter.

CHAPTER 2

BACKGROUND AND PASSIVITY ASSESSMENT

This chapter commences with a brief review of some basic concepts and definitions from linear systems and networks, focusing on making explicit the assumptions that underly the ensuing exposure as well as steering the discussion to the point of view of linear systems as n-ports (n-terminals). Since the treatment is by no means encyclopedic, and is meant mostly to set out the notation, terminology and context pertaining to the research, fine mathematical details should be omitted to avoid heavy notations and long derivations. The second part of the chapter is devoted to passivity assessment methods which are essentially mathematical criteria that permit distinguishing passive systems from non-passive ones. The exposure on assessment is a consolidation of the techniques currently in vogue in the technical literature.

2.1 Background

The starting point for this discussion about passivity entails some further clarification on how transformers actually enter into the discussion. When considering black-box modelling, the experimental data upon a model is further derived sets the way in which the whole process develops. A decision has therefore to be made as to the measurement setup for data acquisition which assuredly depends on the available instrumentation. As usual, one can either have at his disposal time or frequency domain measurements thus leading to a time or frequency domain modelling approach, this research concentrates exclusively on the latter. The justification for proceeding in this manner is a rather simple one: the data underpinning this research happens to be in the frequency domain.

The measurements that serve as input to this investigation are indeed frequency response measurements comprising a set of experimental data having the following usual format:

$$\{(\omega_k, |H(\omega_k)|, \phi(\omega_k)) \mid k = 1, \ldots, K\}, \tag{2.1}$$

where ω_k, $|H(\omega_k)|$ and $\phi(\omega_k)$ correspond to angular frequency, magnitude and phase of the frequency response. This initial step leads to a set of tabulated frequency responses of the structure under investigation. Such system description contains all the needed information concerning the sinusoidal steady-state response of the system (or network) function for several frequency points. Assuming the Laplace Transform is used to describe a given continuous-time linear time-invariant (CT-LTI) system, its network function $H(s)$ is, by definition in Chen (2012) and Desoer & Kuh (1969), an algebraic function of s (the complex frequency or Laplace variable), relating the Laplace Transform of the input to that of the output with a zero initial state (initially inert system). In a more general setting, a linear system is, from a mathematical viewpoint, one whose behaviour is governed by linear equations, whether linear algebraic, linear difference equations, or linear differential equations. This general description of linear systems contained in Cheng (1972) paves the way for a definition of network function irrespective of the Laplace Transform: the network function is a relation between input (excitation) and output (response) by a linear differential equation or linear difference equation whether in the continuous or discrete time respectively.

According to Desoer & Kuh (1969), amplitudes and relative phases of a given sinusoidal input and output are easily measurable quantities and they are related to the network function via the frequency response, since the latter is nothing but the earlier, namely $H(s)$, evaluated at $s = jw_k$. This is how it is possible to experimentally determine the network function of a system without any knowledge of its topology and its constituent element values, that is to say, a black-box description. As a result, this analysis is bound by the experimental data to be linear. Hitherto, the words system and network have been used interchangeably, even though the emphasis has lain in the latter. Henceforth, to make the wording unambiguous and tailor it to the purposes of this work, the network or, more precisely, the n-port viewpoint of a linear system as in reference Wohlers (1969) is considered. In electric network theory, the word *port* has a

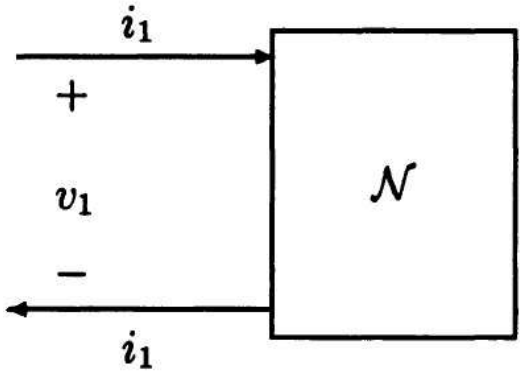

Figure 2.1: The schematic of a LTI one-port.

special meaning. In a somewhat non-rigorous but customary sense, Kuo (1966) defines a port as a pair of terminals in which the current into one terminal equals that of the other terminal and is completely specified when the voltage-current relationship at the terminals of the port is given. Figure (2.1) is an illustration of this concept. As long as linearity is concerned, whether this *one*-port comprises a single resistor or a number of interconnected RLC lumped components is of little importance for it can be completely described by an easily measurable set of experimental data similar to that of equation (2.1).

However useful and effective this concept of *one*-port may be, it is essentially applicable to scalar problems but it can, nonetheless, be further expanded to the more general multivariable case thus leading to the n-port concept. This generalization is not only of theoretical interest but also of a practical one for there exist many electrical devices requiring more than just two terminals (*one*-port) for their mathematical description, e.g. a power transformer.

For establishing the n-port concept, one simply builds on the former, scalar concept: instead of a single pair of terminals there are as many as n pairs, or alternatively $2n$ terminals, and their associated port variables, e.g. voltages across the terminals V_i and input/output currents I_i, as depicted in Figure (2.2). Again, each pair of terminals recursively obeying the conservation of current: the current flowing into one terminal of the pair equals the current flowing out of the other terminal. As succinctly remarked in Allen (2004) and Chua & Lam (1973), any n-port is completely characterized by a

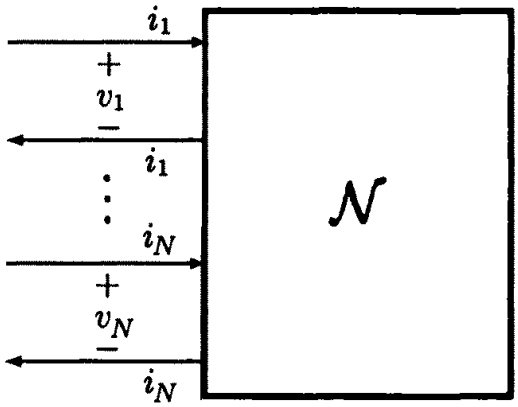

Figure 2.2: The schematic of a LTI n-port.

collection of voltage vectors and current vectors that can appear on its ports.

In reference Huelsman (2011), there is a discussion connecting this idea of n-port and its associated $2n$ variables expressed as pairs of column vectors, with n rows each, related via a $n \times n$ square matrix.

Hence, it is possible to conceive of a $n \times n \times K$ matrix whose every entry consists of a set of data just as the one described by equation (2.1). In fact, as in Horn & Johnson (2012), a matrix is itself a set which allows one to rewrite equation (2.1) in set-theoretic notation. Since describing the data in this set formalism is cumbersome (to which one is referred to Boyd & Chua (1982), Wohlers (1969) and Chua & Lam (1973)), and the majority of the literature uses matrix notation, this set-theoretic notation is not herein pursued. The frequency responses for a n-port at every frequency w_k are essentially linear mappings between collections of inputs (independent variables which may be varied at one's discretion) and outputs (dependent variables which are determined by the network once inputs are specified), i.e. the system acts as a mathematical operator. In summary, one can make fruitful use of the compact matrix notation for describing a LTI network and the Multi-input-Multi-output (MIMO) network function describing a n-port can be thus compactly written as:

$$f(x) = \mathbf{A}x, \tag{2.2}$$

such that $\mathbf{A}$ is a $n \times n$ matrix which defines f as a vector-valued linear function,

$f : \mathbb{C}^n \rightarrow \mathbb{C}^n$, that maps n-vectors to n-vectors. The description of a LTI n-port as a general function $f(x) = \mathbf{A}x$, dissociated from physical quantities, has far-reaching benefits, since, as in Boyd & Chua (1982), this general framework may assume many forms that have special names such as impedance, admittance, hybrid, scattering and transmission.

Each of these special cases are treated as specific formalisms, representations or parameters associated with n-ports, according to Allen (2004), Wohlers (1969), Kuo (1966) and Chua & Lam (1973). Consequently, the form of the system function depends on whether the excitation is a voltage or current source - or even some linear combination thereof - and whether the response is a specified current or voltage, i.e. it depends on what is assigned as port variables (input-output) and how they are grouped, as pointed out in Kuo (1966).

The following subsections contain a discussion on these specific forms of system functions, reflecting the various choices of input-output quantities, thus assigning different physical meanings to equation (2.2) by different groupings of the voltage and current variables. Owing to the context, this exposure focuses on the frequency-domain, though all results are equally valid for the time-domain.

2.1.1 Admittance Y-Parameters

When the general system matrix $\mathbf{A}$ (equation (2.2)) linearly maps input voltage vectors into output current vectors, it is assigned a specific symbol or notation, namely $\mathbf{Y}$, and is called the admittance matrix. In this particular setting, equation (2.2) can be rewritten in the frequency-domain as:

$$\mathbf{i}(jw) \;=\; \mathbf{Y}(jw)\,\mathbf{v}(jw), \tag{2.3}$$

which is usually referred to in the literature as the admittance representation (or formulation/formalism in a more mathematically inclined discussion, e.g. Allen (2004), Wohlers (1969) or Chua & Lam (1973)) of a network. Similarly, the entries of the matrix $\mathbf{Y}$ are

known to be the admittance parameters associated with the n-port. Equation (2.3), which is in fact a compact notation for a set of linear equations, can be written out as follows:

$$\begin{bmatrix} I_1(jw) \\ I_2(jw) \\ \vdots \\ I_n(jw) \end{bmatrix} = \begin{bmatrix} Y_{11}(jw) & Y_{12}(jw) & \cdots & Y_{1n}(jw) \\ Y_{21}(jw) & Y_{22}(jw) & \cdots & Y_{2n}(jw) \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1}(jw) & Y_{n2}(jw) & \dots & Y_{nn}(jw) \end{bmatrix} \times \begin{bmatrix} V_1(jw) \\ V_2(jw) \\ \vdots \\ V_n(jw) \end{bmatrix}, \tag{2.4}$$

such that each $I_i(jw)$ and $V_l(jw)$ correspond to the current and voltage at ports i and l. Every $Y_{il}(jw)$ entry of the admittance matrix is determined by setting the port voltages to zero (by short-circuits), except at the lth port, while measuring the short-circuit current at the ith port; this statement can be framed more pragmatically as:

$$Y_{il}(jw) = \frac{\text{response}}{\text{excitation}} = \frac{I_i(jw)}{V_l(jw)}\Big|_{V_k(jw)=0 \mid k \neq l}. \tag{2.5}$$

Equation (2.5) corresponds to the procedure used in testing and measurement conditions. The diagonal entries $Y_{ii}(jw)$, obtained when both the excitation and response are measured at the same port (or between the same pair of terminals), are referred to as the short-circuit driving-point admittances whereas the otherwise obtained off-diagonal entries $Y_{il}(jw)$, such that $i \neq l$, as short-circuit transfer admittances.

Valuable information and further details on the measurement setup for admittance matrices can be found in Gustavsen (2004*b*) and Gustavsen (2004*a*), specially considering the context of power transformers, providing useful schematics as well as touching some numerical issues. A thorough study of the impact of the measurement setup, the measuring cables to be more precise, on the admittance matrix's entries can be found in Sans et al. (2012) and Sans (2013).

2.1.2 Impedance Z-Parameters

By interchanging the input and output vectors of equation (2.3), i.e., the system now linearly maps input current vectors into output voltage vectors, the resulting matrix

corresponds to the inverse mapping of the previous formulation and is also assigned a specific symbol, namely $\mathbf{Z}$, such that $\mathbf{Z} = \mathbf{Y}^{-1}$. Similarly, matrix $\mathbf{Z}$ is technically called the impedance matrix and produces the following linear mapping:

$$\mathbf{v}(jw) \;=\; \mathbf{Z}(jw)\,\mathbf{i}(jw), \tag{2.6}$$

whose entries form the impedance parameters associated with the n-port represented by $\mathbf{Z}$ and is equivalent to:

$$\begin{bmatrix} V_1(jw) \\ V_2(jw) \\ \vdots \\ V_n(jw) \end{bmatrix} = \begin{bmatrix} Z_{11}(jw) & Z_{12}(jw) & \cdots & Z_{1n}(jw) \\ Z_{21}(jw) & Z_{22}(jw) & \cdots & Z_{2n}(jw) \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1}(jw) & Z_{n2}(jw) & \dots & Z_{nn}(jw) \end{bmatrix} \times \begin{bmatrix} I_1(jw) \\ I_2(jw) \\ \vdots \\ I_n(jw) \end{bmatrix}. \tag{2.7}$$

Each entry $Z_{il}(jw)$ of the impedance matrix is determined by the ratio of the voltage obtained at the ith port due to a current applied to the lth port and the remaining ports open-circuited (currents are zero). Thus,

$$Z_{il}(jw) = \frac{\text{response}}{\text{excitation}} = \frac{V_i(jw)}{I_l(jw)}\Big|_{I_k(jw)=0\,|\,k\neq l}. \tag{2.8}$$

The diagonal entries $Z_{ii}(jw)$ may be referred to as the open-circuit driving-point impedances whereas the off-diagonals as the open-circuit transfer impedances.

2.1.3 Hybrid

According to Huelsman (2011), for the general system matrix of equation (2.2) to be a set of hybrid parameters, it is simply required that the entries of the output vector $f(x)$ be neither all voltages nor all current variables. Therefore, there is no such thing as the hybrid parameters for a given n-port, but rather various hybrid parameters. In one possible choice of hybrid representation for a n-port, the first $\frac{n}{2}$ coordinates of the n-dimensional input vector can be driving currents whereas the remaining coordinates driving voltages. Conversely, for the n-dimensional output vector, the first $\frac{n}{2}$ coordinates are chosen to

be voltages and the remaining ones are currents. Equation (2.2) is a mapping of input current-voltage hybrid vectors into output voltage-current hybrid vectors, thus called a hybrid matrix. For this particular choice the overall structure assumes the following matrix form:

$$\begin{bmatrix} \mathbf{v}_1(jw) \\ \mathbf{i}_2(jw) \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11}(jw) & \mathbf{h}_{12}(jw) \\ \mathbf{h}_{21}(jw) & \mathbf{h}_{22}(jw) \end{bmatrix} \times \begin{bmatrix} \mathbf{i}_1(jw) \\ \mathbf{v}_2(jw) \end{bmatrix}, \tag{2.9}$$

such that $\mathbf{h}_{11}(jw)$, $\mathbf{h}_{12}(jw)$, $\mathbf{h}_{21}(jw)$ and $\mathbf{h}_{22}(jw)$ are block matrices of dimension $(\frac{n}{2})\times(\frac{n}{2})$ and the input-output vectors are formed by stacking the corresponding vector partitions:

$$\mathbf{v}_1 = \begin{bmatrix} V_1(jw) \\ V_2(jw) \\ \vdots \\ V_{\frac{n}{2}}(jw) \end{bmatrix}; \mathbf{i}_2 = \begin{bmatrix} I_{\frac{n}{2}+1}(jw) \\ I_{\frac{n}{2}+2}(jw) \\ \vdots \\ I_n(jw) \end{bmatrix}; \mathbf{i}_1 = \begin{bmatrix} I_1(jw) \\ I_2(jw) \\ \vdots \\ I_{\frac{n}{2}}(jw) \end{bmatrix}; \mathbf{v}_2 = \begin{bmatrix} V_{\frac{n}{2}+1}(jw) \\ V_{\frac{n}{2}+2}(jw) \\ \vdots \\ V_n(jw) \end{bmatrix}.$$

A closer examination of equation (2.9), reveals that the blocks $\mathbf{h}_{11}(jw)$, $\mathbf{h}_{12}(jw)$, $\mathbf{h}_{21}(jw)$ and $\mathbf{h}_{22}(jw)$ correspond to impedances, voltage-ratio transfer functions, current-ratio transfer functions and admittances. When reduced to a special case of a 2-port, equation (2.9) is usually used in describing simple transistor circuits.

As mentioned, equation (2.9) is a particular choice of hybrid parameters. The input-output vectors could also have an alternating pattern, such as:

$$f(x) = \begin{bmatrix} V_1(jw) \\ I_2(jw) \\ \vdots \\ V_{n-1}(jw) \\ I_n(jw) \end{bmatrix}; \qquad x = \begin{bmatrix} I_1(jw) \\ V_2(jw) \\ \vdots \\ I_{n-1}(jw) \\ V_n(jw) \end{bmatrix}.$$

There are many other possibilities, including those with the condition that both variables from at least one of the ports be part of the input-output vectors, but there can be

impeding exceptions. In fact, as mentioned in Huelsman (2011), since a n-port comprises $2n$ variables (to every port is assigned a current and a voltage), there are c possible ways (number of combinations, without considering the permutations) in which these variables once they have been selected, such that:

$$c = \frac{(2n)!}{(n!)^2}, \tag{2.10}$$

where n is the number of ports. Consequently, there are c different sets of parameters for a given n-port. For the simple case of a 2-port, c equals six. These are the usual *z-parameters*, *y-parameters*, *h-parameters*, *g-parameters*, *Transmission-parameters* and *Inverse Transmission-parameters*. For three and four ports the number of sets is even larger: 20 and 70. In fact, it is even possible to form new variables by means of linear combinations of the voltage-current variables. This representation constitutes a formulation in its own right, namely the scattering parameters.

2.1.4 Scattering S-Parameters

In this conceptually different formulation, each coordinate of the input vector is itself a linear combination of the current and voltage associated to a port. Reference Kuo (1966) stresses that this network characterization is a powerful analytical tool, originally used by transmission engineers. Hence, when equation (2.2) comprises a linear mapping between linear combinations of the port variables to other linear combinations of the port variables, the matrix is called the scattering matrix $\mathbf{S}$ and the following notation is used:

$$\mathbf{v}^{-}(jw) \;=\; \mathbf{S}(jw)\;\mathbf{v}^{+}(jw). \tag{2.11}$$

Equation (2.11) is the compact form of:

$$\begin{bmatrix} V_1^-(jw) \\ V_2^-(jw) \\ \vdots \\ V_n^-(jw) \end{bmatrix} = \begin{bmatrix} S_{11}(jw) & S_{12}(jw) & \cdots & S_{1n}(jw) \\ S_{21}(jw) & S_{22}(jw) & \cdots & S_{2n}(jw) \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1}(jw) & S_{n2}(jw) & \dots & S_{nn}(jw) \end{bmatrix} \times \begin{bmatrix} V_1^+(jw) \\ V_2^+(jw) \\ \vdots \\ V_n^+(jw) \end{bmatrix}, \tag{2.12}$$

where the pluses and minuses, superscripts in the input-output vectors, reflect the original transmission line terminology and allude to incident and reflected voltage waves, or forward- and backward-traveling waves. The input-output column vectors $\mathbf{V}^-(jw)$ and $\mathbf{V}^+(jw)$ in equation(2.11), are the following linear combinations of the port variables, as defined in Kuo (1966) and Huelsman (2011):

$$\begin{aligned} \mathbf{v}^+(jw) &= \tfrac{1}{2}(\mathbf{v}^n(jw) + \mathbf{i}^n(jw)) \\ \mathbf{v}^-(jw) &= \tfrac{1}{2}(\mathbf{v}^n(jw) - \mathbf{i}^n(jw)) \end{aligned}, \tag{2.13}$$

such that $\mathbf{v}^n(jw)$ and $\mathbf{i}^n(jw)$ are the normalized (indicated by the superscript n) port voltages and current. Therefore, it is assigned a pair of variables, also called the scattering variables ($V_i^+(jw)$ as input and $V_i^-(jw)$ as output), to each ith port and the variables in the pair are themselves proportional to the sum and difference (linear combinations) of the port's own voltages and currents. By using a constant factor, called the reference impedance, the normalization does not affect the linearity between the port variables and their normalized combination (equation 2.13). Each port has its own normalization constant r_{oi} and the normalized voltage and current coordinates are chosen such that:

$$\begin{aligned} V_i^n(jw) &= \frac{V_i(jw)}{(r_{oi})^{\frac{1}{2}}} \\ I_i^n(jw) &= I_i(jw).(r_{oi})^{\frac{1}{2}} \\ Z_i^n(jw) &= \frac{V_i^n(jw)}{I_i^n(jw)} = \frac{Z_i(jw)}{r_{oi}(jw)} \end{aligned}, \tag{2.14}$$

the reference impedance (can be chosen at one's own discretion) is taken to the square root to allow the definition of the normalized impedance as the ratio of the normalized voltages and currents. For clarity, assume a special scalar case of equation (2.11), i.e., a

one-port (Figure 2.1):

$$V^{-}(jw) \;=\; S(jw)\, V^{+}(jw). \tag{2.15}$$

The scattering parameter (unique since it is a scalar simplification) can be defined as:

$$S(jw) = \frac{V^{-}(jw)}{V^{+}(jw)} \;=\; \frac{V^{n}(jw) - I^{n}(jw)}{V^{n}(jw) + I^{n}(jw)} \;=\; \frac{Z^{n}(jw) - 1}{Z^{n}(jw) + 1}, \tag{2.16}$$

this interpretation of the scattering parameter (as a ratio containing the normalized impedance) is very useful since it reveals the advantages of the scattering formulation over the other ones. Indeed, if the normalized impedance in equation (2.16) is chosen to equal unity (matched impedances), $S(jw)$ is zero.

The multi-variable analogous is of equation (2.15) is, according to Huelsman (2011):

$$\mathbf{S}(jw) = \; [\mathbf{Z}^{n}(jw) - \mathbf{I}][\mathbf{Z}^{n}(jw) + \mathbf{I}]^{-1}, \tag{2.17}$$

where $\mathbf{Z}^{n}(jw)$ is a diagonal matrix whose entries correspond to the normalized impedances at the various ports and $\mathbf{I}$ is the $n \times n$ identity matrix. Each entry $S_{il}(jw)$ of the scattering matrix is determined by the ratio of the reflected quantity at the ith port due to the incident quantity at the lth port and the remaining ports with matched impedances. Thus,

$$S_{il}(jw) = \frac{\text{response}}{\text{excitation}} = \frac{V_{i}^{-}(jw)}{V_{l}^{+}(jw)}\Big|_{Z_{k}^{n}(jw)=1 \,|\, k \neq l}. \tag{2.18}$$

Since the entries $S_{il}(jw)$ relate scattered or reflected voltage waves from the network to incident voltage waves upon the network, they have been appropriately named scattering parameters. The diagonal entries $S_{ii}(jw)$ correspond to reflection coefficients whereas off-diagonal entries $S_{ij}(jw)$ to forward (below the diagonal) or reverse (above the diagonal) transmission coefficients.

Further details on the scattering parameters can be found in the classical references Kurokawa (1965) and Carlin (1956).

2.1.5 Discussion on Background

These background concepts serve the purpose of elucidating some characteristics of the data to be used in the following discussion and also to provide an illustration of some basic applications of matrices (Linear Algebra) to circuit theory. Such applications will be further expanded and serve as useful tools in the study of the passivity property.

The customary sets of network parameters and their basic properties have been discussed. For all previous network representations or parameters, the input variables can be chosen to be the port voltages, currents or even a linear combination thereof. In the case of the admittance and impedance representations, the coordinates of the input vector are either all voltages or all currents, respectively. For the hybrid representation, the input vector has at least one current coordinate and at least one voltage coordinate, thus allowing for various different choices and permutations. The scattering representation is defined in terms of linear combinations of the currents and voltages.

The difference from one parameter set to another lies in the choice of the independent and dependent variables. Nonetheless, there are some practical issues dictating the parameter choice which is best suited to the application at hand. According to Gustavsen & Silva (2013) and Zhongyuan et al. (2013), there arise a number of difficulties when measuring admittances and impedances at frequencies above 1 MHz. Equations (2.5) and (2.8) require short- and open-circuit conditions for determining the admittance, impedance and hybrid parameters which can be exceedingly difficult to be accurately achieved over a broad band of frequencies since short-circuits behave like "inductors" while open-circuits present leakage and "stray capacitances". Such difficulties can be circumvented by employing the scattering parameters, since these do not depend on open or short circuit conditions to characterize a network; instead, matched loads are used, as in equation (2.18). These terminations provide numerous practical advantages over the open-circuit and short-circuit terminations when higher frequencies are used. Moreover, from a theoretical standpoint, at higher frequencies the lumped-parameter assumption for a linear network does not exactly hold. Therefore, the scattering parameters of a network characterization are necessary so that the characterization be accurate also at

higher frequencies. In Carlin (1956), the author emphasizes that all passive networks possess scattering parameters and, at higher frequencies, incident and reflected parameters play dominant roles while the usual voltage-current description is of lesser importance.

Nevertheless, the decisive factor in choosing the network characterization is the available instrumentation. Scattering-parameter measurements require equipment that is usually not readily available to measure the total voltages and currents at network ports.

