## FEDERAL UNIVERSITY OF PARANÁ

# A FRAMEWORK FOR OPTIMAL SCHEDULING OF ELECTRIC VEHICLES AND DEMAND RESPONSE TO SMART GRIDS 

## ROLANDO ARTURO SILVA QUIÑONEZ

## A FRAMEWORK FOR OPTIMAL SCHEDULING OF ELECTRIC VEHICLES AND DEMAND RESPONSE TO SMART GRIDS

Dissertation submitted to the Graduate Program in Electrical Engineering from the Federal University of Paraná, as partial requirement for the degree of Master of Science in Electrical Engineering.

Advisor: Prof. Dr. Clodomiro Unsihuay Vila response to smart grids/ Rolando Arturo Silva Quiñonez. - Curitiba, 2015.

159 f. : il. color. ; 30 cm.

Dissertation - Federal University of Paraná, Exact Sciences Sector, Electrical Engineering Postgraduate Program, 2015.

Preceptor: Clodomiro Unsihuay Vila .
Bibliography: p. 108-114.

1. Smart power grids. 2. Electric vehicles. 3. Electric currents. 4. Linear programming. 5. Electric power distribution. I. Universidade Federal do Paraná. II.Vila, Clodomiro Unsihuay. III. Title.

## CERTIFICATE OF APPROVAL

## ROLANDO ARTURO SILVA QUIÑONEZ

## A FRAMEWORK FOR OPTIMAL SCHEDULING OF ELECTRIC VEHICLES AND DEMAND RESPONSE TO SMART GRIDS

This dissertation has been accepted and approved in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering at the Federal University of Paraná, Brazil.


Prof. Dr. Roberto Cayétano Lotero - Dissertation Reader State University of West Paraná


Prof. Dr. Alexandre Rasi Aoki - Dissertation Reader Federal University of Paraná


Prof. Dr. Odilon Luiz Tortelli - Dissertation Reader
Federal University of Paraná

Curitiba, February 26, 2015.

## DEDICATION

This dissertation is dedicated to my God who knew how to guide me in the right road, give me the strength to keep me going and not give in problems arising, teaching me how to address the adversity without losing dignity or falter in the attempt.
To my family who for them I am what I am. To my parents for their encouragement, advice, understanding, love, support in difficult times, and for helping me with the necessary resources to study. They gave me everything I am as a person, values, principles, character, effort, perseverance and courage to achieve my goals. I also dedicate this dissertation to my friends who have supported me throughout the process.

## ACKNOWLEDGEMENTS

First and foremost, I have to thank my parents for their love and support throughout my life. Thank you both for giving me strength to pursuing my dreams.

I would like to sincerely thank to the coordinator of the Electrical Engineering Department, Prof. Dra. Thelma, for her guidance, support throughout this process, and especially for her confidence and friendship. I would also like to thank Dr. Alexandre Rasi Aoki, Dr. Odilon Luís Tortelli and Dr. Roberto Cayetano Lotero for serving as members on my dissertation committee. His comments and questions were very positive in the completion of the manuscript. I learned from their deep discussions and interpretation of some results presented in this dissertation. Also, I would like to thank my advisor, Dr. Clodomiro, for his guide.

To all my friends, thank you for your understanding and encouragement. Your friendship makes my life a wonderful experience. I cannot list all the names here, but you are always on my mind.
"Happiness is when what you think, what you say, and what you do are in harmony."

Mahatma Ghandi


#### Abstract

Microgrids and distribution systems holds a very important position in the power system since are the main points of link between bulk power and consumers. Due to the convergence of several trends in the energy sector, smart grids are emerging as a solution for the modernization of the electric grid, integrating large shares of distributed and intermittent renewable energy sources, energy storage and electric vehicles, as well as the promise to give consumers more control on their energy consumption. This dissertation presents a computational method for an optimal scheduling for demand response and electric vehicle for a typical house under a microgrid environment and its impact on the electric power distribution network along a 24 hours-daily horizon. The proposed method is a System of Systems (SoS) based framework for optimally operating active distribution grids. The proposed SoS framework defines both; distribution company (DISCO) and microgrids (MGs) as autonomous systems, and recognizes the exchange information process among them. The proposed mathematical algorithm uses a separated optimization process that aim maximizing the benefit of each independent system. A hierarchical optimization algorithm is presented to coordinate the independent systems and to find the optimal operating point of the SoS-based active distribution grid. Therefore, the proposed model considers a special emphasis on variables that impact the total behavior of the system for both the energy supplier and the final consumer. The proposed optimization problem is divided in two sub-problems: the demand response problem-based time for a daily horizon, that is modeled as a mixed integer linear programming and the second problem is the electric distribution network problem that is modeled with an optimal power flow. The proposed method allows an optimized schedule of demand response to end-user level and considers their impacts on the distribution network simultaneously. It also examined the additional benefits of demand response programming, micro-distributed generation, integration of electric vehicles modeling in the microgrid. A case of study containing three scenarios reveals the usefulness and effectiveness of the proposed model. Firstly showing the normal operation without any type of optimization, secondly conducting an optimization process inside of the microgrid with the insertion of the electric vehicle just modeled as a load and managing the distribution system, thirdly inserting renewable distributed generation, electric vehicles for both process: charging and discharging. Finally, making the optimization process for the junction of the microgrid and the distribution network.


Key-words: Smart grid. Electric Vehicle. Optimal power flow. Mixed integer linear programming. Distribution system.


#### Abstract

RESUMO

As microredes e os sistemas de distribuição têm uma posição de importância no sistema de energia, sendo os principais pontos de ligação entre a energia em grandes quantidades e os consumidores finais. Devido à convergência de várias tendências próprias do setor, as redes inteligentes estão emergindo como uma solução para a modernização da rede elétrica, integrando percentagens cada vez maiores de fontes de energia renováveis e intermitentes, armazenamento de energia e veículos elétricos, assim como o compromisso de oferecer aos consumidores maior controle no seu consumo de energia. Esta dissertação apresenta uma metodologia computacional que estabelece um agendamento ótimo para a resposta à demanda no caso de uma casa padrão inserida em um ambiente de microrede, além de determinar o seu impacto sobre a rede de distribuição de energia elétrica ao longo de um horizonte de 24 horas por dia. Este trabalho apresenta também um Sistema de Sistemas (SoS) contextualizado para uma operação ótima de redes de distribuição ativas. O SoS proposto define tanto a companhia de distribuição (DISCO) e a microrede (MGs) como sistemas autônomos, e reconhece o processo de troca de informações entre eles. O algoritmo matemático proposto utiliza um processo de otimização separado que visa maximizar o benefício de cada sistema independente. Adicionalmente, um algoritmo de optimização hierárquica é apresentado para coordenar os sistemas independentes e para determinar o ponto de funcionamento óptimo da rede ativa de distribuição baseada no SoS. O modelo proposto considera uma ênfase especial sobre as variáveis que afetam o comportamento total do sistema, tanto para o fornecedor de energia como para o consumidor final. O problema de otimização proposto é dividido em dois itens: o problema da resposta à demanda baseado no tempo para um horizonte diário, que é modelado como uma programação linear inteira mista e o segundo problema é o problema de rede radial que é modelado com um fluxo de potência ótimo. A metodologia proposta permite um agendamento otimizado para a resposta à demanda ao nível de usuário final e considera simultaneamente seu impacto na rede de distribuição. Além, nesta dissertação foram analisados os benefícios da programação da resposta à demanda, micro-geração distribuída e integração de veículos elétricos modelado na microrede. Um caso de estúdio contendo três distintos cenários revela a utilidade e eficácia do modelo proposto. Em primeiro lugar, mostrando o funcionamento normal, sem qualquer tipo de otimização, em segundo lugar, conduzindo um processo de otimização dentro da microrede com a inserção do veículo elétrico apenas modelado como uma carga e gerenciando o sistema de distribuição, e em terceiro lugar considerando a inserção de geração renovável distribuída, e processos tanto de carga como descarga dos veículos elétricos. Finalmente, foi efetuado um processo de otimização para a junção da microrede e da rede de distribuição.


Palavras-chave: Rede inteligente. Veículo elétrico. Fluxo de potência ótimo. Programação linear inteira mista. Sistema de distribuição.

## LIST OF FIGURES

FIGURE 1 - Future electric smart grid architecture [10] ..... 24
FIGURE 2 - Comparison of electricity storage technologies [22] ..... 28
FIGURE 3-24 hour schedule for a TOU tariff with different priced blocks [31] ..... 36
FIGURE 4 - Critical peak pricing tariff, covered by a flat-tariff [31] ..... 36
FIGURE 5 - Representation of a real time pricing tariff [31] ..... 37
FIGURE 6 - Schematic architecture for a BEVs, HEV and PHEV [34] ..... 37
FIGURE 7 - Basic PHEV drive train series (EV) vs Parallel design [34] ..... 38
FIGURE 8 - International electric vehicle sales targets, 2010-20 [40]. ..... 41
FIGURE 9 - International sales estimation of EV and PHEV, 2010-2015 [40]. ..... 42
FIGURE 10 - Typical electric network system [64] ..... 61
FIGURE 11 - 33 buses radial distribution system ..... 62
FIGURE 12 - Block diagram for the electric vehicle model ..... 50
FIGURE 13 - General injecting profile for EV ..... 56
FIGURE 14 - Typical generation profile for a solar panel ..... 57
FIGURE 15 - DG example of the penetration levels in the microgrids. ..... 57
FIGURE 16 - Distribution system with passive MG integration adapted of [82] ..... 63
FIGURE 17 - Characteristics between the MGs and the distribution network [82] ..... 64
FIGURE 18 - Distribution system with active microgrid integration [82]. ..... 65
FIGURE 19 - White Tariff in UM/kWh. ..... 67
FIGURE 20 - Proposed flowchart for the optimization problem solution ..... 66
FIGURE 21 - System architecture for scenario 'A' ..... 70
FIGURE 22 - Power demand of a house for the scenario $A$. ..... 71
FIGURE 23 - Energy cost for a typical house used on scenario A ..... 71
FIGURE 24 -Total active power profile for buses with low consumers, ..... 72
FIGURE 25 - Total active power profile for buses with medium consumers ..... 72
FIGURE 26 - Total active power profile for buses with high consumers. ..... 72
FIGURE 27 - Voltage magnitude for primary branch on sce A ..... 75
FIGURE 28 - Voltage magnitude for buses on secondary ' $A$ ' branch for sce. A ..... 76
FIGURE 29 - Lambda values for the primary branch for scenario $A$. ..... 77
FIGURE 30 - Lambda values for the secondary ' $A$ ' branch for scenario $A$ ..... 78
FIGURE 31 - Active power losses for primary branch for scenario A ..... 79
FIGURE 32 - Active power losses for secondary 'A' branch for scenario A. ..... 80
FIGURE 33 - Active generation for scenario A ..... 81
FIGURE 34 - Objective function for the final OPF performed in scenario A ..... 81
FIGURE 35 - System architecture for scenario B ..... 83
FIGURE 36 - Power demand of a typical house for the scenario B ..... 84
FIGURE 37 - Energy cost for a typical house used on scenario A ..... 84
FIGURE 38 - Voltage magnitude for primary branch in scenario B ..... 85
FIGURE 39 - Voltage magnitude for secondary branch 'A' in scenario B ..... 86
FIGURE 40 - Lambda values for primary branch in scenario B ..... 87
FIGURE 41 - Lambda values for secondary ' $A$ ' branch in scenario $B$ ..... 88
FIGURE 42 - Active power losses for primary branch in scenario $B$ ..... 90
FIGURE 43 - Active power losses for secondary ' $A$ ' branch in scenario B ..... 90
FIGURE 44 - Active power generation for scenario $B$ ..... 91
FIGURE 45 - Objective function for the final OPF performed in scenario 'B' ..... 91
FIGURE 46 - System architecture for scenario C ..... 93
FIGURE 47 - Power demand for a typical house for scenario C ..... 94
FIGURE 48 - Energy cost for a typical house used on scenario C ..... 94
FIGURE 49 - Voltage magnitude for primary branch in scenario $C$ ..... 95
FIGURE 50 - Voltage magnitude for secondary branch 'A' in scenario C ..... 96
FIGURE 51 - Lambda values for primary branch in scenario C ..... 97
FIGURE 52 - Lambda values for secondary 'A' branch in scenario C ..... 98
FIGURE 53 - Active power losses for primary branch in scenario C ..... 99
FIGURE 54 - Active power losses for secondary ' $A$ ' branch in scenario C ..... 99
FIGURE 55 - Active power generation for scenario C ..... 100
FIGURE 56-Objective function for the final OPF performed in scenario 'C' ..... 100
FIGURE 57 - Voltage magnitude for buses on secondary 'B’ branch for sce. A. ..... 129
FIGURE 58 - Voltage magnitude for buses on secondary 'C' branch for sce. A ..... 129
FIGURE 59 - Lambda values for the secondary ' $B$ ' branch for scenario $A$. ..... 130
FIGURE 60 - Lambda values for the secondary ' $C$ ' branch for scenario $A$ ..... 130
FIGURE 61 - Active power losses for secondary 'B' branch for scenario A ..... 131
FIGURE 62 - Active power losses for secondary 'C' branch for scenario A ..... 131
FIGURE 63 - Voltage magnitude for secondary branch ' $B$ ' in scenario $B$ ..... 132
FIGURE 64 - Voltage magnitude for secondary branch 'C' in scenario B ..... 132
FIGURE 65 - Lambda values for secondary ' $B$ ' branch in scenario $B$ ..... 133
FIGURE 66 - Lambda values for secondary 'C' branch in scenario B ..... 133
FIGURE 67 - Active power losses for secondary ‘B’ branch in scenario B ..... 134
FIGURE 68 - Active power losses for secondary 'C’ branch in scenario B ..... 134
FIGURE 69 - Voltage magnitude for secondary branch ' $B$ ' in scenario $C$ ..... 135
FIGURE 70 - Voltage magnitude for secondary branch ' $C$ ' in scenario $C$ ..... 135
FIGURE 71 - Lambda values for secondary 'B' branch in scenario C ..... 136
FIGURE 72 - Lambda values for secondary ' C ' branch in scenario C ..... 136
FIGURE 73 - Active power losses for secondary ‘B’ branch in scenario C ..... 137
FIGURE 74 - Active power losses for secondary 'C’ branch in scenario C ..... 137

## LIST OF TABLES

TABLE 1 - Comparison between Smart Grids architecture and existing grid [13] ..... 25
TABLE 2 - U.S. Smart Grid Characteristics [15] ..... 26
TABLE 3 - Benefits of demand response [29] ..... 32
TABLE 4 - Costs of demand response [29] ..... 33
TABLE 5 - Branch nomenclature ..... 74
TABLE 6 - Primary variables analyzed for scenario A ..... 82
TABLE 7 - Primary variables analyzed for scenario B ..... 92
TABLE 8 - Primary variables analyzed for scenario C ..... 101
TABLE 9 - Iterative convergence values for the optimization processes ..... 102
TABLE 10 - Daily energy saving for specific microgrid ..... 104
TABLE 11 - Energy comparative costs (UM/day) for specific microgrid ..... 104
TABLE 12 - Comparison of objective functions \& total active power losses ..... 104

## LIST OF ACRONYMS

| AMI | Advanced Metering Infrastructure |
| :--- | :--- |
| AMR | Automatic Meter Reading |
| BEV | Battery Electric Vehicle |
| CPP | Critical Peak Pricing |
| DG | Distributed Generation |
| DISCO | Distribution Company |
| DR | Demand Response |
| DSM | Demand Side Management |
| EINL | Elastic Interruptible Load |
| EV | Electric Vehicle |
| HAN | Home Area Network |
| HVAC | Heating Ventilation Air Conditioning |
| IBP | Incentive Based Program |
| ICE | Internal Combustion Engine |
| IL | Inelastic Loads |
| LAN | Local Area Network |
| LSE | Load Serving Entity |
| PBP | Price Based Program |
| PHEV | Plug-in Hybrid Electric Vehicle |
| RTP | Real Time Pricing |
| SCADA | Supervisory Control And Data Acquisition |
| SG | Smart Grid |
| SoS | System of Systems |
| TOU | Time Of Use |
| V2G | Vehicle 2 (to) Grid |
| VPP | Virtual Power Plant |
| WAN | Wide Area Network |

## NOMENCLATURE

| Consumption ${ }_{t}^{\text {Max_Hourly }}$ | Maximum consumption at hour $t$ in W |
| :---: | :---: |
| Cycles ${ }_{j}^{\text {EV_Charging }}$ | : Number of charging cycles for the EV j |
| Cycles ${ }_{j}^{\text {EV_Discharging }}$ | : Number of discharging cycles for the EV j |
| Cycles ${ }_{k}^{\text {Elast_Load }}$ | : Number of cycles for the elastic interruptible load k |
| Cycles ${ }_{k}^{\text {Ine_Load }}$ | : Number of cycles for the inelastic load k |
| Demand ${ }_{t}^{\text {Total }}$ | : Total demand of the house (including elastic Interruptible and Inelastic loads) at hour $t(\mathrm{~W})$ |
| Duration ${ }_{j}^{\text {EV_Charging }}$ | Duration of the charging cycle of the EV j (hour) |
| Duration ${ }_{j}^{\text {EV_Discharging }}$ | : Duration of the discharging cycle of the EV j (hour) |
| Duration ${ }_{k}^{\text {Elast_Load }}$ | : Duration of the cycle of use for the elastic interruptible load $k$ (hour) |
| Duration ${ }_{k}^{\text {Ine_Load }}$ | : Duration of the cycle of use for the inelastic load k (hour) |
| Energy ${ }_{t}^{\text {Buy_Cost }}$ | : Energy purchase price |
| Energy ${ }_{t}^{\text {Sale_Cost }}$ | : Energy sale price |
| $E V_{j,(t-1)}^{\text {Energy_Stored }}$ | : Initial state of charge, Stored energy in the battery of vehicle j, at hour ( $\mathrm{t}-1$ ) (Wh) |
| $E V_{j, t}^{\text {Energy_Stored }}$ | : State of charge, Stored energy in the battery of vehicle j, at hourt (Wh) |
| $E V_{j, t}^{\text {Energy_trip }}$ | : Energy consumption of vehicle j for traveling at hour $\mathrm{t}(\mathrm{Wh})$ |
| $E V_{j, t}^{\text {Energy_Trip }}$ | : Energy consumption for traveling in time t (Wh) |
| $E V_{j, t}^{\text {Max_Power_Charged }}$ | : Rated hourly max power charged for vehicle j , at hour t (W) |
| $E V_{j, t}^{\text {Max_Power_Discharged }}$ | : Rated hourly max power injected or discharged for vehicle j, at hour $\mathrm{t}(\mathrm{W})$ |
| $E V_{j, t}^{\text {Power_Charged }}$ | : Power charged for vehicle j at hour t (W) |
| $E V_{j, t}^{\text {Power_Discharged }}$ | : Power discharged from vehicle j at hour t (W) |
| $\text { Gen }_{t}^{\text {Energy_Type }}$ | Energy generation from any source (Solar, Wind) at hour $t$ (Wh) |
| $N_{\text {Elast_Load }}$ | : Number of elastic interruptible loads |


| $N_{E V}$ | Number of electric vehicles |
| :---: | :---: |
| $N_{\text {Ine_Load }}$ | : Number of inelastic loads |
| Net ${ }_{t}^{\text {Energy_Consumed }}$ | : Net energy consumption at hour $t(\mathrm{~Wh})$ |
| $P_{k, t}^{\text {Elast_Load }}$ | : Power consumption of elastic interruptible load $k$ at hour $t(W)$ |
| $P_{k, t}^{\text {Ine_Load }}$ | : Power consumption of inelastic load $k$ at hour $t$ (W) |
| $X_{j, t}^{E V}$ | : EV charging status, Binary variable that takes the following values $\left\{\begin{array}{c}0 \text { if } E V \text { is not being charged } \\ 1 \text { if } E V \text { is being charged }\end{array}\right.$; |
| $X_{k,(t-1)}^{\text {Elast Load }}$ | : Elastic interruptible load status for load $k$ at hour ( $\mathrm{t}-1$ ), Binary variable that takes, 0 if is not operating or 1 if it is |
| $X_{k, t}^{\text {Elast_Load }}$ | : Elastic load status for elastic inerruptible load $k$ at hour $t$ that presents the following behavior. $\left\{\begin{array}{c}0 \text { if is not operating } \\ 1 \text { if it is operating }\end{array}\right.$ |
| $Y_{j, t}^{E V}$ | : EV Discharging or injection status, Binary variable that takes the following values $\left\{\begin{array}{l}0 \text { if } \mathrm{EV} \text { is not discharging } \\ 1 \text { if } \mathrm{EV} \text { is discharging }\end{array}\right.$ |
| $Y_{k, t}^{A u x_{-} O N}$ | : Binary variable for device $k$ (Elastic interruptible load), that is equal to $\left\{\begin{array}{l}1 \text { if the device is ON at hour } t \\ 0 \text { if the device is OFF at hour } t\end{array}\right.$ |
| $Z_{k, t}^{\text {Aux_OFF }}$ | : Binary variable for device $k$ (Elastic interruptible load), that is equal to $\left\{\begin{array}{l}1 \text { if the device is OFF at hour } t \\ 0 \text { if the device is } \mathrm{ON} \text { at hour } \mathrm{t}\end{array}\right.$ |
| $\beta_{j}^{\max }$ | : Energy battery maximum limit of EV j (Wh) |
| $\beta_{j}^{\text {min }}$ | : Energy battery minimum limit of EV j (Wh) |
| $\eta_{j}^{\text {EV_Charging }}$ | : Charging efficiency of electric vehicle j |
| $\eta_{j}^{E V \_D i s c h a r g i n g}$ | : Discharging efficiency of electric vehicle j |
| $\psi_{j}^{\max }$ | : Maximum percentage limit of battery capacity for EV j |
| $\psi_{j}^{\text {min }}$ | : Minimum percentage limit of battery capacity for EV j |
| $l$ | : Auxiliary index |
| MUT | : Minimum up time for elastic interruptible loads |
| N_Energy_Type | : Number of micro generators units in the house |

## CONTENTS

1 INTRODUCTION ..... 19
1.1 BACKGROUND AND MOTIVATION ..... 19
1.2 OBJECTIVE AND SCOPE ..... 21
1.2.1 General Objective ..... 21
1.2.2 Specific Objectives ..... 21
1.3 CONTRIBUTIONS ..... 22
1.3.1 The Integrated Microgrid and Distribution Network Modeling ..... 22
1.3.2 Electric Vehicle Modeling ..... 22
1.3.3 Residential Loads Management and Classification Model ..... 22
1.4 OUTLINE OF THE DISSERTATION ..... 23
2 STATE OF ART ..... 24
2.1 SMART GRID (SG) ..... 24
2.1.1 Smart Grids Benefits ..... 24
2.1.2 Enabling Technologies ..... 27
2.1.2.1 Distributed Generation ..... 27
2.1.2.2 Energy Storage ..... 28
2.1.2.3 Power Electronic ..... 28
2.1.2.4 Control Automation and Monitoring ..... 29
2.1.2.5 Communication Systems ..... 29
2.2 DEMAND RESPONSE (DR) ..... 30
2.2.1 Demand Response Benefits and Costs ..... 31
2.2.2 Demand Response Classification ..... 34
2.2.2.1 Incentive-Based Demand Response Programs ..... 34
2.2.2.2 Price-based Demand Response Programs ..... 35
2.3 ELECTRIC VEHICLE (EV) ..... 37
2.3.1 Current EV Technology ..... 38
2.3.2 Vehicle to Grid (V2G) System ..... 39
2.4 SYSTEM OF SYSTEMS ..... 42
2.4.1 The Notion of a System ..... 43
2.4.2 System of Systems Description ..... 44
2.4.3 System of Systems Research Fields ..... 45
2.4.4 Applications ..... 46
3 METHODOLOGY ..... 47
3.1 LOADS MODELING ..... 47
3.2 ELECTRIC VEHICLE MODELING ..... 50
3.3 INTEGRATED LOAD AND EV FORMULATION ..... 54
3.4 DISTRIBUTED GENERATION ..... 56
3.5 MIXED INTEGER LINEAR PROGRAMMING (MILP) PROBLEM FORMULATION FOR MICROGRIDS (MGS) ..... 58
3.6 Tested Distribution System ..... 61
3.7 OPTIMAL POWER FLOW PROBLEM FORMULATION FOR DISTRIBUTION NETWORK USING MATPOWER ..... 62
3.8 SYSTEM OF SYSTEMS BASED METHOD SOLUTION ..... 62
4 EXPERIMENTAL RESULTS, DISCUSSION AND SIMULATIONS ..... 67
4.1 CASE STUDIES ..... 69
4.1.1 Scenario A: "Base Case" ..... 70
4.1.1.1 Buses Voltage Magnitude for Scenario A (pu) ..... 74
4.1.1.2 Lambda Values for Scenario A (UM/MWh) ..... 77
4.1.1.3 Branches Active Power Losses for Scenario A (kW) ..... 79
4.1.1.4 Active Power Generation for Scenario A (MW) ..... 81
4.1.1.5 Objective function for OPF in Scenario A ..... 81
4.1.2 Scenario B: "Optimization of loads and charging of EVs scheduling" ..... 83
4.1.2.1 Buses Voltage Magnitude for Scenario B (pu) ..... 84
4.1.2.2 Lambda for Scenario B (UM/MWh) ..... 87
4.1.2.3 Branches Active Power Losses for Scenario B (kW) ..... 89
4.1.2.4 Active Power Generation for Scenario B (MW) ..... 91
4.1.2.5 OPF Objective Function for Scenario B ..... 91
4.1.3 Scenario C: "Optimization of Loads considering Micro-DG, EV charging and injection " ..... 93
4.1.3.1 Voltage Magnitude for Scenario C (pu). ..... 95
4.1.3.2 Lambda Values for Scenario C (UM/MWh) ..... 97
4.1.3.3 Branches Active Power Losses for Scenario C (kW) ..... 98
4.1.3.4 Active Power Generation for Scenario C ..... 100
4.1.3.5 OPF Objective Function for Scenario C ..... 100
4.2 FINAL ANALYSIS OF THE RESULTS ..... 102
5 CONCLUSION AND FUTURE WORK. ..... 106
5.1 CONCLUDING REMARKS ..... 106
5.2 FUTURE WORKS ..... 107
REFERENCES ..... 108
APPENDIX A - EV PHEV \& models on the actual market ..... 115
APPENDIX B - Modified case for the 33 buses radial distribution network ..... 117
APPENDIX C - Matrix structure for MG resolution applying MILP ..... 120
APPENDIX D - Mixed integer linear programming formulation ..... 122
APPENDIX E - Electrical Equipment Consumption ..... 123
APPENDIX F - Optimal Power Flow Formulation using Matpower ..... 126
APPENDIX G - Pseudocode for SoS application and resolution ..... 128
APPENDIX H - Complementary for scenarios A, B and C ..... 129
APPENDIX I - Numerical Values for Graphics from Scenarios A, B and C. ..... 138

## 1 <br> INTRODUCTION

## 1.1

BACKGROUND AND MOTIVATION

Three main topics have motivated this study: Smart grid, Demand Response \& Electric Vehicles. These topics are related, as their technologies are advancing relatively fast. Smart grid has brought several challenges and also opportunities in the electric infrastructure that allows the operation of the conventional electric power systems to be more reliable, consistent and efficient.

This concept and technology, firstly makes possible at different levels of the electrical system the incorporation of diverse distributed energy sources, smart sensors, frequency monitoring devices, intelligent substation and distribution equipment. It is also found at the costumer level the use of smart appliances. At the same time, software to handle all the information that comes from the parts of the system interaction is present, initially these software are required to quantifiably evaluate, monitor the progress and plan the operability of the smart grid [1].

With the implementation of this concept, it is possible to perform load control, applying innovative and diverse demand response algorithms. One of the main outcomes of the use of smart grids is the relieving stress condition of the power system. Although, the term Demand Side Management (DSM) has been widely used since the 1980's [2], and several studies have been conducted on this path, the term Demand Response has recently been introduced together with the smart grid concept. Demand response (DR) is a customer action to control loads in order to meet certain peak reduction and energy savings aims. With demand response, the customer chooses which loads are to be controlled and for how long. This is the main difference from Demand Side Management, where the electric supplier controls the loads and the final user has no control over the process. In this context have also appeared some scientific knowledge gaps in the implementation of smart grids that have being treated in [3] , [4] .

In addition to demand response and smart grid technology, appears on the scenario the electric vehicle. The automobile, as normally known, is the basic mode of propulsion for the human and has worked since its invention with a gasoline fueled internal combustion engine. Throughout the past 100 years, various social and
international political factors have encouraged interest in other technologies, but the gasoline powered internal combustion engine has continued to dominate the automobile horizon. However, recent policies have placed a new emphasis on the sale of the concept of an electric vehicle with no pollution levels residues. In pursuance of this objective, electric vehicle have emerge as reliable and viable option, firstly, just as a vehicle that need to be charged and more recent models with features that allow them to inject energy to the grid [5].

The approach of the electric vehicle as energy storage source, is based on the ability to storage energy in its battery when the energy has a low price and also to inject energy to the grid when the energy presents a higher price [6]. This feature allows researchers on this field to experiment whether or not is viable to schedule or manage the interaction of the electric vehicles with the system that is supporting them [7]. The automobile sector is constantly looking for a way to make "cleaner" technologies and economically competitive with the traditional internal combustion engine [8].

This dissertation emphasizes in the interaction of the elements inside of the microgrid (Micro Distributed Generation, Electric Vehicles and Demand Response) and how they affect the behavior of the distribution system. Load profile and comparative curves are also the target in order to show the variations that appears while the distribution system and the microgrid are under stress conditions.

### 1.2 OBJECTIVE AND SCOPE

### 1.2.1 General Objective

The main objective in this dissertation is to present a computational method to optimal scheduling for demand response and electric vehicle of a typical house in a microgrid environment and its impact on the electric power distribution network along a 24 hours horizon. The mathematical model takes in consideration the insertion of micro distributed renewable energy generation and its interaction with the electric power distribution network using the System of Systems (SoS) concept.

### 1.2.2 Specific Objectives

To develop a solving technique for the integrated (Microgrid + Distribution network) optimization process based on the SoS concept.

To develop the computational model for residential controllable (Elastic interruptible) and uncontrollable (Inelastic) loads that will be the base for the demand response algorithm, applied to the appliance level along the 24 hours horizon.

To study the daily demand variation curve due the optimal shifting behavior of the controllable loads and validate the optimization process based on its objective function.

To develop an operational behavior model for the electric vehicle that represents the storage, charge and discharge characteristics for the majority of the electric vehicles (BEVs, HEVs, PHEVs) available at this time. Also, several brands and models will be described.

To expand the inputs as much as possible for the electric vehicle modeling, for instance, introducing driving patterns, electric vehicle storage and charge/discharge state after driving, starting hour trip, end hour trip, charging start time, charging rate and develop the load profile of the electric vehicle at the microgrid level.

To analyze the total behavior and interaction of the parts of the microgrid and also insert efficiently sources of micro distributed renewable generation, mainly solar sources, and analyze the way in which this feature of the system affects the micro grid system itself and the distribution system.

### 1.3 CONTRIBUTIONS

### 1.3.1 The Integrated Microgrid and Distribution Network Modeling

The first contribution in this dissertation is the presentation of a novel algorithm for the interaction among the components of the microgrid (that for the purpose of this dissertation is not used in its isolated form) and the distribution network. In this section is proposed an optimal scheduling for demand response and electric vehicle of a typical house in a microgrid environment and their impact on the electric power distribution network along a 24 hours horizon. A hierarchical optimization algorithm is presented to coordinate the independent systems and to find the optimal operating point of the SoS-based active distribution grid. Two different tools are used in order to get this objective: A Mixed Integer Linear Programming (MILP) model for the microgrid and for the distribution network an optimal power flow (OPF) is applied. Both process are iterative and interlinked. With this new method, that optimize each part, is ensured that the system will be also optimized, as seen in the final results, graphs and tables on chapter 4. The proposed method is a computationally improved in comparison with existing methods about optimization of the distribution network under a smart grid environment. The proposed algorithm is tested on a radial distribution network with generators connected into the buses and proved its efficiency solving together both systems, the microgrid level and the distribution level.

### 1.3.2 Electric Vehicle Modeling

The second contribution in this dissertation includes the development of a model for managing the operation of the electric vehicle along a 24 hours horizon, through an algorithm that controls the amount of energy charged/discharged hour by hour, the amount of charging cycles and the duration of the cycles.

### 1.3.3 Residential Loads Management and Classification Model

The third contribution in this dissertation includes the residential load modeling for every type of load. In this part of the work, residential customers can influence their load demand by shifting loads from high tariff to low tariffs, reducing the overall load
demand along the 24 hours horizon. Additionally, they can locally produce electricity using distributed generation. The benefits for the customers are mainly reductions of the energy bill. However, customers have to change their consumption habits and as a consequence their living comfort may be decreased. The classification proposed in this dissertation is such that the customer can decide, first of all how many loads will participate in the optimization process; configure the basic settings for every load to optimize. Also, enable or not the possible working hours or left to the algorithm to choose after the optimization process, whether or not operate according to the surrounding system conditions.

### 1.4 OUTLINE OF THE DISSERTATION

The dissertation is divided into the following Chapters, which include separates brief introductions.

Chapter 2 introduces to the readers the main topics and concepts that will be addressed on this dissertation through a brief outline to the subjects smart grids, electric vehicles, radial distribution system, demand response and system of system.

Chapter 3 contains an approach over the method that is applied in each part of the system; a brief explanation of the optimization tools is performed. Subsequently, a general framework is derived from the sum of all the tools that are explained. Structure, restrictions and equations are detailed. Finally the complete system arises. Boundaries, scope, and expected results are explained.

Chapter 4 introduces all the tested scenarios and provides a comparative chart with the results for each scenario, pointing out the main improvements with the method applied over the complete system.

Last but not least, the conclusions for every part of the system that is used in this dissertation are stated in Chapter 5. Applicability of the method proposed, summary of the main findings are given, an outlook for possible future work are also provided in this chapter.

This chapter condenses the state of art of all the topics that are included in this dissertation. A thorough investigation was conducted in order to provide the basic information and research that have been done in these areas; smart grid (SG), demand response (DR), electric vehicles (EVs), distributed generation (DG) and the distribution network. Several knowledge gaps are identified and then discussed.

### 2.1 SMART GRID (SG)

The term "Smart Grid" can be described as bi-directional electric and communications network that improves the reliability, security, and efficiency of the electric system for small to large scale generation, transmission, distribution, and storage [9], [10]. FIGURE 1 shows how a smart grid looks like in the present and near future


FIGURE 1 - Future electric smart grid architecture [11]

### 2.1.1 Smart Grids Benefits

Smart grid can provide several benefits compared with the existing grid. The present grid is planned in a hierarchical system with centralized power plants on the top and the supplied customers on the bottom through distribution stations [12]. This structure is built on one-way communication (top to bottom) and has no real-time information of the demand. As a result the system is dimensioned to enable sufficient supply at demand peaks [13]. The roll-out of smart grid technology also implies a fundamental re-engineering of the electricity services industry, although typical usage of the term is focused on the technical infrastructure [14]. The smart grid represents the full suite of current and proposed responses to the challenges of electricity supply. Due to the diverse range of factors, there are numerous competing taxonomies and no agreement on a universal definition.

The advantages of Smart Grid compared with the existing grid are illustrated in TABLE 1

TABLE 1 - Comparison between Smart Grids architecture and existing grid [13]

| Existing Grid | Perfect Smart Grid |
| :--- | :--- |
| Electromechanical | Digital |
| One-Way Communication | Two-Way Communication |
| Centralized Generation | Distributed Generation |
| Hierarchical | Network |
| Few Sensors | Sensors Throughout |
| Blind | Self-Monitoring |
| Manual Restoration | Self-Healing |
| Failures and Blackouts | Adaptive and Islanding |
| Manual Check/Test | Remote Check/Test |
| Limited Control | Pervasive Control |
| Few Customer Choices | Many Customer Choices |

Beside of the technological advantages that this concept presents, every system is also characterized for some features that appear intrinsic in its operation. For instance, the system studied and analyzed in [15], stated that the U.S. smart grid is characterized by the following features that appear in TABLE 2. It is clear that the technological advances, helps to improve each of those features, but in this case the smart grid strategy is motivated by concepts of innovation with regard to social and environmental reforms for an interactive economy.

TABLE 2 - U.S. Smart Grid Characteristics [15]
Increased digital information and controls

Dynamic optimization of grid operations, including cyber security

Deployment of distributed resources, including renewable resources Smart Grid
Characteristics

Incorporation of demand-side resources and demand response

Deployment of "smart" technologies and integration of "smart" appliances and consumer devices

Deployment of storage and peak-shaving technology, including plug-in hybrid electric vehicle (PHEV)

Provision of timely information and control options to consumers
Standard development for communication and interoperability of equipment

Identification and lowering of unreasonable barriers to adopt smart grid technology, practices, and services

According to [16], the benefits of smart grid can be briefly resumed as follows:

Digitalization: The smart transmission and distribution grid will employ a unique, digital platform for fast and reliable sensing, measurement, communication, computation, control, protection, visualization, and maintenance of the entire transmission system. This is the fundamental feature that will facilitate the realization of the other smart features. This platform is featured with user-friendly visualization for sensitive situation awareness and a high tolerance for man-made errors.

Flexibility: The flexibility for the future smart transmission and distribution grid is featured in four aspects: 1) expandability for future development with the penetration of innovative and diverse generation technologies; 2) adaptability to various geographical locations and climates; 3) multiple control strategies for the coordination of decentralized control schemes among substations and control centers; and 4) seamless compatibility with various market operation styles and plug-and-play capability to accommodate progressive technology upgrades with hardware and software components.

Intelligence: Intelligent technologies and human expertise will be incorporated and embedded in the smart transmission and distribution grid. Self-awareness of the system operation state will be available with the aid of online time-domain analysis such as voltage/angular stability and security analysis. Self-healing will be achieved to enhance the security of transmission and distribution grid via coordinated protection and intelligent control schemes.

Resiliency: The smart transmission and distribution grid will be capable of delivering electricity to customers securely and reliably in the case of any external or internal disturbances or hazards. A fast self-healing capability will enable the system to reconfigure itself dynamically to recover from attacks, natural disasters, blackouts, or network component failures. Online computation and analysis will enable the fast and flexible network operation and controls such as intentional islanding in the event of an emergency.

Sustainability: The sustainability of the smart transmission and distribution grid is featured as sufficiency, efficiency, and environment-friendly.

### 2.1.2 Enabling Technologies

In order to implement this technology, all over the actual grid, some other technologies must be enabled and matured [17], [18]:

### 2.1.2.1 Distributed Generation

Distributed Generation refers to small rated electricity sources that are normally decentralized and situated close to end-user facilities on the distribution side of the electric grid. Is available to include conventional as well as renewable energy sources. The interconnection of distributed generation to the grid offers a diversity of benefits, including on demand power quality of supply and enhanced reliability. However, the interconnection of distributed generation is a challenge due to the safety, control, and protection issues associated with its bidirectional flows of electricity. Standard of the distributed generation inter-connectivity and technical information are stated in [19]. Some extra information about this type of generation is available in [20] .

### 2.1.2.2 Energy Storage

Electricity is a fragile product that must be consumed within a very short period of time that normally is not easy stored, mainly in excessive amounts. Alternatively, it may be converted into other forms such as mechanical or electrochemical energy [21]. Storage technologies allow these procedures and are among the wanted features for the smart grid.

FIGURE 2 shows a comparison among batteries storage based on their materials and rated power and storage duration. Better characteristics can provide to the grid the following advantages: makes the grid more efficient; enables load leveling and peak shaving, while it reduces dependence on spinning reserve; improves grid reliability and power quality; provides ancillary services; supplying reactive power for voltage regulation and supports transmission and distribution (T\&D) [22].


FIGURE 2 - Comparison of electricity storage technologies [23].

### 2.1.2.3 Power Electronic

Power electronics is essential in the expansion of smart grids because a greater penetration of renewable and alternative energy sources needs complex power systems converter. Usually, a power converter is an interface between the smart grid and local power sources [24]. Solar photovoltaic and wind energy systems play an important function as alternative sources for incorporation in smart grids, that are progressively being installed in residential and commercial locations (typically with a power range of a few kilowatts). Some important characteristics for power electronics systems in smart grids are: High efficiency, Optimal energy transfer, Bidirectional power flow, High reliability, Electromagnetic interference filtering, Smart metering, Real-time information, Communications, Fault tolerance/self-healing.

### 2.1.2.4 Control Automation and Monitoring

A smart grid is an extremely complex nonlinear dynamic network of distributedenergy properties with bidirectional power flow and information that presents several theoretical and practical challenges. Sensing and control systems are key problems that need to be addressed to make it more intelligent and equip it with self-healing, self-organizing, and self-configuring capabilities. This necessitates more complex control, sensing, and computer-oriented monitoring than in the current network, human operators execute these critical tasks. Consequently, some modern control techniques have been claimed to be the best fit for smart grids, for example, agent-oriented programming, multiagent system and implementing computational intelligence into distributed system operation [25]; however, most of these are yet to transcend the investigation domain into large-scale implementation.

### 2.1.2.5 Communication Systems

Self-healing systems have sought to be incorporated into power systems, particularly as the complexity and interactions of several market players significantly increase the risk for large-scale failures. Reconfiguring the system in islanded mode may require hit it to unknown rate and amount of data exchange, two-way communication links, and advanced central computing facilities. Decentralized intelligent control could enable islands to accommodate their native load and generation in a more reliable and efficient way. Local controllers may ensure that each
island is operating within the security limits, safeguarding the electricity supply to their customers [26].

### 2.2 DEMAND RESPONSE (DR)

The re-modeling of the present network towards a smarter opens opportunities for end users in adjusting their demand in order to relieve the grid and minimize costs. This section describes demand response and presents several management alternatives with focus on the benefits for operators and final users. According to the U.S. Department of Energy (DEO) demand response is defined as:
"Changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized" [27].

Nowadays, most consumers in power market are not exposed to actual power prices and consequently have no incentive to react to the power market condition. An electricity tariff with an hourly power price might now help to encourage customers to think about the scheduling of their electricity utilization. However, since most of the customers are unaccustomed to acting at critical times this might not be enough. Creating consciousness it may help to introduce incentives when reducing load at critical times. It is also possible start introducing penalty fees for not reducing load at critical times or both. Demand response can understands both reduction and increase of electrical usage depending on market conditions[28].

### 2.2.1 Demand Response Benefits and Costs

The benefits can be separated in direct, collateral and other benefits (TABLE 3 from the DEO). Direct benefits regard all members who implement demand response; collateral and other benefits regard all members of the electric system. The collateral benefits have system-wide impacts which that encourage policy makers to incentive demand response.

Demand Response costs can be divided into participants and system costs and within those two groups into initial and ongoing costs. The U.S. Department of Energy published an overview explaining the costs in the mentioned groups, which can be seen in TABLE 4 [29].

## TABLE 3 - Benefits of demand response [29]

| Type of <br> Benefits | Recipient (s) | Benefit |
| :---: | :---: | :--- |
| Direct | Customers <br> undertaking <br> Benefits <br> demand response <br> actions | Financial benefits | | Reliability benefitsBill savings <br> Incentive Payments (IBP) |
| :--- |

## TABLE 4 - Costs of demand response [29]



### 2.2.2 Demand Response Classification

"Demand response contains all planned electricity consumption pattern changes by end-use customers that are intended to modify the timing level of instantaneous demand, or total electricity consumption" [30] [31]. DR programs can be organized in two groups: incentive-based demand response (IBP) and price-based demand response (PBP). The U.S. Department of Energy recommends introducing demand response with incentive-based programs. But, if the end-users are used to demand response, the Department recommends to transfer the program to pricebased DR [29]. There are several tariff and program selection within these two classes. The most commonly implemented ones are described below [31].

### 2.2.2.1 Incentive-Based Demand Response Programs

The IBP are divided in two classes, classical and market-based programs. The classical programs include direct control and interruptible/curtailable programs and the market-based programs include demand bidding, emergency DR programs, capacity market and ancillary services market. In the classical programs end-users are rewarded for participating in DR by a bill credit or discount rates. In market-based programs end-users are rewarded with money depending on the amount of load reduction during DR events [31].

Direct Control: This program is mainly offered to residential or small business customers. The operator has the opportunity to shut down electrical devices such as space heating/cooling or water heating of customers on a short period of time.

Interruptible/Curtailable Programs: This program is usually offered to large industry or commercial customers. In the case of system contingencies, customers in this program are asked to reduced demand to a predefined value and receive payment for this. If customers do not respond as contracted, they can face penalty charges or removal from the program.

Demand Bidding: This program is also most suitable for larger customers like industry or commercial customers. Participants can bid in the wholesale market for load reduction at a price they are willing to reduce load for. If the bid goes through they must reduce load or face penalty charges. There are also cases where the operator is offering a utility-set price and the customer is defining how much load can be reduced for this price (Buyback program).

Emergency Demand Response: In case of a reliability-triggered event, customers are offered incentive-based payments, for measured load reduction by the operator. If the customers are not reducing load they may or may not face penalty charges [29].

Capacity Market: Customers in this program have agreed to a predefined load reduction in case of system contingencies. They are usually notified on a day- ahead basis and face penalties if they not respond. They get upfront reservation payments, based on capacity market price and optionally a further payment in case of the actual contingency event based on the load reduction [29].

Ancillary Services Market: In this program customers have the opportunity to bid as operating reserve in the transmission operator (balancing) market. If the bid goes through they are paid the market price to be on stand-by. In the case of a DR event the system operator calls the customer to reduce load [29].

### 2.2.2.2 Price-based Demand Response Programs

Price-Based Demand Response Programs: The PBP has dynamic pricing rates in which electricity prices are not flat. The tariffs follow the fluctuations of real-time costs of electricity. The major idea is to flatten the demand curve by offering high prices at peak demand times and low prices at off-peak times [31].

Time of Use (TOU): TOU tariffs are unit prices divided into different blocks during a 24 h day. These blocks are usually Off-peak, peak and shoulder (which is prior to and after the peak). Each block has its own price, which is the average of production and distribution costs at this time. These prices are typically pre- determined for several
months and vary during the seasons. FIGURE 3 shows a 24 h schedule of a TOU tariff with OFF-peak, peak and shoulder blocks.


FIGURE 3-24 hour schedule for a TOU tariff with different priced blocks [32]

Critical Peak Pricing (CPP): CPP is a rate which is only used during contingencies or high wholesale prices a few days or hours a year [32]. This tariff is usually superimposed on another tariff (e.g. TOU or flat) and customers are informed on a day-ahead basis, (See, FIGURE 4).