Alternatively, when faced with limited resources and lack of the appropriate apparatus, the experimenter might supposedly consider, take admittance measurements and then, reliant on $\mathbf{Z} = \mathbf{Y}^{-1}$, convert them into impedance ones. As far as pure algebra is concerned, parameter conversion poses no problem but for the obvious assumptions of matrix invertibility and so on. However, in the realm of numerical matrix analysis and estimation, this can be problematic. In reference Gustavsen & Silva (2013), the author stresses that, in addition to the initial parameter estimation error, a parameter conversion can contribute to even larger errors. This conversion issue can be as subtle as eigenvalue deterioration, e.g., in Gustavsen (2014), there is discussion on reducing the error magnification due to inaccurately measured small eigenvalues. Should the operation $\mathbf{Z} = \mathbf{Y}^{-1}$ be necessarily carried out, small eigenvalues of $\mathbf{Y}$ become large eigenvalues of $\mathbf{Z}$ thus enlarging the overall error as well. Therefore, parameter conversion is usually discouraged.

In the context of power transformers, it is customary to adopt the admittance representation, the reason why favouring admittance matrices in this discussion. Seen as an n-port, the data-acquisition process for a transformer comprises experimentally determining the frequency-responses for all n^2 entries of the admittance matrix at every frequency in the sequence $\{w_k\}_{k=1}^{K}$ of all measured frequencies. This implies the acquisition, storage and manipulation of as many as n^2K numbers in *floating point format*, or elements of the measurement set in a more stilted mathematical language. In fact, there is a factor of 2 dropped in this count, since every entry consists of magnitude and phase responses (or every entry is a complex number with real and imaginary parts), but in this sort of analysis one is mostly interested in a rough estimate of the order of magnitude. Spe-

cially favourable to the acquisition process, the experimenter need not determine n^2K admittances, since power transformers are special cases of reciprocal networks. In matrix language, this translates as the admittance matrix $\mathbf{Y}$ of a power transformer being a symmetric one. According to Huelsman (2011), reciprocal networks allow the interchange of input (excitation) and output (response) while their ratio remains unchanged. Hence, reciprocity implies $Y_{il}(jw) = Y_{li}(jw)$, i.e., if the excitation is applied at the ith port and the corresponding response measured at the l-port, or alternatively in reversed order, the same resulting ratio (admittance) is obtained. Therefore, the experiment is only performed for the diagonal entries and those above it (or below), substantially reducing the number of measurements.

Another relevant point to be made before proceeding to the next topic concerns the distinction between a n-port and a n-terminal formulation. There has been hitherto only a tepid mention of n-terminals in the opening paragraph of this chapter. The difference between the two does not entail the form (the representation) of the network to which the terminals give access to, but rather how they are externally interconnected. According to Huelsman (2011), since the port voltages are specified with respect to the potential difference between its own two terminals, voltage differences between potentials of terminals belonging to different ports, or any terminal with respect to a reference (grounding point), are essentially meaningless. On the other hand, a terminal representation does not apply when any external component is connected to a single network terminal but is isolated from the other terminals.

In fact, the measurement setup can require a reference node, common or grounded terminal (as in Gustavsen (2004*b*)), thus qualifying the transformer as a n-terminal network. The point in carrying the discussion on the underlying fundamentals in terms of n-ports is well grounded in basically two facts: virtually all the literature of active and passive network theory is developed considering the network on a port basis and, for this research purposes, an n-port can be reduced to a $(n + 1)$-terminal as a special case. A discussion on the equivalence between n-ports and n-terminal networks can be found in Huelsman (2011). Furthermore, the matrix representation and the physical significance

of the variables is identical.

2.2 Transfer Matrix Synthesis

Each of the representations discussed serve as experimental frequency response data from a LTI n-port which is further processed so as to obtain a frequency dependent equivalent (FDE): an algebraic expression (model structure) used to express the dynamics of the LTI n-port as a function of frequency. A survey of available techniques for obtaining such equivalents as well as their structures can be found in Annakkage et al. (2012) and references therein. These equivalents are parametric approximations that supposedly reproduce the data upon their parameters were estimated and predict outcomes for new "unseen" data, i.e., the FDE must also possess a generalization ability. The algebraic expressions herein used to approximate a frequency-dependent transfer matrix $\mathbf{H}(jw)$ are expressed as follows:

$$\mathbf{H}(jw) \;\cong\; \mathbf{D} + \sum_{m=1}^{N} \mathbf{R}_m \Phi_m(jw) \tag{2.19a}$$

$$\mathbf{H}(jw) \;\cong\; \mathbf{D} + \mathbf{C}(jw\mathbf{I} - \mathbf{A}\,)^{-1}\mathbf{B} \tag{2.19b}$$

where $\cong$ denotes an approximation in the least-squares sense. In equation (2.19), the left-hand equivalent comprises a truncated series expansion of a polynomial rational function whereas the right-hand one is a modal equivalent associated with a state-space realization. Approximations of the forms defined in 2.19 can be obtained by means of iterative methods such as the Vector Fitting Algorithm Gustavsen & Semlyen (1999), which is a Sanathanan-Koerner-type algorithm based on the pioneering reference Sanathann & Koerner (1963). Assuming a n-port (n-terminal) LTI system with a N-th order realization, the matrices that constitute the model parameters (to be evaluated) have the following dimensions:

$$\mathbf{H}_{n\times n}, \quad \mathbf{A}_{N\times N}, \quad \mathbf{B}_{N\times n}, \quad \mathbf{C}_{n\times N},$$
$$\mathbf{D}_{n\times n}, \quad \mathbf{R}_{n\times n}.$$

Succeeding parameter estimation, the end result is a model with the aforementioned structures which allow efficient time domain simulations by discretization methods, numerical recursive convolution and time-domain trapezoidal integration of state-space differential equations. According to Anderson & Vongpanitlerd (2006), a state-space realization usually leads to faster and easier computation of time-domain integrations compared with Laplace transforms and their inverses. Furthermore, the state-space approach furnishes a clearer understanding of the internal content of a network description for both analysis and synthesis problems fully underpinned by matrix analysis. The classical Laplace-transform approach places stronger reliance on complex variable analysis instead.

Whichever model structure of equation (2.19) is employed during the initial parameter estimation, there may be a need for conversion of one parameter set into the other. This poses no major difficulty for these parameters are interchangeable, some illustrations of the conversion procedure can be found in Gustavsen & Semlyen (2004) and Gustavsen & Semlyen (2009*a*).

The state-space realization with matrices **A**, **B**, **C**, **D** provides the following time-domain characterization:

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) &+ \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) &+ \mathbf{D}\mathbf{u}(t) \end{cases} , \tag{2.20}$$

where $\mathbf{y}(t)$ and $\mathbf{u}(t)$ are $n-$dimensional time-domain output and input vectors at the terminals and the n-port can be referred to as the system $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$. Equation (2.20) denotes a finite dimensional state representation with $(\mathbf{A}, \mathbf{B})$ controllable, all eigenvalues of $\mathbf{A}$ have negative real part, $\mathbf{D} + \mathbf{D}^T > 0$ and $\mathbf{x}(0) = 0$ (initially inert system). According to Chen (2012), by taking the Laplace Transform of equation (2.20) and after some rearrangements, it yields:

$$\mathbf{H}(s) \;=\; \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A}\,)^{-1}\mathbf{B}\,, \tag{2.21}$$

which is precisely the right-hand side of equation (2.19) and $\mathbf{H}(s)$ is referred to as the transfer matrix of the lumped system $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$, whose transfer function is also as-

sumed rational. The left-hand side of equation (2.21) is also algebraic equivalent to equation (2.2) in the Laplace domain. By way of illustration, should $\mathbf{y}(t)$ and $\mathbf{u}(t)$ in equation (2.20) be respectively $\mathbf{i}(t)$ and $\mathbf{v}(t)$ ($n-$dimensional time-domain current and voltage vectors at the terminals) then $\mathbf{H}(s)$ turns out to be the admittance representation $\mathbf{Y}(s)$. In this sense, a transfer matrix can be understood as more general multi-variable network function. Therefore, the mathematical structures herein employed are restricted to transfer matrices and state-space realizations, the earlier being a input-output (black-box) description whereas the latter can be viewed as a composition of a state-transition mapping and a memoryless read-out function. Henceforth, equation (2.19) and their respective parameter sets associated with any port representation can be referred to as a system under all the assumptions hitherto made.

2.3 Assessment Techniques

Passivity assessment constitutes a system analysis problem intended for determining the inability of a given system to generate energy: the distinguishing feature of passive systems. This analysis can be made rather rigorous with precise mathematical definitions, which in turn depend on the description adopted for the n-port network. According to Willems (1972), "Dissipative systems are of particular interest in engineering and physics. The dissipation hypothesis, which distinguishes such systems from general dynamic systems, results in a fundamental constraint on their dynamical behavior." Power transformers are a typical example of a passive/dissipative system in which part of the electrical energy is dissipated in its windings and core in the form of heat. The terminology dissipative is more formal and can be used as a generalization of the concept of passivity. As early as 1931, besides coining the term positive-real function, Otto Brune (Brune (1931)) first established the association between passivity and positive-real functions. In fact, Brune proved that a rational positive-real function could be realized as a immittance (impedance or admittance) representation of a network consisting of resistors, capacitors, inductors and transformers, i.e., admittance and impedance parameter matrices of passive electrical networks are positive real matrix rational functions.

In 1954 a proof that extends this association to the multi-variable case was provided in Raisbeck (1954). From the late 1950's through the 1960's, electrical engineers had a running debate on the axioms of network theory. As in reference Allen (2004), the referred debate centered on listing the minimal properties of an n-port to obtain the standard formalisms of traditional circuit theory. More formal proofs appeared such as the classical proof D. C. Youla & Carlin (1959) and others based on distribution theory (as in Wohlers (1969) and Zemanian (1963)). Such advanced mathematical background prompted a search for alternative means to prove and assess the passivity property. While positive-realness is a necessary condition, it is not sufficient to establish passivity. In reference Boyd & Chua (1982) a formal proof using general coordinates (which can be derived for any of the representations of the last section) of the passivity criteria is provided, employing Measure Theory but no Distributions. Some methods for assessing passivity of LTI n-ports have been perfected while others have arisen. What follows is a compilation of existing passivity assessment criteria currently being employed to LTI n-ports, starting with classical transfer function/matrix criteria and later evolving to modern methods.

2.3.1 Frequency Sweeping test

Considering a LTI n-port defined in terms of equation (2.21), such that the transfer matrix $\mathbf{H}(s)$ assumes any of the specific representations (as earlier defined: admittance, impedance and etc.) and the following definition:

$$\mathbf{\Phi}(s) = \begin{cases} \mathbf{H}^H(s) + \mathbf{H}(s) & \text{(hybrid representations)} \\ \mathbf{I} - \mathbf{H}^H(s)\mathbf{H}(s) & \text{(scattering representations)} \end{cases}, \tag{2.22}$$

where the superscript ^{H} denotes Hermitian transpose, the LTI n-port thus characterized by $\mathbf{H}(s)$ is positive-real matrix of the complex variable s, as stated in reference Triverio et al. (2007), if the following conditions are satisfied in the open right-half s-plane (i.e., for $\mathrm{Re}(s) > 0$):

1) each $H_{il}(s)$ in $\mathbf{H}(s)$ is defined and analytic;

2) $\mathbf{\Phi}(s) > 0$;

3) $\mathbf{H}(s^*) = \mathbf{H}^*(s)$;

such that the superscript * denotes complex conjugate. The matrix transfer function of a linear time-invariant system that is positive real satisfies analyticity, conjugacy and positive-definiteness. When $\mathbf{H}(s)$ comprises a scattering representation, and since the second condition poses a unitary bound for $\mathbf{\Phi}(s)$, these three conditions are equivalent to requiring bounded-realness, a terminology that appropriately generalizes the fact that each port has reflection coefficient smaller than one, meaning the reflected or transmitted waves can only be a fraction of the incident wave (when considering the incident and reflected waves interpretation for the scattering parameters).

As noted in Triverio et al. (2007), condition one also implies BIBO (Bounded-Input-Bounded-Output) stability, while the second one is a multi-variable generalization for the fact that any passive one-port impedance/admittance has a positive real part. The third condition also ensures that the system impulse response be real. Despite being a side issue that will be later addressed, it is noteworthy that conditions one and three can be even satisfied by default as a direct consequence of the adopted model structure. Satisfying the second condition can be numerically critical, though. Conditions one through three have to be satisfied for all s in the right-half plane ($\mathrm{Re}(s) > 0$) however, according to Wohlers (1969) and Triverio et al. (2007), the lumped-parameter assumption which implies a rational transfer matrix permits testing only along the imaginary axis (i.e., $s = jw$).

Consequently, a simple passivity assessment test broadly used in the literature is:

$$\mathbf{\Phi}(jw) > 0 \qquad \forall\, \omega, \tag{2.23}$$

namely checking positive realness for hybrid representations or bounded realness for scattering ones. In the context of measured port data, equation (2.23) is evaluated only at each frequency in the finite sequence $\{w_k\}_{k=1}^{K}$ of all measured frequencies (as opposed to all the infinite imaginary axis), thus the terminology frequency-sweeping assessment.

Under the working assumption of a symmetric system, in agreement with a power-

transformer application, this frequency-sweeping assessment test can be further tailored to simpler forms. Then, let the LTI n-port measured data to be represented in terms of its admittance matrix, thus $\mathbf{H}(jw) = \mathbf{Y}(jw)$, which is the customary representation for power transformers. Therefore, the positive-realness condition as given by equation (2.23) for all measured frequencies yields:

$$\mathbf{Y}^H(jw_k) + \mathbf{Y}(jw_k) > 0 \qquad \longleftrightarrow \qquad \mathbf{G}(jw_k) > 0. \tag{2.24}$$

In such circumstances, passivity requires that the conductance matrix ($\mathbf{G}(jw_k) = \text{Re}[\mathbf{Y}(jw_k)]$) be positive real. Should the inequality in equation (2.24) hold, then the following inequality also holds:

$$\mathbf{P}_{pot}(jw_k) = \text{Re}\{\mathbf{v}^*(jw_k)\mathbf{Y}(jw_k)\mathbf{v}(jw_k)\} = \text{Re}\{\mathbf{v}^*(jw_k)\mathbf{G}(jw_k)\mathbf{v}(jw_k)\} > 0. \tag{2.25}$$

In the quadratic form of equation (2.25) whose associated physical interpretation is that of absorbed active power, $\mathbf{v}(jw_k)$ is a voltage vector. It is a well-established fact from linear algebra, as can be found in references such as Golub & Loan (2012) and Horn & Johnson (2012), that the eigenvalues of a positive real quadratic form such as that in equation (2.25) are all strictly positive. If all eigenvalues of $\mathbf{G}(jw_k)$ are strictly positive, so is the smallest one. Thus, the passivity assessment condition can take the even simpler form:

$$\lambda_{min}(\mathbf{G}(jw_k)) > 0, \tag{2.26}$$

where $\lambda_{min}(\,.\,)$ denotes smallest eigenvalues of a matrix. Stated in other terms, if the smallest eigenvalue of $\mathbf{G}(jw_k)$ is negative, the passivity condition is said to be locally violated. The violation is local in the sense that once a violation is confirmed at frequency w_k there is no implication so as to violations at frequencies w_{k-1} nor w_{k+1}, i.e., passivity has to be verified in the frequency sweeping manner. As a matter of fact, equations (2.24)

through (2.26) are equivalent. According to Triverio et al. (2007), if $\mathbf{H}(jw) = \mathbf{S}(jw)$, the LTI n-port is a scattering representation and similar assessment criteria can be derived. In this case, the positive-realness condition for $\mathbf{\Phi}(jw_k)$ produces:

$$\mathbf{I} - \mathbf{S}^H(jw_k)\mathbf{S}(jw_k) > 0. \tag{2.27}$$

If $\mathbf{\Phi}(jw_k)$ is a positive-real matrix, then $\mathbf{S}(jw_k)$ is bounded-real or unitary bounded, following the usual literature terminology. From equation (2.27), the requirement for passivity is that the eigenvalues of the Gram matrix $\mathbf{S}^H(jw_k)\mathbf{S}(jw_k)$ be smaller than those of the identity, and therefore smaller than one. Since the square roots of the non-zero eigenvalues of the associated Gram matrix are the singular values of the original matrix $\mathbf{S}(jw_k)$, it has become standard procedure in the literature to establish the passivity criterion in terms of the singular values of $\mathbf{S}(jw_k)$ rather than the eigenvalues of the associated Gram matrix. Thus,

$$\sigma_{max}(\mathbf{S}(jw_k)) \leq 1, \tag{2.28}$$

where $\sigma_{max}(\,.\,)$ denotes largest singular value of a matrix. Equations (2.27) and (2.28) are the analogues, for the scattering representation, of equations (2.24) and (2.26), respectively. Hence, if the largest singular value of $\mathbf{S}(jw_k)$ is larger than one, the nonexpansivity (scattering analogous of passivity) condition is also said to be locally violated.

Table (2.3.1) summarizes all the discussion on frequency-sweeping assessment techniques for LTI n-ports.

Table 2.1: Physicality Constraints

Representation	Y (Z or hybrid)	S	H
Symmetry	$\mathbf{Y} = \mathbf{Y}^T$	$\mathbf{S} = \mathbf{S}^T$	-
Realness	$\mathbf{Y}(s) = \mathbf{Y}^*(-s)$	$\mathbf{S}(s) = \mathbf{S}^*(-s)$	$\mathbf{H}(s) = \mathbf{H}^*(-s)$
Passivity	$\lambda_{min} > 0$	$\sigma_{max} < 1$	-
Matrix Dimension	$n \times n$	$n \times n$	$n_{output} \times n_{input}$

2.3.2 LMI-based test

This assessment method relies upon a connection between the feasibility of linear matrix inequalities associated with a state-space description and the positive-realness (bounded-realness) condition of the transfer matrix, this latter concept has been previously defined for the frequency-sweep assessment. The LTI system transfer matrix system is positive real if its state space realization satisfies the Positive Real Lemma (PRL). These concepts originate in Control Theory and are associated with the solution to the Lur'e equation. In reference Kalman (1964) this connection is first established for Single-Input-Single-Output (SISO) systems, the extension of this result to the multi-variable case (MIMO) appeared later in Anderson (1967*b*). Considering a LTI n-port as characterized by equation (2.21), a system thus characterized is passive only if the following LMI, whose variable $\mathbf{P}$ is a $n \times n$ matrix, is feasible:

$$\begin{bmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{B} - \mathbf{C}^T \\ \mathbf{B}^T\mathbf{P} - \mathbf{C} & -\mathbf{D}^T - \mathbf{D} \end{bmatrix} \leq 0, \tag{2.29}$$

with feasibility meaning that the solution set to equation (2.29) is nonempty, i.e., there exists at least one matrix $\mathbf{P}$ such that it is symmetric and positive-definite ($\mathbf{P} = \mathbf{P}^T > 0$). Otherwise, the system is said to be non-passive. Feasibility of the LMI is a sufficient condition for system passivity, since it implies that $\mathbf{H}^H(s) + \mathbf{H}(s) > 0$ for all $\mathrm{Re}(s) > 0$, i.e., the system is globally passive. Thus, in this sense, feasibility and passivity are equivalent statements. According to Curtain (1999) and Boyd et al. (1994), the result in equation (2.29), the positive-real lemma (PRL), is also usually called the Yakubovich-Kalman-Popov-Anderson or Kalman-Yakubovich lemma to reflect its authorship. The nonexpansivity condition for the scattering representation in terms of a LMI test, derived in Anderson (1967*a*), is called the bounded-real lemma (BRL) in the variable $\mathbf{P}$ and its associated matrix inequality is:

$$\begin{bmatrix} \mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{B} & \mathbf{C}^T \\ \mathbf{B}^T\mathbf{P} & -\mathbf{I} & \mathbf{D}^T \\ \mathbf{C} & \mathbf{D} & -\mathbf{I} \end{bmatrix} \leq 0, \tag{2.30}$$

such that $\mathbf{P} = \mathbf{P}^T > 0$. If the BRL is satisfied, then $\mathbf{S}(s)$ is a bounded-real matrix: $\mathbf{S}^H(s)\mathbf{S}(s) < \mathbf{I}$ for all $\mathrm{Re}(s) > 0$ and the system is globally nonexpansive. As noted in Boyd et al. (1994), this fact can also be expressed as $\|\mathbf{S}\|_\infty \leq 1$. In reference Curtain (1999), one can find some extensions of these results to the infinite-dimensional case as well as further perspectives.

2.3.3 Hamiltonian test

The spectral properties of certain Hamiltonian matrices provide yet another connection to the positive/bounded-realness of the transfer matrix. A proof of this connection is established in reference Boyd et al. (1989), whence the following definition of a Hamiltonian matrix $\mathbf{M}$ is a simple restatement of:

$$\mathbf{J}^{-1}\mathbf{M}\,\mathbf{J} = -\mathbf{M}^T \qquad \text{where} \qquad \mathbf{J} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix}, \tag{2.31}$$

i.e., a matrix $\mathbf{M}$ satisfying equation (2.31), with $\mathbf{J}^{-1} = \mathbf{J}^T = -\mathbf{J}$, is a Hamiltonian Matrix. Besides, as emphasized in Grivet-Talocia (2004), its spectrum is symmetric with respect to both the real and imaginary axis and its characteristic polynomial is real and even-ordered. Therefore, if λ_i is an eigenvalue of $\mathbf{M}$ ($\lambda_i \in \{\lambda(\mathbf{M})\}$) then, due to the spectral symmetry, $-\lambda_i$ is also an eigenvalue of $\mathbf{M}$: $-\lambda_i \in \{\lambda(\mathbf{M})\}$.

Assuming a LTI n-port characterized as in equation (2.21), the system associated Hamiltonian matrices are formed as follows:

$$\mathbf{M}_\delta = \begin{bmatrix} \mathbf{A}+\mathbf{B}[2\delta\,\mathbf{I}-\mathbf{D}-\mathbf{D}^T]^{-1}\mathbf{C} & \mathbf{B}[2\delta\,\mathbf{I}-\mathbf{D}-\mathbf{D}^T]^{-1}\mathbf{B}^T \\ -\mathbf{C}^T[2\delta\,\mathbf{I}-\mathbf{D}-\mathbf{D}^T]^{-1}\mathbf{C} & -\mathbf{A}^T-\mathbf{C}^T[2\delta\,\mathbf{I}-\mathbf{D}-\mathbf{D}^T]^{-1}\mathbf{B}^T \end{bmatrix}, \tag{2.32}$$

for hybrid representations and

$$\mathbf{M}_{\gamma} = \begin{bmatrix} \mathbf{A} - \mathbf{B}[\mathbf{D}^T\mathbf{D} - \gamma^2\,\mathbf{I}]^{-1}\mathbf{D}^T\mathbf{C} & -\gamma\mathbf{B}[\mathbf{D}^T\mathbf{D} - \gamma^2\,\mathbf{I}]^{-1}\mathbf{B}^T \\ \gamma\mathbf{C}^T[\mathbf{D}\mathbf{D}^T - \gamma^2\,\mathbf{I}]^{-1}\mathbf{C} & -\mathbf{A}^T + \mathbf{C}^T\mathbf{D}[\mathbf{D}^T\mathbf{D} - \gamma^2\,\mathbf{I}]^{-1}\mathbf{B}^T \end{bmatrix}, \tag{2.33}$$

for the scattering representation whose respective parameters (δ and γ) are both strictly positive and δ is not an eigenvalue of $(\mathbf{D} + \mathbf{D}^T)/2$ whereas γ is not a singular value of $\mathbf{D}$. Thus, the following connection can be established:

$$\begin{cases} \delta \in \{\lambda(\mathbf{H}(jw_k))\} \longleftrightarrow jw_k \in \{\lambda(M_{\delta})\} & \text{(for hybrid representations)} \\ \gamma \in \{\sigma(\mathbf{H}(jw_k))\} \longleftrightarrow jw_k \in \{\lambda(M_{\gamma})\} & \text{(for scattering representations)} \end{cases}, \tag{2.34}$$

such that the parameters δ and γ are respectively eigenvalues and singular values of the corresponding transfer matrix if and only if the associated hamiltonian matrix has purely imaginary eigenvalues, as proved in Boyd et al. (1989). According to Grivet-Talocia (2004), this result permits one to determine the exact frequencies at which the eigenvalues and singular values cross or are set in the neighbourhood of a given threshold which correspond to the values assigned to parameters δ and γ. If the hamiltonian matrix has no purely imaginary eigenvalue, the assigned threshold is not crossed nor "touched". The threshold of interest is zero ($\delta = 0$) for hybrid representations and one ($\gamma = 1$) for the scattering case. In this particular setting, purely imaginary eigenvalues of the corresponding hamiltonian matrix (if any) should provide evidence as to the violation of the positive-realness condition posed by equation (2.23). Therefore, a LTI n-port given in terms of equation (2.21) is passive, or nonexpansive, under this Hamiltonian criterion, if and only if:

$$\mathbf{M}_{(\delta=0)} = \begin{bmatrix} \mathbf{A} - \mathbf{B}(\mathbf{D} + \mathbf{D}^T)^{-1}\mathbf{C} & \mathbf{B}(\mathbf{D} + \mathbf{D}^T)^{-1}\mathbf{B}^T \\ -\mathbf{C}^T(\mathbf{D} + \mathbf{D}^T)^{-1}\mathbf{C} & -\mathbf{A}^T + \mathbf{C}^T(\mathbf{D} + \mathbf{D}^T)^{-1}\mathbf{B}^T \end{bmatrix}, \tag{2.35}$$

for hybrid representations and

$$\mathbf{M}_{(\gamma=1)} = \begin{bmatrix} \mathbf{A} + \mathbf{B}(\mathbf{I} - \mathbf{D}^T\mathbf{D})^{-1}\mathbf{D}^T\mathbf{C} & \mathbf{B}(\mathbf{I} - \mathbf{D}^T\mathbf{D})^{-1}\mathbf{B}^T \\ -\mathbf{C}^T(\mathbf{I} - \mathbf{D}^T\mathbf{D})^{-1}\mathbf{C} & -\mathbf{A}^T - \mathbf{C}^T\mathbf{D}(\mathbf{I} - \mathbf{D}^T\mathbf{D})^{-1}\mathbf{B}^T \end{bmatrix} \tag{2.36}$$

for the scattering representation have no purely imaginary eigenvalues. By construction, $\mathbf{M}$ is a matrix of size $2n \times 2n$, twice the size of the state dynamics matrix and, as stressed in references Grivet-Talocia & Ubolli (2008) and Grivet-Talocia (2004), this passivity assessment is frequency-independent with the resultant Hamiltonian matrices also being constant since its parameters are the matrices $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ associated to the LTI system. Hence, it is possible to conclude that the absence of purely imaginary eigenvalues in the referred set implies feasibility of the LMI's, equations (2.29) and (2.30), as well as positive realness of equation (2.22). On the other hand, if there exists at least one purely imaginary eigenvalue, the system is non-passive.