FIGURE 4 - Critical peak pricing tariff, covered by a flat-tariff [32]

Real Time Pricing (RTP): RTP is a tariff with hourly changing prices reflecting real costs of the wholesale market. see FIGURE 5. Customers are informed about the prices on a day-ahead or hour-ahead basis. This is the most direct and transparent tariff and suitable for competitive electricity markets [11].


FIGURE 5 - Representation of a real time pricing tariff [32]

### 2.3 ELECTRIC VEHICLE (EV)

According to the National Renewable Energy Laboratory (NREL) [33], a plug-in hybrid-electric vehicle (PHEV) is a vehicle with the ability to recharge its electrochemical energy storage with electricity from an off-board source. FIGURE 6 [34] shows the pictures of battery electric vehicle (BEV, i.e. EV), HEV and PHEV.


FIGURE 6 - Schematic architecture for a BEVs, HEV and PHEV [35]

FIGURE 7, illustrate two schematic for a typical EV architecture: series and parallel. Series drive train architecture powers the vehicle only by an electric motor using electricity from a battery. The battery is charged from an electrical outlet, or by the gasoline engine via a generator. A parallel drive train adds a direct connection between the engine and the wheels, adding the potential to power the vehicle by electricity and gasoline simultaneously and by gasoline only.


FIGURE 7 - Basic PHEV drive train series (EV) vs Parallel design [35]

### 2.3.1 Current EV Technology

The electric network as a power source for vehicles offers many advantages over traditional fossil fuel-powered vehicles. However, it has two main disadvantages that call into question its applicability for use in personal automotive transportation. In [36] is done the classification of these disadvantages as storage and recharging. "Storing it [electricity] is more bulky and expensive (Batteries versus a sheet metal gas tank) and refueling is slow". These difficulties suggest that EVs, when related to gasoline vehicles, will initially experience from higher costs, reduced range and refueling challenges when commuting/traveling significant distances [36].

Present electric vehicle technology is completely dependent on internal battery storage systems; when used in EV applications, batteries are generally the heaviest component of the powertrain, accounting for over one third of a vehicles weight in some cases [36]. Battery recharging for electric vehicles is also now restricted mostly to households, as few charging stations exist along roads and highways, different those available for fossil fuel-powered vehicles. Battery technology is continuously
improving, however, and will likely decrease in weight and size, with simultaneous increases to capacity and recharge speed with new research and development [37].

### 2.3.2 Vehicle to Grid (V2G) System

Besides of the normal charging process, Vehicle-to-grid (V2G) technology is one of various potential energy storage technologies that can be modified to support flexible energy systems through the enhanced use of renewable energy sources (e.g. wind and solar) [38]. V2G technology, via a real-time signal, combines the automobile (specifically the vehicles battery) with existing grid utility systems [39], providing EVs the capacity to transport power from the vehicles Energy Storage System to the grid, coordinated in part by the needs of the electric system [38]. This two-way communication system enables functions to better management of the electricity resources and control peak energy demand requirements placed on the grid [39]. V2G systems could also offer financial welfares to holders, thus reducing the overall costs of acquiring an electric vehicle. Vehicle to Grid systems allow vehicle holders to generate revenue from selling power back to the grid [39]. As identified by [40] "The batteries in these vehicles can store cheaper valley value power and deliver it back to the grid during daytime hours when demand for electricity and prices are highest".

The plug-in hybrid electric vehicles (PHEV) and battery electric vehicles (BEV) are the two major types of electric vehicles. Several models of these electric vehicles are now commercially available in the market. The PHEVs are equipped with a combination of battery storage system chargeable from the grid and conventional internal combustion engine (ICE) [41]. A battery electric vehicle (BEV) is a type of electric vehicle (EV) that uses chemical energy stored in rechargeable battery packs. BEVs use electric motors and motor controllers instead of internal combustion engines (ICEs) for propulsion. Some of the models available in the actual market for PHEV \& BEV appear in APPENDIX A with their characteristics.

The market for purely electric vehicles is in its infancy. The Nissan Leaf was the first to become available in the U.S., with Ford, Toyota, and Honda rolling out models in 2011 and 2012. The Nissan Leaf sold 8,720 in its first 11 months [42]. Nissan expects to sell over 10,000 of the Leaf within the first year of rollout [42]. The Tesla Model S, a luxury BEV, received considerable attention including Motor Trend's "Car
of the Year" award in 2012. In the long term, pike research projects that BEVs will account for $0.8 \%$ of U.S. car sales by 2017 [43].

The market for PEVs and HEVs is more developed, nevertheless, still has to reach a rapid deployment. Hybrids have been retrofitted for plug-in capability since they were introduced in the early 2000s. The Chevy Volt was the first HEV on the market, but it was soon followed by Toyota and Ford models in 2011 and 2012 [44]. Through August 2012, 13,479 Volts were sold [45]. The Volt topped Consumer Reports' Owner Satisfaction Survey for both 2011 and 2012, with 92 percent of owners saying they would make the same purchase again [46]. Although PEVs and HEVs do not have the same mileage range limitations that battery electric vehicles do, they tend to be more expensive than battery electric vehicles and gas cars because they must incorporate both gas and electric power systems. Pike Research projects estimates that by 2017 plug-in hybrids will comprise 1.2\% of U.S. car sales [47].

Since BEVs, PEVs, and HEVs are still relatively news, it difficult to accurately project how fast these markets will expand. The performance of the traditional hybrid market can give some insight into how these emerging markets will mature. While these markets will face different challenges than traditional hybrids (notably adequate charging infrastructure), many of the same market and consumer demands will play a role in how fast these younger markets develop.

Several insistent targets have been set worldwide by many nations for the widespread use and acceptance of the plug-in hybrids and battery electric vehicles. FIGURE 8 [41] portrays the international sales targets set by various countries for electric vehicles by 2020. Most of these announcements were made in the last year, which establishes the importance given to electric vehicle deployment in the international level. If these targets were achieved, 4 million electric vehicles would be sold by 2020.


FIGURE 8 - International electric vehicle sales targets, 2010-20 [41].

By September 2014, cumulative global sales of plug-in electric passenger cars and utility vans was over 604,000 units, consisting of 356,232 all-electric cars and utility vans, and 247,700 plug-in hybrids [48]. The global stock of PEVs grew from 100,000 units on the road in 2011, to more than 180,000 units in 2012, to 405,000 at the beginning of 2014. During 2012, sales of pure electric cars were led by Japan with a $28 \%$ market share of global sales, followed by the United States with a $26 \%$ share, China with $16 \%$, France with $11 \%$, and Norway with 7\%. Plug-in hybrid sales in 2012 were led by the United States with a $70 \%$ share of global sales, followed by Japan with a $12 \%$, and the Netherlands with $8 \%$ [41].

In terms of global sales of LDV electric vehicles (Light Duty Vehicle) estimated by the International Energy Agency roadmap for the period 2010-2050 appear in FIGURE 9 [41]. The sale targets set for electric vehicles by 2050 are expected to meet a share of $50 \%$ of the total cars available worldwide.

But, for a full-scale adoption of electric vehicles, there are several challenges to be attended. The major issues like vehicle range; battery energy density and battery life are projected to progress further in the coming years with modern technologies and technical breakthroughs. The current high purchase price of electric vehicles could be made reasonable to the end user by applying different government subsidies, rebates and incentive schemes. The batteries of electric vehicles are normally expected to plug-in and charge at home during the off- peak hours (night hours) and when the electricity prices are low. This corresponds to slow charging of batteries which may take 6-8 hours [49].

The charging infrastructure, battery charging or swapping stations and smart grids for coordinated charging have to be mobilized in joint with the targets of EVs set by the utilities and the respective governments. International standards play a key role in reducing research and development costs and place a strong foundation for innovation and rapid implementation and deployment of a product in the market. Some of the international standards which are relevant to EVs that deals with the important aspects like vehicular communications, EV charging/discharging, power transfer with grid and battery performance are the SAE standards (SAE J1772, SAE J2847 etc.) and IEC standards (IEC 61851, IEC 62196) [50].


FIGURE 9 - International sales estimation of EV and PHEV, 2010-2015 [41].

### 2.4 SYSTEM OF SYSTEMS

The original notions of system, a word derived from Latin systēma, dates back to Plato and Aristotle. Its meaning and intent was connecting a composition of interrelated and inter-working parts or components. The principal characteristic of a system is its ability to emerge with new features starting from its initial configuration, a property that is manifested through the composition (purposeful or un-intended) and interworking of parts that cannot be readily traced or reduced to the parts in isolation. In the words of Aristotle in Metaphysica, "the whole is greater than the sum of its parts", a notion often referred to as strong emergence. The components in a system give rise to a structure, relationship that in turn generates behavior and emergence. The general characteristics of systems were formalized in models and principles by biologist Ludwig Von Bertalanffy [51] during middle of 20th century.

The concept of "System of Systems (SoS)" is a more recent notion that is gaining prominence in science and technology largely due to its highly desirable properties and emergence. In a SoS, components are replaced by complex and largely autonomous systems that render emergence in a more collaborative and resilient manner [52].

These notions underpin our approach to understanding of complex natural phenomena such as global warming as well as empower us to engineer increasingly advanced products and services through systems engineering [53].

### 2.4.1 The Notion of a System

General systems are characterized by the philosophers Plato and Fredrich Hegel as integrated whole in which [54]:

The whole is more than the sum of the parts.
The whole defines the nature of the parts.
The parts cannot be understood by studying the whole
The parts are dynamically interrelated or interdependent.

In general terms, a system should ideally be characterized through recognition and characterization of its principal emergent properties in preference to the traditional focus on structure or intrinsic behavior. Surprisingly, many definitions of systems and their properties are devoid of systematic structure thus lacking a systems approach.

The concept of "System of Systems" (SoS), a federation of autonomous complex systems that in turn collaborate to generate an emergent property at macro level, has gained prominence in the last ten years [55].

In general, System of systems it's a group of task-oriented or dedicated systems that collect their resources and capabilities together to generate a different and new, more complex system which offers more functionality and performance than simply the summation of the component systems.

Presently, System of Systems is a critical research discipline for which frames of reference, thought processes, quantitative analysis, tools, and design methods. The method for defining, abstracting, modeling, and analyzing system of systems problems is typically referred as system of systems engineering [56].

The SoS paradigm [57] offers many desirable properties chiefly enhanced functionality, resilience and adaptability that is being exploited and developed in many diverse fields from air traffic control to future combat systems. In the same vein as general systems above, the definition of SoS is best pursued as a class [58].

### 2.4.2 System of Systems Description

Emerging next generation smart environments such as Smart Grids, Smart Cities, and Smart Enterprises are complex systems that require a complete and holistic knowledge of their operations for effective decision-making. Multiple systems currently operate within these environments and real-time decision support will require a System of Systems (SoS) approach to provide a functional view of the entire environment to understand, optimize, and reinvent processes. The required system of systems will need to connect systems that cross organizational boundaries, that come from multiple domains, (i.e. finance, manufacturing, facilities, IT, water, traffic, waste, etc.) and operate at different levels (i.e. region, district, neighborhood, building, business function, individual) [59].

The differing SoS types present different interoperability requirements. On the one hand a Directed SoS with dedicated resources and central coordination can implement an Integrated or Unified approach to interoperability. On the other extreme a Virtual SoS has no central authority or resources and may require a Federated approach to interoperability, in such scenarios a constitute system may not even be aware they are involved in a SoS.

The existing system design mind-set views interoperability as an external responsibility, and outsources interoperability to external systems. Within SoS, interoperability needs to be a fundamental requirement to their design and operation. The key challenge is to simplify interoperability without increasing complexity, hierarchy, control, or acquisition cost. This will require a change in mind-set of system design to embrace interoperability concerns. An effective interoperability approach for SoS will minimize complexity and ensure constituent systems do not need to be reengineered as other constituent systems are added, removed, modified, or replaced. To this end, a common interoperability infrastructure is needed to support flexible SoS information interoperability. However, improving the conceptual and technical
interoperability of systems is an important step to support organizational interoperability.

A SoS is an expression often used to describe the internet, a defense communications network, a smart electrical grid, or other complex assembly of distributed, stand-alone parts operating as an integrated entity. [60] Have determined that systems of systems generally have three distinguishing traits: physically distributed systems, prime dependency of overall system functionality on the linkages between the distributed systems, and system heterogeneity. According to [61] modern systems of systems have five common characteristics: operational independence of the individual systems, managerial independence of the systems, geographical distribution, emergent behavior, and evolutionary development. From these descriptions it is possible to converge on a generalized conception of a SoS as comprising a collection of dispersed, independent, current and developing systems that function holistically through SoS defined interfaces and performance parameters to achieve a new level of performance and capability.

### 2.4.3 System of Systems Research Fields

Current research into effective approaches to system of systems problems includes:

- Establishment of an effective frame of reference
- Crafting of a unifying lexicon
- Developing effective methodologies to visualize and communicate complex systems
- Study of designing architecture
- Interoperability
- Formal modeling language with integrated tools platform
- Study of various modeling, simulation, and analysis techniques
- Network theory
- Agent-based modeling
- General system theory
- Probabilistic robust design (including uncertainty modeling/management)
- Object oriented simulation and programming
- Multi-objective optimization
- Study of various numerical and visual tools for capturing the interaction of system requirements, concepts, and technologies


### 2.4.4 Applications

Systems of systems, while still being investigated predominantly in the defense sector, is also seeing application in such fields as national air and auto transportation and space exploration. Other applications where it can be applied include health care, design of the Internet, software integration, and energy management.

Within system of systems significant technical challenges exist in terms of information interoperability that require overcoming conceptual (syntax and semantics) and technological barriers

This section is emphasized on the different methodologies, optimization tools and mathematical techniques that were used in this dissertation. Starting from the load classification process that decides which of the householder loads will enter into the first optimization process inside the microgrid. Then, depending on the cases that are analyzed on this dissertation, the EV and the DG will be included or not.

On the first scenario, named as Scenario A, that will be described on chapter 4, the loads are treated without any kind of restriction; normal householder behavior is analyzed and used as a comparative base for others scenarios ' B ' and ' C '.

Right after load classification process for the elastic interruptible loads, these are filtered through a set of restrictions with the charging/discharging modeling for the EV. In addition, injection from the EV (discharging) and sources of GD (Solar, Eolic) are included according to the scenario studied. Then, matrices with equality and inequality constraints are formed, in order to start the iterative process into the optimization algorithm for the microgrid.

Finally a SoS architecture is used in order to solve the integrated framework, that in this case is made up with the distribution system and the microgrids with all of its components.

### 3.1 LOADS MODELING

In this dissertation, the scheduling or optimal allocation of residential loads aims to minimize the daily or monthly cost due the electricity tariff from the distribution concessionaire. In this problem the decision variables are just the equipment's treated as elastic interruptible loads, scattered and represented along the 24 hours corresponding to the hours of the day in which the residential electrical appliances or devices are scheduled.

The scheduling of the electrical devices, reflecting the average consumption habits of residential users will be inserted on the study and entered as constraints of the problem, except for the scenario ' $A$ ', Base case, that is used as the main comparison pattern for the scenarios proposed. The following formulation is intended to be as general as possible in order to facilitate the future introduction of variables to
optimize with this method. A full list with all the elastic and inelastic loads is detailed in APPENDIX D.

For the calculation of the net consumption is considered that the residence has its own micro generation (solar or wind), it is assumed that these plants have a zero operating/fuel costs. It should be noted that this dissertation is not considering the investment cost of these micro generation units, and studies of economic feasibility. This work considers a future scenario for the popularization of micro generation (which in Brazil still depends on increased government incentives).

In addition to the micro generation, the use of electric vehicles as an energy source, restricted to peak periods, will also be considered in the model. This restriction will be imposed in order to minimize the number of charge and discharge cycles of the vehicle, which directly affect the battery life of the EV. So, besides the solar energy generation, the residence will also take in consideration the energy available in the electric vehicle battery bank.

Therefore, the net energy consumed from the DISCO, $N e t_{t}^{\text {Energy_Consumed }}$ is formulated as follow:

$$
\begin{gather*}
\text { Net }_{t}^{\text {Energy_Consumed }}=\text { Demand }_{t}^{\text {Total }}-\text { Gen }_{t}^{\text {Total }}  \tag{Eq 1}\\
\forall t=1 \ldots 24
\end{gather*}
$$

Where:

| Net ${ }_{t}^{\text {Energy_Consumed }}$ : | Net energy consumption from DISCO at hour $t$ (in Wh); |
| :---: | :---: |
| Demand ${ }_{t}^{\text {Total }}$ | Total demand of the house (including elastic and inelastic loads) at hour $t$ (in Wh) |
| Gen ${ }_{t}^{\text {Energy_Type }}$ | Power injection from any source (Solar, Wind) at hour $t$ (in Wh) |

The equations for this section starts with the definition for the total load consumption Eq 2, that is formed for the algebraic sum of inelastic and elastic interruptible loads. It should be noted that a total of 36 inelastic loads (See APPENDIX E) are analyzed for all the scenarios and only the elastic interruptible loads enter into the optimization process.

$$
\begin{gathered}
\text { Demand }_{t}^{\text {Total }}=\sum_{k=1}^{N_{\text {Ine_Load }}} P_{k, t}^{\text {Ine_Load }} * \text { Duration }_{k}^{\text {Ine }_{\text {Load }}} * \text { Cycles }_{k}^{\text {Ine }_{\text {Load }}} \\
+\sum_{k=1}^{N_{\text {Elast_Load }}} P_{k, t}^{\text {Elast_Load }} * X_{k, t}^{\text {Elast_Load }} \\
\forall t=1 \ldots 24
\end{gathered}
$$

Eq 3, represents the total generation that considers all the micro energy DG types (solar, wind, etc), for the system.

$$
\begin{gathered}
\operatorname{Gen}_{t}^{\text {Total }}=\sum_{k=1}^{\text {N_Energy_Type }} \operatorname{Gen}_{t, k}^{\text {Energy_Type }} \\
\forall t=1 \ldots 24
\end{gathered}
$$

Where:

$$
P_{k, t}^{\text {Ine_Load }}
$$

$$
P_{k, t}^{\text {Elast_Load }}
$$

Duration ${ }_{k}^{\text {Ine_Load }}$ : Duration of the cycle of use for the inelastic load $k$ (in hours);
Duration ${ }_{k}^{\text {Elast_Load }}$ : Duration of the cycle of use for the elastic interruptible load $k$ (in hours);
Cycles ${ }_{k}^{\text {Ine_Load }} \quad:$ Number of cycles for the inelastic load $k$
Cycles ${ }_{k}^{\text {Elast_Load }}$ : Number of cycles for the elastic interruptible load $k$;
$X_{k, t}^{\text {Elast_Load }} \quad:$ Elastic load status for elastic interruptible load $k$ at hour $t$ that presents the following behavior $\left\{\begin{array}{c}0 \text { if is not operating } \\ 1 \text { if it is operating }\end{array}\right.$;
$N_{\text {Ine_Load }} \quad:$ Number of inelastic loads;
$N_{\text {Elast_Load }} \quad$ : Number of elastic interruptible loads.
N_Energy_Type : Number of micro generators units in the house.

### 3.2 ELECTRIC VEHICLE MODELING

To explore the impacts of EVs penetration into the microgrids on the distribution network system, it is important to get a thorough understanding of the EV individual charging pattern. Several characteristics were presented in Erro! Fonte de referência ão encontrada.. This section describes the EV modeling method applied to the injection and charging procedure. In order to reach this objective the following parameters are essential: The charging/discharging power rate, the battery total capacity as shown FIGURE 10.


FIGURE 10 - Block diagram for the electric vehicle model

The battery energy balance for each vehicle is now considered in Eq 4 as formulated in [51]. The state of charge variable $E V_{j, t}^{\text {Energy_Stored }}$ represents the stored energy in the EV j at time t .

$$
\begin{aligned}
& E V_{j, t}^{\text {Energy_Stored }} \\
& \quad=E V_{j,(t-1)}^{\text {Energy_Stored }}+\eta_{j}^{E V_{-} \text {Charging }} \times E V_{j, t}^{\text {Power_Charged }} \\
& \quad-E V_{j, t}^{\text {Energy_Trip }}-\left[\frac{1}{\left.\eta_{j}^{E V_{-} \text {Discharging }}\right]}\right. \\
& \quad \times E V_{j, t}^{\text {Power_Discharged }} \\
& \quad \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\}
\end{aligned}
$$

Where

| $E V_{j, t}^{\text {Energy_Stored }}$ | : State of charge, Stored energy in the battery of vehicle $j$, at |
| :---: | :--- |
|  | hour $t(\mathrm{~Wh})$ |
| $E V_{j,(t-1)}^{\text {Energy_Stored }}$ | $:$ State of charge, Stored energy in the battery of vehicle $j$, at |
|  | hour $(t-1)(\mathrm{Wh})$. |
| $E V_{j, t}^{\text {Power_Charged }}$ | : Power charged of vehicle $j$ at hour $t(\mathrm{~Wh}) ;$ |
| $E V_{j, t}^{\text {Power_Discharged }}$ | : Power discharged of vehicle $j$ at hour $t(\mathrm{~Wh}) ;$ |
| $E V_{j, t}^{\text {Energy_trip }}$ | : Energy consumption of vehicle $j$ for traveling at hour $t(\mathrm{~Wh}) ;$ |
| $\eta_{j}^{E V_{-} \text {Charging }}$ | : Charging efficiency of electric vehicle $j ;$ |
| $\eta_{j}^{E V_{-} \text {Discharging }}$ | $:$ Injecting or Discharging efficiency of electric vehicle $j$. |
| $N_{E V}$ | $:$ Number of electric vehicles; |

The discharge and charge limits $E V_{j, t}^{\text {Power_Discharged }}$ and $E V_{j, t}^{\text {Power_Charged }}$ for each EV considering the battery discharge rate are given in Eq. 5 and Eq. 6.

$$
\begin{gather*}
E V_{j, t}^{\text {Power_Discharged }} \leq E V_{j, t}^{\text {Max_Power_Discharged }} \times Y_{j, t}^{E V}  \tag{Eq 5}\\
\forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
E V_{j, t}^{\text {Power_Charged }} \leq E V_{j, t}^{\text {Max_Power_Charged }} \times X_{j, t}^{E V}  \tag{Eq 6}\\
\forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\}
\end{gather*}
$$

Where:
$\begin{array}{ll}E V_{j, t}^{\text {Max_Power_Charged }} & \text { : Rated hourly max power charged for vehicle } j \text {, at hour } t \\ & (\mathrm{~W}) ;\end{array}$
$E V_{j, t}^{\text {Max_Power_Discharged }} \quad:$ Rated hourly max power injected or discharged for vehicle $j$, at hour $t(\mathrm{~W})$;

| $X_{j, t}^{E V}$ | $:$ EV charging status, Binary variable that takes the following |
| :--- | :--- |
|  | values $\left\{\begin{array}{c}0 \text { if EV is not being charged } \\ 1 \text { if } \mathrm{EV} \text { is being charged }\end{array} ;\right.$ |
| $Y_{j, t}^{E V}$ | $:$ EV Discharging or injection status, Binary variable that takes |
|  | the following values $\left\{\begin{array}{c}0 \text { if EV is not discharging } \\ 1 \text { if } \mathrm{EV} \text { is discharging }\end{array}\right.$ |

Eq 7 appears to restrict the charging or discharging of the EV, both processes can not be executed at the same time.

$$
\begin{gather*}
X_{j, t}^{E V}+Y_{j, t}^{E V} \leq 1  \tag{Eq 7}\\
\forall t \in\{1, \ldots, T\} ; \forall j \in\left\{1, \ldots, N_{E V}\right\} ; \quad X, Y \in\{0,1\}
\end{gather*}
$$

Reduction of battery life due to the reaching of a maximum limit $\beta_{j}^{\max }$ and $a$ minimum limit $\beta_{\mathrm{j}}^{\text {min }}$ of charging are restricted by Eq 8 and Eq 9, to prevent loss of the battery characteristics. These limits are defined on the battery capacity limit for each EV as in Eq 10 and Eq 11. The EV battery discharge and charge limits consideres respectively, the battery state of charge and the battery capacity and the previous period of stored energy are given by Eq 12 and Eq 13.

$$
\begin{array}{cc}
E V_{j, t}^{\text {Energy_Stored }} \leq \beta_{j}^{\max } \quad \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} & \text { Eq } 8 \\
E V_{j, t}^{\text {Energy_Stored }} \geq \beta_{j}^{\min } \quad \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} & \text { Eq } 9 \\
\beta_{j}^{\text {max }}=\psi_{j}^{\max } \times E V_{j}^{\text {Battery_Capacity }} \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} & \text { Eq } 10 \\
\quad \beta_{j}^{\text {min }}=\psi_{j}^{\text {min }} \times E V_{j}^{\text {Battery_Capacity }} \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} & \text { Eq } 11 \tag{Eq 11}
\end{array}
$$

$$
\begin{array}{cc}
{\left[\frac{1}{\eta_{j}^{\text {EV_Discharging }}}\right] \times E V_{j, t}^{\text {Power_Discharged }} \leq E V_{j,(t-1)}^{\text {Energy_Stored }}}  \tag{Eq 12}\\
\forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} & \text { Eq } 12 \\
\eta_{j}^{E V_{-} \text {Charging }} \times E V_{j, t}^{\text {Power_Charged }} \leq \beta_{j}^{\text {max }}-E V_{j,(t-1)}^{\text {Energy_Stored }} & \\
\forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} & \text { Eq } 13
\end{array}
$$

Where:

| $\beta_{j}^{\text {max }}$ | $:$ Energy battery maximum limit of EV $\mathrm{j}(\mathrm{Wh}) ;$ |
| :--- | :--- |
| $\beta_{j}^{\text {min }}$ | $:$ Energy battery minimum limit of EV $\mathrm{j}(\mathrm{Wh})$. |
| $\psi_{j}^{\text {max }}$ | $:$ Maximum percentage limit of battery capacity for EV $\mathrm{j} ;$ |
| $\psi_{j}^{\text {min }}$ | $:$ Minimum percentage limit of battery capacity for EV j. |

### 3.3 INTEGRATED LOAD AND EV FORMULATION

One of the main contributions of this dissertation is the modeling of the loads using Unit Commitment theories where the minimum operational time or minimum uptime for every load must be satisfied for an optimal operation.

This part of the model is formulated as a deterministic optimization problem where estimated revenue is maximized using the $0 / 1$ mixed integer linear programming. The effectiveness of the proposed model in optimizing load scheduling is verified through the scenarios proposed and further analyzed with detailed discussion.

Eq14 to Eq 17 presents the self-scheduling load problem of determining the unit commitment status for the loads operation before submitting it in a day head pattern to the next step of the optimization process.

Eq14 shows the restriction hour by hour for the maximum amount of energy that can be consumed by the house that is being optimized. Eq 15 presents the restriction in order to reach the minimum operating time for the elastic loads, Eq 16 and Eq 17 are placed in order to respect the amount of cycles and the duration of every cycle for the charging and discharging process of the EV.

$$
\begin{gather*}
\sum_{k=1}^{N_{E l a s t \_ \text {Load }} P_{k, t}^{\text {Elast_Load }} * X_{k, t}^{\text {Elast_Load }}+\sum_{j=1}^{N_{E V}} \eta_{j}^{\text {EV_Charging }} * E V_{j, t}^{\text {Power_Charged }} * X_{j, t}^{E V}} \begin{array}{c}
\leq \text { Consumption }{ }_{t}^{\text {Max_Hourly }} \\
\forall t=1 \ldots 24 \\
\sum_{k=1}^{24} X_{k, t}^{\text {Elast_Load }} \leq \text { Duration }_{k}^{\text {Elast_Load }} * \text { Cycles }_{k}^{\text {Elast_Load }} \\
\forall k=1 \ldots N_{\text {Elast_Load }} \\
\sum_{t=1}^{24} X_{j, t}^{E V} \leq \text { Duration }_{j}^{E V \_C h a r g i n g ~} * \text { Cycles }_{j}^{\text {EV_Charging }} \\
\forall j=1 \ldots N_{E V}
\end{array} . \tag{Eq14}
\end{gather*}
$$

$$
\begin{gather*}
\sum_{t=1}^{24} Y_{j, t}^{E V} \leq \text { Duration }_{j}^{E V \_ \text {Discharging }} * \text { Cycles }_{j}^{\text {EV_Discharging }}  \tag{Eq 17}\\
\forall j=1 \ldots N_{E V}
\end{gather*}
$$

Where

| Consumption $_{t}^{\text {Max_Hourly }}$ | : Maximum consumption at hour $t$ in Wh. |
| :---: | :--- |
| Duration $_{k}^{\text {Elast_Load }}$ | : Duration of the of cycle of the elastic interruptible load $k$ |
|  | ( hour); |
| Cycles $_{k}^{\text {Elast_Load }}$ | : Number of cycles for the elastic interruptible load $k ;$ |
| Duration $_{j}^{\text {EV_Discharging }}$ | : Duration of the discharging cycle of the EV $j$ ( hour); |
| Cycles $_{j}^{\text {EV_Discharging }}$ | : Number of discharging cycles for the EV $j$. |
| Duration $_{j}^{\text {EV_Charging }}$ | : Duration of the charging cycle of the EV $j$ ( hour); |
| Cycles $_{j}^{\text {EV_Charging }}$ | : Number of charging cycles for the EV $j ;$ |

From Eq 18 to Eq 20 auxiliary variables are declared to respect the minimum up time or minimum time of use for each load, at the same time restriction boundaries of the initial and final time of use are introduced.

$$
\begin{gather*}
X_{k, t}^{\text {Elast_Load }}+X_{k,(t-1)}^{\text {Elast_Load }}=Y_{k, t}^{\text {Aux_oN }}-Z_{k, t}^{\text {Aux_OFF }}  \tag{Eq 18}\\
\forall t=1 \ldots 24 \forall k=1 \ldots N_{\text {Elast_Load }} \\
Y_{k, t}^{\text {Aux_oN }}-Z_{k, t}^{\text {Aux_ofF }} \leq 1  \tag{Eq 19}\\
\forall t=1 \ldots 24 \forall k=1 \ldots N_{\text {Elast_Load }} \\
Y_{k, t}^{\text {Aux_oN }}+\sum_{l=1}^{M U T-1} Z_{k,(t+l)}^{\text {Aux_OFF }} \leq 1 \\
\forall t=1 \ldots 24 \forall k=1 \ldots N_{\text {Elast_Load }} \forall l=1 \ldots M U T-1
\end{gather*}
$$

Eq 20

Where
$Y_{k, t}^{\text {Aux_ON }} \quad:$ Binary variable for device $k$ (Elastic interruptible load),
$\left\{\begin{array}{l}1 \text { if the device is ON at hour } t \\ 0 \text { if the device is OFF at hour } \mathrm{t}\end{array}\right.$;

| $Z_{k, t}^{\text {Aux_OFF }}$ | $:$ Binary variable for device $k$ (Elastic interruptible load), |
| :--- | :--- |
|  | $\left\{\begin{array}{l}1 \text { if the device is OFF at hour } \mathrm{t} \\ 0 \text { if the device is ON at hour } \mathrm{t}\end{array}\right.$ |
| $X_{k,(t-1)}^{\text {Elast_Load }}$ | $:$ Elastic load status for elastic interruptible load $k$ at hour $t$, Binary |
| $M U T$ | variable that takes, 0 if is not operating or 1 if it is; |
| $l$ | $:$ Minimum up time for every elastic load; |
| $l$ | $:$ Auxiliary index. |

### 3.4 DISTRIBUTED GENERATION

Implementing DGs in the migrogrids has several benefits, but at the same time it faces many restrictions and limitations. DG units, being scalable, can be built to meet immediate needs and later be scaled upwards in capacity to meet future demand growth [62]. Scalability allows DG units to reduce their capital and operations costs and thus large capital is not tied up in investments or in their support infrastructure. However, on the other hand, installing DG in the microgrid can also increase the complexity of the system [63].

For this dissertation only two types of distributed generation were used, the solar and the injection process from the EV in the energy discharging period of the battery, resulted from the optimization process. On FIGURE 11 a not optimized EV injecting profile.


FIGURE 11 - General power profile for EV

In FIGURE 12 is exemplified a typical daily generation profile for a photovoltaic panel, that will be added on the scenario 'C'. Also a part of the microgrid structure is shown in FIGURE 13 with different DG insertion levels.


FIGURE 12 - Typical generation profile of a solar panel


FIGURE 13-DG example of the penetration levels in the microgrids

## 3.5 <br> MIXED INTEGER <br> LINEAR <br> PROGRAMMING (MILP) <br> PROBLEM FORMULATION FOR MICROGRIDS (MGS)

In this dissertation it is assumed that microgrids are represented by sets of residencies and each residency has a set of loads, EVs and DGs, as was formulated in the previous sections. Distribution power flow constraints are not considered between residencies of the same microgrid. The overall and integrated formulation of the microgrids problem is modeled as a Mixed Integer Linear Programming problem. A typical MILP formulation is explained in APPENDIX C, that was presented in [64]. Due to practical and didactical reasons, in the following formulation it is assumed that the microgrid is conformed only by one house. Then, the formulation to a MG formed by several residential houses is generalized.

Objective function:

Minimize:

$$
\begin{align*}
\text { Energy }_{t}^{\text {Buy_Cost }} & *\left(\sum_{t=1}^{24} \text { Demand }_{t}^{\text {Total }}+\sum_{t=1}^{24} \sum_{j=1}^{N_{E V}} E V_{j, t}^{\text {Power_Charged }}\right) \\
& + \text { Energy }_{t}^{\text {Sale_Cost }}  \tag{Eq 21}\\
& *\left(-\sum_{t=1}^{24} \sum_{j=1}^{N_{E V}} E V_{j, t}^{\text {Power_Discharged }}-\text { Gen }_{t}^{\text {Total }}\right)
\end{align*}
$$

Subject to the following constraints:

$$
\begin{gathered}
\text { Demand }_{t}^{\text {Total }}=\sum_{k=1}^{N_{\text {Ine_Load }}} P_{k, t}^{\text {Ine_Load }} * \text { Duration }_{k}^{\text {Ine_Load }} * \text { Cycles }_{k}^{\text {Ine_Load }} \\
+\sum_{k=1}^{N_{\text {Elast_Load }}} P_{k, t}^{\text {Elast_Load }} * X_{k, t}^{\text {Elast_Load }} \\
\forall t=1 \ldots 24
\end{gathered}
$$

$$
\begin{aligned}
& E V_{j, t}^{\text {Energy_Stored }} \\
& =E V_{j,(t-1)}^{\text {Energy_Stored }}+\eta_{j}^{E V_{-} \text {Charging }} \times E V_{j, t}^{\text {Power_Charged }} \\
& -E V_{j, t}^{\text {Energy_Trip }}-\left[\frac{1}{\eta_{j}^{\text {EV_Discharging }}}\right] \times E V_{j, t}^{\text {Power_Discharged }} \\
& \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& E V_{j, t}^{\text {Power_Discharged }} \leq E V_{j, t}^{\text {Max_Power_Discharged }} \times Y_{j, t}^{E V} \\
& \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& E V_{j, t}^{\text {Power_Charged }} \leq E V_{j, t}^{\text {Max_Power_Charged }} \times X_{j, t}^{E V} \\
& \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& X_{j, t}^{E V}+Y_{j, t}^{E V} \leq 1 \\
& \forall t \in\{1, \ldots, T\} ; \forall j \in\left\{1, \ldots, N_{v}\right\} ; X, Y \in\{0,1\} \\
& E V_{j, t}^{\text {Energy_Stored }} \leq \beta_{j}^{\max } \quad \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& E V_{j, t}^{\text {Energy_Stored }} \geq \beta_{j}^{\text {min }} \quad \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& \beta_{j}^{\max }=\psi_{j}^{\max } \times E V_{j}^{\text {Battery_Capacity }} \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& \beta_{j}^{\text {min }}=\psi_{j}^{\text {min }} \times E V_{j}^{\text {Battery_Capacity }} \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& {\left[\frac{1}{\eta_{j}^{\text {EV_Discharging }}}\right] \times E V_{j, t}^{\text {Power_Discharged }} \leq E V_{j,(t-1)}^{\text {Energy_Stored }}} \\
& \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& \eta_{j}^{\text {EV_Charging }} \times E V_{j, t}^{\text {Power_Charged }} \leq \beta_{j}^{\max }-E V_{j,(t-1)}^{\text {Energy_Stored }} \\
& \forall t \in\{1, \ldots, T\} ; \quad \forall j \in\left\{1, \ldots, N_{E V}\right\} \\
& \sum_{k=1}^{N_{\text {Elast_Load }}} P_{k, t}^{\text {Elast_Load }} * X_{k, t}^{\text {Elast_Load }}+\sum_{j=1}^{N_{E V}} \eta_{j}^{\text {EV_Charging }} * E V_{j, t}^{\text {Power_Charged }} * X_{j, t}^{E V} \\
& \leq \text { Consumption }{ }_{t}^{\text {Max_Hourly }} \\
& \forall t=1 \ldots 24 \\
& \sum_{k=1}^{24} X_{k, t}^{\text {Elast_Load }} \leq \text { Duration }_{k}^{\text {Elast_Load }} * \text { Cycles }_{k}^{\text {Elast_Load }} \\
& \forall k=1 \ldots N_{\text {Elast_Load }}
\end{aligned}
$$

$$
\begin{align*}
& \sum_{t=1}^{24} X_{j, t}^{E V} \leq \text { Duration }_{j}^{E V_{-} \text {Charging }} * \text { Cycles }_{j}^{E V \_ \text {Charging }}  \tag{Eq 16}\\
& \forall j=1 \ldots N_{E V} \\
& \sum_{t=1}^{24} Y_{j, t}^{E V} \leq \text { Duration }_{j}^{E V \_D i s c h a r g i n g} * \text { Cycles }_{j}^{E V_{-} \text {Discharging }} \\
& \forall j=1 \ldots N_{E V} \\
& X_{k, t}^{\text {Elast_Load }}+X_{k,(t-1)}^{\text {Elast_Load }}=Y_{k, t}^{\text {Aux_ON }}-Z_{k, t}^{\text {Aux_OFF }} \\
& \forall t=1 \ldots 24 \forall k=1 \ldots N_{\text {Elast_Load }} \\
& Y_{k, t}^{\text {Aux_ON }}-Z_{k, t}^{\text {Aux_OFF }} \leq 1 \\
& \forall t=1 \ldots 24 \forall k=1 \ldots N_{\text {Elast_Load }} \\
& Y_{k, t}^{A u x_{-} O N}+\sum_{l=1}^{M U T-1} Z_{k,(t+l)}^{\text {Aux_OFF }} \leq 1 \\
& \forall t=1 \ldots 24 \forall k=1 \ldots N_{\text {Elast_Load }} \forall l=1 \ldots M U T-1 \\
& \left\{X_{k, t}^{\text {Elast_Load }}, Y_{k, t}^{\text {Aux_ON }}, Z_{k, t}^{\text {Aux_OFF }}, X_{j, t}^{E V}, Y_{j, t}^{E V}\right\} \text { Binary variables }
\end{align*}
$$

Where:

$$
\begin{array}{ll}
\text { Energy }_{t}^{\text {Buy_Cost }} & \text { Energy purchase price } \\
\text { Energy }_{t}^{\text {Sale_Cost }} & \text { Energy selling price }
\end{array}
$$

### 3.6 Tested Distribution System

As clients keep increasing day by day, the distribution system has become more complex. Therefore, it is important to improve the reliability of the system. Reliable electric power system serves client loads without interruptions in power supply [65], and has the ability to deliver uninterrupted services to clients [66]. Electricity is produced and delivered to customers through three main subsystems; generation, transmission and distribution, as presented in FIGURE 14.


FIGURE 14 - Typical electric network system [65]

The generation subsystem is responsible for the electrical energy production. This can be generated by conversion of any energy sources available, kinetic, potential, mechanical, nuclear etc.... [67].

Normally, typical distribution system designs are divided into three different types: radial, loop and mesh systems. However, in practice, all these designs are
normally combined or incorporated together [68]. Due to its flexibility and configurability a 33 buses radial system have been considered in this dissertation to demonstrate the efficiency of the proposed algorithm that is presented in the next chapter. FIGURE 15 presents the physical structure of the system, details and system data are available in [69].


FIGURE 15-33 buses radial distribution system

### 3.7 OPTIMAL POWER FLOW PROBLEM FORMULATION FOR DISTRIBUTION NETWORK USING MATPOWER

For most of the optimal power flow problems a methodical answer is difficult to find. The most popular numerical methods for solving the optimal power flow problem are interior points and Newton's (or Newton-Raphson's) method. For the purposes of this dissertation the interior point used in Matpower [5], under a matlab environment, is used to solve the iterative optimal power flow over the radial network.

Matpower includes code to solve the optimal power flow problem. The standard version of each takes the form, from [70], that is included in APPENDIX E.

In modernized power systems, the transmission and distribution grids are separately used by independent system operator (ISO) and distribution companies (DISCOs). As the working condition of one-grid impacts the decisions made by operators of other grids, the ISO and DISCOs should cooperate and collaborate with each other in order to run the entire power system in a safe and economic way, according to the concept of system of systems (SoS) engineering [71].

Then, is presented a system of systems (SoS) based framework for optimally operating active distribution grids. The proposed SoS framework presented in this dissertation defines both distribution company (DISCO) and microgrids (MGs) as autonomous systems, and recognizes the process of information interchange among them as showed in FIGURE 16. As the DISCO and MGs are physically connected, the operational condition of one might impact the operating point of other systems [72] as showed in FIGURE 17 and FIGURE 18.


FIGURE 16 - Distribution system with passive MG integration adapted of [56]


FIGURE 17 - Characteristics between the MGs and the distribution network [56].

The proposed mathematical algorithm uses a separated optimization process that aim maximizing the benefit of each independent system. A hierarchically algorithm, first running an optimal power flow (OPF) for the general operating system values for the distribution system using Matpower tool and then creating a loop that involves the MILP microgrids (MGS) problem. A hierarchical optimization algorithm is formulated to coordinate the independent systems and to find the optimal operating point of the SoS-based active distribution grid. The mathematical and graphical results show the efficiency of the proposed SoS framework and resolution method.


FIGURE 18 - Distribution system with active microgrid integration [56].

FIGURE 19 presents the flowchart that is used to solve the optimization problem stated in this work. Firstly are define the tolerance convergence values that controls the iterative optimization process for every part of the system, are also defined the active and reactive power hour by hour, the white tariff is then entered in the simulation accompanied with the objective function restriction values for the OPF and MILP. The iterative processes for the MGs and for the distribution system are then started, respecting the tolerance values. A pseudocode is also included in APPENDIX F.


FIGURE 19 Proposed flowchart for the optimization problem solution.

## 4 EXPERIMENTAL RESULTS, DISCUSSION AND SIMULATIONS

The algorithms, models and optimization procedures tested in this dissertation were discussed and analyzed in the previous chapter. Also, the database, the preliminary basic configuration for every scenario, the parameters and the variables needed also were introduced. In this section will be examined in detail the variables needed in order to correctly implement and apply the system of system modeling for the entire network.

Is also analyzed the behavior of the OPF for the radial distribution system and the introduction of renewable energy, acting in the same scenario with the inclusion of the EV as a V2G. Active and reactive power consumption, consumption costs and generation are analyzed and explained. All the results will be discussed and related. The proposed method is implemented using MATLAB as main programmatic tool. The algorithms were performed and ran partly on an Intel(R) Core(TM) i3-2375M 1.50 GHz and also was used an 1.7 GHz Intel Core i7, OS X 10.9.5 (13F34). Beside, the MATPOWER (runopf) simulator is used to test the distribution systems to evaluate the results coming from the MILP program for the microgrid.

For the sake of simplicity, is used a white tariff with UM (Units of Money) values, without any numerical alteration in scale from the original white tariff in $\mathrm{R} \$$ (Brazilian Real) or (US Dollar) as presented in FIGURE 20.


FIGURE 20 - White Tariff in UM/kWh

Based on the 33 buses radial distribution system, a classification is performed according to its physical positioning. All the buses are then classified into low consumers branch, medium consumer branch, high consumers and optimization branch. Four generators (See APPENDIX B) are distributed along the system as it's seen in FIGURE 21. Low consumers are allocated from buses 19 to 22 (Secondary branch 'A'), medium consumers from buses 26 to 33 (Secondary branch 'C'), high consumer from buses 23 to 25 (Secondary branch ' $B$ '), finally in the main line from bus 1 to 18 (Primary branch) are placed the microgrids that are being part of the optimization process. The total behavior of this base configuration is discussed on the following tables.

The classification proposed for the branches will be kept along all the scenarios; this sorting was effectuated with the primary goal of showing in better detail the results of all the system. The complexity and the amount of data was not facilitating the displaying of the results. Three secondary branches and one primary are introduced and manipulated. The reference of each graphic appears on every table or graphic from now on.

On FIGURE 22 is presented the load profile for individual householders that will be introduced into every microgrid, in this graphics appear the consumption of the inelastic loads and the consumption of the elastic interruptible loads, also appear the amount of energy charged by the EV. FIGURE 23 shows its respective energy costs. In the meantime the energy costs are calculated based on the white tariff with UM/kWh values. Notice that a set of 50 houses is used and inserted in the buses that are being part of the optimization process, values from FIGURE 22 are multiplied by a factor of 50, simulating an equal behavior for all the users, in order to insert it into the respective buses.

### 4.1 CASE STUDIES

On this section the results of implementing the proposed system of system strategy at various user penetration levels is presented. Also, the introduction of the EV as a V2G or G2V is considered, for the 33 buses distribution system (See APPENDIX B). Different configurations are proposed to show the results of the method proposed. The "scenario A" that is the basic comparison pattern for the others scenarios, the "scenario B" that is considering the EV just as a load that is included in the optimization process, The "scenario $C$ " will consider the insertion of a photovoltaic source also the EV in its both characteristics (Charging and Injecting energy).

Some results are presented in a 2D format and the others in 3D format that visibly, displays all the changes on the distribution network, according to the scenario that is under analysis.

### 4.1.1 Scenario A: "Base Case"

In this scenario the main idea is to show the basic configuration for the system studied. A 33 buses radial distribution system is analyzed and reconfigured in order to implement the algorithm proposed.


FIGURE 21 - System architecture for scenario ' $A$ '


FIGURE 22 - Power demand of a house for the scenario A.


FIGURE 23 - Energy cost for a typical house used on scenario A

FIGURE 24 - FIGURE 26 presents the total load profile for every type of user (Low, Medium, High) on a 24 -hour horizon [73].


FIGURE 24-Total active power profile for buses with low consume.


FIGURE 25 - Total active power profile for buses with medium consume.


FIGURE 26 - Total active power profile for buses with high consume.