The mere existence of purely imaginary eigenvalues of the Hamiltonian matrix is sufficient for distinguishing passive from non-passive behavior. However, further information can be obtained. By way of illustration, assume a Hamiltonian matrix (associated to a system as described by equation (2.21) whose k positive imaginary eigenvalues form the following set:

$$\Omega \ = \{\lambda_1 = j\omega_1, \lambda_2 = j\omega_2, \ldots, \lambda_k = j\omega_k\}. \tag{2.37}$$

such that the eigenvalues are sorted in ascending order and $k \leq \frac{n}{2}$. Each of these eigenvalues correspond to the frequencies at which the positive-realness threshold of equation (2.22) is reached, i.e., if:

$$\mathbf{\Phi}(jw_i) \not\succ 0 \qquad \forall\ \lambda_i = j\omega_i \ \in \ \Omega. \tag{2.38}$$

Nonetheless, since the eigenvalues or singular values are a continuous function of frequency, it is possible to characterize how the boundary is reached, that is to say, whether non-passive (or passive) behavior is found at frequencies higher than some jw_i or other-

wise.

Using a first-order approximation based on Perturbation Theory, the author in Grivet-Talocia (2004) derives a continuous function of frequency for the eigenvalues or singular values whose slope at the threshold can be evaluated, e.g., the frequencies at which the eigenvalues of $\mathbf{G}(jw)$ change sign. Hence, the elements in the set Ω define breakpoints between passive and non-passive behavior depending on the slope at each jw_i thus eliminating the need for sweeping along the frequency axis. As a result, passivity can be characterized in terms of bands of violation. With the set Ω and knowledge of the slopes at each of its k breakpoints, or crossover frequencies, the whole spectrum of frequencies can be divided into intervals, e.g., $(0, \omega_1), (\omega_2, \omega_3), ..., (\omega_k, \infty)$, such that the behavior within each (ω_{i-1}, ω_i) can be specified. It is even possible to quantify the maximum violation in each band (ω_{i-1}, ω_i) using the bisection algorithm, as proposed in Boyd et al. (1989). Alternatively, bands of violation can also be identified more pragmatically by evaluating the positive-realness criterium, equation (2.23), at the midpoint frequency between each successive element in the set Ω, e.g., $(\omega_{i-1}+\omega_i)/2$. Hence, if the positive-realness criterium fails to hold at the midpoint frequency $(\omega_{i-1} + \omega_i)/2$, then (ω_{i-1}, ω_i) defines a violation band. Furthermore, evaluate the same criterium at $w_1/2$ (from DC to w_1) and $2w_n$ (from w_n to ∞, asymptotic behavior).

Finer details, generalizations and further technical assumptions can be found in Grivet-Talocia (2004), Boyd et al. (1989), Golub & Loan (2012) and references therein.

2.3.4 Half-size Test Matrix

Some technical difficulties arise when computing the purely imaginary eigenvalues of a Hamiltonian matrix that define frequency boundaries for passivity violations: (i) the eigenvalues are not purely imaginary but have a small real component due to numerical noise and (ii) the Hamiltonian matrix is twice as large as the associated state-dynamics matrix, thus leading to a time-consuming eigenvalue computation for large models. As a matter of fact, the eigenvalue computational complexity grows cubically with the matrix dimension.

Considering its symmetrical spectrum, the Hamiltonian eigenvalues appear in pairs and quadruples. Therefore, there is redundancy in computing the same information twice. Only for symmetrical systems, large computational savings can be obtained so as to establish the crossover frequencies and the passivity violating intervals with matrices whose size are half that of the Hamiltonian matrix, thus the name Half-size test matrices. These test matrices haven been proposed in references Gustavsen & Semlyen (2009*a*) and Gustavsen & Semlyen (2008) for admittance (impedance) and scattering representations respectively. Owing to some incorrect statements in Gustavsen & Semlyen (2009*a*), the authors made a revision to their own claims in Gustavsen & Semlyen (2009*b*), but a read of a further discussion on this issue as presented in Wong & Zhang (2010) is also recommended. A reduction in the size of the test matrix by a factor of two corresponds to the computational complexity to fall off by a factor of nearly eight (due to the cubic complexity relation), thus making the eigenvalue computation less of an issue. The numerical noise problem in the eigenvalues is also compensated since the eigenvalues of the half-size matrix are noiseless purely real and whose square roots correspond to the crossover frequencies that determine the breakpoints between passive and non-passive behavior hence also corresponding to the eigenvalues of the Hamiltonian matrix, with no redundancy. Assuming a LTI n-port expressed as in equation (2.21), the Half-size test matrices are:

$$\mathbf{T} = \mathbf{A}[\mathbf{B}\mathbf{D}^{-1}\mathbf{C} - \mathbf{A}], \tag{2.39}$$

for admittance (impedance) representation and

$$\mathbf{T} = [\mathbf{A} - \mathbf{B}[\mathbf{D} - \mathbf{I})]^{-1}\mathbf{C}][\mathbf{A} - \mathbf{B}[\mathbf{D} + \mathbf{I}]^{-1}\mathbf{C}], \tag{2.40}$$

for the scattering case. In a procedure similar to that of the Hamiltonian matrix, the crossover frequencies can be identified as the square root of the positive-real and negative-real eigenvalues of $\mathbf{T}$, for the admittance and scattering representations respectively. Therefore, the admittance Half-size test equivalent of the set given in equation (2.37) yields:

$$\Omega \ = \{\lambda_1 = \omega_1^2, \lambda_2 = \omega_2^2, \ldots, \lambda_k = \omega_k^2\}. \tag{2.41}$$

The rest of the procedure to determine the violating frequency bands follows the same steps as explained for the Hamiltonian matrix. A performance comparison between the half-size test and hamiltonian test matrices is made in Gustavsen & Semlyen (2009*a*).

2.3.5 Discussion on Assessment

These assessment techniques or passivity criteria have been ordered as they have historically appeared in the literature and permit passivity characterization at different levels of detail. Deciding upon which one to use is not at one's full discretion. For instance, assessing passivity before a model is synthesized rules out any choice of assessment technique based on model parameters. Hence, passivity assessment on raw data can only be accomplished with a search over measured frequencies, i.e., the sweeping technique. As for the parameter-based techniques, they assume a state-space realization to assemble the test matrices, which is by no means the only model structure usually deployed. Rational models constitute yet another commonly used structure in the context of frequency-dependent behaviour, specially the ubiquitous *pole-residue* rational form. Even though the majority of passivity criteria is formulated in terms of $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ parameter set, a model initially synthesized in rational form can be converted into a state-space realization so that the test matrices be correctly assembled, an example of the conversion procedure can be found in Gustavsen & Semlyen (2004). Each assessment technique has its own pros and cons and the right choice of assessment must also best reflect one's interest in assessing passivity, e.g., a simple passivity certificate or a more detailed and refined passivity characterization so as to identify some localized equipment pathology.

Applicable to both raw data and model, the frequency-sweeping tests constitute the simplest form of assessing passivity, at least conceptually. They are frequency-dependent in the sense that the test must be performed at every single frequency point: theoretically in the whole continuum of the right-half s-plane, $\mathrm{Re}(s) > 0$, and experimentally for the

finite set $\{w_k\}_{k=1}^{K}$ of measured frequencies. Searching for positive-real matrices over a discrete set of measured frequencies imposes major limits on what conclusions can be reached due to inexorable experimental obstacles such as upper and lower bounds of the acquisition process and coarse sampling intervals. Owing to these two illustrative difficulties, one remains ignorant of what the system behavior can be between successive frequency samples as well as beyond the upper and lower bounds. As a result, if no negative eigenvalue is found at any frequency, test is inconclusive for positive-realness is only a necessary condition for passivity (not sufficient). Nonetheless, if this same procedure returns a single negative eigenvalue at a single frequency point, the system is assuredly non-passive since the necessary condition is not fulfilled. Therefore, this test is remarkably effective in furnishing a non-passivity certificate, but ineffective otherwise.

The LMI-based tests are very efficient in furnishing a global passivity certificate, in fact this test provides a single bit of information: the system is either passive or otherwise, no further information can be retrieved. It is a frequency-independent test for there is no search over frequencies and only applicable once a model has been synthesized, since it is parameter-dependent. The passivity certificate is issued as soon as a single matrix $\mathbf{P}$ is found, this constitutes a feasibility problem that can be formulated as in Boyd & Vandenberghe (2004) and numerically solved with CVX, a package for specifying and solving convex programs CVX Research (2012),Grant & Boyd (2008) with core solver SEDUMI Sturm (1999). If there is no such $\mathbf{P}$, i.e., the solution set to the LMI is empty, the problem is infeasible and the system is non-passive but there are no means so as to specify where along the frequency axis the violations occur.

The Hamiltonian test is the most detailed among all the assessment tests. It involves no search over frequencies, thus frequency-independent, but it does require models parameters, hence a parameter-dependent test. Its appealing feature lies precisely in the possibility of specifying where along the frequency axis violations occur, if they ever do. Recalling that the purely imaginary eigenvalues of the Hamiltonian matrix are sorted in ascending order and the continuous bands of frequencies arranged in ascending order from $\omega = 0$ up to $\omega \to \infty$, i.e., $(0, \omega_1), (\omega_2, \omega_3), ..., (\omega_n, \infty)$; the first $(0, \omega_1)$ and last (ω_n, ∞)

bands allow passivity assessment in d.c. and asymptotic terms, therefore a global continuous assessment (as opposed to finite discrete frequencies). Furthermore, as in reference Grivet-Talocia (2004), these results are not compromised in case of inaccurate sampling.

In many respects the Half-size test is similar to the Hamiltonian test. As matter of fact, according to Gustavsen & Semlyen (2008) and Gustavsen & Semlyen (2009*a*), the motivations for deriving it were specially the Hamiltonian matrix's long eigenvalue computation time that can be very time-consuming for large models and also their sensitivity to noise resulting in small real components as opposed to the purely imaginary form. The eigenvalue computation can actually dominate the total computation time of further modeling steps, e.g., parameter estimation or passivity enforcement. Half-size tests are frequency-independent and parameter-dependent as well, they can reduce the eigenvalue computation time by a factor of nearly eight but have a major restriction: they are only applicable to symmetric models.

The frequency-independent methods are more reliable, namely the LMI-based, Hamiltonian and Half-size tests, since the usual problems associated with the sweeping method such as determining how fine a search over the frequencies should be do not arise. However, the only choice for assessing passivity before parameter estimation is the sweeping assessment which can effectively provide a passivity violation certificate if there is at least a single negative eigenvalues (or a singular value larger than one) for at least one frequency point. Otherwise, knowledge of passivity violations remain in obscurity and the test is inconclusive.

Even though most of the literature on passivity assessment concentrates on symmetrical models on the grounds that physical systems result in symmetrical port matrices there is something of note regarding any model symmetry assumption. For instance, the simplifying correspondence established in equation (2.24) and the arguments of equations (2.25) and (2.26) are strictly valid provided the system is symmetric, as noted in Gustavsen & Semlyen (2009*a*) and Wong & Zhang (2010). However, the positive-realness condition for the general form of equation (2.23) must hold for a passive system, irrespective of any symmetry assumptions. The Half-size tests are valid exclusively for symmetric systems

whereas the LMI-based and Hamiltonian tests apply even when the system is devoid of symmetry. In the case of power transformers, symmetry is generally not an issue but care must be taken when synthesizing a model for some strategies aimed at reducing the computational time of parameter estimation can break the symmetry of the original data. A practical illustration of this difficulty comprises the use of *pole-residue* modeling obtained via the algorithm known as Vector Fitting (VF), as in Gustavsen & Semlyen (1999), employing its column-wise fitting procedure aiming at computational savings thus resulting in slightly unsymmetrical models. Furthermore, conversion from the rational to the state-space forms requires a specific procedure so that symmetry is not broken, as demonstrated in Gustavsen & Semlyen (2004) and Gustavsen & Semlyen (2008).

It is of paramount importance to interpret the common occurrence of inconsistent time-domain simulations of intrinsically passive systems. Examples of such difficulties abound in the literature and their root causes are usually associated with unmodeled characteristic features such as stability, causality, and chiefly passivity, both in the data used in model derivation or the model itself. Obviously, these arguments favouring passivity as a desired or characteristic feature exclude the cases concerning data and models of explicit sources such as generators. These passivity assessment techniques are essentially passivity conditions expressed as mathematical criteria that through various equivalent algebraic conditions provide means to distinguish passive systems from non-passive ones within the broader general set of all linear systems. Even though this research dwells on the linearity assumption, most of the results can be extended, *mutatis mutandis*, to time-varying and infinite dimensional systems as well.

CHAPTER 3

PRE-PROCESSING APPROACH

This chapter concerns the formulation of a novel pre-processing approach as a means to enforce passivity on data that has been invariably corrupted during the acquisition process. The vast majority of efforts on passivity enforcement have favoured model parameter perturbation, requiring an initial approximation to be modified until passivity is eventually attained, on the grounds that passivity may not be preserved during model parameter estimation, i.e., passive data do not imply passive models. Conversely, what follows is a pre-processing formulation in terms of data passivity enforcement to serve the very purpose of recovering the characteristic passive behavior of data from degradation sustained during acquisition, preceding any parameter estimation.

3.1 Pre-Processing Strategies

Since data is discrete and finite, the pre-processing is formulated as a point-by-point frequency-domain counter-acting perturbation matrix that when added to the corrupted non-passive measurement matrix is capable of enforce positive-real conditions, i.e., recover the n-port original dissipative (passive) properties. This statement translates as:

$$\hat{\mathbf{G}}(jw_k) = \widetilde{\mathbf{G}}(jw_k) + \mathbf{\Delta}(jw_k) > 0. \tag{3.1}$$

where w_k denotes a discrete measurement frequency in the set $\{w_k\}_{k=1}^{K}$ of all measured frequencies, $\hat{\mathbf{G}}(jw_k)$ is the perturbed positive-definite conductance matrix, $\widetilde{\mathbf{G}}(jw_k)$ is the unperturbed data, $\mathbf{\Delta}(jw_k)$ is the perturbation matrix and $\widetilde{\mathbf{G}}(jw_k)$ is found to violate the passivity criteria at a given measurement frequency w_k. Perturbation matrices are used only in case passivity violations occur, such that:

$$\boldsymbol{\Delta}(jw_k) = \begin{cases} 0 & \text{if} \quad \nexists \quad \lambda\{\widetilde{\mathbf{G}}(jw_k)\} < 0 \\ \boldsymbol{\Delta}(jw_k) & \text{if} \quad \exists \quad \lambda\{\widetilde{\mathbf{G}}(jw_k)\} < 0 \end{cases} \tag{3.2}$$

where $\lambda\{\widetilde{\mathbf{G}}(jw_k)\}$ is the set of eigenvalues of $\widetilde{\mathbf{G}}(jw_k)$ at frequency w_k. This procedure aims at suppressing any passivity violations corrupting the original data, specially large violations, which would otherwise be propagated through the identification algorithm and render invalid the core assumption of small violations that the enforcement algorithms work on, possibly resulting in larger accuracy degradation or even lack of convergence. Therefore, the objective is to determine a matrix (a data perturbation matrix) that adds to the raw, unprocessed, corrupted measurement data and enforces positive-real constraints, and yet causes - in some sense - the least possible deviation from the original data. Four distinct criteria to find a perturbation matrix are herein proposed so that the perturbed data fulfills the positive-realness condition.

3.1.1 Uniform Eigenvalue-Shift Perturbation - UESP

This first approach uses a special perturbation matrix to satisfy equation (3.1), namely a diagonal matrix that shifts up all the eigenvalues of $\widetilde{\mathbf{G}}(jw_k)$ by the absolute value of its most negative eigenvalue at every violating frequency:

$$\boldsymbol{\Delta}(jw_k) = \mu(jw_k)\mathbf{I}, \tag{3.3}$$

where $\mathbf{I}$ is the identity matrix and the parameter $\mu(jw_k) \in \mathbb{R}$ is defined, at every frequency, according to equation (3.2). This procedure has a least effect in the sense that all eigenvectors are invariant under the pre-processing and the nullspace is forced to coincide with the direction of the eigenvector associated with the largest passivity violation, the rank is reduced by one, see Horn & Johnson (2012). Furthermore, the perturbation matrix has structure: its diagonal (thus sparse), symmetric and positive-definite by construction. By preserving the eigenvectors during the pre-processing, any previous diagonalization (as in Gustavsen (2014)) of the data matrix $\widetilde{\mathbf{G}}(jw_k)$ still holds for the pre-processed (thus

passive) data.

This could be interpreted as a correction for a uniform corruption of all eigenvalues during data acquisition and not only the violating (negative) eigenvalues. As far as the conventional wisdom goes, only negative eigenvalues are unacceptable, thus only *ad hoc* corrections are made. However, even positive eigenvalues are possibly less positive (the system appears to be less dissipative) due to data corrupting agents.

3.1.2 Least Diagonal Perturbation - LDP

In the previous strategy, all eigenvalues are increased by the same amount, whereas the set of eigenvectors remain invariant. However, it can be modified so that its impact on the the diagonal entries be least in a different sense, namely a minimum trace criterium. Consequently, a different perturbation matrix $\mathbf{\Delta}(jw_k)$ can be formed while retaining its diagonal (thus sparse), symmetric and positive-definite structure. As a result, we formulate the following optimization problem to obtain such perturbation matrix:

$$\begin{aligned} \min_{\mathbf{\Delta}} . \quad & \mathbf{tr}(\mathbf{\Delta}(jw_k)) \\ s.to \quad & \widetilde{\mathbf{G}}(jw_k) + \mathbf{\Delta}(jw_k) > 0 \\ & \mathbf{\Delta}(jw_k) \qquad diagonal \\ & \mathbf{\Delta}(jw_k) > 0 \end{aligned} \tag{3.4}$$

where **tr** denotes the trace of the perturbation matrix, i.e., the sum of its diagonal elements, Horn & Johnson (2012). In (3.4), the perturbation matrix is the smallest (as measured by its trace), positive-definite, diagonal matrix to enforce positive-realness. In preceding algebraic formulation (UESP), the perturbation matrix $\mathbf{\Delta}(jw_k)$ is by construction positive definite with repeated eigenvalues, whereas in alternative this formulation the perturbation matrix is allowed to have different positive eigenvalues.

3.1.3 Least-Eigenvalue Perturbation - LEP

The preceding formulations result in perturbation matrices that are exclusively diagonal. As a consequence, only the diagonal entries are perturbed. By suppressing the require-

ment that the perturbation matrix be diagonal, the possibilities to obtain a perturbation matrix $\mathbf{\Delta}(jw_k)$ resulting in a least impact on measurements increase, for even off-diagonal entries can be allocated to compensate passivity violations. Therefore, we require that the perturbation matrix $\mathbf{\Delta}(jw_k)$ to ensure positive-realness has the least possible maximum eigenvalue. The problem is then to find a symmetric $\mathbf{\Delta}(jw_k)$ whose $\lambda_{max}(jw_k)$ is minimized and still satisfies the positive-realness condition (3.1):

$$\begin{aligned} \min_{\mathbf{\Delta}} . \quad & \lambda_{max}(\mathbf{\Delta}(jw_k)) \\ s.to \quad & \widetilde{\mathbf{G}}(jw_k) + \mathbf{\Delta}(jw_k) > 0 \end{aligned} \tag{3.5}$$

where $\lambda_{max}(\mathbf{\Delta}(jw_k))$ is the maximum eigenvalue of the perturbation matrix. As in Boyd & Vandenberghe (2004), equation (3.5) with a non-differentiable and highly non-linear objective can be reewritten as an equivalent Semidefinite Program (SDP embedding):

$$\begin{aligned} \min_{\mu,\mathbf{\Delta}} . \quad & \mu \\ s.to \quad & \mathbf{\Delta}(jw_k) < \mu\mathbf{I} \\ & \widetilde{\mathbf{G}}(jw_k) + \mathbf{\Delta}(jw_k) > 0 \end{aligned} \tag{3.6}$$

the formulation is now affine in the variables and can be reliably solved. This formulation has a least effect on $\widetilde{\mathbf{G}}(jw_k)$ in the sense that the perturbation being added ($\mathbf{\Delta}(jw_k)$) has its largest eigenvalue minimized.

3.1.4 Least-Norm Perturbation - LNP

Alternatively, a symmetric $\mathbf{\Delta}(jw_k)$ could also have a different least effect: it can be required to have the smallest euclidean norm, as long as (3.1) be satisfied. Therefore, we reformulate the optimization objective to obtain a Least-Norm Perturbation matrix, as follows:

$$\begin{aligned} \min_{\mathbf{\Delta}} . \quad & |(\mathbf{\Delta}(jw_k))|_2 \\ s.to \quad & \widetilde{\mathbf{G}}(jw_k) + \mathbf{\Delta}(jw_k) > 0 \end{aligned} \tag{3.7}$$

Equation (3.7), a constrained norm minimization with a non-differentiable and non-

linear objective, which is also SDP-representable, i.e. we can write its equivalent Semidefinite Program (SDP embedding), or LMI representation, as (also refer to Boyd & Vandenberghe (2004)):

$$\begin{array}{ll} \min\limits_{\mu,\mathbf{\Delta}}. & \mu \\ s.to & \widetilde{\mathbf{G}}(jw_k) + \mathbf{\Delta}(jw_k) > 0 \\ & \begin{bmatrix} \mu\mathbf{I} & \mathbf{\Delta}(jw_k) \\ \mathbf{\Delta}^T(jw_k) & \mu\mathbf{I} \end{bmatrix} > 0, \end{array} \tag{3.8}$$

which is now a LMI representation of problem (3.7) and can be reliably solved.

3.1.5 Weighted LEP and LNP

Both the LEP and LNP formulations furnish full perturbation matrices as opposed to the strictly diagonal (by construction) matrices of the UESP and LDP formulations. However, since the measured transformer admittance matrices happen to have their largest entries in the diagonals, this same pattern repeats for the perturbation matrices. This can cause the deviations for the diagonal entries of the perturbed measurement to be larger than the corresponding deviations for the non-diagonal entries.