All the simulations are executed in a 24 hours line time, and all the data from every bus and branch is collected and displayed. Basically in this dissertation are shown the voltage variation in every bus hour-by-hour, Operating Marginal Cost or Lambda values (UM/MWh), active power losses in the branches and the generation for the generator buses, and in the end, all the graphics are discussed. According to the characteristic and previous configuration presented for the scenario ' A ', graphically are shown its variations and response for "scenario B" and "scenario C". Subsequently, the parameters of interest for this study are briefly discussed hour by hour. The graphics are presented in the following order: Buses voltage magnitude, Lambda values, Active power losses, and Generation.

In general, the format chosen in order to give a better understanding of the results is a 3D format that in its ' $Y$ axis' presents: the voltage magnate in (pu), the lambda values obtained from the optimization of the distribution system and the active power losses for every bus. In its 'X axis' presents the total 24 hours horizon and finally, in its ' $Z$ axis' appears every bus or branch of the system with its own characteristics. Also all the numerical data that appears in all the graphics of this dissertation can be found in the APPENDIX I.

Due to the big amount of data tabulated the total system is divided into 4 different set of branches, it is possible to see in FIGURE 21, that one optional sorting due to the system physical structure is to group the buses respect to the branch in which they are located. For instance the buses in the first superior branch of the system can be part of the group that in this dissertation we call it as 'Secondary branch ' $A$ '. The second superior line from the top to the bottom and the longest one that appear on the system will be grouped, and called in this dissertation as 'Primary branch'. The third proposed division for the system will group the buses that are located in the branch where appear the buses from number 23 to 25 , for this branch the name that will receive in this dissertation will be 'Secondary branch 'B'. Finally the fourth proposed division is presented and introduced with the buses that go from bus number 26 till bus 33, that are graphically located in the inferior line of the system.

For this scenario 'A', in the case of the lambda values or Operating Marginal Cost for the system and the active power losses, the same classification structure is used in each case. On this scenario will be emphasized where the maximum values of each of the parameters plotted are located in the branches; i.e., for the scenario where voltage magnitude is analyzed, will be identified where the maximum values
appears and at what time they appeared. Exactly the same process will be done for the graphics where the lambda values are analyzed and also for the active power losses.

From now on the classification effectuated will be used at will for all the plots in 3D, always accompanied of the scenario where are they being used and its respective units. TABLE 5 presents the nomenclature defined for the branches that is mainly used in the analysis of the losses for the three different scenarios.

TABLE 5-Branch nomenclature

| Branch \# | From | To |
| :---: | :---: | :---: |
| 1 | 1 | 2 |
| 2 | 2 | 3 |
| 3 | 3 | 4 |
| 4 | 4 | 5 |
| 5 | 5 | 6 |
| 6 | 6 | 7 |
| 7 | 7 | 8 |
| 8 | 8 | 9 |
| 9 | 9 | 10 |
| 10 | 10 | 11 |
| 11 | 11 | 12 |
| 12 | 12 | 13 |
| 13 | 13 | 14 |
| 14 | 14 | 15 |
| 15 | 15 | 16 |
| 16 | 16 | 17 |
| 17 | 17 | 18 |
| 18 | 2 | 19 |
| 19 | 19 | 20 |
| 20 | 20 | 21 |
| 21 | 21 | 22 |
| 22 | 3 | 23 |
| 23 | 23 | 24 |
| 24 | 24 | 25 |
| 25 | 6 | 26 |
| 26 | 26 | 27 |
| 27 | 27 | 28 |
| 28 | 28 | 29 |
| 29 | 29 | 30 |
| 30 | 30 | 31 |
| 31 | 31 | 32 |
| 32 | 32 | 33 |

4.1.1.1 Buses Voltage Magnitude for Scenario A (pu)

FIGURE 27 show a mostly constant behavior along the 24 hours for buses 1 , 2 and 3 , small or not variation at all is displayed. For buses 4 to 7 , variations in the order of 1.05 pu to 1.1 pu are found. From buses 8 to 18 minimum values are located around 19 and 21 hours, with a minimum value of 0.90 pu , and with a 1.08 pu constant values from 12:00 am until 6:00 am.


FIGURE 27 - Voltage magnitude for primary branch on sce A

FIGURE 28 shows the secondary 'A' branch voltage magnitude for each of its buses. For bus 22 an almost constant behavior is observed from 7:00 am till 12: 00 pm due that is directly connected to generator number 2, for the buses 2, 19, 20, 21 and almost symmetric and proportional behavior is shown.

FIGURE 58 shows the secondary 'B' branch voltage magnitude for each of its buses, this branch is presenting its lowest voltage values in the bus 25 and the higher for this branch are presented in bus 23 where generator number 3 is connected.

FIGURE 59 presents the voltage magnitude for the secondary branch ' C ', in this case the highest and more consistent voltage magnitude appears in the bus 26 almost for all the 24 hours, the lowest voltage magnitude are presented in bus 32 .


FIGURE 28 - Voltage magnitude for buses on secondary 'A' branch for sce. A

### 4.1.1.2 Lambda Values for Scenario A (UM/MWh)

3D graphic describing the behavior of the operational cost into every bus of the primary branch appears in FIGURE 29 where a symmetric behavior according to the amount of load connected to the system along the day is seen. Lambda values for the primary branch in this scenario get its higher values around 18 hours to 20 hours, this is due to the amount of loads that are being allocated at that hour into the system and the price of the energy at that hour. In this branch for all the buses hour by hour are observed similar behaviors with small variations among buses.


FIGURE 29 - Lambda values for the primary branch for scenario A.

FIGURE 30 presents the Lambda values for the secondary ' $A$ ' branch, for this scenario; apparently no higher distinctive value in this graph can be found, the max value is around $643.76 \mathrm{UM} / \mathrm{MWh}$ at 20 hours, almost a parametric behavior along the day for every hour can be observed.

FIGURE 60 shows the Lambda values for the secondary 'B' branch for this scenario, this graph presents a behavior that changes according to the amount of loads optimized in the 24 hours horizon. In comparison with FIGURE 61 where Lambda values for the secondary 'C' branch are showed, almost the same behavior
is found, with similar variations during the day. This happens mainly to the energy prices and the high and medium consumer loads.


FIGURE 30 - Lambda values for the secondary 'A' branch for scenario A

### 4.1.1.3 Branches Active Power Losses for Scenario A (kW)

In the following graphs are presented the losses in the different branches of the system (See TABLE 5), according to the previous classification effectuated for the buses. FIGURE 31 presents the Active power losses for the primary branch, in this scenario presents its higher losses among branches 9-10 due to allocation of the generator number 1, reaching its peak value with 288.97 kW around 20 hours, in this figure several branches appear with almost no considerable losses throughout the day.

FIGURE 32, where Active power losses for secondary ' $A$ ' branch are presented for scenario "A", can be seen from the graph that is presenting its higher branch losses right through the branches from bus19 to bus 20 at 20 hours with 30.12 kW, mainly due to the distance and operational cost of the generator number 2 in the secondary branch A.

FIGURE 62, where Active power losses for secondary 'B' branch are presented for scenario A, presenting its higher branch losses through the branches 22 and 23, around 20 hours, For the same hour FIGURE 63, where Active power losses for secondary 'C' branch is presented, shows branch from bus 27 to bus 28 also with the maximum value for this branch.



FIGURE 32 - Active power losses for secondary ' $A$ ' branch for scenario $A$

### 4.1.1.4 Active Power Generation for Scenario A (MW)

FIGURE 33 presents the GD in distribution system and how much those generators are producing hour by hour. A generation peak is found around 20 hours, as was expected to be, due to the loads placed at that hour. Due to its generation prices can be notice that the generator placed on bus 9 and the substation that is transmitting energy into the bus 1 are the generators that are producing most of the energy consumed by the system.


FIGURE 33 - Active generation for scenario A

### 4.1.1.5 Objective function for OPF in Scenario A

FIGURE 34 shows the variation of the objective function for the final OPF with the optimal values for this scenario along the 24 hours.


FIGURE 34-Objective function for the final OPF performed in scenario A

TABLE 6 - Primary variables analyzed for scenario A

|  |  | Voltage Magnitude |  |  |  | P Losses (1^2*R) |  | Lambda P |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min |  | Max |  | Max |  | Min |  | Max |  |
| Hour | Objective <br> Function | $\begin{array}{r} \hline \text { Value } \\ \text { (pu) } \end{array}$ | Bus | Value <br> (pu) | Bus | Value (MW) | $\begin{gathered} \hline \text { Branches } \\ \text { From-To } \end{gathered}$ | Value (UM/MWh) | Bus | Value (UM/MWh) | Bus |
| 1 | 220.46 | 1.058 | 33 | 1.1 | 9 | 0.01 | 27-28 | 117.98 | 9 | 126.52 | 33 |
| 2 | 160.17 | 1.062 | 33 | 1.1 | 9 | 0.01 | 27-28 | 100.67 | 9 | 107.21 | 33 |
| 3 | 135 | 1.066 | 33 | 1.1 | 9 | 0.01 | 27-28 | 92.46 | 9 | 97.85 | 33 |
| 4 | 91.26 | 1.074 | 33 | 1.1 | 9 | 0.01 | 27-28 | 76.2 | 9 | 79.53 | 33 |
| 5 | 98.33 | 1.072 | 33 | 1.1 | 9 | 0.01 | 27-28 | 79.08 | 9 | 82.79 | 33 |
| 6 | 140.5 | 1.074 | 33 | 1.1 | 9 | 0.01 | 27-28 | 94.58 | 9 | 98.64 | 33 |
| 7 | 777.82 | 1.044 | 33 | 1.1 | 22 | 0.01 | 27-28 | 218.39 | 22 | 241.63 | 33 |
| 8 | 1305.26 | 1.001 | 18 | 1.1 | 22 | 0.04 | 9-10 | 277.16 | 22 | 334.02 | 18 |
| 9 | 488.5 | 1.058 | 18 | 1.1 | 22 | 0.01 | 9-10 | 174.55 | 22 | 187.51 | 18 |
| 10 | 408.09 | 1.067 | 33 | 1.1 | 22 | 0.01 | 27-28 | 160.86 | 22 | 170.22 | 33 |
| 11 | 904.44 | 1.026 | 18 | 1.1 | 22 | 0.02 | 9-10 | 234.03 | 22 | 267.43 | 18 |
| 12 | 1445.65 | 0.995 | 18 | 1.1 | 22 | 0.05 | 9-10 | 290.81 | 22 | 355.16 | 18 |
| 13 | 506.43 | 1.057 | 33 | 1.1 | 22 | 0.01 | 27-28 | 178.49 | 22 | 192.33 | 33 |
| 14 | 560.38 | 1.055 | 33 | 1.1 | 22 | 0.01 | 27-28 | 187.15 | 22 | 202.4 | 33 |
| 15 | 731.71 | 1.049 | 18 | 1.1 | 22 | 0.01 | 9-10 | 212.28 | 22 | 232.39 | 18 |
| 16 | 502.78 | 1.057 | 33 | 1.1 | 9 | 0.01 | 27-28 | 178.51 | 9 | 192 | 33 |
| 17 | 1084.17 | 1.026 | 33 | 1.1 | 22 | 0.03 | 27-28 | 256.89 | 22 | 294.55 | 33 |
| 18 | 3010.82 | 0.976 | 18 | 1.1 | 22 | 0.07 | 9-10 | 406.92 | 22 | 532.48 | 18 |
| 19 | 5519.7 | 0.95 | 18 | 1.1 | 22 | 0.13 | 9-10 | 538.07 | 22 | 767.55 | 18 |
| 20 | 7839.44 | 0.9 | 18 | 1.1 | 1 | 0.29 | 9-10 | 593.1 | 22 | 1227.24 | 18 |
| 21 | 5640.3 | 0.946 | 18 | 1.1 | 22 | 0.13 | 9-10 | 536.64 | 22 | 781.25 | 18 |
| 22 | 5058.57 | 0.903 | 18 | 1.1 | 22 | 0.2 | 9-10 | 505.39 | 22 | 810.38 | 18 |
| 23 | 1287.71 | 1.025 | 33 | 1.1 | 22 | 0.03 | 27-28 | 280.29 | 22 | 321.78 | 33 |
| 24 | 842.5 | 1.044 | 33 | 1.1 | 22 | 0.02 | 2-3 | 227.21 | 22 | 251.5 | 33 |

Is observed from the graphics and TABLE 6 that the objective function for the system reaches its lowest value at 3 hours and its higher value at 20 hours as it seen, due to the operative cost of the system, the bus with the minimum voltage value appears at the same hour that the objective function reaches its higher value, with a 0.9 pu in bus 18, For that hour a 1.1 pu voltage value appears as max value in bus 1. The higher values for the losses were focalized on two branches, the first one that goes from bus 27 to bus 28 and the other one that goes from bus 9 to bus 10, in this branch the peak for the losses of the system were also reached at 20 hours, almost 0.29 MW. Basically the lowest lambda values were obtained on the buses 9 and 22, almost varying its frequency along the proposed horizon.

### 4.1.2 Scenario B: "Optimization of loads and charging of EVs scheduling"

Departing from the scenario ' $A$ ' the main idea for the scenario ' $B$ ' is to show the changes due to the reallocation and optimization realized for the basic configuration of the system studied. Once again the 33 buses distribution system is analyzed in order to implement the algorithm proposed. Based on the 33 buses radial distribution system, a classification is performed according to its physical positioning as in the scenario $A$, but with the difference that in this case the charging periods for the electrical vehicle are optimized. It is not anymore charged without restrictions. All the buses for low consumers branch, medium consumer branch, high consumers and optimization branch, are loaded with the same demand, but for the primary branch a previous MILP optimization is effectuated in every microgrid. The system used for this scenario ' $B$ ' is presented in FIGURE 35. In the same way that in the scenario ' $A$ ', once again Low consumers are allocated from buses 19 to 22, secondary ' $A$ ' branch, medium consumers from buses 26 to 33 in what we call as secondary ' $C$ ' branch, high consumer from buses 23 to 25 in what we call as secondary ' B ' branch, finally in the main branch from bus 1 to 18 where the primary branch is placed with the microgrid to optimize, FIGURE 36 presents the total power demand profile and FIGURE 37 the energy costs. The behavior and variations for this scenario are shown in the following graphs and tables.


FIGURE 35 - System architecture for scenario B


FIGURE 36 - Power demand of a typical house for the scenario B.


FIGURE 37 - Energy cost for a typical house used on scenario B

### 4.1.2.1 Buses Voltage Magnitude for Scenario B (pu)

FIGURE 38 shows more voltage variations in comparison with that is showed in FIGURE 27, where the base case is presented. In this scenario buses 17 and 18 presents the lowest voltage value for the primary branch, around 2:00 am and 4:00 am with a 0.93 pu value, due to the shifting of the loads after the optimization process and the distance to closer generator that is located in bus number 9 . Higher and more constant values in this case are presented in buses 1, 2, 3 and 4 due to the voltage stability that provides the sub-station.


FIGURE 38 - Voltage magnitude for primary branch in scenario $B$

FIGURE 39 is presenting variations in the voltage from 1.090 pu to 1.1 pu , the lowest voltage magnitude is found at bus number 2 around 10:00 am, highest values are found at bus 22 with an almost stable value at 1.1 pu all day long. FIGURE 64 presents the voltage magnitude for secondary branch ' $B$ ' in scenario $B$, in this figure can be appreciated the pattern modification in the graphic in comparison with the scenario 'A'. The lowest voltage magnitude is observed in bus 25 around 19:00 hours, buses 23 and 3 are presenting a similar graphical behavior; the highest voltage magnitude is 1.09 that is presented at bus 3 . FIGURE 65 presents the voltage magnitude for secondary branch ' $C$ ' in scenario $B$, in this branch the higher voltage
magnitude values are located in bus number 6 with a 1.08 pu , the lowest values for this simulation are found at bus number 33, around 21:00 hours with 0.96 pu value, this variations occurs mainly due to the proximity or the distance between the buses and the generators. For buses 26 and 27 a similar graphic can be observed, due to the stability provided by the generator number 4 in bus 27 .


FIGURE 39 - Voltage magnitude for secondary branch ' $A$ ' in scenario $B$

### 4.1.2.2 Lambda for Scenario B (UM/MWh)

As can be seen in FIGURE 40, after the optimization process a better peak reduction and a proportional distribution according to the amount of loads optimized by the algorithm can be observed. The highest operational values are found in bus 18 with an amount of $479.19 \mathrm{UM} / \mathrm{MWh}$ in comparison with the peak for the scenario ' $A$ ', this almost represents a 41 \% peak reduction in its congested hour (21:00), mainly due to its load optimization.


FIGURE 40 - Lambda values for primary branch in scenario $B$

FIGURE 41 presents the Lambda values for secondary 'A' branch in scenario B, for this branch the higher values are found around 19:00 and 21:00 hours with a maximum value close to $435.5 \mathrm{UM} / \mathrm{MWh}$, the lowest values can be observed around 10:00 hours with a cost of 123.59 UM/MWh in almost all the branch. In comparison with its counterpart in the Scenario 'A' a peak reduction of almost $32 \%$ can be appreciated, but a rising of almost $60 \%$ can be observed in its minimum value, that in Scenario 'A' was at 4:00 am.

FIGURE 66 presents the Lambda values for secondary 'B' branch in scenario $B$, in this case the lowest values appear in bus 23 at 10:00 hours close to the 124.87 UM/MWh, highest values will appear on bus 25 at 21 hours with a value of 467.37

UM/MWh. Comparing the highest and the lowest values for this scenario with the scenario ' $A$ ', can be noticed that a reduction of almost $35 \%$ in the peak of the graphic was reached, but a $38 \%$ increasing was perceived for the lowest value that was kept in bus 23.

FIGURE 67 presents Lambda values for secondary ' $C$ ' branch in scenario $B$, in this case, lowest values are always found in bus number 26, while the highest values for the scenario ' $A$ ' were allocated over the bus 33 and for the scenario ' $B$ ' over the bus number 6.


FIGURE 41 - Lambda values for secondary 'A' branch in scenario B

### 4.1.2.3 Branches Active Power Losses for Scenario B (kW)

In the following graphs are presented the losses in the different branches of the system, according to the previous classification effectuated for the buses (See TABLE 5). FIGURE 42 presents the active power losses for the primary branch in this scenario, with its higher value among buses 9-10 but with a considerable shifting peak due to the optimization process that has been effectuated till this point. Several branches in this figure appear with almost no considerable losses through the day.

FIGURE 43, where active power losses for secondary ' $A$ ' branch, appears presenting its higher branch losses right through the branches from bus 19 to bus 20 with a peak of 11.31 kW at 21 hours, for this branch other power losses peak is found around 1:00 am, but with a value of 8.98 kW . The lowest losses are presented in branch 18 where a maximum peak for this branch of 1.09 kW can be seen, that in comparison with scenario 'A' have presented a considerable reduction from 2.51 kW , due to the shifting load as result of the optimization process..

FIGURE 68, where active power losses for secondary ' B ' branch are presented, this scenario shows its higher branch losses through the branches 23 with maximum losses of 37.29 kW at 20:00 hours, in comparison with the scenario ' $A$ ' for the same hour that initially was working with a value of 37 kW an increase of 0.29 kW cab be appreciated for this hour. The lowest losses values remain for both cases at branch 18, which at the beginning of the day, presents a 1.28 kW at 7:00 am as maximum value.

FIGURE 69 where, active power losses for secondary ' $C$ ' branch are presented. From this figure can be observed that no losses are presented in branch 32 along the day, almost the same behavior that was seen for this branch in the scenario ' $A$ '. The highest values remains in the branch 27 with total losses of 126.71 kW at 20 hours, a reduction for this branch can be observed from its original value in case 'A' of 129.66 kW .


FIGURE 42 - Active power losses for primary branch in scenario B


FIGURE 43 - Active power losses for secondary 'A' branch in scenario B

### 4.1.2.4 Active Power Generation for Scenario B (MW)

In comparison with FIGURE 33, FIGURE 44 presents a better distribution in the generation hours and also in the peaks reduction for all the generators. Must be reminded that the load through the secondary branches is the same with almost no alterations in comparison with the scenario "A". This graphic is used as pattern to know how the losses, lambda values, and voltage magnitude can vary.


FIGURE 44 - Active power generation for scenario B

### 4.1.2.5 OPF Objective Function for Scenario B

FIGURE 45 shows the variation of the objective function for the final OPF with the optimal values for this scenario along the 24 hours.


FIGURE 45-Objective function for the final OPF performed in scenario 'B'

TABLE 7 - Primary variables analyzed for scenario B

| Hour |  | Voltage Magnitude (pu) |  |  |  | Active Power <br> Losses (MW)Max |  | Lambda P (UM/MWh) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min |  | Max |  |  |  | Min |  | Max |  |
|  | Objective Function | Value (pu) | Bus | Value (pu) | Bus | Value <br> (MW) | Branches From-To | Value (UM/MWh) | Bus | Value (UM/MWh) | Bus |
| 1 | 1223.78 | 0.988 | 18 | 1.10 | 22 | 0.05 | 9-10 | 266.49 | 22 | 330.37 | 18 |
| 2 | 1829.16 | 0.939 | 18 | 1.10 | 22 | 0.10 | 9-10 | 315.99 | 22 | 443.61 | 18 |
| 3 | 1525.68 | 0.953 | 18 | 1.10 | 22 | 0.08 | 9-10 | 291.6 | 22 | 393.97 | 18 |
| 4 | 1578.73 | 0.938 | 18 | 1.10 | 22 | 0.10 | 9-10 | 294.09 | 22 | 413.16 | 18 |
| 5 | 1038.66 | 0.978 | 18 | 1.10 | 22 | 0.06 | 9-10 | 244.83 | 22 | 309.97 | 18 |
| 6 | 1553.51 | 0.947 | 18 | 1.10 | 22 | 0.09 | 9-10 | 293.8 | 22 | 403.07 | 18 |
| 7 | 458.69 | 1.051 | 33 | 1.10 | 22 | 0.01 | 27-28 | 170.12 | 22 | 185.26 | 33 |
| 8 | 405.24 | 1.05 | 33 | 1.10 | 9 | 0.01 | 27-28 | 160.09 | 9 | 174.05 | 33 |
| 9 | 275.17 | 1.067 | 33 | 1.10 | 9 | 0.01 | 27-28 | 132.27 | 9 | 139.62 | 33 |
| 10 | 233.32 | 1.064 | 33 | 1.10 | 9 | 0.01 | 27-28 | 121.2 | 9 | 128.63 | 33 |
| 11 | 423.46 | 1.06 | 33 | 1.10 | 22 | 0.01 | 27-28 | 163.66 | 22 | 175.3 | 33 |
| 12 | 1547.68 | 0.988 | 18 | 1.10 | 22 | 0.05 | 9-10 | 299.78 | 22 | 371.94 | 18 |
| 13 | 506.43 | 1.057 | 33 | 1.10 | 22 | 0.01 | 27-28 | 178.49 | 22 | 192.33 | 33 |
| 14 | 560.38 | 1.055 | 33 | 1.10 | 22 | 0.01 | 27-28 | 187.15 | 22 | 202.4 | 33 |
| 15 | 519.39 | 1.055 | 33 | 1.10 | 22 | 0.01 | 27-28 | 180.55 | 22 | 195.41 | 33 |
| 16 | 502.78 | 1.057 | 33 | 1.10 | 9 | 0.01 | 27-28 | 178.51 | 9 | 192 | 33 |
| 17 | 1084.17 | 1.026 | 33 | 1.10 | 22 | 0.03 | 27-28 | 256.89 | 22 | 294.55 | 33 |
| 18 | 1649.46 | 0.992 | 33 | 1.10 | 22 | 0.07 | 27-28 | 312.45 | 22 | 385.92 | 33 |
| 19 | 2866.62 | 0.982 | 33 | 1.10 | 22 | 0.09 | 27-28 | 411.5 | 22 | 519.11 | 33 |
| 20 | 2571.17 | 1.003 | 33 | 1.10 | 22 | 0.06 | 27-28 | 390.9 | 22 | 473.59 | 33 |
| 21 | 3036.63 | 0.961 | 33 | 1.10 | 22 | 0.13 | 27-28 | 415.2 | 22 | 557.36 | 33 |
| 22 | 1713.44 | 1.012 | 33 | 1.10 | 22 | 0.05 | 27-28 | 312.23 | 22 | 380.14 | 33 |
| 23 | 1287.71 | 1.025 | 33 | 1.10 | 22 | 0.03 | 27-28 | 280.29 | 22 | 321.78 | 33 |
| 24 | 842.5 | 1.044 | 33 | 1.10 | 22 | 0.02 | 2-3 | 227.21 | 22 | 251.5 | 22 |

Is observed from the graphics and TABLE 7 that the objective function for the system reaches its lowest value at 10 hours and its higher value at 21 hours. The bus with minimum voltage values appears at 4 hours with a 0.938 pu in bus 18 , for that exactly hour a 1.1 pu voltage value appear as max value in bus 22. The higher values for the losses were focalized on two branches, the first one, that goes from bus 9 to bus 10 and the other one, that goes from bus 27 to bus 28 , and is in this branch that the peak for the losses of the system is reached at 21 hours, almost 0.13 MW. Basically the lowest lambda values were obtained on the buses 9 and 22, almost varying its frequency of appearance along the day.