The problem can be circumvented by adding a penalty term to the scalar objective function. This is accomplished adding a quadratic (thus convex) penalty function to the objective so that the pre-processing algorithm finds a perturbation matrix that smoothly allocates its entries, between diagonal and off-diagonal entries, such that the positive-realness is attained with a compensation that still satisfies equation (3.1). Equation (3.6) for the LEP formulation is reworked into its weighted version:

$$\begin{array}{ll} \min\limits_{\mu,\mathbf{\Delta}}. & \mu(jw_k) + \beta \sum_{i=1}^{n} (G_{ii}^2(jw)) \\ s.to & \mathbf{\Delta}(jw_k) < \mu(jw_k)\mathbf{I} \\ & \widetilde{\mathbf{G}}(jw_k) + \mathbf{\Delta}(jw_k) > 0 \end{array} \tag{3.9}$$

where β is scalar factor that can be tunned and $\sum_{i=1}^{n} (G_{ii}^2(jw))$ is the sum penalizing the objective function for the allocation of large entries to the diagonal of the perturbation

matrix. Similarly, the LNP (3.8) is also reworked into its weighted version, thus the problem can be recast as:

$$\begin{aligned} \min_{\mu,\boldsymbol{\Delta}} . \quad & \mu(jw_k) + \beta \sum_{i=1}^{n} (G_{ii}^2(jw)) \\ s.to \quad & \widetilde{\mathbf{G}}(jw_k) + \boldsymbol{\Delta}(jw_k) > 0 \\ & \begin{bmatrix} \mu(jw_k)\mathbf{I} & \boldsymbol{\Delta}(jw_k) \\ \boldsymbol{\Delta}^T(jw_k) & \mu(jw_k)\mathbf{I} \end{bmatrix} > 0. \end{aligned} \tag{3.10}$$

3.1.6 Algorithm

There follows a pseudo-code to better illustrate the overall modeling flow while placing the pre-processing approach into a detailed perspective.

INPUT: $\widetilde{\mathbf{G}}(jw) \leftarrow$ frequency-domain input data
for all $w_k \in \{w_k\}_{k=1}^{K}$ **do**
Preprocessing $(\widetilde{\mathbf{G}}(jw_k))$ {
if $\exists \quad \lambda(\widetilde{\mathbf{G}}(jw_k)) < 0$ **then**
$\widetilde{\mathbf{G}}(jw_k) \leftarrow$ preprocessed data
else
$\widetilde{\mathbf{G}}(jw_k)$ remains unchanged
end if
}
end for
do System Identification
do Passivity Enforcement
OUTPUT: Model

3.2 Discussion

This exposition of the pre-processing strategy is to be further employed and analyzed for actual power transformers in chapter 5. The advantage of pre-processing resides in accuracy gains that can be achieved by relieving parameter adjustment for the compensation

of in-band violations, thus resulting in a better overall compromise. Other instances of pre-processing in the context of power transformers exist and, as matter of fact, any data adjustment can be regarded as a pre-processing. In reference Morched et al. (1993), for instance, diagonalization of a transformer admittance matrix $\mathbf{Y}(jw)$ with a real similarity transformation matrix is used to obtain computation time gains. Similarity transformation matrices are in general complex and frequency-dependent. In order to achieve a real similarity transformation matrix, the authors in the aforementioned reference considered the admittance matrix as individual blocks, each block pertaining to a transformer winding. Diagonal and off-diagonal entries are then averaged so that blocks are balanced, thus permitting block diagonalization with real matrices. This approach however finds limitations in terms range of application for symmetry within transformer admittance matrices depends on winding configuration. Another simplistic pre-processing approach using diagonalization employing Fortescue transformation can be found in reference Gustavsen & Semlyen (1998). A more recent example of pre-processing devised to prevent increased error magnification in time-domain simulations due to inaccurate representation of small eigenvalues of the admittance matrix can be found in Gustavsen (2014). The author introduces a special similarity transformation matrix to avoid error magnification simulations with arbitrary terminations are of interest. This pre-processing called *Mode-Revealing Transformation* is simpler and faster an alternative to the *Modal Vector Fitting*, as in Gustavsen & Heitz (2008).

Pre-processing as herein formulated comprises an original formulation also intended for better accuracy, but the basic principle is to enforce passivity on data thus relieving modal parameter perturbation to achieve a better global compromise.

CHAPTER 4

POST-PROCESSING APPROACH

This chapter concerns the post-processing approach thus marking the dividing line between data and model passivity, the former has been discussed in the preceding chapter whereas the latter is hereupon discussed. Post-processing is intended for ensuring that the model behavior be strictly passive under any circumstances and it basically hinges on either deriving a passive model from a non-passive one or directly devising a passive model. However subtle the difference between these alternatives may seem, it is important that they be distinguished. Therefore, the first part of the chapter is committed to perturbative model passivity enforcement (estimate parameters then enforce passivity) and the second part focuses on non-perturbative passive approximation (enforce passivity as parameters are estimated). This discussion is independent of any previous attempts at taming passivity during the pre-processing stage, whether they have proved fruitful or not, and even the general case for which no pre-processing is used.

4.1 Perturbative Passivity Enforcement

Passivity enforcement is a procedure whose sole purpose is to ascertain that the passivity criteria be fulfilled by computing a correction or perturbation to an existing model of a given frequency response. In the search for a passive model to a passive system, it is precisely passivity violation as identified by some assessment method what prompts the enforcement task to be performed. As passivity has remained an unmodeled effect during parameter estimation, the estimated parameters do not reflect the exact system dynamics and must therefore be changed insofar as passive behavior be recovered. Model parameters are usually evaluated by means of some minimizing criteria so as to yield an optimal approximation to data, hence any parameter perturbation must be minimal as well. Typically, not all but only some parameters are actually perturbed to compensate

passivity violations specially those permitting some further structure simplification to be favourably exploited, e.g., linearity. In general terms, a parameter perturbation comprises computing a $\mathbf{\Delta}$ to be incorporated into a set of parameters $\mathbf{\Theta}$, i.e.,

$$\widehat{\mathbf{\Theta}}' = \widehat{\mathbf{\Theta}} + \mathbf{\Delta}, \tag{4.1}$$

such that the computed perturbation simultaneously renders the model passive and has a least impact on the original model:

$$\min \|\mathbf{\Delta}\|. \tag{4.2}$$

As a result, any compensation made to the original parameters is minimal in terms of the resulting model approximation error.

4.1.1 Discrete-frequency Passivity Enforcement

This particular enforcement strategy presupposes an existing accurate model in some functional form whose parameters serve as the starting point for further developments. As the discrete-frequency qualifier indicates, this procedure entails formulating a problem constraint that ensures the direct enforcement of the passivity condition for all or some judiciously selected frequency samples at which passivity is violated so as to recover a global passive behavior. Since the method relies on discrete enforcement, it is therefore unable to guarantee that a globally passive behavior be attained. Nevertheless, to overcome this difficulty, the enforcement algorithm can be embedded into an iterative procedure associated with the right assessment methods so that strategic frequencies be chosen and a global solution is thus obtained. As a consequence, the difference between existing formulations in the literature lies in the choice of the adaptive frequency sampling employed and the optimization strategy used to formulate an objective function as well its constraints. Common implementations include Linear Programming (LP) in Saraswat et al. (2004), Quadratic Programming (QP) in Gustavsen & Semlyen (2001) and Second-order Cone Programming (SOCP) in Grivet-Talocia & Ubolli (2008). The problem constraints must

establish a functional correspondence between parameter and eigenvalue perturbation, such correspondence is usually a non-linear one thus leading to another iterative procedure in order to attain a satisfactory result. In terms of problem formulation, parameter perturbations resulting in a minimal change to the initial approximation can be computed by some optimization-based algorithm.

First developed in Gustavsen & Semlyen (2001), the passivity enforcement method accompanying the Vector Fitting (VF) algorithm Gustavsen & Semlyen (1999) is based on Quadratic Programming. The overall package structure, i.e., estimation and enforcement was packed into a single routine in Gustavsen (2002) and Gustavsen (2007). In this particular setting, the existing initial approximation is structured as a *pole-residue* model, as parameterized by VF algorithm:

$$\mathbf{H}(jw) \;\cong\; \widehat{\mathbf{H}}(jw) = \sum_{m=1}^{N} \frac{\mathbf{R}_m}{jw - a_m} + \mathbf{D} + s\mathbf{E}, \tag{4.3}$$

whose parameters $\mathbf{R}_m$, a_m, $\mathbf{D}$ and $\mathbf{E}$ can be modified so that the passivity criteria be fully satisfied and the terms $\mathbf{D}$ and $\mathbf{E}$, that define the system's high-frequency asymptotic properties, possibly equal zero as stressed in Gustavsen (2007). By keeping the parameters in the denominator unchanged (the system's poles a_m), the computed values of the initial approximation and its parameters are linearly related; consequently, incremental changes made to its parameters are also linearly related to incremental changes to the approximation values. Since power transformers are this research's target application, it is assumed that $\widehat{\mathbf{H}}(jw) = \widehat{\mathbf{Y}}(jw)$, the transformer's admittance matrix is approximated with a *pole-residue* model. The symmetry of the admittance matrix is easily attained during the identification process since it implies that $\mathbf{R}_m$, $\mathbf{D}$ and $\mathbf{E}$ be all symmetric.

The following formulation of passivity enforcement was initially devised in Gustavsen & Semlyen (2001) and later refined in Gustavsen (2007), and is chiefly based on residue perturbation. Hence, the objective is to change the models' parameters so that the approximation remains as accurate as possible and the passivity criteria is fully satisfied, thus:

$$\begin{aligned} \Delta\mathbf{G}(jw) &= \sum_{m=1}^{N} \frac{\Delta\mathbf{R}_m}{jw - a_m} + \Delta\mathbf{D} \approx 0 \\ \widehat{\mathbf{G}}(jw) &+ \Delta\mathbf{G}(jw) > 0 \end{aligned} \tag{4.4}$$

which can be brought to a standard form QP, such as:

$$\begin{aligned} &\text{minimize} \quad (1/2)\Delta\mathbf{x}^T(\mathbf{K}^T\mathbf{K})\Delta\mathbf{x} - (\mathbf{K}^T\mathbf{b})^T\Delta\mathbf{x} \\ &\text{subject to} \quad \mathbf{F}\Delta\mathbf{x} < \mathbf{c} \end{aligned}, \tag{4.5}$$

where $\Delta\mathbf{x}$ is formed by stacking the parameters $(\Delta\mathbf{R}_m, \Delta\mathbf{D})$ to be perturbed, $\mathbf{K}$ is a matrix built upon a linear relation between parameters and admittances increments and $\mathbf{F}$ comprises the constraint matrix which establishes a linear relation between parameter change and eigenvalue change. Equation (4.5) defines a constrained least squares fitting whose constraints are directly related to the passivity requirement. Consequently, any correction made to the initial approximation is minimal in terms of the resulting fitting error, since it ensures passivity with a smallest possible increase in a least-squares sense, which can also be manipulated by some weighting scheme. The problem possibly has to be iteratively solved due to the actual nonlinear relation between its parameters and the associated eigenvalues, which has been linearized. For a general case comprising an admittance matrix of size $n \times n$ and an approximation order of N, the vector $\Delta\mathbf{x}$ containing the $n^2(N+1)$ stacked parameters is formed as:

$$\Delta\mathbf{x}^T = [\Delta R_{11,1} \ldots \Delta R_{11,N}\ \Delta D_{11}\ R_{21,1} \ldots \Delta R_{21,N}\ \Delta D_{21,1} \ldots \Delta R_{nn,1} \ldots \Delta R_{nn,N}\ \Delta D_{nn}]^T .$$

According to Gustavsen & Semlyen (2001), the vector $\Delta\mathbf{x}$ is linearly related to the incremental changes to every entry $\Delta\hat{g}_{ij}(jw_k)$ of the model $\mathrm{Re}(\widehat{\mathbf{Y}}(jw_k))$ at every frequency point w_k by a matrix $\mathbf{K}_k(jw_k)$, i.e., $\Delta\widehat{\mathbf{g}}(jw_k) = \mathbf{K}_k(jw_k)\Delta\mathbf{x}$ such that $\Delta\widehat{\mathbf{g}}(jw_k)$ comprises a vector whose entries are the stacked columns of $\Delta\widehat{\mathbf{G}}(jw_k)$.

The matrix $\mathbf{K}_k(jw_k)$ that establishes the linear relation is based on the assumptions that the set of poles $\{a_m\}$ initially estimated are held fixed while the other parameters are readjusted. Thus, $\mathbf{K}_k(s = jw_k)$ equals:

$$\begin{bmatrix} \frac{1}{s-a_1} & \dots & \frac{1}{s-a_N} & 1 & 0 & \dots & 0 & 0 & \dots & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & 0 & \frac{1}{s-a_1} & \dots & \frac{1}{s-a_N} & 1 & 0 & \dots & \dots & 0 & 0 \\ \vdots & \dots & \vdots & \vdots & 0 & \dots & 0 & 0 & \ddots & 0 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & 0 & \frac{1}{s-a_1} & \dots & \frac{1}{s-a_N} & 1 \end{bmatrix}.$$

Therefore, the matrix $\mathbf{K}$ in equation (4.5) is formed by stacking all $\mathbf{K}_k(jw_k)$ corresponding to every individual frequency sample w_k, notice that the matrix $\mathbf{K}$ is due to an assumed linearization between parameter change and conductance change. The vector $\mathbf{b}$ in equation (4.5) contains the stacked columns of the difference matrix between the measured and estimated admittance matrix: $\mathbf{Y}(jw) - \widehat{\mathbf{Y}}(jw)$. As for the matrix $\mathbf{F}$, it defines the linearization between the vector of parameter perturbation $\Delta\mathbf{x}$ and the vector of eigenvalue perturbation $\mathbf{c}$, such that $\mathbf{F} = \mathbf{QK}$. The matrix $\mathbf{K}$ is built as earlier discussed whereas matrix $\mathbf{Q}$ consists of the stacked $\mathbf{Q}_k(jw_k)$ matrices for every violating frequency containing the following arrangements of the normalized (so that their 2-norm is unitary) eigenvectors of the approximated conductance matrix at each w_k:

$$\mathbf{Q}_k(jw_k) = \begin{bmatrix} v_1^T & 0 & \dots & 0 \\ 0 & v_2^T & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & v_n^T \end{bmatrix} \begin{bmatrix} v_1^T & \dots & 0 \\ \vdots & \ddots & 0 \\ 0 & \dots & v_1^T \\ v_2^T & \dots & 0 \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ 0 & \dots & v_{n-1}^T \\ v_n^T & \dots & 0 \\ \vdots & \ddots & 0 \\ 0 & \dots & v_n^T \end{bmatrix}.$$

such that $v_1, \dots, v_n$ are the normalized eigenvectors of the associated diagonalization of the measured admittance matrix, i.e., the columns of the matrix of eigenvector $\mathbf{T}$ in

$\mathbf{Y}(jw_k) = \mathbf{T}(jw_k)\mathbf{\Lambda}(jw_k)\mathbf{T}^{-1}(jw_k)$. Matrices $\mathbf{K}$ and $\mathbf{F}$ in equation (4.5) owe their sizes to the number of measured frequencies and the number of violating frequencies respectively, the higher the number of frequencies the larger the matrices, hence implying a longer computation time. In its earliest formulation in reference Gustavsen & Semlyen (2001), all frequency samples found to violate the passivity criteria are involved in the process of adjusting the residues, with an impact on the size of matrix $\mathbf{F}$ and the vector $\mathbf{b}$ of equation (4.5). This process was later refined in Gustavsen (2007) for which the authors use the Hamiltonian matrix to define strategic frequencies within every violation band instead of sweeping the whole measured spectrum and including all violating frequencies in the constraint equations. With the Hamiltonian assessment method, the constraint equations are significantly reduced in size by only including the frequencies for which the passivity violation is the largest within every band instead of the entire violating band. Nonetheless, assessing passivity with the Hamiltonian matrix requires that the model be converted from the pole-residue form to the state-space form, conversion details can be found in Gustavsen & Semlyen (2004) and Gustavsen & Semlyen (2009*a*).

The main drawback of being selective in choosing the frequencies to constitute the problem constraint is that unpredicted violations can be created at frequencies other than those initially assessed, which is why the robust iterative procedure proposed in the paper Gustavsen (2007) are used. Its basic strategy is to include in the constraint matrix $\mathbf{F}$ additional samples chiefly outside the fitting band which are assigned a small weight so that the enforcement scheme does not concentrate within the bounds of the fitting range.

However, the problem reduction via the Hamiltonian assessment greatly improves computation times since matrix $\mathbf{F}$ is full but only contains as many rows as ones chooses for the enforcement. On the other hand, matrix $\mathbf{K}$ is sparse thus permitting this feature to be favourably exploited. One last remark about the Hamiltonian assessment method is that it allows to include frequency points beyond the fitting range, not possible with in the earlier formulation using sweeping assessment methods, thus allowing a global passivity enforcement to be attained in an iterative scheme.

It is noted that the vector $\Delta\mathbf{x}$ does not include any ΔE_{ij}. As discussed in Cipparrone

et al. (2001), even if the improper term $\mathbf{E}$ is non zero, it does not exert influence on the matrix $\mathrm{Re}(\widehat{\mathbf{Y}}(jw_k))$ whose eigenvalues should be perturbed. Therefore any ΔE_{ij} is not related to the passivity requirement, it can however be enforced to be positive-definite for circuit realization purposes. As a matter of fact, in its earliest formulation published in Gustavsen & Semlyen (2001), the vector $\Delta\mathbf{x}$ contained only the residue terms and passivity was enforced separately to both $\mathbf{D}$ and $\mathbf{E}$ with an algebraic procedure based on matrix diagonalization. For this reason, this particular passivity enforcement scheme is known as Residue Perturbation (RP).

The RP scheme enforces passivity while minimizing the approximation change to the measurements in an element-wise fashion. There is however a sensitivity problem concerning n-port systems (e.g., power transformers) with a large spread in the admittance matrix eigenvalues, i.e., a significant disparity in the magnitudes of the eigenvalues. This poses a difficulty in cases requiring matrix inversion due to testing a model with terminal conditions other than those assumed for parameter estimation. A common example is the inversion of an estimated admittance matrix so that a open-circuit terminal configuration be simulated or even using current sources as input to the model instead voltage sources. Care must be taken in matrix inversion processes, specially where the maximum-to-minimum eigenvalue ratio is large. As consequence of linear algebra, small eigenvalues of $\widehat{\mathbf{Y}}(jw)$ become large eigenvalues of $\widehat{\mathbf{Z}}(jw) = \widehat{\mathbf{Y}}^{-1}(jw)$ giving rise to numerical issues. Therefore, parameter perturbations should take into account this relative impact on each system mode.

As a result, a Modal Perturbation (MP) was proposed in Gustavsen (2008*b*). Basically, this MP approach comprises a reformulation of equation (4.4) so that the perturbation size of the admittance eigenvalues is inversely proportional to the eigenvalue size. The reformulated objective writes as follows:

$$\begin{gathered}\left(\sum_{m=1}^{N}\frac{\Delta\mathbf{R}_m}{jw-a_m}+\Delta\mathbf{D}\right)\left(\frac{\mathbf{t}_i(jw)}{|\lambda_i(jw)|}\right)\approx 0\\ \widehat{\mathbf{G}}(jw)+\Delta\mathbf{G}(jw)>0\end{gathered} \tag{4.6}$$

where $\mathbf{t}_i(jw)$ and $|\lambda_i(jw)|$ are the eigenvectors and eigenvalues associated with $\mathbf{Y}(jw) =$

$\mathbf{T}(jw)\mathbf{\Lambda}(jw)\mathbf{T}^{-1}(jw)$. This reformulation basically includes a weighting scheme for which the sizes of the perturbations are inversely proportional to that of the corresponding eigenvalue magnitudes, thereby avoiding uneven perturbational effects over small modes (eigenvalues) such that they are perturbed relatively to their size, thus preserving the overall model behavior when using hybrid terminations. The relative weighting scheme comes at the cost of a relative higher computation time for enforcement, thus its main drawback as compared with the preceding RP and thereby limiting the MP approach to small and medium-scale problems associated with the need of parameter conversion and arbitrary terminal configurations. This modal perturbation resulted in a reformulation of the vector fitting main algorithm (VF) for special uses addressing arbitrary terminal conditions, as in reference Gustavsen & Heitz (2008).

Even though both the RP and MP approaches have a small perturbational effect over the initial parameter, they are relatively demanding in terms computation time. In Gustavsen (2008*a*), the author proposes another refinement to RP and MP enforcement methods which significantly reduces the computation time at the expense of a slightly poorer approximation, i.e, a larger modelling error. The idea is to reduce the number of variables in the vector $\Delta\mathbf{x}$ of the QP formulation in equation (4.5). The reduction is attained by perturbing only the eigenvalues of each matrix $\Delta\mathbf{R}_m$ and $\Delta\mathbf{D}$ (also $\Delta\mathbf{E}$, if applicable) as opposed to the earlier strategies that perturb each individual entry of these matrices. This procedure leads to the Fast Residue Perturbation (FRP) and the Fast Modal Perturbation (FMP) methods, both described in Gustavsen (2008*a*).

Therefore, the discrete-enforcement scheme as formulated by equation (4.5) comes in four variations, namely the Residue Perturbation (RP), Modal Perturbation (MP), Fast Residue Perturbation (FRP) and Fast Modal Perturbation (FMP). A performance comparative can be found in reference Gustavsen (2008*a*).

4.1.2 Hamiltonian Perturbation Methods

This passivity enforcement scheme is a natural continuation of the Hamiltonian assessment method for which the initial approximation is structured as a *state-space* model.

Its applicability encompasses all the LTI n-port representations earlier discussed, i.e., any state-space realization whose input-output transfer matrix is in admittance, impedance, hybrid, or scattering form. Basically, the process entails the determination of a compensation to the state-space matrices $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ such that the perturbed system is rendered passive at the cost of a minimal change to system's input-output characteristic response. Under the Hamiltonian perspective, a given system is passive if its associated Hamiltonian matrix has no purely imaginary eigenvalues, thus meaning that the compensated state-space realization obtained from this passivity enforcement must have an associated Hamiltonian matrix whose set of eigenvalues does not contain a single purely imaginary eigenvalue. In Grivet-Talocia (2004), a perturbation scheme is devised so as to displace these purely imaginary eigenvalues off the imaginary axis (a passivity requirement) and yet cause a least change to the system's input-output characteristic response. This objective is pursued by applying a perturbation to associated matrix $\mathbf{C}$ only, leaving the remaining matrices unchanged. According to Gustavsen & Semlyen (2004), the matrix $\mathbf{C}$ of a *state-space* model corresponds to all residue matrices of a *pole-residue* model and, in this sense, this Hamiltonian perturbation scheme is also referred to in the literature as a residue perturbation.

In order to guarantee that the deviation of the perturbed system's input-output response from those of the original system be minimal, the cost function in the variable $\Delta\mathbf{C}$ used as criteria for a least deviation minimizes the energy associated with the change of each matrix element's impulse responses. The task involves computing a $\Delta\mathbf{C}$ that perturbs the initially estimated state matrix $\mathbf{C}$, i.e., $\mathbf{C}' = \mathbf{C} + \Delta\mathbf{C}$, such that the perturbed system $\{\mathbf{A}, \mathbf{B}, \mathbf{C}', \mathbf{D}\}$ be passive while minimizing the cumulative energy of the induced perturbation on the system's impulse responses:

$$E = \sum_{i,j=1}^{n} \int_{0}^{\infty} |(\Delta h)_{i,j}(t)|^2 dt = \mathrm{tr}(\Delta\mathbf{C}\,\mathbf{W}\,\Delta\mathbf{C}^T) \tag{4.7}$$

where tr(.) denotes the trace operator that takes as argument a square matrix and $\mathbf{W}$ is the system controllability Gramian (defined in Chen (2012)), computed as the unique,

symmetric and positive definite solution of the Lyapunov equation $\mathbf{AW}+\mathbf{WA}^T = -\mathbf{BB}^T$. The right-hand side of equation (4.7) can be further simplified by means of a suitable coordinate change, namely $\Delta\mathbf{C}_k = \Delta\mathbf{C}\,\mathbf{K}^T$, such that $\mathbf{K}^T$ is derived from the Cholesky factorization of the Gramian ($\mathbf{W} = \mathbf{K}^T\mathbf{K}$), thus yielding:

$$E = \mathrm{tr}(\Delta\mathbf{C}_k\Delta\mathbf{C}_k^T) = ||\Delta\mathbf{C}_k||_F^2 = ||\mathrm{vec}(\Delta\mathbf{C}_k)||_2^2 \tag{4.8}$$

with vec(.) denoting an operator that stacks the columns of a matrix into a single column vector and the subscript $_F$ standing for the Frobenius norm. According to Grivet-Talocia (2004), equation (4.8) establishes a correspondence between the minimization of the system impulse-response perturbations and the perturbation of the state matrix $\mathbf{C}$ which is minimal in this sense, using the appropriate coordinate system.