### 4.1.3 Scenario C: "Optimization of Loads considering Micro-DG, EV charging and injection "

In this scenario (See FIGURE 46) the insertion of the EV, as a V2G, and MicroDG are considered as main target for the optimization process. The load profile and costs for a typical house that is situated inside of the microgrid are shown in FIGURE 47 and FIGURE 48, graphs and tables are shown to describe the buses voltage profile, branch losses and lambda power values. Finally a resume is done with a comparative for the buses behavior throughout the complete optimization process.


FIGURE 46 - System architecture for scenario C

FIGURE 47 shows the displacement final profile for the loads, EV and the Micro-DG. In comparison with the scenario 'A' a visible shifting from the optimized loads in scenario ' $C$ ' due to the optimized scheduling process previously done using the proposed model it can be visualized. Also in this figure can be appreciated in which hours of the day the house is consuming or generating and injecting to the grid, the points where the house is autonomous of the energy consumption from the grid appears at 10 and 18 hours. Also the energy injection coming from the EV on
peak hours reduce the total energy consumption in those hours, where a bigger amount of energy is needed.


FIGURE 47 - Power demand for a typical house for scenario C

FIGURE 48 presents the cost related to the inelastic loads, elastic loads, EV and solar panel. Negative values represent an injection point into the system, positive values represents a direct consumption by the different type of loads.


FIGURE 48 - Energy cost for a typical house used on scenario C

### 4.1.3.1 Voltage Magnitude for Scenario C (pu)

FIGURE 49 present the Voltage magnitude for primary branch in scenario C. In this figure can be noticed that a decreasing of the voltage values in buses 15, 16, 17 and 18, from 0 hours till 6 hour occurs, this happen due to the shifting of the loads to this hours where the energy price is lower. Buses from 1 to 6 present more stability along the day with almost no voltage variations at all. In comparison with the scenario ' $A$ ' for this branch the characteristics of the curve have presented bigger changes in the periods from 00:00 to 6:00 am and also from 18:00 to 22:00 hours


FIGURE 49- Voltage magnitude for primary branch in scenario C

In FIGURE 50, FIGURE 70 and FIGURE 71 are graphed the voltage magnitude variation along the three secondary branches showing its highest and lowest values respectively, and at what period of time are this happening.

For secondary branch ' $A$ ' in scenario ' $C$ ', the voltage magnitude in all its busses is presented in the following figure, where minimum voltage magnitude are found at 20 hours at bus 19, as was seen in scenario ' A ' this bus was working with 1.09 pu voltage magnitude and due to the optimization process has suffered a
decreasing in its magnitude to 1.07 pu , for the rest of the buses can be appreciated a similar behavior, higher voltage magnitude on scenario ' $A$ ' at night hours and lower voltage magnitude at night hours in scenario ' C '.


FIGURE 50 - Voltage magnitude for secondary branch ' A ' in scenario C

### 4.1.3.2 Lambda Values for Scenario C (UM/MWh)

In FIGURE 51, appear the results for the operational cost values, which are showing a considerable displacement from the scenario 'A'. For this scenario a maximum peak of $443 \mathrm{UM} / \mathrm{MWh}$ at bus 18 can be seen, in comparison with scenario ' $A$ ' where was possible to find values in the order of $785 \mathrm{UM} / \mathrm{MWh}$ at nights hours, where a higher energy price take place.

FIGURE 52, presents the resulting operational values for the secondary ' $A$ ' branch in scenario $C$, peak reduction at night hours and a better distribution in the cost along the day in comparison with the scenario ' A ', can be appreciated. Taking as example bus number 2, this bus have presented a decreasing behavior, in the operational costs, since started working at 643,76 UM/MWh in scenario 'A' and for the scenario ' $C$ ' has a working point in 336 UM/MWh at 20:00 hours.


FIGURE 51 - Lambda values for primary branch in scenario C


FIGURE 52 - Lambda values for secondary ' $A$ ' branch in scenario C

### 4.1.3.3 Branches Active Power Losses for Scenario C (kW)

FIGURE 53, FIGURE 54, FIGURE 74 and FIGURE 75 presents the active power losses for the branches of the system.

For the primary branch can be observed that branches that use to not have losses in scenario 'A' are now in scenario 'C' presenting. In the first scenario most of the losses were just active at night, meanwhile in this last scenario, can be observed from the graphics that a displacement in these values have occurred. According to the figure that represents the active power losses for the scenario ' C ', can also be observed that the highest losses values are found in the primary branch and in the secondary ‘C’ branch.

Branches 17, 26 and 31 have the characteristics that are not presenting considerable losses at any time, for this scenario. Maximum losses appear at branch 27 and branch number 9.


FIGURE 53 - Active power losses for primary branch in scenario $C$


FIGURE 54 - Active power losses for secondary ' $A$ ' branch in scenario $C$

### 4.1.3.4 Active Power Generation for Scenario C

The following graph shows the active power generation for scenario $C$.


FIGURE 55 - Active power generation for scenario C

### 4.1.3.5 OPF Objective Function for Scenario C

The following figure shows the objective function variation for scenario C .


FIGURE 56-Objective function for the final OPF performed in scenario ' C '

TABLE 8 - Primary variables analyzed for scenario C

| Hour |  | Voltage Magnitude (pu) |  |  |  | Active Power Losses (MW) |  |  | Lambda P (UM/MWh) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min |  | Max |  | Max |  | Min |  | Max |  |
|  | Objective Function | Value (pu) | Bus | $\begin{gathered} \text { Value } \\ (\mathrm{pu}) \end{gathered}$ | Bus | Value (MW) <br> (MW) | Branches From-To | Value (UM/MWh) | Bus | Value (UM/MWh) | Bus |
| 1 | 1223.78 | 0.988 | 18 | 1.1 | 22 | 0.05 | 9-10 | 266.49 | 22 | 330.37 | 18 |
| 2 | 1829.16 | 0.939 | 18 | 1.1 | 22 | 0.10 | 9-10 | 315.99 | 22 | 443.61 | 18 |
| 3 | 1525.68 | 0.953 | 18 | 1.1 | 22 | 0.08 | 9-10 | 291.6 | 22 | 393.97 | 18 |
| 4 | 1578.73 | 0.938 | 18 | 1.1 | 22 | 0.10 | 9-10 | 294.09 | 22 | 413.16 | 18 |
| 5 | 1038.66 | 0.978 | 18 | 1.1 | 22 | 0.06 | 9-10 | 244.83 | 22 | 309.97 | 18 |
| 6 | 1537.69 | 0.984 | 18 | 1.1 | 22 | 0.09 | 9-10 | 292.51 | 22 | 400.19 | 18 |
| 7 | 435.5 | 1.051 | 33 | 1.1 | 9 | 0.01 | 27-28 | 166.19 | 22 | 180.48 | 33 |
| 8 | 342.11 | 1.048 | 33 | 1.1 | 9 | 0.01 | 27-28 | 146.55 | 9 | 159.93 | 33 |
| 9 | 342.11 | 1.064 | 33 | 1.1 | 9 | 0.01 | 7-8 | 111.36 | 9 | 118.24 | 33 |
| 10 | 144.78 | 1.057 | 33 | 1.1 | 18 | 0.01 | 7-8 | 94.02 | 18 | 101.34 | 33 |
| 11 | 286.8 | 1.057 | 33 | 1.1 | 9 | 0.01 | 27-28 | 134.25 | 9 | 144.21 | 33 |
| 12 | 1258.91 | 1.009 | 18 | 1.1 | 22 | 0.04 | 9-10 | 273.27 | 22 | 324.01 | 18 |
| 13 | 362.19 | 1.057 | 33 | 1.1 | 9 | 0.01 | 27-28 | 151.1 | 9 | 162.55 | 33 |
| 14 | 429.82 | 1.058 | 33 | 1.1 | 22 | 0.01 | 27-28 | 165.06 | 22 | 177.13 | 33 |
| 15 | 431.54 | 1.056 | 33 | 1.1 | 9 | 0.01 | 27-28 | 165.52 | 22 | 178.02 | 33 |
| 16 | 451.79 | 1.055 | 33 | 1.1 | 9 | 0.01 | 27-28 | 168.76 | 9 | 182.01 | 33 |
| 17 | 1035.71 | 1.026 | 33 | 1.1 | 22 | 0.03 | 27-28 | 251.5 | 22 | 287.83 | 33 |
| 18 | 708.29 | 0.982 | 33 | 1.1 | 18 | 0.07 | 27-28 | 198.66 | 18 | 253.00 | 33 |
| 19 | 1583.14 | 0.972 | 33 | 1.1 | 9 | 0.09 | 27-28 | 302.27 | 9 | 386.07 | 33 |
| 20 | 1373.75 | 0.992 | 33 | 1.1 | 9 | 0.06 | 27-28 | 283.04 | 9 | 346.19 | 33 |
| 21 | 1712.78 | 0.956 | 33 | 1.1 | 9 | 0.13 | 27-28 | 316.19 | 9 | 418.08 | 33 |
| 22 | 771 | 1.002 | 33 | 1.1 | 18 | 0.05 | 27-28 | 209.25 | 18 | 254.95 | 33 |
| 23 | 1287.71 | 1.025 | 33 | 1.1 | 22 | 0.03 | 27-28 | 280.29 | 22 | 321.78 | 33 |
| 24 | 842.5 | 1.044 | 33 | 1.1 | 22 | 0.02 | 2-3 | 227.21 | 22 | 251.5 | 33 |

Is observed from the graphics that the objective function for the system reaches its lowest value at 10 hours and its higher value around 2 hours. The bus with the minimum voltage value appears at 4 hours with a 0.938 pu in bus 18 , for that exactly hour a 1.1 pu voltage value appear as max value for this parameter in bus 22. The highest values for the losses were focalized on two branches, the first one that goes from bus 9 to bus 10 and the other one that goes from bus 27 to bus 28, and is in this branch that the peak for the losses of the system is reached at 21 hours, almost 0.13 MW . Basically the lowest lambda values were obtained on the buses 9 , 18 and 22 , almost varying its frequency of showing along the proposed horizon.

### 4.2 FINAL ANALYSIS OF THE RESULTS

In this part of the chapter a compilation of the output data from the algorithm is effectuated, principally comparing the results of the scenarios among them.

TABLE 9 presents the convergence values for the two iterative processes that are effectuated for the algorithm proposed. Four different convergence targets were evaluated for $\varepsilon 1$ and $\varepsilon 2$. For each of these were tested values in the order of: 0.1, $0.01,0.001$ and 0.001 . For this dissertation was chosen the following configuration $\varepsilon 1=0,01$ and $\varepsilon 2=0,01$. The number of iterations in order to reach the convergence limit can be seen in the third column with its respective time spent in the operations.

TABLE 9 - Iterative convergence values for the optimization processes

| Value <br> of $\varepsilon_{1}$ | Value <br> of $\varepsilon_{2}$ | Iterations | $\boldsymbol{T}$ <br> (sec.) |
| :---: | :---: | :---: | :---: |
| 0.1 | 0.1 | 3 | 45 |
| 0.01 | 0.01 | 6 | 69 |
| 0.001 | 0.001 | 19 | 89 |
| 0.0001 | 0.0001 | 37 | 227 |

In
TABLE 10 a comparison of the power consumed in every scenario and energy reductions are presented, negative values means that an energy injection is taking place. For the three different scenarios similar configurations are set for the amount of inelastic loads and elastic loads, 2.35 MW and 2.30 MW respectively. In terms of amount of power consumed or saved, the scenarios ' $A$ ' \& ' $B$ ' presents the same characteristics, this is because no external sources of energy are considered for this scenarios. For the scenario ' C ', the EV is considered as an energy source that is available in hours where the energy price is higher, also is considered the energy coming from the solar panel, which is normally available in morning hours. Can be seen on this table that a relief for the power system, due to the combination of the EV injection and the solar panel generation is taking place with a reduction of 1.30 MW daily for all the microgrid.

TABLE 11, the equivalent operational costs are presented using the white tariff as base for the three scenarios. This table resume the monetary behavior for the system and includes the energy purchase cost for the inelastic loads, consumption costs for the elastic loads with the inclusion or not of the EV, depending of the scenario and barely used as a rechargeable battery. Also are analyzed the cost that are involved with the energy generation coming from the EV and the solar panel that was used for the scenario ' $C$ '. Due to the target of this dissertation, can be appreciated that in this case the value assigned to the inelastic loads remains constant for the three scenarios. It is worth noting that for scenarios ' B ' \& ' C ' the operational costs for the elastic interruptible loads are the same, this happens because of the total loads taken in account in the optimization process, for both cases, is the same. For scenario ' $C$ ' are presented the costs, that in this scenario, that value represents a direct income to the user, who is able to inject energy to the grid or even to consume it at will. For the scenario 'B' a reduction of UM 353.86 is the result for a simple microgrid after have applied the algorithm proposed, almost a $40 \%$ comparing it with the scenario ' $A$ '. For the scenario ' $C$ ' a reduction of UM 649.17 can be seen, that in comparison with the scenario 'A' represents a reduction of $73 \%$, showing the usefulness of the method applied.

Finally, TABLE 12 compares the objective function values and the active power losses and how its behavior is along the day for the three scenarios. Also presents the percentage reduction for both cases. An initial objective function with a value of $38,759.99 \mathrm{UM}$, which is the result for the optimization process in the scenario 'A'. Scenarios 'B' \& 'C', with the algorithm proposed presents improvements in their objective functions, $24 \%$ and $41 \%$ respectively. The initial losses for the system were calculated in scenario 'A' giving as result a total of $5,991.45 \mathrm{~kW} /$ day, which includes the losses due to the electrical equipment's and the EV as a rechargeable battery with typical behaviors. The algorithm proposed applied to the scenario 'B' and scenario ' $C$ ' gave as results for the electric power losses a reduction $14 \%$ and $18 \%$ respectively, in comparison with scenario ' $A$ '.

As can be seen, TABLE 10, TABLE 11 and TABLE 12 present an executive brief of the main results of this dissertation.

TABLE 10 - Daily energy saving for specific microgrid

| Sce | Inelastic <br> Loads <br> (MWh/day) | Elastic Loads <br> + EV Charging <br> (MWh/day) | EV <br> Injection <br> (MWh/day) | Solar Panel <br> Generation <br> (MWh/day) | Total <br> Energy <br> (MWh/day) | Energy <br> Reduction <br> (MW/day) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 2.35 | 2.30 | - | - | 4.65 | - |
| B | 2.35 | 2.30 | - | - | 4.65 | - |
| C | 2.35 | 2.30 | -0.94 | -0.36 | 3.36 | 1.30 |

TABLE 11 - Energy comparative costs (UM/day) for specific microgrid

| Sce | Inelastic <br> Costs <br> (UM/day) | Elastic Costs <br> + EV Charge <br> Costs <br> (UM/day) | EV Injection Costs (UM/day) | Solar Panel <br> Generation <br> Cost <br> (UM/day) | Total <br> Costs <br> (UM/day) | Reduction <br> (UM/day) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 379.81 | 511.81 | - | - | 891.62 | - |
| B | 379.81 | 157.95 | - | - | 537.76 | 353.86 |
| C | 379.81 | 157.95 | -269.35 | -25.96 | 242.45 | 649.17 |

TABLE 12 - Comparison of objective functions \& total active power losses

| Sce | Total Objective <br> Function (UM/day) | Objective <br> Function <br> Reduction (\%) | Total Active <br> Power Losses <br> $(\mathbf{k W} /$ day $)$ | Active Power <br> Losses <br> Reduction (\%) |
| :---: | :---: | :---: | :---: | :---: |
|  | $38,759.99$ | - | $5,991.45$ | - |
| B | $29,233.76$ | 24.58 | $5,101.91$ | 14.85 |
| C | $22,534.13$ | 41.86 | $4,889.99$ | 18.38 |

As can be seen in FIGURE 57 is shown the three objective functions plotted over the same axes, in order to compare its behavior through the three scenarios. For scenario "A" a peak at night can be observed with a value close to the 8000 $\mathrm{UM} / \mathrm{hr}$, for scenario "B" a maximum value of $3000 \mathrm{UM} / \mathrm{hr}$ and for scenario "C" a maximum value of $2000 \mathrm{UM} / \mathrm{hr}$ is


FIGURE 57 Objective Functions for Scenarios A, B and C. CONCLUSION AND FUTURE WORK

A summary of this dissertation and the main findings are presented in this Chapter. An analytical evaluation of the method used is also given along with proposals for future investigations.

### 5.1 CONCLUDING REMARKS

In this dissertation a computational method for an optimal scheduling for demand response and electric vehicle in a typical house under a microgrid environment and its impact on the electric power distribution networks along a 24 hours-daily horizon has been presented. The proposed method is a system of systems (SoS) based framework for optimally operating an active distribution grid. The proposed SoS framework defines both distribution company (DISCO) and microgrids (MGs) as autonomous systems, and recognizes the exchange information process among them. A hierarchical optimization algorithm is presented to coordinate the independent systems in order find the optimal operating point of the SoS-based active distribution grid.

The proposed method allows an optimized scheduling of demand response to end-user in the microgrid level, with the development of the computational model for residential controllable (Elastic interruptible) and uncontrollable (Inelastic) loads, also were considered their impacts on the distribution network simultaneously. Also examine the additional benefits of demand response programming, distributed generation, integration of electric vehicles modeling in the microgrid. A case of study containing three scenarios, have revealed the usefulness and effectiveness of the proposed model.

This dissertation also analyses current researches on System of Systems, smart grid, load modeling, electric vehicle and demand response. A demand response scheme is proposed to manage residence and microgrid jointly. Residential loads, as well as EVs and solar panels, are modeled, into the different scenarios. Scenarios of study are executed at diverse load scheduling levels. The focus is to set the best possible optimized values that will enter into the distribution network. Also to study the
daily demand variation curve due the optimal shifting behavior of the controllable loads and validate the optimization process based on its objective function.

In specific, this dissertation examined the interaction of the microgrid, its elements and the distribution network, also examined the scheduling operations problems inside the microgrid and how this affects into the radial distribution network systems.

A case of study containing three scenarios reveals the usefulness and effectiveness of the proposed model. The reduction of the active power losses and the reduction of the objective function value are some of the main contributions of the algorithm proposed, besides of the savings that were observed in every simulation.

### 5.2 FUTURE WORKS

Future investigations and research work will be underlined on numerous features of the recommended method and its respective applications, due to the amplitude of the study field.

An economic dispatch could be run for future scenarios with and without the Electric Vehicle demand and varying the types of renewable energy, the outputs of this study could be used to analyze the total system costs with and without EV.

The quantification of benefits and costs related to EV coordination for ancillary services and local grid support are not addressed. Such analysis can bring to light which of the different concepts applied on the SoS, are the most immediately applicable, considering the needs of the different places where the algorithm could be applied.

Although the use of smart appliances was not considered in the proposed dissertation, it could be included with such price-based DR methods. Mixing additional elements in the grid is another aspect that could provide interesting results. First, integrating distributed energy storage at the distribution level would provide additional elasticity, and could contribute to temporarily reduce the net load during demand peaks, by serving as a buffer and a complement or competitor to DR. Second, the integration of larger shares of DG resources could enable microgrid islanding. Although distribution PV resources are currently considered, larger DG sources could be added, and their impact on the operation of the system evaluated.

## REFERENCES

[1] A. Singhal and R. P. Saxena, "Software models for Smart Grid," 2012 First Int. Work. Softw. Eng. Challenges Smart Grid, vol. 3, no. 2, pp. 42-45, Jun. 2012.
[2] H. Zhong, Q. Xia, Y. Xia, C. Kang, L. Xie, W. He, and H. Zhang, "Integrated dispatch of generation and load: A pathway towards smart grids," Electr. Power Syst. Res., vol. 3, no. 4, p. 8, Apr. 2014.
[3] D. Wang, S. Ge, H. Jia, C. Wang, S. Member, Y. Zhou, N. Lu, and X. Kong, "A Demand Response and Battery Storage Coordination Algorithm for Providing Microgrid Tie-Line Smoothing Services," IEEE Trans. Sustain. ENERGY, pp. 1-11, 2014.
[4] Z. Fan, "A Distributed Demand Response Algorithm and Its Application to PHEV Charging in Smart Grids," IEEE Trans. Smart Grid, vol. 3, no. 3, pp. 1280-1290, Sep. 2012.
[5] C. Wen, J. Chen, J. Teng, and S. Member, "Decentralized Plug-in Electric Vehicle Charging Selection Algorithm in Power Systems," IEEE Intell. Syst., vol. 3, no. 4, pp. 1779-1789, 2012.
[6] S. Han, S. Member, S. Han, and K. Sezaki, "Development of an Optimal Vehicle-to-Grid Aggregator for Frequency Regulation," IEEE Trans. Smart Grid, vol. 1, no. 1, pp. 65-72, 2010.
[7] U. K. Madawala, S. Member, and D. J. Thrimawithana, "A Bidirectional Inductive Power Interface for Electric Vehicles in V2G Systems," IEEE Trans. Ind. Electron., vol. 58, no. 10, pp. 4789-4796, 2011.
[8] W. Gu, H. Yu, W. Liu, J. Zhu, and X. Xu, "Demand Response and Economic Dispatch of Power Systems Considering Large-Scale Plug-in Hybrid Electric Vehicles/Electric Vehicles (PHEVs/EVs): A Review," IEEE Power Energy, vol. 6, no. 9, pp. 4394-4417, Aug. 2013.
[9] S. M. Amin and B. F. Wollenberg, "Toward a smart grid," Power Energy Mag. IEEE, vol. 3, no. 5, pp. 34-41, 2005.
[10] Electric Power Research Institute, "Smart Grid Resource Center," 2011. [Online]. Available: http://smartgrid.epri.com/Index.aspx. [Accessed: 23-Nov2014].
[11] M. G. Simões, S. Member, R. Roche, S. Member, E. Kyriakides, S. Suryanarayanan, B. Blunier, K. D. Mcbee, P. H. Nguyen, P. F. Ribeiro, and A. Miraoui, "A Comparison of Smart Grid Technologies and Progresses in Europe and the U . S .," IEEE Trans. Ind. Appl., vol. 48, no. 4, pp. 1154-1162, 2012.
[12] "From Hierarchical to Open Access Electric Power Systems," Proc. IEEE, vol. 95, no. 5, pp. 1060-1084, 2007.
[13] H Farhangi, "The path of the smart grid," Power Energy Mag. IEEE, vol. 8, no. 1, pp. 1-8, 2010.
[14] U.S. Department of Energy, "Assessment of Demand Response and Advanced Metering," Washington, 2013.
[15] "110th US Congress, 2007 Energy Independence and Security Act (EISA07), Dec. 2007." 2007.
[16] F. Li, S. Member, W. Qiao, and H. Sun, "Smart Transmission Grid: Vision and Framework," IEEE Trans. Smart Grid, vol. 1, no. 2, pp. 168-177, 2010.
[17] S. Suryanarayanan, J. Mitra, and S. Member, "Enabling Technologies for the Customer-driven Microgrid," IEEE Power Energy, vol. 3, no. 4, pp. 9-11, 2009.
[18] X. Yu, C. Cecati, T. Dillon, and M. G. Simo, "An Industrial Electronics Perspective," IEEE Ind. Electron. Mag., vol. 2, no. 1, pp. 49-63, 2011.
[19] 109th U.S. Congress, 2005 Energy Policy Act (EPAct05). 2005.
[20] F. A. Farret and M. G. Simões, Integration of Alternative Sources of Energy, Wiley. Hoboken, NJ, USA: John Wiley \& Sons, Inc., 2006.
[21] Y. Ozturk, P. Jha, S. Kumar, and G. Lee, "A personalized home energy management system for residential demand response," 4th Int. Conf. Power Eng. Energy Electr. Drives, vol. 5, pp. 1241-1246, May 2013.
[22] G. D. Rodriguez, A. F. Renewable, and I. Generation, "A Utility Perspective of the Role of Energy Storage in the Smart Grid," IEEE Trans. Ind. Appl., vol. 48, no. 4, pp. 4-5, 2010.
[23] "Electricity Storage Association," 2014. [Online]. Available: http://energystorage.org/energy-storage.
[24] R. Carnieletto, D. Iglesias Brandao, F. A. Farret, and M. Simoes, "Smart Grid initiatives," IEEE Ind. Appl., vol. 17, no. 5, pp. 27-35, 2011.
[25] "PEBB—Power electronics building blocks, from concept to reality," Conf. Power Electron, vol. 24, no. 1, pp. 12-16, 2006.
[26] "For the good of the grid," IEEE Power Energy Mag., vol. 6, no. 6, pp. 48-59, 2008.
[27] "Federal Energy Regulatory Comission. Assessment of Demand Response and Advanced Metering. Technical report.," 2010.
[28] A. Leon-garcia, "Price Prediction in Real-Time Electricity," IEEE Trans. Smart Grid, vol. 1, no. 2, pp. 120-133, 2010.
[29] "US Department of Energy Benefits of demand response in electricity markets and recommendations for achieving them - A report to the United States Congress pursuant to Section 1252 of the Energy Policy," 2006.
[30] "The Power to Choose - Demand Response in Liberalized Electricity Markets Mitsubishi i-MiEV," 2013.
[31] M. H. Albadi and E. F. El-Saadany, "A summary of demand response in electricity markets," Electric Power Systems Research, vol. 78, no. 11. pp. 1989-1996, Nov-2008.
[32] "Ergon Energy/Energex. Residential time of use tariff.," 2014. [Online]. Available: https://www.yourpowerqld.com.au/energy-pricing-and-economy-rates/time-of-use-tariff.
[33] C. Status, L. Prospects, K. Challenges, and T. Markel, "Plug-in Hybrid Electric Vehicles Current Status, Long-Term Prospects and Key Challenges," 2006.
[34] "Burbank water and Power," 2014. [Online]. Available: http://www.burbankwaterandpower.com/electric-vehicles\#3.
[35] J. Axsen and A. Burke, "Batteries for Plug-in Hybrid Electric Vehicles (PHEVs ): Goals and the State of Technology circa 2008," 2008.
[36] N. S. Pearre, W. Kempton, R. L. Guensler, and V. V. Elango, "Electric vehicles: How much range is required for a day's driving?," Transp. Res., vol. 19, no. 6, pp. 1171-1184, Dec. 2011.
[37] R. T. Doucette and M. D. McCulloch, "Modeling the prospects of plug-in hybrid electric vehicles to reduce CO2 emissions," Appl. Energy, vol. 88, no. 7, pp. 2315-2323, Jul. 2011.
[38] H. Lund and W. Kempton, "Integration of renewable energy into the transport and electricity sectors through V2G," Energy Policy, vol. 36, no. 9, pp. 35783587, Sep. 2008.
[39] B. K. Sovacool and R. F. Hirsh, "Beyond batteries: An examination of the benefits and barriers to plug-in hybrid electric vehicles (PHEVs) and a vehicle-to-grid (V2G) transition," Energy Policy, vol. 37, no. 3, pp. 1095-1103, Mar. 2009.
[40] D. P. Birnie, "Solar-to-vehicle (S2V) systems for powering commuters of the future," J. Power Sources, vol. 186, no. 2, pp. 539-542, Jan. 2009.
[41] "International Energy Agency Report. Technology roadmap: Electric and plug in hybrid electric vehicles," 2009. [Online]. Available:
http://www.iea.org/publications/freepublications/publication/technology-roadmap-electric-and-plug-in-hybrid-electric-vehicles-evphev.html.
[42] E. Loveday, "Chevy Volt Sales Trump Nissan LEAF in November 2011," 2011. [Online]. Available: http://www.plugincars.com/chevy-volt-sales-nissan-leaf-november-2011-110656.html.
[43] P. Research, "Electric Vehicle Geographic Forecast," 2011. [Online]. Available: http://www.navigantresearch.com/research/electric-vehicle-geographicforecasts.
[44] P.I.A., "Plug-in America Vehicle Tracker," 2015. [Online]. Available: http://www.pluginamerica.org/vehicles.
[45] J. Voelcker, "Green Car Concept.," 2014. [Online]. Available: http://www.greencarreports.com/news.
[46] S. Edelstein, "Digital Trends," 2015. [Online]. Available: http://www.digitaltrends.com/cars/.
[47] P. Research, "Electric Vehicle Geographic Forecasts," 2015. [Online]. Available: http://www.navigantresearch.com/research/electric-vehicle-geographic-forecasts.
[48] HibridCARS, "Global Plug-in Car Sales Now Over 600,000," 2014. [Online]. Available: http://www.hybridcars.com/global-plug-in-car-sales-now-over600000/.
[49] F. C. C. CHAN, "An overview of electric vehicle technology.pdf," IEEE Ind. Electron. Mag., vol. 81, no. 9, pp. 1202 - 1213, 1993.
[50] S. A. E. Ground, V. Standards, and U. Grid, "SAE International, SAE Ground Vehicle Standards," 2010.
[51] S. Guberman, "Reflections on Ludwig Bertalanffy's ‘General System Theory: Foundations, Development, Applications,'" Gestalt Theory, vol. 26, pp. 44-57, 2013.
[52] A. Hessami, "A framework for characterising complex systems and system of systems," Proc. - 2013 IEEE Int. Conf. Syst. Man, Cybern. SMC 2013, pp. 1702-1708, 2013.
[53] N. Karcanias and A. G. Hessami, "System of systems and emergence Part 1: Principles and framework," Int. Conf. Emerg. Trends Eng. Technol. ICETET, pp. 27-32, 2011.
[54] N. Karcanias and A. G. Hessami, "System of systems and emergence Part 2: Synergetic effects and emergence," Int. Conf. Emerg. Trends Eng. Technol. ICETET, pp. 33-38, 2011.
[55] C. H. Azani, "System of systems architecting via natural development principles," 2008 IEEE Int. Conf. Syst. Syst. Eng. SoSE 2008, 2008.
[56] D. Popper, S., Bankes, S., Callaway, R., and DeLaurentis, "System-of-Systems Symposium- Report on a Summer Conversation," Arlington, VA., 2004.
[57] J. Holt, S. Perry, M. Brownsword, and G. L. Ita, "Model-based requirements engineering for system of systems Engineering College of Aarhus," Proc. 2012 7th Int. Conf. Syst. Syst. Eng., pp. 561-566, 2012.
[58] J. J. Simpson and M. J. Simpson, "System of systems complexity identification and control," 2009 IEEE Int. Conf. Syst. Syst. Eng., 2009.
[59] E. Curry, "System of systems information interoperability using a linked dataspace," Proc. - 2012 7th Int. Conf. Syst. Syst. Eng. SoSE 2012, vol. 1, no. 3, pp. 101-106, 2012.
[60] D. B. Agusdinata, "Specification of System of Systems for Policymaking in The Energy Sector," 2006 IEEE/SMC Int. Conf. Syst. Syst. Eng., vol. 8, pp. 1-24, 2006.
[61] A. P. Sage and C. D. Cuppan, "On the Systems Engineering and Management of Systems of Systems and Federations of Systems," Inf. Knowl. Syst. Manag., vol. 2, pp. 325-345, 2001.
[62] T. Ackermann, G. Andersson, and L. Söder, "Distributed generation: a definition," Electr. Power Syst. Res., vol. 57, no. 3, pp. 195-204, Apr. 2001.
[63] L. Kaiyu, "Economic Operation and Planning of Distribution System Sources," 2010.
[64] A. J. Conejo, E. Castillo, R. Minguez, and R. Garcia-Bertrand, Decomposition Techniques in Mathematical Programming, First. New York, NY: Springer, 2008, p. 542.
[65] R. Brown, Electric Power Distribution Reliability, 2nd ed., vol. 20020625. CRC Press, 2009.
[66] H. Ghoreishi, H. Afrakhte, and M. Jabbari ghadi, "Optimal placement of tie points and sectionalizers in radial distribution network in presence of DGs considering load significance," 2013 Smart Grid Conf., vol. 14, pp. 85-96, Dec. 2013.
[67] P. Systems, E. Drives, and G. Kaur, "A new method for load-flow solution of radial distrution networks," THAPAR UNIVERSITY, 2009.
[68] S. Sivanagaraju, N. Visali, V. Sankar, and T. Ramana, "Enhancing Voltage Stability of Radial Distribution Systems by Network Reconfiguration," Electr. Power Components Syst., vol. 33, no. 5, pp. 539-550, Mar. 2005.
[69] M. E. Baran and F. F. Wu, "Network Reconfiguration in Distribution Systems for Loss Reduction and Load Balancing," IEEE Power Eng. Rev., vol. 9, no. 4, pp. 101-102, 1989.
[70] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, "Matpower: Steady- State Operations, Planning and Analysis Tools for Power Systems Research and Education," Power Syst. IEEE Trans., vol. 26, no. 1, pp. 12-19, Jul. 2009.
[71] A. Kargarian, S. Member, Y. Fu, and S. Member, "System of Systems Based Security-Constrained Unit Commitment Incorporating Active Distribution Grids," IEEE Trans. POWER Syst., vol. 29, no. 5, pp. 2489-2498, 2014.
[72] A. K. Marvasti, S. Member, Y. Fu, S. Member, and S. Dormohammadi, "Optimal Operation of Active Distribution Grids : A System of Systems Framework," IEEE Trans. POWER Syst., vol. 5, no. 3, pp. 1228-1237, 2014.
[73] A. A. Francisquini and A. P. Feltrin, "ESTIMAÇÃO DE CURVAS DE CARGA EM PONTOS DE CONSUMO E EM TRANSFORMADORES DE DISTRIBUIÇÃO," UNIVERSIDADE ESTADUAL PAULISTA "JÚLIO DE MESQUITA FILHO," 2006.
[74] "U.S. Environmental Protection Agency and U.S. Department of Energy," 2014. [Online]. Available: http://www.fueleconomy.gov/feg/evsbs.shtml.
[75] "U.S. Environmental Protection Agency and U.S. Department of Energy Nissan Leaf," 2014. [Online]. Available:
http://www.fueleconomy.gov/feg/Find.do?action=sbs\&id=32154.
[76] "Green Car Congress Renault ZOE," 2014. [Online]. Available:
http://www.greencarcongress.com/2012/03/zoe-20120309.html.
[77] "U.S. Environmental Protection Agency and U.S. Department of Energy Honda Fit EV." 2014.
[78] "U.S. Environmental Protection Agency and U.S. Department of Energy Tesla Model S," 2014. [Online]. Available:
http://www.fueleconomy.gov/feg/Find.do?action=sbs\&id=33367\&id=33368.
[79] "Inside EV Chevy Spark EV," 2014. [Online]. Available:
http://insideevs.com/2014-chevy-spark-ev-gets-epa-range-rating-of-82-miles-119-mpge-combined/.
[80] "VWVortex Volkswagen e-Up," 2014. [Online]. Available: http://www.vwvortex.com/artman/publish/article_2661.shtmI?COLLCC=224882 6346\&.
[81] "U.S. Environmental Protection Agency and U.S. Department of Energy Ford Focus Electric," 2014. [Online]. Available:
http://www.fueleconomy.gov/feg/Find.do?action=sbs\&id=32278.
[82] "U.S. Environmental Protection Agency and U.S. Department of Energy BMW i3," 2014. [Online]. Available:
http://www.fueleconomy.gov/feg/Find.do?action=sbs\&id=35207.
[83] "autobloggreen Toyota iQ," 2014. [Online]. Available:
http://www.greencarcongress.com/2012/03/zoe-20120309.html.
[84] "U.S. Environmental Protection Agency and U.S. Department of Energy Fiat 500e," 2014. [Online]. Available:
http://www.fueleconomy.gov/feg/Find.do?action=sbs\&id=33396.
[85] "DAIMLER- DAIMLER Smart EV," 2014. [Online]. Available:
http://www.daimler.com/technology-and-innovation/drive-technologies/battery-electric-drives.
[86] "Toyota Prius Plug-in Hybrid," 2013. [Online]. Available:
http://www.toyota.es/coches/prius-plugin/index.json.
[87] "Mitsubishi Outlander PHEV," 2014. [Online]. Available: http://www.mitsubishi-motors.es/outlander-phev/\#!

## APPENDIX A - EV PHEV \& models on the actual market

| Automotive Brand | Model | Production | Battery <br> Size (kW) | Battery Type | Charging Capacity (kWh) | Certified <br> Battery Range (Km) | Charging <br> Time (h) | Charging Type | Ref |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mitsubishi | i-MiEV | 2009-present | 16 | Lithium-ion | 3.6 | 160 | 7 | SAE J1772-2009 inlet, adapters for domestic AC sockets (110-240 V) | [74] |
| Nissan | Leaf | 2013 | 24 | Lithium-ion | 3.3 | 200 | 8 | 240 V AC on SAE J1772-2009 inlet, adapters for domestic AC sockets (110-240 V) | [75] |
| Renault | Fluence Zoe | 2013-present | 22 | Lithium-ion | 3.5 | 185 | 7 | On-board charger (230 V 15 <br> A), optional upgrade to Zoe's Chameleon charger ( 43 kW ) | [76] |
| Honda | Fit EV | 2013 | 20 | Lithium-ion | 7 | 132 | 3 | 240-volt outlet | [77] |
| Tesla | Model S | 2013-present | 85 | Lithium-ion | 11 | 426 | 4.6 | 85-265 V onboard charger for $1 \varphi 40$ A or $3 \varphi 16$ A [5] on IEC Type 2 inlet[6] | [78] |
| Chevrolet | Spark EV | 2013 | 21,3 | Nano phosphate lithium-ion | 3 | 132 | 7 | 240-volt charging station | [79] |
| Volkswagen | e-Up! | 2010-present | 18,7 | Lithium-ion | 3.3 | 160 | 5 | Regular 230 volt plug | [80] |
| Ford | Focus Electric | 2011-present | 23 | Lithium-ion | 6.6 | 122 | 4 | Onboard charger on SAE-J1772-2009 inlet | [81] |


| Automotive Brand | Model | Production | Battery <br> Size (kW) | Battery Type | Charging <br> Capacity (kWh) | Certified <br> Battery Range (Km) | Charging <br> Time (h) | Charging Type | Ref |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BMW | i3 | 2014 | 18.8 | Lithium-ion | 7.4 | 240 | 3 | On-board charger on IEC Combo AC, optional Combo DC | [82] |
| Toyota | iQ | 2013 | 12 | Lithium-ion | 4 | 85 | 3 | On-board charger on IEC Combo AC, optional Combo DC | [83] |
| Fiat | 500e | 2013/2014 | 24 | Lithium iron phosphate | 6 | 120 | 4 | Level 2 (240 volt) on-board charging module | [84] |
| Daimler AG | Smart ED | 2009-present | 16.5 | Lithium-ion | 3.3 | 135 | 4 | On-board charger | [85] |
| Toyota | Prius | 2011-present | 4.4 | Lithium-ion | 1.3 | 870 | 3 | On-board charger | [86] |
| Mitsubishi | Outlander | 2012-present | 12 | Lithium-ion | 3.5 | 990 | 3 | On-board charger | [87] |

## APPENDIX B - Modified case for the 33 buses radial distribution network.

```
function mpc = T_case33
% CASE33 Power flow data for 33 bus modified, 4 generator case.
% Please see CASEFORMAT for details on the case file format.
%% MATPOWER Case Format : Version 2
mpc.version = '2';
%%----- Power Flow Data
%% system MVA base
mpc.baseMVA = 100;
Vbase = 12.66*(10^3)
Sbase = 100*(10^6);
Zbase = (Vbase^2)/Sbase;
%% bus data
% BUS_TYPE (1 = PQ. Load Bus
    2 = PV. Generation Bus
    3 = ref. Swing Bus
    4 = isolated
bus i type Pd Qd Gs Bs area Vm
\begin{tabular}{llllllll}
1 & 3 & 0.0000 & 0.0000 & 0 & 0 & 1 & 1.00 \\
2 & 1 & 0.1000 & 0.0600 & 0 & 0 & 1 & 1.00 \\
3 & 1 & 0.0900 & 0.0400 & 0 & 0 & 1 & 1.00 \\
4 & 1 & 0.1200 & 0.0800 & 0 & 0 & 1 & 1.00 \\
5 & 1 & 0.0600 & 0.0300 & 0 & 0 & 1 & 1.00 \\
6 & 1 & 0.0600 & 0.0200 & 0 & 0 & 1 & 1.00 \\
7 & 1 & 0.2000 & 0.1000 & 0 & 0 & 1 & 1.00 \\
8 & 1 & 0.2000 & 0.1000 & 0 & 0 & 1 & 1.00 \\
9 & 2 & 0.0600 & 0.0200 & 0 & 0 & 1 & 1.00 \\
10 & 1 & 0.0600 & 0.0200 & 0 & 0 & 1 & 1.00 \\
11 & 1 & 0.0450 & 0.0300 & 0 & 0 & 1 & 1.00 \\
12 & 1 & 0.0600 & 0.0350 & 0 & 0 & 1 & 1.00 \\
13 & 1 & 0.0600 & 0.0350 & 0 & 0 & 1 & 1.00 \\
14 & 1 & 0.1200 & 0.0800 & 0 & 0 & 1 & 1.00 \\
15 & 1 & 0.0600 & 0.0100 & 0 & 0 & 1 & 1.00 \\
16 & 1 & 0.0600 & 0.0200 & 0 & 0 & 1 & 1.00
\end{tabular}
\begin{tabular}{llll}
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\) \\
12.66 & 1 & 1.10 & \(0.90 ;\)
\end{tabular}
```

| 17 | 1 | 0.0600 | 0.0200 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | 1 | 0.0900 | 0.0400 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 19 | 1 | 0.0900 | 0.0400 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 20 | 1 | 0.0900 | 0.0400 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 21 | 1 | 0.0900 | 0.0400 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 22 | 2 | 0.0900 | 0.0400 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 23 | 2 | 0.0900 | 0.0500 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 24 | 1 | 0.4200 | 0.2000 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 25 | 1 | 0.4200 | 0.2000 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 26 | 1 | 0.0600 | 0.0250 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 27 | 2 | 0.0600 | 0.0250 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 28 | 1 | 0.0600 | 0.0200 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 29 | 1 | 0.1200 | 0.0700 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 30 | 1 | 0.2000 | 0.6000 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 31 | 1 | 0.1500 | 0.0700 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 32 | 1 | 0.2100 | 0.1000 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |
| 33 | 1 | 0.0600 | 0.0400 | 0 | 0 | 1 | 1.00 | 0 | 12.66 | 1 | 1.10 | 0.90; |

];
\%\% generator data
\% bus Pg Qg Qma
ramp_10 ramp_30 ramp_q ap mpc.gen $=$

| 1 | 7 | 0 | 7 | -7 | 1 | 100 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 22 | 2 | 0 | 2 | -2 | 1 | 100 |
| 27 | 3 | 0 | 3 | -3 | 1 | 100 |
| 23 | 2 | 0 | 3 | -3 | 1 | 100 |
| 9 | 7 | 0 | 7 | -7 | 1 | 100 |


| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 ;$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 ;$ |  |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 ;$ |  |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 ;$ | $\%$ |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 ;$ |  |

\%\% branch data
\%fbus tbus
mpc.branch = [

| 1 | 2 | $0.0922 /$ Zbase |
| :--- | :--- | :--- |
| 2 | 3 | $0.4930 /$ Zbase |
| 3 | 4 | $0.3660 /$ Zbase |
| 4 | 5 | $0.3811 /$ Zbase |
| 5 | 6 | $0.8190 /$ Zbase |
| 6 | 7 | $0.1872 /$ Zbase |
| 7 | 8 | $1.7114 /$ Zbase |
| 8 | 9 | $1.0300 /$ Zbase |
| 9 | 10 | $1.0400 /$ Zbase |

$0.0477 / Z b a s e$
$0.2511 / Z b a s e$
$0.1840 / Z b a s e$
$0.1941 / Z b a s e$
$0.0700 / Z b a s e$
$0.6188 / Z b a s e$
$1.2351 / Z b a s e$
$0.7400 / Z b a s e$
b rateA rateB rateC ratio angle status angmin angmax


APPENDIX C - Matrix structure for MG resolution applying MILP


## APPENDIX D - Mixed integer linear programming formulation.

Mixed-integer linear programming (MILP) problems can be solved in a centralized fashion using the powerful solvers nowadays available. Branch and cut techniques that have been developed during the last decade of the twen- tieth century allow us, using personal computers, to solve problems at least two orders of magnitude larger than those problems solvable before the devel- opment of such branch and cut techniques [42]. Alternatively, MILP problems can be decomposed to separate integer and continuous variables, which is equivalent to considering the integer variables as complicating variables. The resulting continuous subproblem may be decomposed by blocks. In this case, such decomposable structure can be usually exploited computationally to develop efficient algorithms. This situation often arises in practice, particularly, in long-term multiperiod investment planning problems. Investment decisions are integer while operation decisions are continuous and often separable by a time period.

The case of complicating constraints in MILP problems is not so common in practice. A decomposition technique similar to the Dantzig-Wolfe decom- position for such type of problems is denominated "Branch and Price." This rather specific decomposition technique is computationally involved and is not addressed in this book.