Again, the relation between the residues (free variables) and the passivity condition is a nonlinear one, thus a first-order perturbation (linearization) will be applied to the system's original Hamiltonian matrix ($\mathbf{M}_{\delta=0}$ in the hybrid case and $\mathbf{M}_{\gamma=1}$ for scattering representations) so as to displace its imaginary eigenvalues, in an iterative scheme, hence forcing them to eventually move off the imaginary axis. The perturbed Hamiltonian matrix whose set of eigenvalues contains no purely imaginary element can be written as:

$$\begin{aligned}\mathbf{M}'_{\delta=0} &\cong \mathbf{M}_{\delta=0} + \Delta\mathbf{M}_{\delta=0}\\ \mathbf{M}'_{\gamma=1} &\cong \mathbf{M}_{\gamma=1} + \Delta\mathbf{M}_{\gamma=1}\end{aligned}$$

for both hybrid and scattering representations, respectively. The first-order terms are themselves Hamiltonian matrices in terms of the free variable $\Delta\mathbf{C}$, assembled as follows:

$$\Delta\mathbf{M}_{\delta=0} = \begin{bmatrix} \mathbf{B}\mathbf{Q}_\delta^{-1}\Delta\mathbf{C} & \mathbf{0} \\ -\mathbf{C}^T\mathbf{Q}_\delta^{-1}\Delta\mathbf{C} - \Delta\mathbf{C}^T\mathbf{Q}_\delta^{-1}\mathbf{C} & -\Delta\mathbf{C}^T\mathbf{Q}_\delta^{-1}\mathbf{B}^T \end{bmatrix}, \tag{4.9}$$

with $\mathbf{Q}_\delta = 2\delta\,\mathbf{I} - \mathbf{D} - \mathbf{D}^T$ for hybrid representations and

$$\Delta\mathbf{M}_{\gamma=1} = \begin{bmatrix} -\mathbf{B}\mathbf{R}_\gamma^{-1}\mathbf{D}^T\Delta\mathbf{C} & \mathbf{0} \\ \mathbf{C}^T\mathbf{S}_\gamma^{-1}\Delta\mathbf{C} + \Delta\mathbf{C}^T\mathbf{S}_\gamma^{-1}\mathbf{C} & \Delta\mathbf{C}^T\mathbf{D}\mathbf{R}_\gamma^{-1}\mathbf{B}^T \end{bmatrix}, \tag{4.10}$$

with $\mathbf{R}_\gamma = \mathbf{D}^T\mathbf{D} - \gamma^2\mathbf{I}$ and $\mathbf{S}_\gamma = \mathbf{D}\mathbf{D}^T - \gamma^2\mathbf{I}$ for scattering representations. By changing the initial parameter matrix $\mathbf{C}$, the first order perturbation induces a displacement of each imaginary eigenvalue jw_i which is given by the following linear expression:

$$j\Delta w_i = \frac{\mathbf{v}_i^H \mathbf{J}\Delta\mathbf{M}\mathbf{v}_i}{\mathbf{v}_i^H \mathbf{J}\mathbf{v}_i} \tag{4.11}$$

where $\mathbf{v}_i$ are the eigenvectors associated to the purely imaginary eigenvalues jw_i of the original unperturbed Hamiltonian matrix and the matrix $\mathbf{J}$ as defined in equation (2.31). Equation (4.11) contains the prescribed amount by which the eigenvalues can be displaced under the first-order perturbation scheme and its numerator is a linear function of the free variable $\Delta\mathbf{C}$, as in equations (4.9) and (4.10) depending on the representation adopted. This linear relation is used to derive a linear equality between parameter perturbation and eigenvalue displacement. According to Grivet-Talocia (2004), this linear relation can be written as:

$$\mathrm{Re}[\mathbf{v}_{i1}^T \otimes \mathbf{z}_i^H]\mathrm{vec}(\Delta\mathbf{C}_k) = -\,\mathrm{Im}[\mathbf{v}_{i1}^H \mathbf{v}_{i2}](\Delta w_i) \tag{4.12}$$

where the original Hamiltonian matrix eigenvector $\mathbf{v}_i$ has been partitioned as $\mathbf{v}_i = [\mathbf{v}_{i1}^T \ \mathbf{v}_{i2}^T]^T$, $\otimes$ is the Kronecker product (defined in Loan (2000)) and $\mathbf{z}_i = -\mathbf{Q}_\delta^{-1}\mathbf{B}^T\mathbf{v}_{i2} - \mathbf{Q}_\delta^{-1}\mathbf{C}\mathbf{v}_{i1}$ is used for hybrid representations whereas $\mathbf{z}_i = \mathbf{D}\mathbf{R}_\gamma^{-1}\mathbf{B}^T\mathbf{v}_{i2} + \mathbf{S}_\gamma^{-1}\mathbf{C}\mathbf{v}_{i1}$ for the scattering representations. Equation (4.12) establishes a linear constraint to every violating eigenvalue jw_i of the original unperturbed Hamiltonian matrix. Therefore, by minimizing the system energy criterion devised in equation (4.8) constrained by the equalities given by equation (4.12), the following optimization problem is formulated:

$$\begin{aligned} &\text{minimize} && ||\mathrm{vec}(\Delta\mathbf{C}_k)||_2 \\ &\text{subject to} && \mathbf{Z}\mathrm{vec}(\Delta\mathbf{C}_k) = \mathbf{y} \end{aligned}, \tag{4.13}$$

such that $\mathbf{Z}$ and $\mathbf{y}$ are formed by the left- and right-hand sides of the constraint equation (4.12) for every purely imaginary eigenvalue in the set Ω. As observed in Grivet-Talocia & Ubolli (2008), the above formulation is not convex which implies that an iterative

application of equation (4.13) may fail to converge, its solution however can be obtained for every iteration via standard pseudo-inverse methods.

4.2 Non-perturbative Passive Enforcement

This second class of passivity enforcement differs from its predecessor in that no perturbation is applied, instead these methods enable the identification of *a priori* guaranteed passive models by construction, thus meaning that the customary passivity assessment and the consequential enforcement based on parameter-perturbation are no longer required. Both methods herein presented are technically underpinned by convex optimization methods and a prescribed set of system poles, i.e., a fixed-denominator strategy will be used.

4.2.1 Convex Optimization Methods I - PRL/BRL

Originally devised in Coelho et al. (2001) and Coelho et al. (2004), this method assumes *a priori* knowledge of the system poles. In a *state-space* context, this statement corresponds to having as starting point a prescribed state-dynamics matrix $\mathbf{A}$. In practice, this initial information is derived from a system-identification procedure attaining satisfactory accuracy such that all parameters are discarded, with exception of the system poles. Since the authors in the aforementioned references claim that any convenient rational approximation algorithm can be used to generate this initially prescribed pole structure, no reference to any specific algorithm is herein made, even though some possible choices have been mentioned earlier in this text. The early assumption made on the matrix $\mathbf{A}$ of a state-space characterization as given by equation (2.21) leads to a linear model, for the denominator is fixed. As opposed to the previous methods based on additive compensation of the transfer function, the objective is no longer to seek for a minimal perturbation so that the model be rendered passive but rather compute (re-compute) the remaining parameters from scratch and under passive constraints. Therefore, the problem is twofold: pole estimation and the subsequent estimation of the remaining parameters

(chiefly the residues) under the passivity constraints. This latter part is then formulated as a convex programming problem since the passivity constraint as represented by the positive(bounded)-real lemma is a linear matrix inequality (LMI), thus convex.

Therefore, suppose a transformer admittance matrix with a given initial pole set, a guaranteed passive approximation for a LTI n-port in hybrid representation can be attained by solving the following problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{K} |\Xi_{il}(jw_k)| \left| Y_{il}(jw_k) - \hat{Y}_{il}(jw_k) \right|^2 \\ \text{subject to} \quad & \begin{bmatrix} -\mathbf{A}^T\mathbf{P} - \mathbf{P}\mathbf{A} & -\mathbf{P}\mathbf{B} + \mathbf{C}^T \\ -\mathbf{B}^T\mathbf{P} + \mathbf{C} & \mathbf{D}^T + \mathbf{D} \end{bmatrix} \geq 0 \\ & \mathbf{P} \geq 0 \end{aligned} \tag{4.14}$$

where $\Xi_{il}(jw_k)$ is a weighting function, $Y_{il}(jw_k)$ correspond to data measurement whereas $\hat{Y}_{il}(jw_k) = \mathbf{D}_{il} + \mathbf{C}_i[jw_k\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B}_l$ is the corresponding model approximation for every matrix entry, the difference between the two which is to be minimized for all entries at all frequency points is the objective function. For simplicity, each weighting function $\Xi_{il}(jw_k)$ could assumed to be $\Xi_{il}(jw_k) = 1$ without any loss of generality. This particular optimization problem is also formulated in Oliveira et al. (2014). Considering the use of a controllable canonical form for the system $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ and the *a priori* knowledge of $\mathbf{A}$, the remaining system matrices $\mathbf{C}$ and $\mathbf{D}$ as well as the matrix $\mathbf{K}$ in the LMI (Positive-real Lemma, as in equation (2.29)) constitute the problem variables. The assumption of a controllable form for the re-estimation of parameters is not a restricting one, since it reflects the fact that only pole preservation is actually relevant for this procedure, irrespective of any specific coordinate system.

Hence, the objective and constraint functions of equation (4.14) are linear (affine) in the variables. Basically, the matrix numerator $\mathbf{C}$ and the constant term $\mathbf{D}$ are recalculated so that the PRL is satisfied and the error between the measurements and the model approximation is minimized. The formulation for scattering representations is identical, except that the LMI constraint should be replaced by the Bounded-real Lemma (BRL),

equation (2.30). Convexity ensures that the final result is the best passive approximation of the data, given the pole set. As in Coelho et al. (2004), equation (4.14) can be rewritten in an equivalent form. Let $\mathbf{e}_i$ denote ith column of the identity matrix as well as matrices $\mathbf{F}_{il} \in \mathbb{R}^{2K\times n}$ and $\mathcal{Y}_{il} \in \mathbb{R}^{2K}$ be defined as follows:

$$\mathbf{F}_{il}(k,:) = \begin{cases} \Xi_{il}(jw_k)\,\mathrm{Re}[\mathbf{J}(k)] & k \leq K \\ \Xi_{il}(jw_k)\,\mathrm{Im}[\mathbf{J}(k-K)] & k > K \end{cases}$$

$$\mathcal{Y}_{il}(k) \;= \begin{cases} \Xi_{il}(jw_k)\,\mathrm{Re}[\mathbf{Y}(k)] & k \leq K \\ \Xi_{il}(jw_k)\,\mathrm{Im}[\mathbf{Y}(k-K)] & k > K \end{cases} \tag{4.15}$$

such that $\mathbf{J}(k) = [\mathbf{B}_l^T[jw_k\mathbf{I} - \mathbf{A}]^{-1}\mathbf{e}_{(l)}^T]$ is a row vector, $\mathcal{Y}_{il}(k)$ contains the real and imaginary parts of each admittance entry measurement (i,l) for all jw_k and each k comprises the sequence of measured frequencies. Hence, these definitions permit the rewriting of the objective function in equation (4.14) as:

$$\sum_{i=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{K} |\Xi_{il}(jw_k)| \left|Y_{il}(jw_k) - \hat{Y}_{il}(jw_k)\right|^2 = \sum_{i=1}^{n}\sum_{l=1}^{n} ||\mathbf{F}_{il}\mathbf{X}_i - \mathcal{Y}_{il}|| \tag{4.16}$$

such that $\mathbf{X}_i$ is the ith column of the matrix $\mathbf{X}$ containing all model parameters to be estimated:

$$\mathbf{X} = \begin{bmatrix} \mathbf{C}^T \\ \mathbf{D}^T \end{bmatrix}.$$

With the standard QR factorization, every $\mathbf{F}_{il}$ becomes $\mathbf{F}_{il} = \mathbf{Q}_{il}\mathbf{R}_{il}$, where $\mathbf{Q}_{il}$ is an orthogonal matrix ($\mathbf{Q}_{il}^T\mathbf{Q}_{il} = \mathbf{I}$) and $\mathbf{R}_{il}$ full rank. Thus,

$$||\mathbf{F}_{il}\mathbf{X}_i - \mathcal{Y}_{il}|| = [\mathbf{F}_{il}\mathbf{X}_i - \mathcal{Y}_{il}]^T[\mathbf{F}_{il}\mathbf{X}_i - \mathcal{Y}_{il}] = \mathbf{E}_{il}^T\mathbf{E}_{il} - \delta_{il}^2, \tag{4.17}$$

such that $\delta_{il}^2 = \mathcal{Y}_{il}^T[\mathbf{I} - \mathbf{Q}_{il}\mathbf{Q}_{il}^T]\mathcal{Y}_{il}$ and $\mathbf{E}_{il} = [\mathbf{R}_{il}\mathbf{X}_i - \mathbf{Q}_{il}^T\mathcal{Y}_{il}]$. It is noteworthy that all these matrices are real by construction (according to equation (4.15)), thus leading to real parameter matrices as solutions. Therefore, the equivalent formulation of problem (4.14)

is:

$$\begin{aligned}
&\text{minimize} && t(\mathbf{C},\mathbf{D},\mathbf{K}) \\
&\text{subject to} && \begin{bmatrix} -\mathbf{A}^T\mathbf{P}-\mathbf{P}\mathbf{A} & -\mathbf{P}\mathbf{B}+\mathbf{C}^T \\ -\mathbf{B}^T\mathbf{P}+\mathbf{C} & \mathbf{D}^T+\mathbf{D} \end{bmatrix} \geq 0 \\
& && \mathbf{P} \geq 0 \\
& && \left.\begin{array}{l} \mathbf{E}_{il}^T\mathbf{E}_{il}-\delta_{il}^2 \leq t_{il} \\ t \geq 0 \\ t_{il} \leq t \end{array}\right| 1 \leq i,l \leq n
\end{aligned} \tag{4.18}$$

which is a convex problem in its *epigraph* form. This formulation is due to Coelho et al. (2004) and *epigraph* forms are explained in Boyd & Vandenberghe (2004). Other optimization goals could have been established to solve this problem, nevertheless. The error constraint, namely $\mathbf{E}_{il} = [\mathbf{R}_{il}\mathbf{X}_i - \mathbf{Q}_{il}^T\mathcal{Y}_{il}]$, can be further rewritten in terms of its semi-definite equivalent formulation via the Schur complement and therefore be replaced in equation (4.18) by the following LMI:

$$\begin{bmatrix} t_{i,l}-\delta_{i,l}^2 & \mathbf{E}_{il}^T \\ \mathbf{E}_{il} & \mathbf{I} \end{bmatrix} \geq 0. \tag{4.19}$$

This problem can be solved with CVX, a package for specifying and solving convex programs CVX Research (2012),Grant & Boyd (2008) with core solver SEDUMI Sturm (1999). *Ad hoc* problem reformulations can be done, specially structure-exploiting ones thus leading to better efficiency. In Coelho et al. (2004), the explicit positive-real lemma LMI constraint and its accompanying matrix variable $\mathbf{P}$ are removed. Furthermore, the constraint $\mathbf{E}_{il}^T\mathbf{E}_{il} - \delta_{il}^2 \leq t_{i,l}$ is a second-order cone which allows specific SCOP reformulations as well. The formulation herein presented can be employed to any MIMO system with no specific assumption on matrix structure. Many approximation methods fit MIMO systems in parts, i.e., a matrix can have its columns fit independently as if each column consisting of SIMO systems so that the overall MIMO system is eventually obtained by column concatenation. This strategy is well illustrated in Coelho et al. (2001), for the

positive-real lemma formulation.

The PRL approach can also be formulated as a perturbative enforcement scheme, an example is provided by Grivet-Talocia & Ubolli (2008). A similar formulation of the PRL constrained problem has been implemented to serve as a test bed for the case studies analyzed in the subsequent chapter. Details can be found in Oliveira et al. (2014), in essence it involves the use of orthonormal basis functions (Takenaka-Malmquist) and the Sanathanan-Koerner iterations for the pole estimation. Basically, the constrained problem of equation (4.14) is reworked into a semi-definite formulation by transforming quadratic inequalities to linear matrix inequalities (LMI's) via Schur complement.

4.2.2 Convex Optimization Methods II - PFVF

Another non-perturbative enforcement method that estimates parameters ensuring *a priori* passive behavior is the Positive Fraction Vector Fitting (PFVF). It first appeared in Tommasi et al. (2008) for single-input single-output (SISO) systems and was later extended to multi-input multi-output (MIMO) systems in Tommasi et al. (2011). Although this method also relies on convex programming for parameter estimation, passivity is not enforced by requiring that the positive(bounded) real lemma be satisfied which was the case in the previous convex approach. The Vector Fitting algorithm is used to determine an initial set of poles for the system's transfer matrix, therefore a fixed-pole structure is also pursued. Considering the *pole-residue* model structure, the transfer matrix is expanded as a truncated series of residue matrices and the associated poles. Essentially, the method obtains an overall passive transfer matrix by constraining each individual term of the series to be positive-real, i.e., each subsystem is passive. Therefore, the PFVF combines pole identification via the Vector Fitting (VF) algorithm with convex constrained residues identification, once the poles are known the problem constraints are easily written in convex form. The overall strategy of the PFVF method defines a sufficient condition to guarantee positive-realness for the overall transfer matrix, but not a necessary condition. Considering a *pole-residue* expansion of the transfer matrix with fixed poles as in equation (4.3), a passive transfer function can be attained by requiring that each model constituent

fraction is positive-real, namely:

$$\mathbf{D} \geq 0 \ , \ \mathbf{E} \geq 0, \ , \ \mathbf{R}_1 \geq 0 \ , \ldots , \ \mathbf{R}_N \geq 0.$$

Since the system poles are known, these constraints comprise LMI constraints, thus convex. In case the set of poles contains complex conjugate pairs $\frac{\mathbf{R}_i}{jw-a_i}$ and $\frac{\mathbf{R}_i^*}{jw-a_i^*}$, they enter the constraint set as two distinct matrices:

$$-[\mathrm{Re}(a_i)\,\mathrm{Re}[\mathbf{R}_i] + \mathrm{Im}(a_i)\,\mathrm{Im}[\mathbf{R}_i]] \geq 0$$
$$-[\mathrm{Re}(a_i)\,\mathrm{Re}[\mathbf{R}_i] - \mathrm{Im}(a_i)\,\mathrm{Im}[\mathbf{R}_i]] \geq 0$$

Hence, a guaranteed passive approximation for a LTI n-port in admittance representation can be realized by solving the following convex problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{K}\left|Y_{il}(jw_k) - \hat{Y}_{il}(jw_k)\right|_2 \\ & \mathbf{D} \geq 0 \\ & \mathbf{E} \geq 0 \\ \text{subject to} \quad & \mathbf{R}_1 \geq 0 \\ & \vdots \\ & \mathbf{R}_N \geq 0 \end{aligned} \tag{4.20}$$

where each $Y_{il}(jw_k)$ corresponds to admittance measurement and $\hat{Y}_{il}(jw_k)$ is a linear function of model parameters. As proposed in Tommasi et al. (2011), this problem is solved with CVX, a package for specifying and solving convex programs CVX Research (2012),Grant & Boyd (2008).

4.3 Discussion

The essence of all passivity enforcement schemes is to use the passivity conditions defined in chapter (2) as problem constraints so that passivity be either recovered as in perturbative schemes or secured as in non-perturbative schemes. All enforcement approaches seek the best compromise by being rigorous and offering global guarantees on positive-realness

at the cost of a small accuracy loss due to further constraining the modeling process. As passivity enforcement enters the modeling process it seems natural to conjecture that sacrificing model optimality for passive behavior might be fruitless. Obviously, under passivity enforcement the problem is further constrained and the unconstrained approximation is shifted from the allegedly optimal solution thus prompting a sense of a degraded solution. Nonetheless, model parameter estimation that exclusively relies on accuracy as a guiding criteria can only provide allegedly optimal approximations since its based on data that is sampled, incomplete, noisy and finite in range thus failing to contain the system's genuine dynamics as also expressed by passive behaviour. Therefore, passivity constraints displace allegedly optimal solutions to genuinely optimal passive solutions. Under this perspective, a poorer data fit is actually a better data fit.

This chapter has been divided into two sections in order to distinguish the striking features between perturbative and non-perturbative enforcement procedures. A key factor contributing to large deviations between a passive approximation and data is the eigenvalue spread, i.e., how large the negative eigenvalues are. Algorithms employing parameter perturbation are more sensitive to the presence of significant violations since they are based on first-order approximations which are valid under the assumption that correcting the eigenvalues only requires small perturbations. This can become remarkably delusional for anyone evaluating model performance for approximations are tested within the frequency range of the available measurements while ignorance prevails as to what the system dynamics is beyond the observations. This is a distinguishing feature between the two perturbative methods: discrete-frequency samples and the Hamiltonian perturbation. The earlier places stronger emphasis on observed data to compensate passivity violations thus keeping good in-band accuracy whereas the second compensates passivity even for out-of-band behavior hence requiring a possibly larger parameter perturbation for the model may have large out-of-band passivity violations, perhaps even larger than the in-band ones, reflecting in a poorer in-band fit for the Hamiltonian method.

Therefore, the best local accuracy must be carefully weighed against the averagely global passive behavior. For perturbative methods, an increased number of iterations may

be required since it is not guaranteed that for every iteration all violating eigenvalues are suppressed in addition to the fact that parameter perturbation can generate new violations, hence the need for a recursively applied process. These limitations obviously influence the number of iterations required before convergence is attained thus leading to increased accuracy degradation or even divergence for some pathological cases with large violating eigenvalues. Specially critical for perturbative methods, the assumption of small accuracy degradation during passivity enforcement is only valid provided passivity violations are small. This is a consequence of using linearization to establish the passivity constraints. Large violations can inflict lack of convergence or large model degradation. Such large violations can be suppressed in a pre-processing procedure based on data passivity enforcement so that the model be more amenable to passivity enforcement which in turn becomes more effective and substantial accuracy gains be accomplished.

Instead of applying parameter perturbation, both convex optimization methods are frequency-independent and rely on fixed-pole parameter estimation permitting the identification of *a priori* guaranteed passive models thus eschewing the passivity check and consequential parameter perturbation as required by most of the other available algorithms. The intrinsic limitations of first-order linearization no longer apply, therefore these methods are less sensitive to the eigenvalue spread. The PRL/BRL approach is the most general of the two convex-based approaches, since the positive-real lemma constraints provide necessary and sufficient conditions for system passivity. The major drawback of the PRL/BRL approach lies in the fact that the number of problem variables considerably increases due to the additional LMI matrix variable $\mathbf{P}$ which suggests its use for problems of moderate complexities.

The PFVF formulation was motivated precisely by the fact that the PRL/BRL approach is relatively involved and requires unacceptable computation time for LTI systems with a large number of ports. Since the PFVF establishes sufficient but not necessary passivity conditions, sub-optimal solutions are usually attained. The number of optimization variables grows linearly with system order for the PFVF, as opposed to the quadratic relation of the PRL/BRL approach (due to the LMI constraint matrix $\mathbf{P}$). Therefore,

computational efficiency and straightforward criteria come at the price of sub-optimality. Choosing which constraints to be applied is a problem that can affect the effectiveness of the enforcement procedure. Constraints must guarantee that the transfer function be positive-real but be only as restrictive as necessary to allow the algorithm to search all possible models that are passive given the pole set. Over-constraints can result in problem infeasibility, i.e., there are no parameters that guarantee passive behavior within some small approximation tolerance. According to Coelho et al. (2004), constraining each subsystem to be positive-real ensures that the overall system be indeed positive-real, but it could also exceedingly over-constrain the problem instead of admitting all passive models and thus leading to inaccurate models.

It is possible to conclude that there is some trade-off between perturbative and non-perturbative algorithms: the former can lead to accuracy degradation or even fail to converge but the computational cost rises favourably with model order, the latter is insensitive to the eigenvalue spread and *a priori* guaranteed to provide a passive result but can be impractical for large models in terms of computation time. A comparative study providing illustrative examples of each enforcement scheme can be found in Gustavsen (2008*b*) and Chiariello et al. (2010).

CHAPTER 5

CASE STUDIES

This chapter is devoted to present results obtained with both pre- and post-processing. In the first half of the chapter, a comparative study reveals in terms of approximation accuracy the positive impact of data pre-processing when large and small passivity violations occur, as opposed to the usual approach wherein no data pre-processing is employed. The second part of the chapter concentrates solely on post-processing, results achieved by the LMI-based (convex) formulation herein established are presented.

In Figures (5.1) and (5.2), it is possible to visualize flowcharts corresponding to the traditional approach (without preprocessing) and the proposed approach (including preprocessing), respectively. These figures provide a perspective of the overall process. Dotted lines are used to indicate the different modeling stages for which a comparison is be made. As a fitness measure, we use the relative index proposed in Chiariello et al. (2010) and defined as follows:

$$\text{error} = \frac{1}{K}\sum_{k=1}^{K}\frac{||\mathbf{Y}_{data}(k) - \mathbf{Y}_{fit}(k)||_F}{||\mathbf{Y}_{data}(k)||_F} \tag{5.1}$$

Another guiding index is the percentage of frequency intervals where violations occur with respect to the overall frequency interval ψ, also defined in Chiariello et al. (2010).

5.1 Pre-processing

In order to illustrate and assess the impact of the preprocessing on the overall modelling procedure, this analysis is underpinned by results obtained with two 6×6 and a 4×4 terminal admittance matrices of operating power transformers with different ratings, thus a total of three transformers are employed in the pre-processing analysis. The data are in full compliance with what is discussed in section (2.1) and references therein.

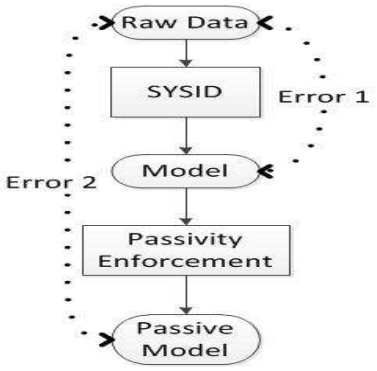

Figure 5.1: Modeling Process: post-processing only.

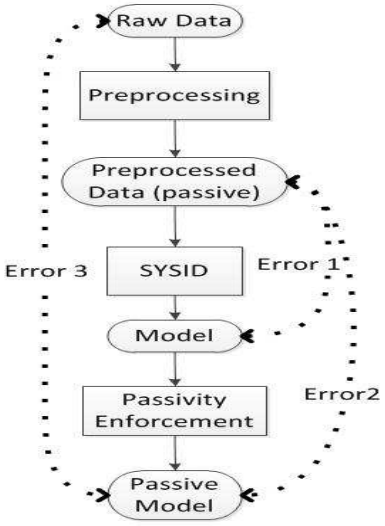

Figure 5.2: Propossed modeling Process: pre- and post-processing

While problem (3.3) is purely algebraic, to solve problems (3.4), (3.9) and (3.10) we used CVX, a package for specifying and solving convex programs CVX Research (2012), Grant & Boyd (2008) with core solver SEDUMI Sturm (1999). For the System Identification step - SYSID - we used Vector Fitting, as in Gustavsen (2002), and its embedded Passivity Enforcement scheme, as described in Gustavsen & Semlyen (2001) and Gustavsen (2007). The number of poles for each approximation has been defined according to the Akaike's Information Criterion, see Chapter 16 of Ljung (1999).

5.1.1 Example 1

In this first example, a 72/6.9kV, 6 MVA, Yd, three-phase power transformer is analyzed, the data comprises 1001 logarithmically spaced samples between 100 Hz and 10 MHz, approximated with 64 poles and an inverse weighting of the admittance magnitude. Figure (5.3) reveals the overall transformer's eigenvalue profile with its zero-crossings (violations) whereas Figure (5.4) contains the raw frequency-response measurements.

Table 5.1 summarizes the ψ index, accounting for the percentage of frequencies at which inequality (2.26) is violated with respect to the full measured band. As one reads along the columns, the ψ index is presented at different modeling steps (according to

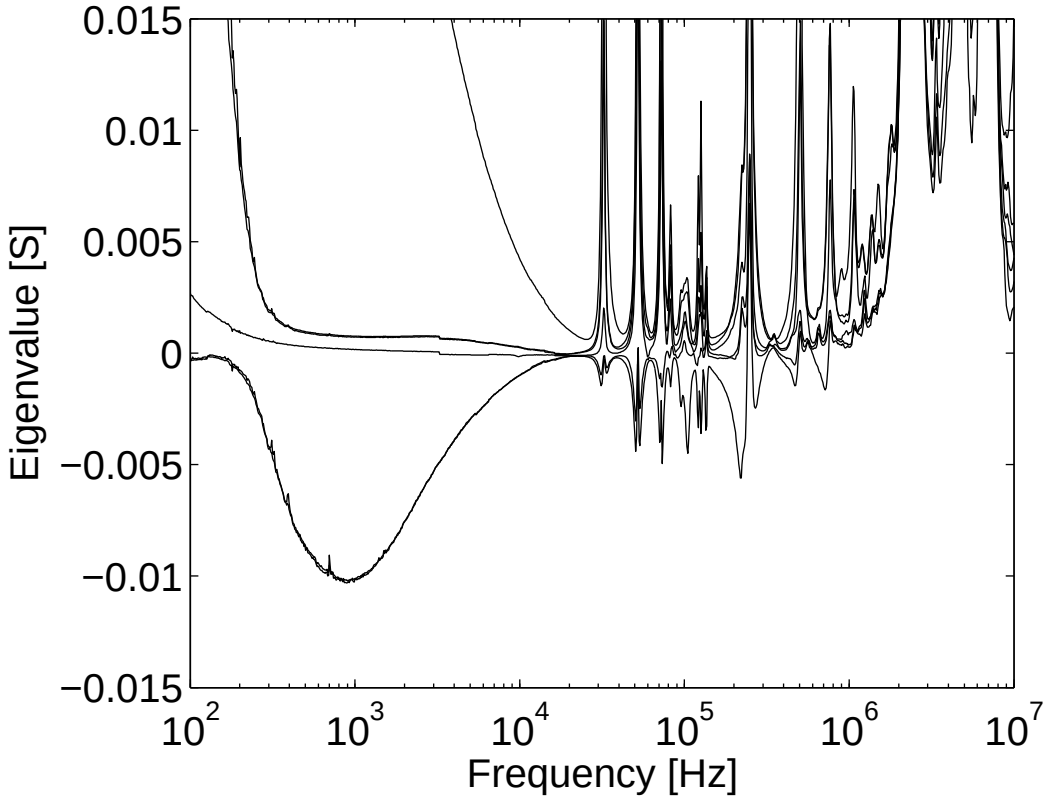

Figure 5.3: Example 1 - $\widetilde{\mathbf{G}}(jw_k)$ Eigenvalues.

flowcharts); rows one to four contain the results obtained with preprocessing (see Figure 5.2) whereas in the bottom row lies the corresponding figures for no preprocessing (see Figure 5.1). Column one shows the percentage index for the original raw-data, as also illustrated in Figure (5.3).

In the second column, the same index is recomputed considering the effect of preprocessing on data which is made passive - with exception of the last row, accounting for the traditional approach (without preprocessing). Column three summarizes the percentage of violation for the unenforced models, i.e., the outputs of the SYSID step. Analysis of column three reveals that the models obtained with preprocessed data have a smaller ψ than the traditional model, thus requiring a smaller number of iterations for the enforcement algorithm to reach convergence.

Table (5.2) consolidates the results in terms of misfit, as defined in equation (5.1), see Figures (5.1) and (5.2) for a reference on all errors. Column one contains Error 1 between the input and output of the identification procedure, i.e., a misfit between data (with and

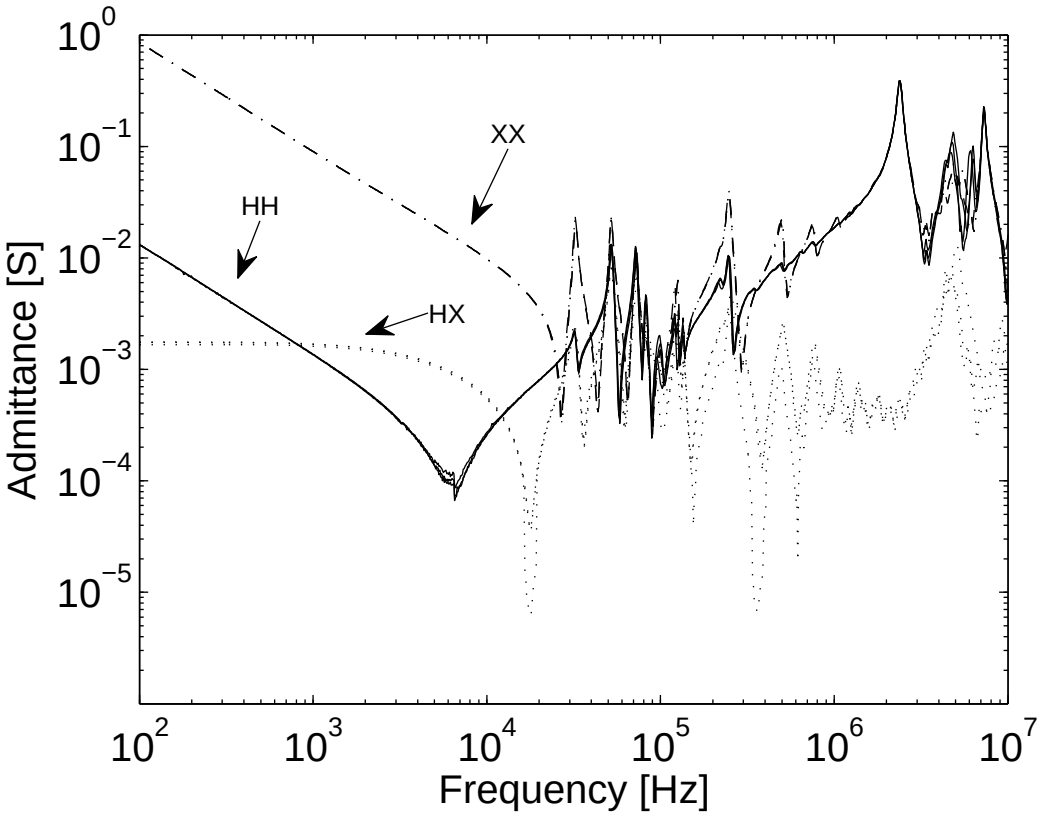

Figure 5.4: Example 1 - Magnitude HH's, XX's and HX's admittance diagonal entries.

Table 5.1: Example 1 - ψ Percentage of Violating frequencies

	Raw Data	Prep. Data	Model	Pass. Model
UESP	73.3	0.0	37.5	0.0
LDP	73.3	0.0	42.8	0.0
LEP	73.3	0.0	37.5	0.0
LNP	73.3	0.0	37.7	0.0
No prep.	73.3	- -	61.0	0.0

Table 5.2: Example 1 - Relative Error

	Error 1	Error 2	Error 3
UESP	0.1139	0.1754	**0.2168**
LDP	0.1235	0.1838	**0.2087**
LEP	0.1028	0.1674	**0.2017**
LNP	0.1024	0.1679	**0.2009**
No prep.	0.1245	**0.2888**	- -

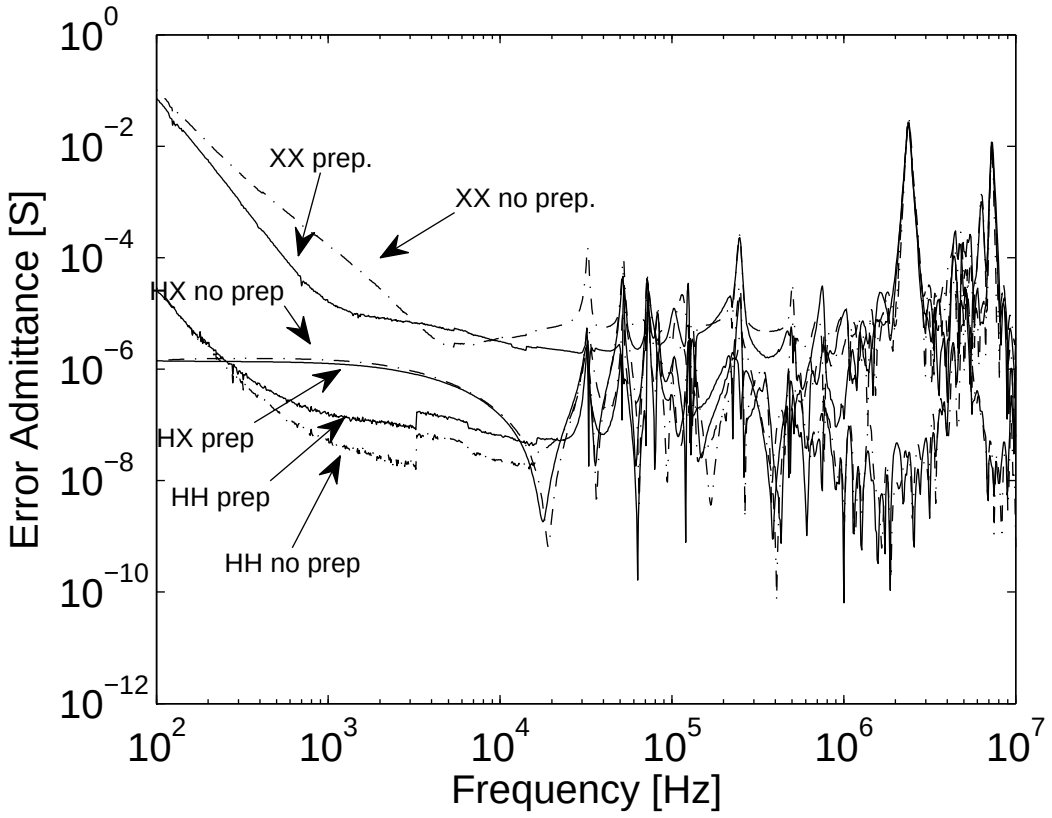

Figure 5.5: Example 1 - HH's, XX's and HX's admittance errors.

without pre-processing) and model. The second column reveals Error 2 between the input to SYSID and the passive model.

The rightmost column is only computed for the preprocessing approach so that the ultimate misfit reflects the overall difference to the raw original data (see Figure 5.2). In the case of preprocessing, the data to serve as input to identification is not the original data, but the passive preprocessed data. Consequently, we compute error 3 (only applicable when preprocessing is applied) so that results be indeed comparable, i.e., the same data base is used. By comparing the final errors (the rightmost column for preprocessed data and the central column in the bottom row for no preprocessing - both in **boldface**) one can see that preprocessing led to improved model accuracy, for any of its alternative formulations. According to Table (5.2), the best performance was attained for this data set was the Least-Norm Perturbation (LNP).

On closer examination, we notice that all preprocessing formulations have achieved similar performance, in terms of misfit. It is somewhat expected that the purely algebraic

formulation UESP performed slightly poorer than the alternative preprocessing formulation, as it is the most conservative in the sense that all eigenvalues are shifted up (in a passivity-seeking direction), thereby leading to a further increase in data passivity. Among the optimization-based strategies, the LDP perturbation has resulted in the largest misfit, this pattern will also repeat in the subsequent examples. Justification for this improved performance on the part of preprocessing lies in the fact that it relieves model parameter perturbation (passivity enforcement in post-processing) of the larger violations, permitting the use of the linearization assumption under which such algorithms work best, as confirmed by Table (5.2). For this example, the weighting used for both the LEP and LNP formulations correspond to a unitary β. Figure (5.5) displays the error curves for the computed frequency-response curves obtained with and without preprocessing. By inspection of Figure (5.3), we see that the majority of passivity violation occurs in the first two log-cycles, confirming the largest differences in the errors curves to occur in that same interval, which is precisely where the preprocessing acts more actively (where the larger violations occur).

5.1.2 Example 2

This second example comprises data of a 537/16 kV, 155MVA, single-phased power transformer represented by its 4×4 admittance matrix, with 1141 logarithmically spaced samples between 20 Hz and 10 MHz, approximated by a model containing 50 poles and a inverse weighting of the admittance magnitude. Figure (5.6) shows the transformer's eigenvalue profile where it is possible to see the presence of large violations and Figure (5.7) contains the raw frequency-response measurements.

The indices ψ and relative error can be visualized in Tables (5.3) and (5.4), respectively. Similarly, Table (5.3) reveals that the unenforced models obtained with preprocessed data violate less than those obtained with the traditional approach. Table (5.4) confirms that preprocessing leads to more accurate models. The best error performance was achieved with the LEP with a weighting factor β equals one.

Figure (5.8) displays the error curves for the computed frequency-response curves

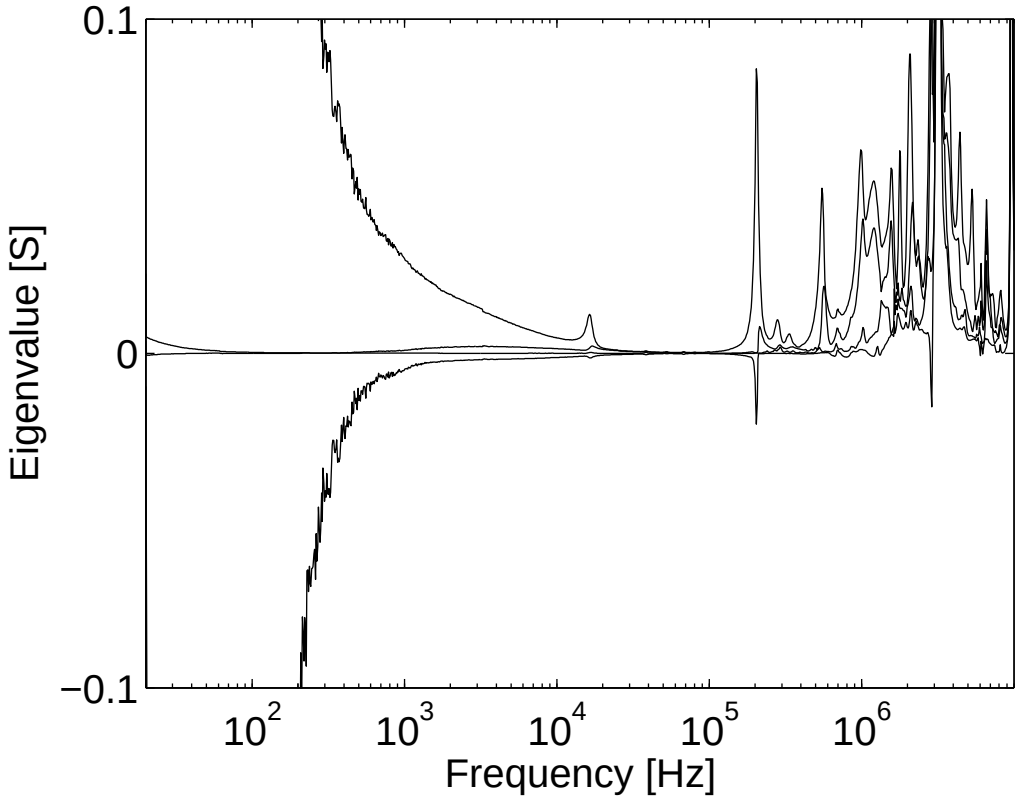

Figure 5.6: Example 2 - $\widetilde{\mathbf{G}}(jw_k)$ Eigenvalues.

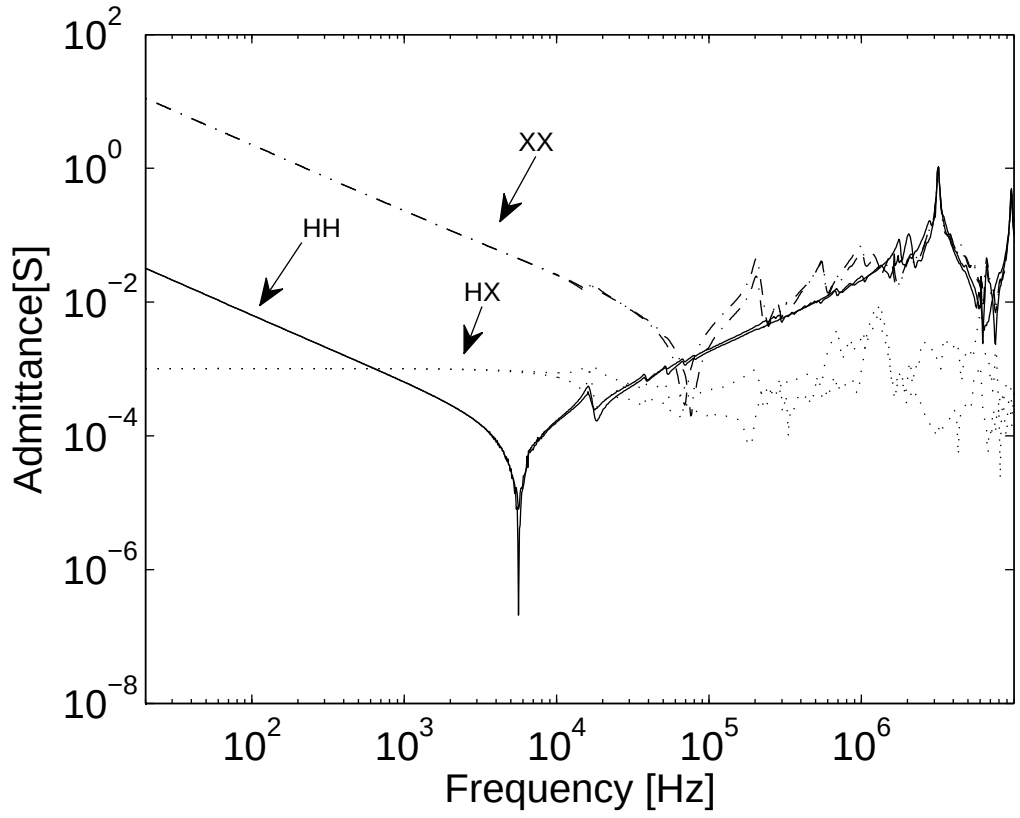

Figure 5.7: Example 2 - Magnitude HH's, XX's and HX's admittance diagonal entries.

Table 5.3: Example 2 - ψ Percentage of Violating frequencies

	Raw Data	Prep. Data	Model	Pass. Model
UESP	68.9	0.0	56.0	0.0
LDP	68.9	0.0	59.6	0.0
LEP	68.9	0.0	59.0	0.0
LNP	68.9	0.0	58.7	0.0
No prep.	68.9	- -	68.2	0.0

Table 5.4: Example 2 - Relative Error

	Error 1	Error 2	Error 3
UESP	0.0627	0.0786	**0.0842**
LDP	0.0534	0.0730	**0.0836**
LEP	0.0406	0.0620	**0.0739**
LNP	0.0538	0.0669	**0.0767**
No prep.	0.0448	**0.0892**	- -

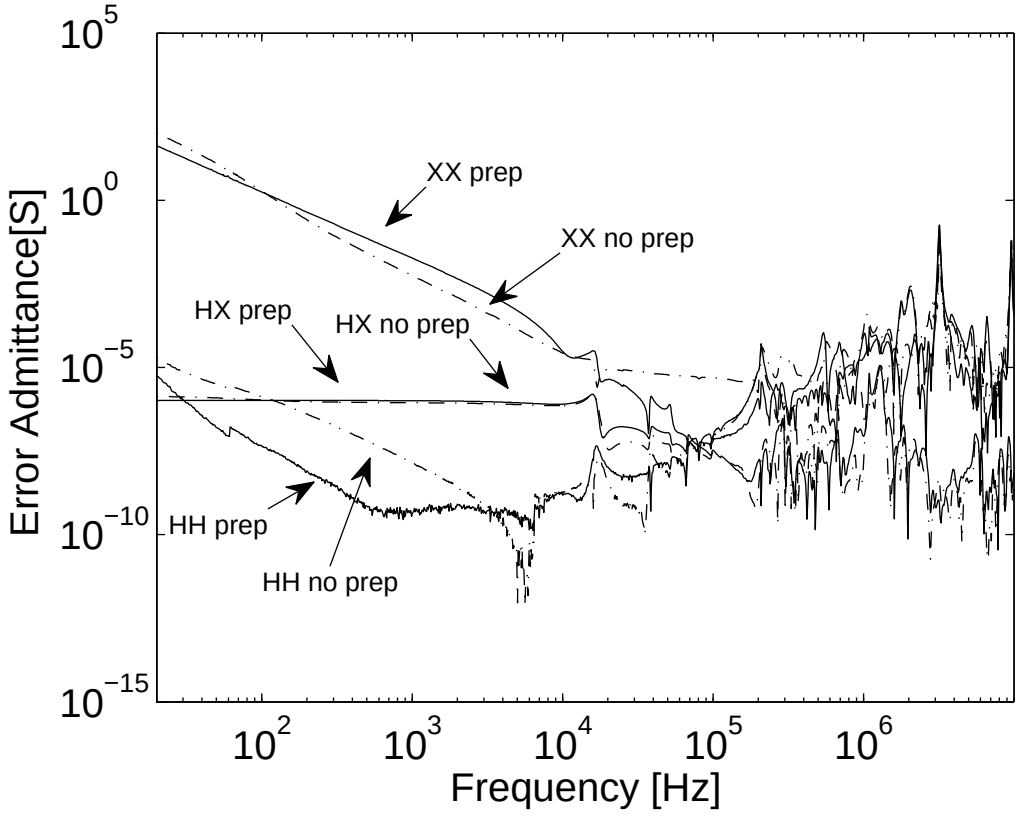

Figure 5.8: Example 2 - Magnitude HH's, XX's and HX's admittance errors.

obtained with and without preprocessing. As before, analysis of Figure (5.6) reveals most of the passivity violation to occur at lower frequencies which corroborates the largest differences in the errors curves to occur in a similar manner.

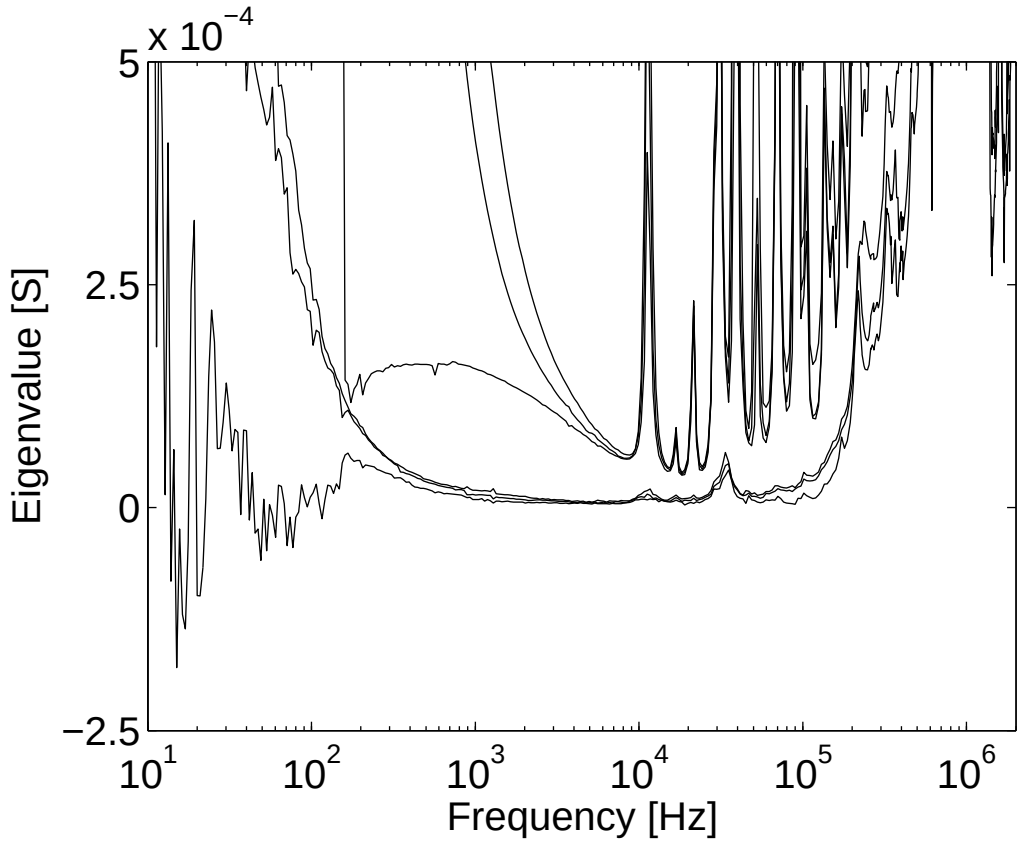

Figure 5.9: Example 3 - $\widetilde{\mathbf{G}}(jw_k)$ Eigenvalues.

5.1.3 Example 3

In this last example, the data comprises a 500/230 kV, 400 MVA, Yd, three-phase power transformer, comprising 600 logarithmically spaced samples between 10 Hz and 2 MHz in a 76-poles approximation and a inverse weighting of the admittance magnitude. Figure (5.9) shows the transformer's eigenvalue profile, the violations are small (not large as opposed to all previous examples), and Figure (5.10) contains the raw frequency-response measurements in a similar manner. Table (5.5) reveals that the unenforced models obtained with preprocessed data violate less than those obtained with the traditional approach, for all preprocessing choices.

Table (5.6) displaying the relative error confirms that preprocessing has contributed to a smaller error hence increasing the model accuracy, even in the case of small passivity violations. In this example, the accuracy gains of pre-processing are modest, as expected. This comes as a result of the small passivity violations which is precisely the requirement for the linearity assumption of the enforcement algorithms. In such circumstances, the

Table 5.5: Example 3 - ψ Percentage of Violating frequencies

	Raw Data	Prep. Data	Model	Pass. Model
UESP	3.3	0.0	8.0	0.0
LDP	3.3	0.0	4.8	0.0
LEP	3.3	0.0	4.6	0.0
LNP	3.3	0.0	4.6	0.0
No prep.	3.3	- -	8.2	0.0

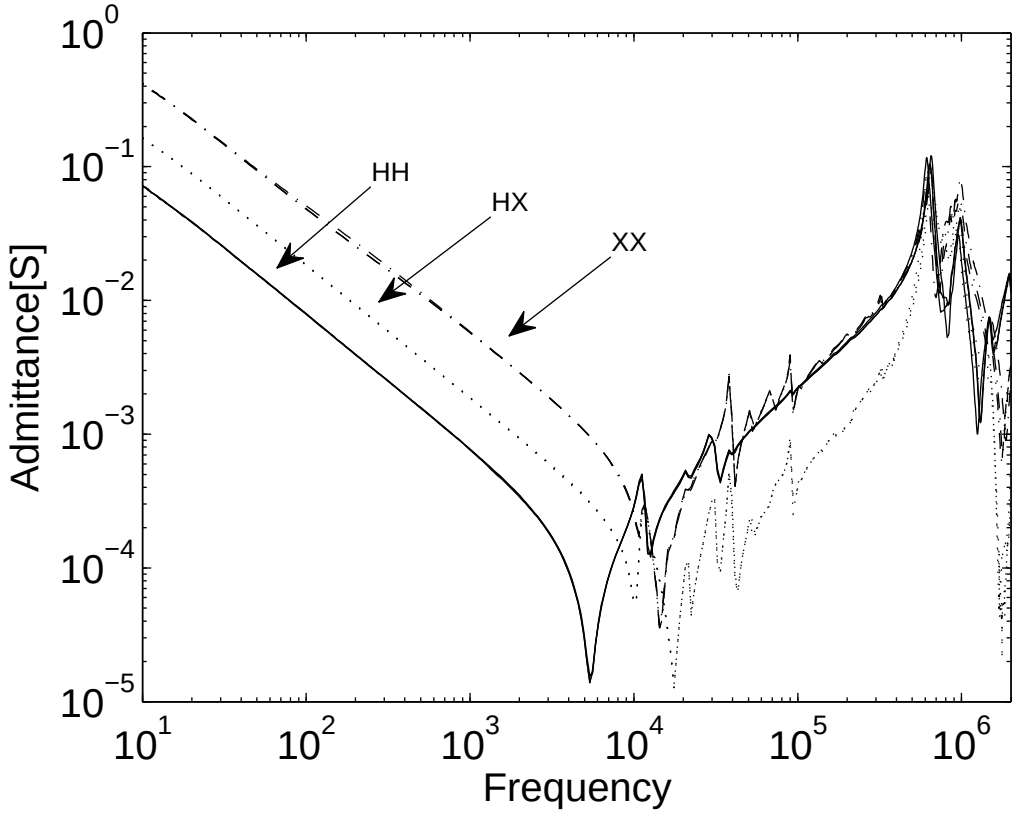

Figure 5.10: Example 3 - Magnitude HH's, XX's and HX's admittance diagonal entries.

enforcement algorithms are working in optimal conditions. However, this example has revealed that even in this less favourable case, pre-processing has increased the accuracy for both the LEP and LNP or did not affect the ultimate misfit as it was the case for UESP and LDP. For this example, the best misfit was achieved with both the LEP and LNP with not weighting.

Figure (5.11) displays the error curves for the computed frequency-response curves obtained with and without preprocessing. The eigenvalues violate more at lower frequencies, as seen in Figure (5.9).

Table 5.6: Example 4 - Relative Error

	Error 1	Error 2	Error 3
UESP	0.0035	0.0035	**0.0035**
LDP	0.0035	0.0035	**0.0035**
LEP	0.0034	0.0034	**0.0034**
LNP	0.0035	0.0034	**0.0034**
No prep.	0.0035	**0.0035**	- -

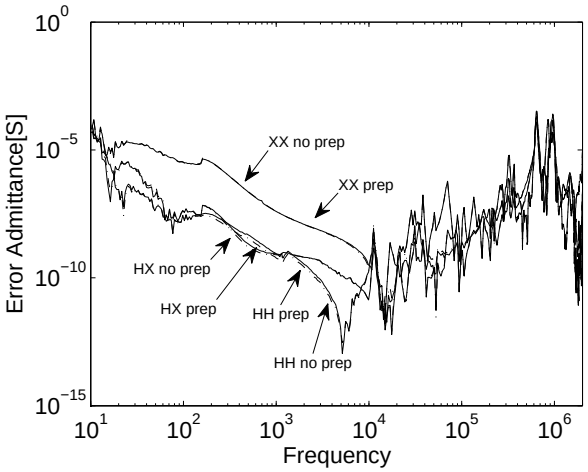

Figure 5.11: Example 4 - Magnitude HH's, XX's and HX's admittance errors.

5.2 Post-processing

The following two examples serve to illustrate an alternative implementation of the convex formulation passivity enforcement scheme base on the positive-real lemma (PRL). These case studies also use data from actual power transformers as measured by their admittance formulations. These results have appeared in Oliveira et al. (2014).

5.2.1 Example 4

The high-voltage terminal admittance of a single-phase 525/18 kV, 256 MVA step-up transformer serves as an example for testing the proposed enforcement procedure. In Oliveira & Mitchell (2013), this particular data set was also employed, nonetheless the emphasis then resided in passivity-related issues and their consequences in transient analysis.

With frequency-response admittance measurements at the high-voltage winding, a black-box model is obtained. In this case $n = 1$. The available measurements $Y(jw_k)$

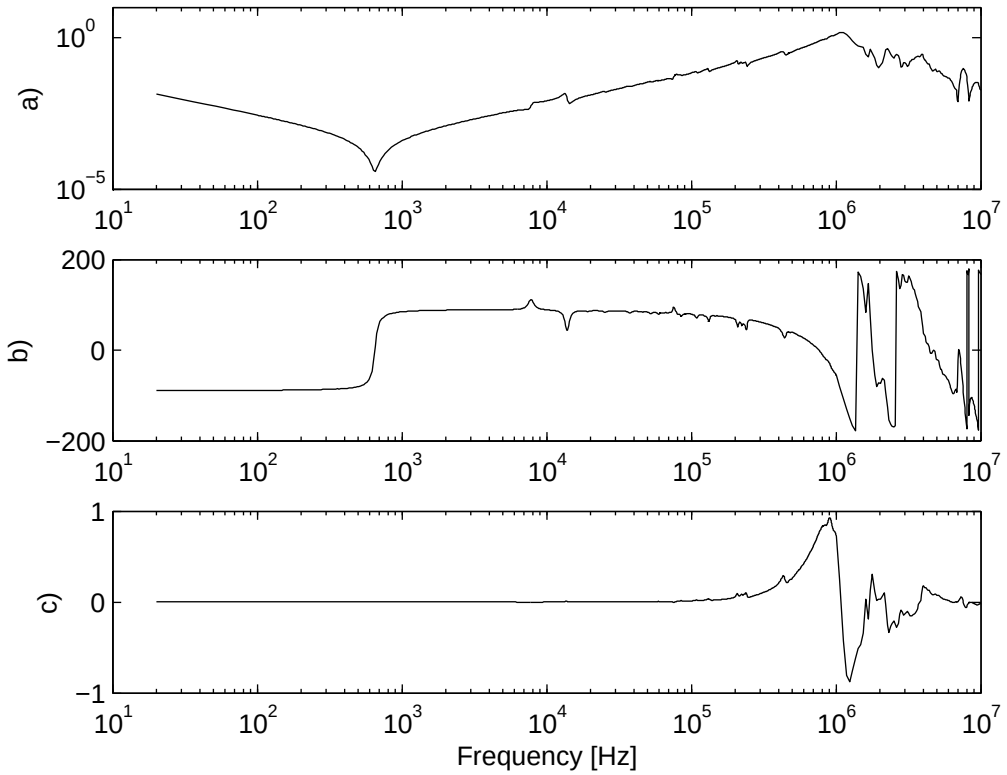

Figure 5.12: High-voltage terminal admittance: (a) magnitude [dB] and (b) angle [degrees]; and, (c) High-voltage terminal conductance [S].

($K = 910$) are depicted in Figure 5.12. In that same figure, the terminal conductance can be visualized (see equation (2.26)).

Measurements depicted Figure 5.12 clearly demonstrate anomalous behavior since the conductance matrix is not positive definite at various frequencies, chiefly in the higher-frequency range, this is assuredly attributable to measurement interference and noise.

A state-space realization for $\mathbf{Y}$ can be obtained with the system identification procedure defined in reference Oliveira et al. (2014), assuming that: $\{\phi_{11m}(t)\}_{m=1}^{\infty}$ are Takenaka-Malmquist basis functions, truncated at the first 10 basis functions ($N = 10$), such that the weighting function $\Xi(jw_k)$ is chosen to be $\Xi(jw_k) = 1/|Y(jw_k)|$.

Within 40 iterations, a model is identified. A comparison between model approximation and measurements can be observed in Figure (5.13). According to Figure (5.12) measurements happen to be non-passive, the anomaly propagates into the estimated model as revealed in Figure (5.13) and confirmed by the presence of purely imaginary eigenvalues

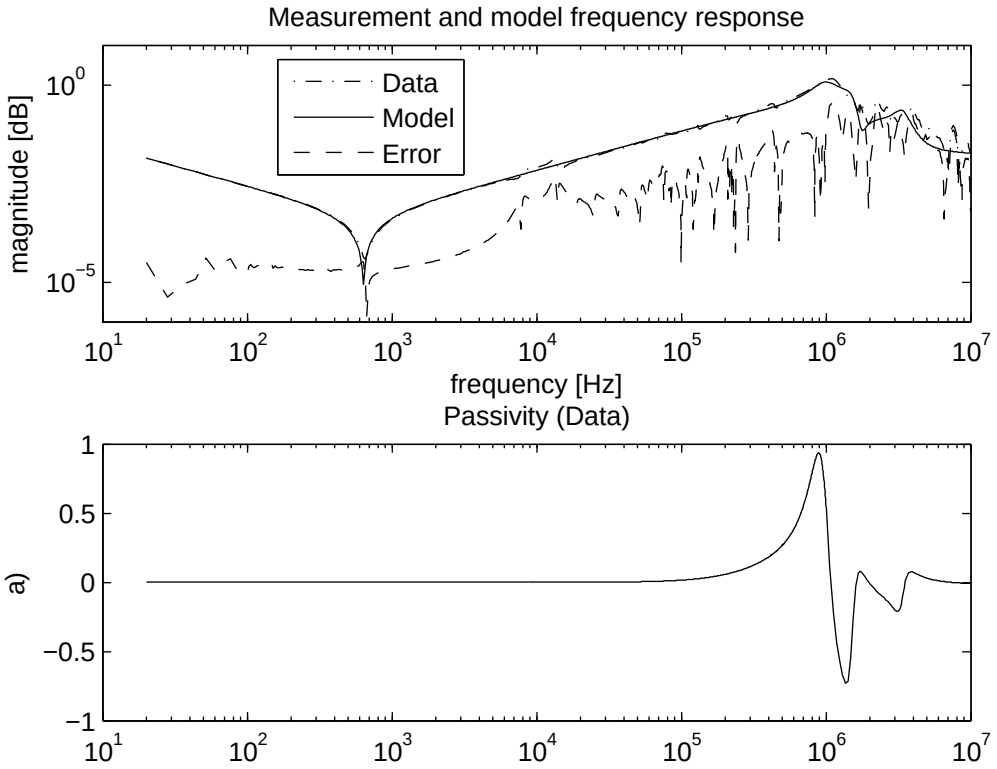

Figure 5.13: Model, data frequency response and modelling error: (a) magnitude [dB] and (b) Model Conductance [S].

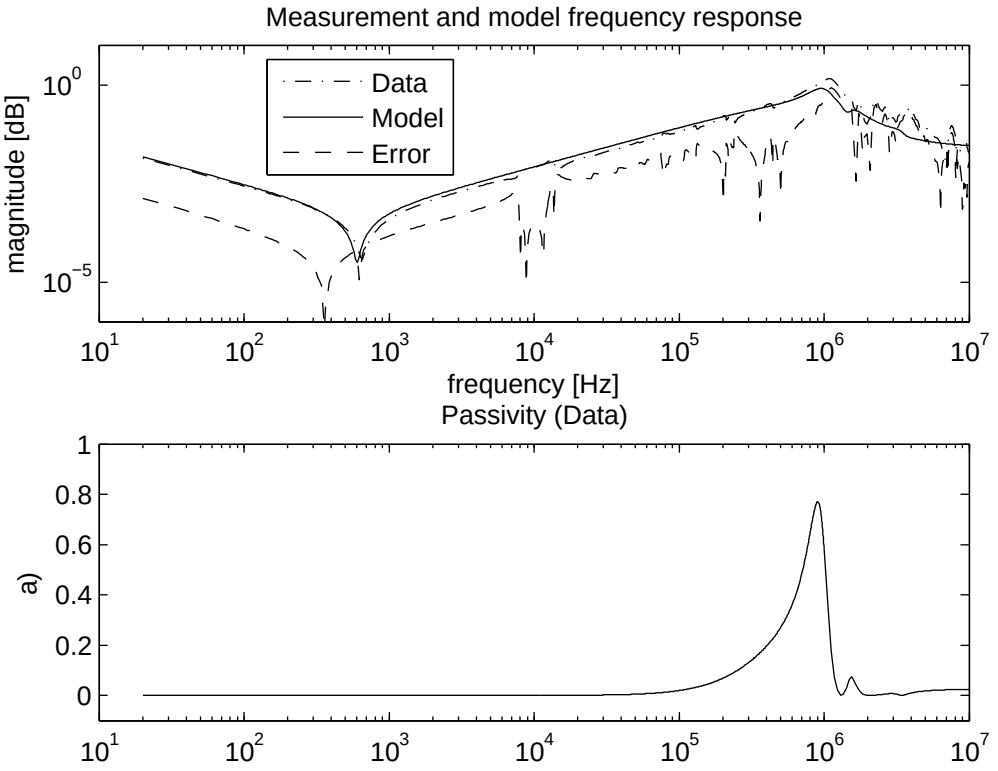

Figure 5.14: Model, data frequency response and modelling error: (a) magnitude [dB] and (b) Conductance [S].

Table 5.7: Example 4 - MSE

	Data	1st model	Enforced model
ψ	12.86%	8.57%	0%
MSE ($\times 10^{-3}$)		3.34	21.5

in the associated Hamiltonian matrix eigenvalues' set.

The procedure proposed in Oliveira et al. (2014) is therefore applied so that matrices **C** and **D** are once more obtained. Figure 5.14 is a comparison of the new model approximation and measurements; it also reveals the model conductance which is clearly positive at all frequencies and the passivity conditions are then satisfied.

The index ψ, proposed in Chiariello et al. (2010), which consists of the percentage rate of violating frequencies with respect to the full frequency range, is computed. Even though frequencies beyond measurements are neglected, the referred index still is an interesting parameter for evaluating passivity violation. Table 5.7 is a summary for the obtained ψ index and model mean-square error (MSE), considering w_k and $k = 1, \ldots, 910$. Row

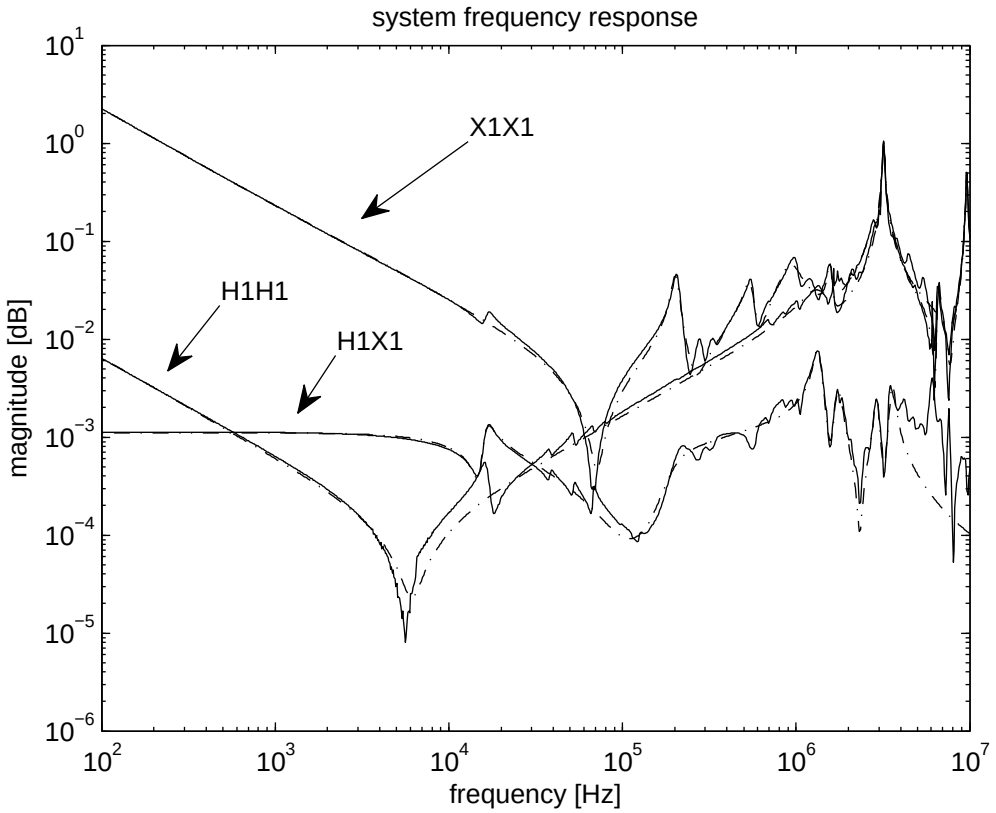

Figure 5.15: Measurement (solid line) and non-passive model (dashed line) admittance matrices (dashed line).

one reveals the degree to which initial-unprocessed data (column 1) is non-passive to be higher than that of the model without enforcement (column 2). Column three confirms that the enforcement is successful and has taken the index to zero. By examining MSE results, it is evident that enforcement penalizes model approximation so as to render it passive.

5.2.2 Example 5

The convex-optimization enforcement procedure is employed to model a two-terminal, single-phased, 537/16 kV, 155 MVA, power transformer admittance matrix. Terminals $H0$ and $X2$ are assumed grounded. For the remaining terminals, i.e. $H1$ and $X1$, $n = 2$. The available admittance measurements $Y(jw_k)$ ($K = 1141$) are displayed in Figure 5.15. Measurement conductance matrix (see equation (2.26)) eigenvalues can be visualized in Figure 5.16 which reveals passivity violation for some frequencies.

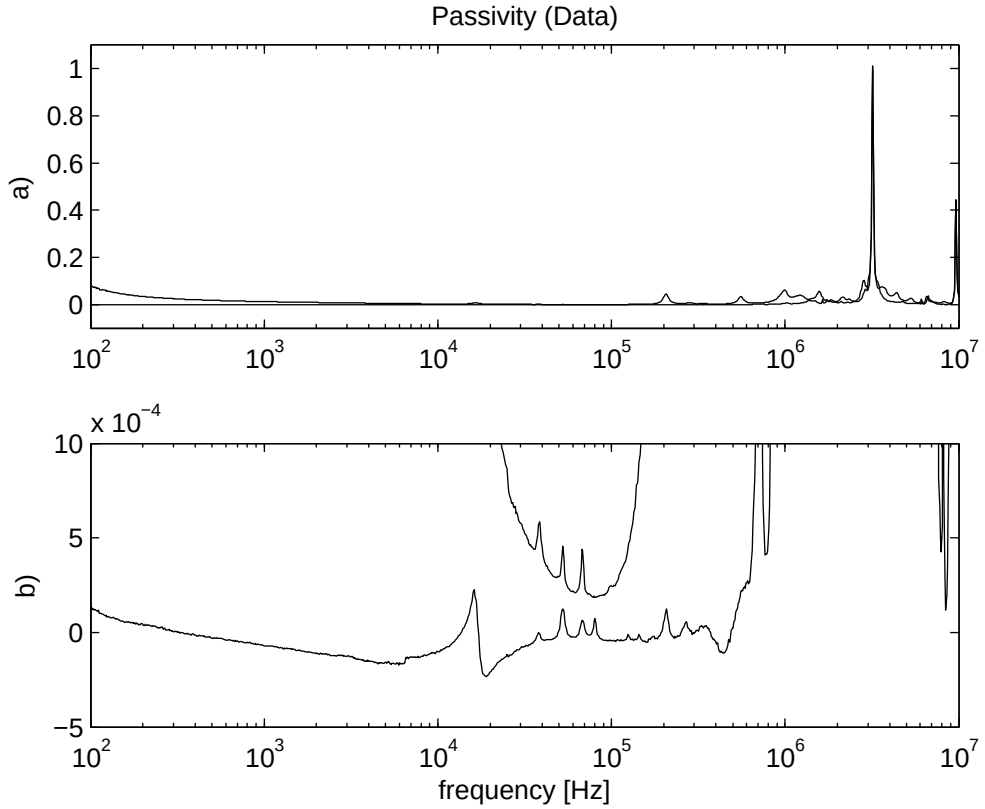

Figure 5.16: Measured conductance matrix eigenvalues, a) full-range b) near-zero amplitude magnification.

A state-space realization for $\mathbf{Y}$ can be obtained with the system identification procedure from reference Oliveira et al. (2014), assuming that: $\{\phi_{11m}(t)\}_{m=1}^{\infty}$ are Takenaka-Malmquist basis functions, restricted to the first 14 functions ($N = 14$), and the weighting function $\Xi(jw_k)$ is chosen to be $\Xi(jw_k) = 1/|Y(jw_k)|$. Within 50 iterations, a model is identified. A comparison between model approximation and measurements can be observed in Figure 5.15. According to Figure (5.16), the identified model is perfectly consistent with non-passive measurements, i.e. both are non-passive. As a result, there is no matrix P such the positive-real lemma is satisfied thus confirming non-passive behavior.

The identified model is then subjected to the LMI-based enforcement procedure, in the variables $\mathbf{C}$ and $\mathbf{D}$, so that the optimal and passive approximation to the data is obtained. Figure 5.17 displays a comparison between the identified passive optimal model and measurements. The optimal model is clearly passive and satisfies the positive-real condition, at the expense of a slightly poorer model approximation.

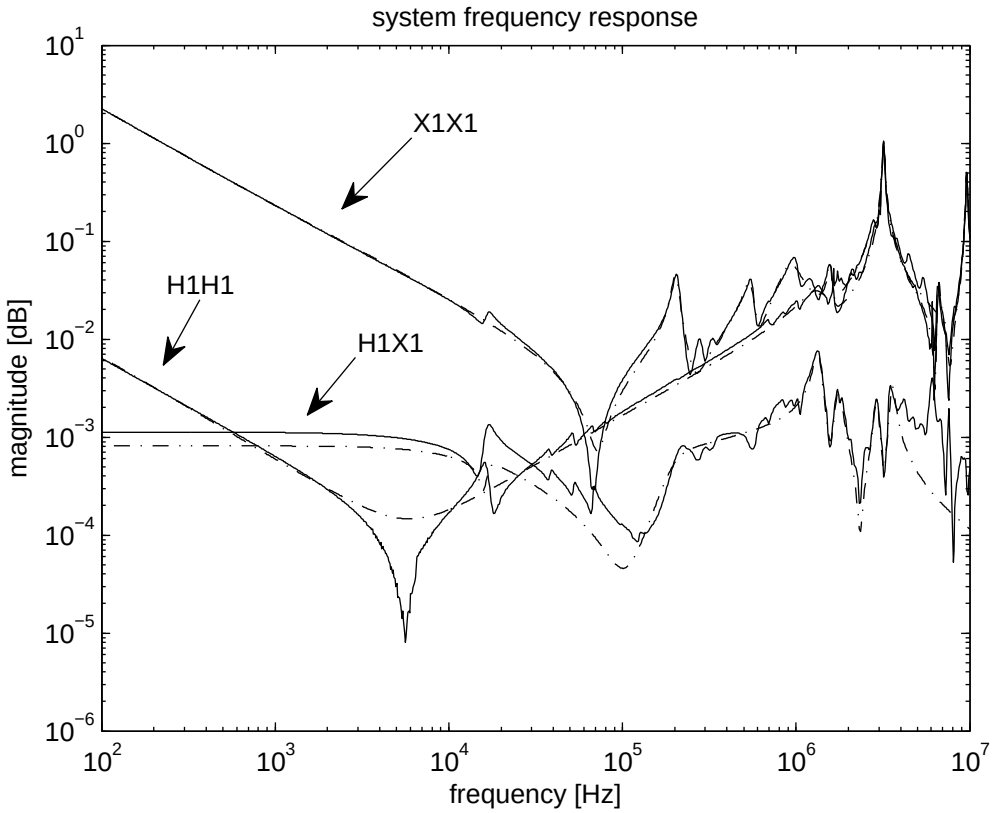

Figure 5.17: Measurement (solid line) and non-passive model(dashed line) admittance matrices.

Table 5.8: Example 5 - MSE

	Data	1st model	Enforced model
ψ	45.62%	42.68%	0%
MSE ($\times 10^{-5}$) $H1H1$		3.98	3.42
MSE ($\times 10^{-7}$) $H1X1$		2.07	2.64
MSE ($\times 10^{-5}$) $X1X1$		8.77	8.81

Table 5.8 is a summary for the obtained ψ index and model mean-square error (MSE), considering w_k and $k = 1, \ldots, 1141$. Data and prior-to-enforcement model are non-passive to a similar degree. The enforced model is unambiguously passive, as expected. There seems to be an apparent degrading for the enforced model when it is compared against the non-passive model. In terms of MSE, terminal $H1H1$ MSE has slightly decreased for the enforced model in spite of the acute misfit at about 5kHz. For terminals $H1X1$ and $X1X1$, MSE has slightly increased.

CHAPTER 6

CONCLUSIONS

Modeling is an intellectual activity brimming over with subtleties which can certainly undermine the whole enterprise if not addressed appropriately. This research is entirely devoted to one vitally important aspect of the subject: passivity. Devising a passive model is a multi-faceted problem which consistently reappears in the context of transient simulations. Many algorithms exist that robustly conceive frequency-domain approximations to an arbitrarily close degree of accuracy, with no guarantee on simulation stability, nevertheless. As passivity constraints enter the modeling problem it grows in both complexity and difficulty. Many different formulations exist to address this issue with varying degrees of reliability and computation time. This particular research has culminated in a original formulation for pre-processing and a reformulation of a convex post-processing which can be employed either independently or jointly.

Pre-processing cannot by itself obviate the need for model passivity enforcement. Its advantage resides in improving model accuracy by providing guaranteed passive data to identification algorithms and subsequent passivity enforcement. This is particularly crucial in case of large passivity violations whereby the linearity assumptions under which the problem constraints reside are no longer valid. Therefore, pre-processing is a good modeling practice. Moreover, by delivering passive data to the identification algorithm, preprocessing actually delivers better-to-emulate data. When adjusting parameters, the model performance is affected globally, on the other hand data passivity compensation partly relieves the global accuracy comprise of the approximation and increased accuracy is achieved. Data perturbation seems to cost less, in terms of model accuracy and approximation, than parameter perturbation. Even though the case studies herein presented consisted exclusively of admittance parameters, it is readily applicable to all n-port representations requiring only trivial modifications for the scattering parameters.

Post-processing is a *sine qua non* for obtaining passive models. The novel convex formulation herein formulated based on the positive-real lemma provides both necessary and sufficient conditions for obtaining a guaranteed passive model. The well-known disadvantage of convex formulations concerns the usual large computation time for high-order models due to the additional LMI variables. The enforcement formulation herein proposed is independent of the algorithm employed to generate the initial approximation, any public domain algorithm can be used, e.g., VF. The method furnishes a high-accuracy guaranteed-passive model and can be implemented in packages available in the public domain.

As a few parting remarks, it is important to highlight what is envisaged as possible extensions of what has been so far proposed. First and foremost, formulating passivity conditions for non-linear model structures remains, to the best of this author's knowledge, an open question. Challenging issues pertaining to the task of enforcing physical realizability, i.e., converting a given model structure into an electrical network with strictly positive elements, still deserves further consideration. This approach seeks models that are structurally passive/dissipative. Infinite-dimensional or Distribution Theory formulations pose yet another challenge to the passivity community, chiefly in terms of numerical implementation in a digital computer. Minor extensions to this research could certainly address the systematic use of weighting schemes to avoid accuracy degradation of specific entries, promising ideas seem to be emerging of modal formulations. Another issue of relevance entails passivity-preserving model-order reduction, specially for convex formulations which require long computation times for high-order models. These techniques are basically intended for eliminating model redundancy or dynamics with negligible effects on the transfer functions aiming at computational gains.

Prospecting for further research areas is an obviously challenging task for one can always do the subject an injustice by offering others too partial a view on the subject, thus ending up luring talents away from it. Hopefully, the saturation point of this vibrant subject of passive models will not be reached anytime soon and many unexpected emergent ideas will arise.

BIBLIOGRAPHY

Aguirre, L. A. (2007), *Introdução à Identificação de Sistemas*, 3a. edn, Editora UFMG.

Akçay, H., Islam, S. M. & Ninness, B. (1998), 'Identification of power transformer models from frequency response data: A case study', *Signal Processing* **68**(3), 307–315.

Allen, J. C. (2004), *H-infinity Engineering and Amplifier Optimization*, 1 edn, Birkhauser.

Anderson, B. D. O. (1967*a*), 'Algebraic description of bounded real matrices', *Electronics Letters* **2**(12), 464–465.

Anderson, B. D. O. (1967*b*), 'A system theory criterion for positive real matrices', *SIAM Journal on Control* **5**(2), 171–182.

Anderson, B. D. O. & Vongpanitlerd, S. (2006), *Network Analysis and Synthesis: A Modern Systems Theory Approach*, Dover Publications.

Annakkage, U. D., Nair, N. K. C., Liang, Y., Gole, A. M., Dinavahi, V., Gustavsen, B., Noda, T., Ghasemi, H., Monti, A., Matar, M., Iravani, R. & Martinez, J. A. (2012), 'Dynamic system equivalents: A survey of available techniques', *IEEE Transactions on Power Delivery* **27**(1), 411–420.

Bechara, R. (2010), Analysis of power transformer failures (in portuguese), Master's thesis, USP/Escola Politécnica.

Boyd, S. P., Balakrishnan, V. & Kabamba, P. (1989), 'A bisection method for computing the H_∞ norm of a transfer matrix and related problems', *Mathematics of Control, Signals, and Systems* **2**, 207–219.

Boyd, S. P. & Chua, L. O. (1982), 'On the passivity criterion for LTI n-ports', *Circuit Theory and Applications* **10**, 323–333.

Boyd, S. P., Ghaoui, L. E., Feron, E. & Balakrishnan, V. (1994), *Linear Matrix Inequalities in System and Control Theory*, SIAM.

Boyd, S. P. & Vandenberghe, L. (2004), *Convex Optimization*, Cambridge University Press.

Brune, O. W. H. O. (1931), Synthesis of a finite two-terminal network whose driving-point impedance is a prescribed function of frequency, PhD thesis, MIT - Department of Electrical Engineering.

Carlin, H. J. (1956), 'The scattering matrix in network theory', *IRE Transactions on Circuit Theory* **3**(2), 88–97.

Chen, C.-T. (2012), *Linear System Theory and Design*, 3 edn, Oxford University Press.

Cheng, D. K. (1972), *Analysis of Linear Systems*, world student series edition edn, Addison-Wesley Publishing Company.

Chiariello, A. G., Magistris, M. D., Tommasi, L. D., Deschrijver, D. & Dhaene, T. (2010), Numerical validation of a procedure for direct identification of passive linear multi-port with convex programming, *in* '14th IEEE Workshop on Signal Propagation on Interconnects', pp. 141–144.

Chua, L. O. & Lam, Y.-F. (1973), 'A theory of algebraic n-ports', *IEEE Transactions on Circuit Theory* **20**(4), 370–382.

Cipparrone, F. A. M., Gustavsen, B. & Semlyen, A. (2001), 'Discussion of enforcing passivity for admittance matrices approximated by rational functions', *IEEE Transactions on Power Systems* **16**(4), 954–955.

Coelho, C. P., Phillips, J. & Silveira, L. M. (2001), A convex programming approach to positive real rational approximation, *in* 'Proceedings of International Conference on Computer Aided Design', Santa Clara CA, USA, pp. 245–251.

Coelho, C. P., Phillips, J. & Silveira, L. M. (2004), 'A convex programming approach for generating guaranteed passive approximations to tabulated frequency data', *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems* **23**(2), 293–301.

Curtain, R. F. (1999), 'Old and new perspectives on the positive-real lemma in systems and control theory', *Zeitschrift für Angewandte Mathematik und Mechanik* **79**(9), 579–590.

CVX Research, I. (2012), 'CVX: Matlab software for disciplined convex programming, version 2.0', http://cvxr.com/cvx.

D. C. Youla, L. J. C. & Carlin, H. J. (1959), 'Bounded real scattering matrices and the foundations of linear passive network theory', *IRE Transactions on Circuit Theory* **12**, 102–124.

Desoer, C. A. & Kuh, E. S. (1969), *Basic Circuit Theory*, international student edition edn, McGraw-Hill.

Golub, G. H. & Loan, C. F. V. (2012), *Matrix Computations*, 4 edn, Johns Hopkins University Press.

Grant, M. & Boyd, S. (2008), Graph implementations for nonsmooth convex programs, *in* V. Blondel, S. Boyd & H. Kimura, eds, 'Recent Advances in Learning and Control', Lecture Notes in Control and Information Sciences, Springer-Verlag Limited, pp. 95–110.

Grivet-Talocia, S. (2004), 'Passivity enforcement via perturbation of hamiltonian matrices', *IEEE Transactions on Circuits and Systems* **51**(9), 1755–1769.

Grivet-Talocia, S. & Ubolli, A. (2008), 'A comparative study of passivity enforcement schemes for linear lumped macromodels', *IEEE Transactions on Advanced Packaging* **31**(4), 673–683.

Gustavsen, B. (2002), 'Computer code for rational approximation of frequency dependent admittance matrices', *IEEE Transactions on Power Delivery* **17**(4), 1093–1098.

Gustavsen, B. (2004*a*), 'Frequency-dependent modeling of power transformers with ungrounded windings', *IEEE Transactions on Power Delivery* **19**, 1328–1334.

Gustavsen, B. (2004*b*), 'Wide band modeling of power transformers', *IEEE Transactions on Power Delivery* **19**, 414–422.

Gustavsen, B. (2007), 'Computer code for passivity enforcement of rational macromodels by residue perturbation', *IEEE Transactions on Advanced Packing* **30**(2), 209–215.

Gustavsen, B. (2008*a*), 'Fast passivity enforcement for pole-residue models by perturbation of residue matrix eigenvalues', *IEEE Transactions on Power Delivery* **23**(4), 2278–2285.

Gustavsen, B. (2008*b*), 'Passivity enforcement of rational models via modal perturbation', *IEEE Transactions on Power Delivery* **23**(2), 768–775.

Gustavsen, B. (2010), 'Study of transformer resonant overvoltages caused by cable-transformer high-frequency interaction', *IEEE Transactions on Power Delivery* **25**(2), 770–779.

Gustavsen, B. (2014), 'Rational modeling of multiport systems via a symmetry and passivity preserving mode-revealing transformation', *IEEE Transactions on Power Delivery* **29**(1), 199–206.

Gustavsen, B. & Heitz, C. (2008), 'Modal vector fitting: A tool for generating rational models of high accuracy with arbitrary terminal conditions', *IEEE Transactions on Advanced Packing* **31**(4), 664–672.

Gustavsen, B. & Semlyen, A. (1998), 'Application of vector fitting to state equation representation of transformers for simulation of electromagnetic transients', *IEEE Transactions on Power Delivery* **13**(3), 834–842.

Gustavsen, B. & Semlyen, A. (1999), 'Rational approximation of frequency domain responses by vector fitting', *IEEE Transactions on Power Delivery* **14**(3), 1052–1061.

Gustavsen, B. & Semlyen, A. (2001), 'Enforcing passivity for admittance matrices approximated by rational functions', *IEEE Transactions on Power Systems* **16**(1), 97–104.

Gustavsen, B. & Semlyen, A. (2004), 'A robust approach for system identification in the frequency domain', *IEEE Transactions on Power Delivery* **19**(3), 1167–1173.

Gustavsen, B. & Semlyen, A. (2008), 'Fast passivity assessment for s-parameter rational models via a half-size test matrix', *IEEE Transactions on Microwave Theory and Techniques* **56**(12), 2701–2708.

Gustavsen, B. & Semlyen, A. (2009*a*), 'A half-size singularity test matrix for fast and reliable passivity assessment of rational models', *IEEE Transactions on Power Delivery* **24**(1), 345–351.

Gustavsen, B. & Semlyen, A. (2009*b*), 'On passivity tests for unsymmetrical models', *IEEE Transactions on Power Delivery* **24**(3), 1739–1741.

Gustavsen, B. & Silva, H. M. J. D. (2013), 'Inclusion of rational models in an electromagnetic transients program: Y-parameters, z-parameters, s-parameters, transfer functions', *IEEE Transactions on Power Delivery* **28**(2), 1164–1174.

Hori, M., Mitani, M., Sakurano, H. & Motoyama, H. (2007), 'Observation and analysis of incident surge voltage waveforms in substations by winter lightning', *IEEE Transactions on Power Delivery* **22**(1).

Horn, R. A. & Johnson, C. R. (2012), *Matrix Analysis*, 2 edn, Cambridge University Press.

Huelsman, L. P. (2011), *Circuits, Matrices and Linear Vector Spaces*, Dover Publications Inc.

Ihlenfeld, L. P. R. K., Oliveira, G. H. C. & Sans, M. R. (2015), 'A data passivity-enforcement preprocessing approach to multiport system modeling', *IEEE Transactions on Power Delivery* .

JWG-A2/C4.39, C. (2014), *Electrical Transient Interaction Between Transformers and the Power System. Part 1: Expertise and Part 2: Case Studies*, Cigre.

Kalman, R. E. (1964), On a new characterization of linear passive systems, Technical Report 64, Research Institute for Advanced Studies - RIAS, Baltimore.

Kuo, F. F. (1966), *Network Analysis and Synthesis*, wiley international edition edn, John Wiley and Sons.

Kurokawa, K. (1965), 'Power waves and the scattering matrix', *IEEE Transactions on Microwave Theory* **13**(2), 194–202.

Ljung, L. (1999), *System Identification: Theory for the user*, 2a. edn, Prentice Hall.

Ljung, L. (2010), 'Perspectives on system identification', *ELSEVIER Annual Reviews in Control* **34**(1), 199–206.

Loan, C. F. V. (2000), 'The ubiquitous kronecker product', *Journal of Computational and Applied Mathematics* **123**, 85–100.

Maestrelli, R. (2010), Funções ortonormais em tempo contínuo com seleção ótima das dinâmicas do modelo na identificação de sistemas no domínio da frequência, Master's thesis, Pontifícia Universidade Católica do Paraná.

Maestrelli, R. & Oliveira, G. H. C. (2010), Método para identificação de sistemas contínuos mal amortecidos no domínio da frequência usando funções ortonormais, *in* '17 Congresso Brasileiro de Automática'.

Morched, A., Marti, L. & Ottevangers, J. (1993), 'A high frequency transformer model for the emtp', *IEEE Transactions on Power Delivery* **8**(3), 1615–1626.

OLIVEIRA, G. H. C., M. R. R. A. C. O. (2009), An application of orthonormal basis functions in power transformers wide band modeling, *in* '7th IEEE International Conference on Control and Automation'.

Oliveira, G. H. C. & Mitchell, S. D. (2013), Comparison of black-box modeling approaches for transient analysis: A gis case study, *in* 'Proceeding of the International Conference on Power System Transients', Vancouver, CA.

Oliveira, G. H. C., Rodier, C. & Ihlenfeld, L. P. R. K. (2014), 'LMI-based method for estimating passive black-box models in power systems transient analysis', *IEEE Transactions on Power Delivery* .

Raisbeck, G. (1954), 'A definition of passive linear networks in terms of time and energy', *Journal of Applied Physics* **25**(12), 1510–1514.

Reginato, B. C. (2008), Ambiente computacional para identificação no domínio do tempo e da frequência usando bases de funções ortonormais, Master's thesis, Pontifícia Universidade Católica do Paraná.

Reginato, B. C. & Oliveira, G. H. C. (2007), On selecting the mimo generalized orthonormal basis functions poles by using particle swarm optimization, *in* 'Proc. of the European Control Conference', pp. 1–6.

Reginato, B. C. & Oliveira, G. H. C. (2008), Algoritmo híbrido para seleção de pólos em modelos formados por bases de funções ortonormais generalizadas, *in* '17 Congresso Brasileiro de Automática'.

Sanathann, C. K. & Koerner, J. (1963), 'Transfer function synthesis as a ratio of two complex polynomials', *IEEE Transactions Automatic Control* **9**(1), 56–58.

Sans, M. R. (2013), Análise de métodos de medição da matriz admitâcia em transformadores de potência, Master's thesis, UFPR/Electrical Engineering Department.

Sans, M. R., Oliveira, G. H. C. & Junior, J. A. T. (2012), Sobre métodos de medição da matriz de admitância para simulação de transitórios elétricos envolvendo transformadores de potência, *in* 'XIX Congresso Brasileiro de Automática'.

Saraswat, D., Achar, R. & Nakhla, M. S. (2004), 'A fast algorithm and practical considerations for passive macromodeling of measured/simulated data', *IEEE Transactions on Advanced Packaging* **27**(1), 57–70.

Shipp, D. D., Dionise, T. J., Lorch, V. & MacFarlane, B. G. (2011), 'Transformer failure

due to circuit-breaker-induced switching transients', *IEEE Trans. on Industry Applications* **47**(2), 707–717.

Sturm, J. F. (1999), 'Using sedumi 1.02, a matlab toolbox for optimization over symmetric cones', *Optimization Methods and Software* pp. 625–653.

Tommasi, L. D., de Magistris, M., Deschrijver, D. & Dhaene, T. (2011), 'An algorithm for direct identification of passive transfer matrices with positive real fractions via convex programming', *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields* **24**(10), 375–386.

Tommasi, L. D., Deschrijver, D. & Dhaene, T. (2008), 'Single-input-single-output-passive macromodeling via positive fractions vector fitting', *Twelfth IEEE Workshop on Signal Propagation on Interconnects* .

Transformers-Switchgear-Committee (2010), *IEEE Guide to Describe the Occurrence and Mitigation of Switching Transients Induced by Transformers, Switching Device, and System Interaction*, number C57.142, IEEE Power & Energy Society.

Triverio, P., Grivet-Talocia, S., Nakhla, M. S., Canavero, F. G. & Achar, R. (2007), 'Stability, causality, and passivity in electrical interconnect models', *IEEE Transactions on Advanced Packaging* **30**(4), 795–808.

Willems, J. C. (1972), 'Dissipative dynamical systems', *Archive for Rational Mechanics and Analysis* **45**, 321–393.

Wohlers, M. R. (1969), *Lumped and distributed passive networks*, 1 edn, Academic Press.

Wong, N. & Zhang, Z. (2010), 'Discussion of a half-size singularity test matrix for fast and reliable passivity assessment of rational models', *IEEE Transactions on Power Delivery* **25**(2), 1212–1213.

Zemanian, A. H. (1963), 'An n-port realizability theory based on the theory of distributions', *IEEE Transactions on Circuit Theory* pp. 265–274.

Zhongyuan, Z., Fangcheng, L. & Guishu, L. (2013), 'High-frequency circuit model of a potential transformer for the very fast transient simulation in gis', *IEEE Transactions on Power Delivery* **23**(4), 1995–1999.