A general MILP problem has the form

Objective function:

$$
\begin{equation*}
\min _{x_{1}, \ldots, x_{n} ; y_{1}, \ldots, y_{m}} \sum_{i=1}^{n} c_{1} x_{1}+\sum_{j=1}^{m} d_{j} y_{j} \tag{Eq 22}
\end{equation*}
$$

Subject to

$$
\begin{align*}
& \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{a} \ell_{1} \mathrm{x}_{1}+\sum_{\mathrm{j}=1}^{\mathrm{m}} \mathrm{e} \ell_{\mathrm{j}} \mathrm{y}_{\mathrm{j}}=\mathrm{b}_{\ell} ; \forall \ell=1, \ldots, \mathrm{q}  \tag{Eq 23}\\
& \mathrm{x}_{\mathrm{i}}^{\text {down }} \leq \mathrm{x}_{\mathrm{i}} \leq \mathrm{x}_{\mathrm{i}}^{\mathrm{up}}, \mathrm{x}_{\mathrm{i}} \in \mathbb{N} ; \mathrm{i}=\{1, \ldots, \mathrm{n}\}  \tag{Eq 24}\\
& \mathrm{y}_{\mathrm{i}}^{\text {down }} \leq \mathrm{y}_{\mathrm{i}} \leq \mathrm{y}_{\mathrm{i}}^{\text {up }}, \mathrm{y}_{\mathrm{i}} \in \mathbb{R} ; \mathrm{j}=\{1, \ldots, \mathrm{~m}\} \tag{Eq 25}
\end{align*}
$$

Note that upper and lower bounds have been imposed on optimization variables. This reflects what happens in most engineering and science problems and simplifies the mathematical treatment required. The most common integer variables in real world applications are binary variables. Note that any integer variable can be substituted by a set of binary variables, as shown below.

$$
\begin{equation*}
x=\left\{a_{1} a_{2}, \ldots, a_{n}\right\} \tag{Eq 26}
\end{equation*}
$$

Can be substituted by n binary variables as follows:

$$
\begin{array}{cc}
x=\sum_{i=1}^{n} a_{i} u_{i} & \text { Eq } 27  \tag{Eq 27}\\
\sum_{i=1}^{n} u_{1}=1 & \text { Eq } 28 \\
u_{i} \in\{0,1\} ; i=\{1, \ldots, n\} &
\end{array}
$$

| Electrical equipment | Type of Load | Consumption kWh/h | Consumption Wh/h |
| :---: | :---: | :---: | :---: |
| Air conditioner (compact or split) $2300 \mathrm{fg} / \mathrm{h}$ | Inelastic | 0.99 | 990 |
| Vacuum (large) | Elastic | 0.35 | 350 |
| Blender (arm) | Inelastic | 0.2 | 200 |
| Heater (large) | Inelastic | 2.5 | 2500 |
| Water Heater Tank | Inelastic | 1.5 | 1500 |
| Heater (medium) | Inelastic | 2 | 2000 |
| Computer | Inelastic | 0.36 | 360 |
| DVD | Inelastic | 0.025 | 25 |
| Audio Equipment (small) | Inelastic | 0.018 | 18 |
| Stereo | Inelastic | 0.075 | 75 |
| Exhaust Fan (kitchen) | Inelastic | 0.12 | 120 |
| Freezer (1100 fg / h) | Inelastic | 0.15 | 150 |
| Microwave | Inelastic | 1.3 | 1300 |
| 100 W Lamp | Inelastic | 0.1 | 100 |
| 60 W Lamp | Inelastic | 0.06 | 60 |
| 75 W Lamp | Inelastic | 0.075 | 75 |
| Automatic Washing Machine | Elastic | 2.2 | 2200 |
| Semiautomatic or Manual Washing | Elastic | 0.7 | 700 |
| Blender | Inelastic | 0.35 | 350 |
| Cleaner Polishes | Elastic | 0.3 | 300 |
| Sewing Machine | Elastic | 0.075 | 75 |
| Iron (for clothes) Automatic | Elastic | 1 | 1000 |
| Iron (clothes) Common | Elastic | 0.55 | 550 |
| Food Processor | Inelastic | 0.25 | 250 |
| Air Purifier | Inelastic | 0.1 | 100 |
| Radio | Inelastic | 0.08 | 80 |
| Tape Recorder (small) | Inelastic | 0.008 | 8 |
| Tape Recorder with CD | Inelastic | 0.038 | 38 |
| Refrigerator (13 feet) | Inelastic | 0.265 | 265 |
| Refrigerator with Freezer ( $1 / 4 \mathrm{HP}$ ) | Inelastic | 0.184 | 184 |
| Refrigerator with Freezer ( $1 / 2 \mathrm{HP}$ ) | Inelastic | 0.368 | 368 |
| Ventilator | Inelastic | 0.065 | 65 |
| Medium Hair Dryer | Elastic | 0.7 | 700 |
| Centrifugal Dryer | Elastic | 0.2 | 200 |
| Dryer with Resistance | Elastic | 2 | 2000 |
| TV (21 ") | Inelastic | 0.115 | 115 |
| TV (29 ") | Inelastic | 0.205 | 205 |
| Toaster | Inelastic | 0.8 | 800 |
| Fluorescent Tube (105 Wh) | Inelastic | 0.135 | 135 |
| Turbofan | Inelastic | 0.184 | 184 |
| Ceiling Fan (small) | Inelastic | 0.1 | 100 |
| Fan (large) | Inelastic | 0.2 | 200 |
| Fan (medium) | Inelastic | 0.1 | 100 |
| Shower | Elastic | 3.5 | 3500 |
| Juicer | Inelastic | 0.25 | 250 |
| Water Pump | Inelastic | 0.4 | 400 |


| Coffee Pot | Inelastic | 0.7 | 700 |
| :--- | ---: | ---: | ---: |
| Dish Washer | Elastic | 1.5 | 1500 |

## APPENDIX F - Optimal Power Flow Formulation using Matpower.

Objective function

$$
\begin{equation*}
\min _{x} f(x) \tag{Eq 29}
\end{equation*}
$$

Subject to

$$
\begin{array}{cc}
g(x)=0 & \text { Eq 30 } \\
h(x) \leq 0 & \text { Eq 31 } \\
x_{\min } \leq x \leq x_{\max } & \text { Eq 32 }
\end{array}
$$

The optimization vector x for the standard AC OPF problem consists of the $\mathrm{n}_{\mathrm{b}} \mathrm{x}$ 1 vectors of voltage angles $\Theta$ and magnitudes $V_{m}$ and the $n_{g} \times 1$ vectors generator real and reactive power injections $\mathrm{P}_{\mathrm{g}}$ and $\mathrm{Q}_{\mathrm{g}}$.

$$
x=\left[\begin{array}{c}
\Theta \\
V_{m} \\
P_{g} \\
Q_{g}
\end{array}\right]
$$

Eq 33

The objective function is simply a summation of individual polynomial cost functions $f_{P}^{i}$ and $f_{Q}^{i}$ of real and reactive power injections, respectively, for each generator:

$$
\begin{equation*}
\min _{\Theta, V_{m}, P_{g}, Q_{g}} \sum_{i=1}^{n_{g}} f_{P}^{i}\left(p_{g}^{i}\right)+f_{P}^{i}\left(q_{g}^{i}\right) \tag{Eq 34}
\end{equation*}
$$

The equality constraints in Eq 30. are simply the full set of $2^{*} n_{b}$ nonlinear real and reactive power balance equations. The inequality constraints Eq 32 consist of two sets of $n_{1}$ branch flow limits as nonlinear functions of the bus voltage angles and magnitudes, one for the from end and one for the to end of each branch:

$$
\begin{align*}
& h_{f}\left(\Theta, V_{m}\right)=\left|F_{f}\left(\Theta, V_{m}\right)\right|-F_{\max } \leq 0 \\
& h_{t}\left(\Theta, V_{m}\right)=\left|F_{t}\left(\Theta, V_{m}\right)\right|-F_{\max } \leq 0 \tag{Eq 36}
\end{align*}
$$

$$
\text { Eq } 35
$$

The flows are typically apparent power flows expressed in MVA, but can be real power or current flows, yielding the following three possible forms for the flow constraints:

$$
\begin{array}{rrr}
S_{f}\left(\Theta, V_{m}\right), & \text { apparent power } \\
F_{f}\left(\Theta, V_{m}\right)=P_{f}\left(\Theta, V_{m}\right), & \text { real power }  \tag{Eq 37}\\
I_{f}\left(\Theta, V_{m}\right), & \text { current }
\end{array}
$$

Where $\mathrm{I}_{\mathrm{f}}$ (Current at the 'from' bus) is defined by the admittance matrix multiplied by the voltage node, $\mathrm{S}_{\mathrm{f}}$ is the complex power injection, $\mathrm{P}_{\mathrm{f}}=\mathrm{R}\left\{\mathrm{S}_{\mathrm{f}}\right\}$ and the vector of flow limits $\mathrm{F}_{\text {max }}$ has the appropriate units for the type of constraints. It is like for $\mathrm{F}_{\mathrm{t}}\left(\Theta, \mathrm{V}_{\mathrm{m}}\right)$. The variable limits Eq 33 include an equality constraint on any reference bus angle and upper and lower limits on all bus voltage magnitudes and real and active generator injections.

$$
\begin{array}{rll}
\theta_{i}^{\text {ref }} \leq \theta_{i} \leq \theta_{i}^{\text {ref }}, & i \in \mathfrak{J}_{\text {ref }} & \text { Eq } 38  \tag{Eq 38}\\
& & \\
v_{m}^{i, \text { min }} \leq v_{m}^{i} \leq v_{m}^{i, \text { max }}, & i=1, \ldots, n_{b} & \text { Eq } 39 \\
p_{g}^{i \text { min }} \leq p_{g}^{i} \leq p_{g}^{i \text { max }}, & i=1, \ldots, n_{g} & \text { Eq } 40 \\
q_{g}^{i \text { min }} \leq q_{g}^{i} \leq q_{g}^{i \text { max }}, & i=1, \ldots, n_{g} & \text { Eq } 41
\end{array}
$$

## APPENDIX G - Pseudocode for SoS application and resolution

Step 0:
Set: iteration count, $i=0$; convergence target error $\varepsilon 1$ and $\varepsilon 2$ at $(0,01)$
Set Initial Values for OPF Objective function Values (OPFOV_i) = Infinite
Set White tariff for each hour.
Set initial Objective Function value for Microgrid (MGOV_i)=Infinite
Define appropriately the horary Active and Reactive Power Demand for every bus ( $P_{-} i$, Q_i)
Set MG counter: $K=1$, hour counter: $t=1$
Step 1: For $t=1: 24$ hours:

- Solve Optimal Power Flow for each P_i, Q_i
- Get CMO_i for each bus and hour $t$.
- Get OPFOV_i

Step 2: For k=1: K microgris number

- Run Microgrid model (MG) using White tariff ;
- Get P_i, Q_i and MGOV_i

Step 3: Test the convergence criteria:

- |OPFOV_i - OPFOV_i_1| $\leq \varepsilon 1$
- $\left|M G O V_{-} i-M G O V_{-} i \_1\right| \leq \varepsilon 2$

If step 3 is true, then go to step 4; else go to step 1.

Step 4 Save optimal values of P_i, Q_i, CMO_i, Loads, EV, etc
Step 5 End.

## APPENDIX H - Complementary for scenarios A, B and C.

Supplementary graphics from Scenario A


FIGURE 58 - Voltage magnitude for buses on secondary 'B' branch for sce. A.


FIGURE 59 - Voltage magnitude for buses on secondary 'C' branch for sce. A


FIGURE 60 - Lambda values for the secondary ' $B$ ' branch for scenario A.


FIGURE 61 - Lambda values for the secondary 'C' branch for scenario A


FIGURE 62 - Active power losses for secondary 'B’ branch for scenario A


FIGURE 63 - Active power losses for secondary ' $C$ ' branch for scenario $A$

Supplementary graphics from Scenario B


FIGURE 64 - Voltage magnitude for secondary branch ' $B$ ' in scenario $B$


FIGURE 65 - Voltage magnitude for secondary branch ' $C$ ' in scenario $B$


FIGURE 66 - Lambda values for secondary ' $B$ ' branch in scenario $B$


FIGURE 67 - Lambda values for secondary 'C' branch in scenario $B$


FIGURE 68 - Active power losses for secondary ' B ' branch in scenario B


FIGURE 69 - Active power losses for secondary 'C' branch in scenario B

Supplementary graphics from Scenario C


FIGURE 70 - Voltage magnitude for secondary branch ' B ' in scenario C


FIGURE 71 - Voltage magnitude for secondary branch ' $C$ ' in scenario $C$


FIGURE 72 - Lambda values for secondary ' $B$ ' branch in scenario $C$


FIGURE 73 - Lambda values for secondary 'C' branch in scenario $C$


FIGURE 74 - Active power losses for secondary ' $B$ ' branch in scenario $C$


FIGURE 75 - Active power losses for secondary 'C' branch in scenario C

## APPENDIX I - Numerical Values for Graphics from Scenarios A, B and C.

| Voltage Magnitude for Scenario A |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Bus1 | 1.0929 | 1.093120613 | 1.092877591 | 1.093790821 | 1.093811028 | 1.094404171 | 1.094936582 | 1.094064245 | 1.095225764 | 1.097275978 | 1.094308735 | 1.093656654 |
| Bus2 | 1.0923 | 1.092592868 | 1.092393484 | 1.093395407 | 1.093399925 | 1.093915782 | 1.093805276 | 1.092624386 | 1.09433383 | 1.09644907 | 1.093105573 | 1.092142181 |
| Bus3 | 1.0885 | 1.089322949 | 1.089377718 | 1.091007455 | 1.090925041 | 1.091109686 | 1.087523512 | 1.084697188 | 1.089266577 | 1.092258705 | 1.086434346 | 1.083737504 |
| Bus4 | 1.0878 | 1.088682008 | 1.0889176 | 1.090751633 | 1.090583683 | 1.090825071 | 1.084688015 | 1.080387677 | 1.087605391 | 1.091037218 | 1.083414586 | 1.079477277 |
| Bus5 | 1.0870 | 1.088055392 | 1.088472895 | 1.090519626 | 1.090262617 | 1.09060874 | 1.082152408 | 1.076598591 | 1.086209142 | 1.090020452 | 1.080813378 | 1.075786066 |
| Bus6 | 1.0856 | 1.086819895 | 1.08761154 | 1.090107573 | 1.089660426 | 1.0903252 | 1.07750981 | 1.069767157 | 1.083869419 | 1.088342826 | 1.076271595 | 1.069246176 |
| Bus7 | 1.0864 | 1.087548976 | 1.088306693 | 1.090663905 | 1.090242038 | 1.090822087 | 1.077742781 | 1.069416412 | 1.084000004 | 1.088701125 | 1.076080432 | 1.068852931 |
| Bus8 | 1.0948 | 1.095239238 | 1.095501983 | 1.096341521 | 1.096189545 | 1.096345059 | 1.082183802 | 1.069119619 | 1.087202074 | 1.093570581 | 1.077081566 | 1.068164674 |
| Bus9 | 1.1000 | 1.099999998 | 1.099944451 | 1.099865161 | 1.099876474 | 1.099905808 | 1.086053383 | 1.070952485 | 1.090086479 | 1.097235445 | 1.079245146 | 1.069897593 |
| Bus10 | 1.0984 | 1.098943484 | 1.099052427 | 1.098973072 | 1.098984394 | 1.097826066 | 1.074961038 | 1.051819927 | 1.081249954 | 1.090531279 | 1.064574093 | 1.049403234 |
| Bus11 | 1.0981 | 1.098783971 | 1.098917748 | 1.098838384 | 1.098849707 | 1.097512071 | 1.073286595 | 1.048932114 | 1.07991599 | 1.08951918 | 1.062359548 | 1.046309928 |
| Bus12 | 1.0977 | 1.098518113 | 1.098693288 | 1.098613907 | 1.098625233 | 1.096988639 | 1.070490297 | 1.044101308 | 1.077689306 | 1.087830487 | 1.058658536 | 1.041133859 |
| Bus13 | 1.0961 | 1.097503363 | 1.097836607 | 1.097757164 | 1.097768499 | 1.094989932 | 1.059772263 | 1.02551778 | 1.069162876 | 1.081370032 | 1.044450284 | 1.021209645 |
| Bus14 | 1.0955 | 1.09714818 | 1.097536759 | 1.097457294 | 1.097468632 | 1.094290227 | 1.056014269 | 1.018991856 | 1.066174524 | 1.079106618 | 1.039465201 | 1.014211002 |
| Bus15 | 1.0951 | 1.096868443 | 1.097300603 | 1.097221121 | 1.097232461 | 1.093739157 | 1.05305488 | 1.013853164 | 1.063821161 | 1.077324108 | 1.035539639 | 1.008700145 |
| Bus16 | 1.0947 | 1.096614251 | 1.097086015 | 1.097006517 | 1.09701786 | 1.093238371 | 1.050363586 | 1.009176634 | 1.061681397 | 1.075703668 | 1.031968589 | 1.003684286 |
| Bus17 | 1.0942 | 1.096274586 | 1.096799276 | 1.096719758 | 1.096731103 | 1.092569102 | 1.046762251 | 1.002910869 | 1.058819036 | 1.073536676 | 1.027187405 | 0.996962383 |
| Bus18 | 1.0940 | 1.096190284 | 1.096728109 | 1.096648586 | 1.096659932 | 1.092403002 | 1.045868804 | 1.001356987 | 1.05810885 | 1.07299897 | 1.026001443 | 0.995295489 |
| Bus19 | 1.0925 | 1.092752874 | 1.092544464 | 1.093495123 | 1.093500259 | 1.094005871 | 1.093996774 | 1.092888217 | 1.094538657 | 1.096440823 | 1.093347033 | 1.092443501 |
| Bus20 | 1.0947 | 1.094630828 | 1.094294161 | 1.094773711 | 1.094807259 | 1.095377594 | 1.096837864 | 1.096481401 | 1.097200236 | 1.097634497 | 1.096612258 | 1.096350902 |
| Bus21 | 1.0954 | 1.095255606 | 1.094871297 | 1.095222215 | 1.095269754 | 1.095901576 | 1.097824672 | 1.09761604 | 1.09809511 | 1.098273568 | 1.097693188 | 1.097546109 |
| Bus22 | 1.0969 | 1.096551249 | 1.096060568 | 1.096188143 | 1.09627172 | 1.097092718 | 1.099999829 | 1.099999847 | 1.099999791 | 1.099999782 | 1.099999824 | 1.099999877 |
| Bus23 | 1.0861 | 1.087179908 | 1.087235453 | 1.089185996 | 1.089129712 | 1.089000637 | 1.085819763 | 1.083670062 | 1.087115062 | 1.09026636 | 1.08476288 | 1.082274759 |
| Bus24 | 1.0802 | 1.082084616 | 1.08227188 | 1.085017808 | 1.084961306 | 1.08404521 | 1.079399281 | 1.077236637 | 1.080702354 | 1.084398281 | 1.078071503 | 1.074837609 |
| Bus25 | 1.0773 | 1.079545216 | 1.079798183 | 1.08294079 | 1.08288418 | 1.081575579 | 1.076198704 | 1.074029593 | 1.077505661 | 1.081473362 | 1.074735728 | 1.071129565 |


| Bus26 | 1.0844 | 1.085636661 | 1.086574417 | 1.089364728 | 1.088837988 | 1.089681911 | 1.076312895 | 1.068934223 | 1.083288053 | 1.087638438 | 1.075656039 | 1.068590655 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus27 | 1.0830 | 1.08432392 | 1.085427691 | 1.088552985 | 1.087933988 | 1.089009443 | 1.075118837 | 1.068252826 | 1.082780093 | 1.086964688 | 1.075179461 | 1.068138267 |
| Bus28 | 1.0733 | 1.075679892 | 1.077768059 | 1.082848563 | 1.081744322 | 1.08330747 | 1.062915555 | 1.055966824 | 1.075101113 | 1.079121809 | 1.065469627 | 1.05635528 |
| Bus29 | 1.0672 | 1.070220552 | 1.072930839 | 1.079246684 | 1.077835894 | 1.079707139 | 1.055205861 | 1.04820476 | 1.070251658 | 1.074168794 | 1.059336557 | 1.048911342 |
| Bus30 | 1.0644 | 1.067718501 | 1.070713985 | 1.077596086 | 1.076044788 | 1.07805725 | 1.051672005 | 1.044646878 | 1.068029195 | 1.071898859 | 1.056525619 | 1.04549934 |
| Bus31 | 1.0599 | 1.063623344 | 1.067086274 | 1.074895976 | 1.073114571 | 1.075358303 | 1.045884147 | 1.038819493 | 1.064392279 | 1.068184169 | 1.051923928 | 1.039911422 |
| Bus32 | 1.0589 | 1.062715177 | 1.066281766 | 1.074297171 | 1.072464738 | 1.074759756 | 1.044600629 | 1.037527211 | 1.06358573 | 1.067360372 | 1.050903438 | 1.03867224 |
| Bus33 | 1.0583 | 1.062170276 | 1.065799068 | 1.073937905 | 1.072074853 | 1.074400645 | 1.043830468 | 1.036751789 | 1.063101807 | 1.0668661 | 1.05029113 | 1.037928686 |


| Voltage Magnitude for Scenario A |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Bus1 | 1.095949734 | 1.095409499 | 1.094611147 | 1.095692081 | 1.094690478 | 1.096188328 | 1.098928145 | 1.099999959 | 1.099999922 | 1.099999848 | 1.095020638 | 1.093902293 |
| Bus2 | 1.095023183 | 1.094440681 | 1.093514401 | 1.094765523 | 1.093330666 | 1.093958927 | 1.09589322 | 1.096933249 | 1.096943969 | 1.097191682 | 1.093530458 | 1.092719727 |
| Bus3 | 1.089882306 | 1.088987531 | 1.087315895 | 1.089862663 | 1.085673856 | 1.081913784 | 1.080020927 | 1.081328948 | 1.080761672 | 1.08278416 | 1.085370138 | 1.085844286 |
| Bus4 | 1.088453728 | 1.087296489 | 1.084961977 | 1.088712302 | 1.082516726 | 1.074738699 | 1.07284468 | 1.07415416 | 1.072069721 | 1.074794932 | 1.082504096 | 1.08338501 |
| Bus5 | 1.087219009 | 1.085832566 | 1.082913511 | 1.087724651 | 1.079629233 | 1.068194246 | 1.066596522 | 1.068475463 | 1.064248556 | 1.067927568 | 1.079937288 | 1.081231995 |
| Bus6 | 1.085062399 | 1.083269988 | 1.079296474 | 1.086072715 | 1.07416967 | 1.055704723 | 1.055148639 | 1.057499566 | 1.049325997 | 1.055440071 | 1.075196339 | 1.077394404 |
| Bus7 | 1.08563543 | 1.083721997 | 1.079518244 | 1.08664167 | 1.074928056 | 1.056123365 | 1.056149608 | 1.063225754 | 1.050476234 | 1.055708265 | 1.076106693 | 1.077764825 |
| Bus8 | 1.092419505 | 1.089639978 | 1.083819948 | 1.094594429 | 1.084021318 | 1.062364206 | 1.065989527 | 1.076589962 | 1.061559875 | 1.055197661 | 1.086649254 | 1.083422612 |
| Bus9 | 1.097233322 | 1.094056846 | 1.087564658 | 1.099999788 | 1.09064846 | 1.068768443 | 1.075397109 | 1.089777749 | 1.071722781 | 1.059092773 | 1.094202702 | 1.087999182 |
| Bus10 | 1.090611414 | 1.086244585 | 1.076862107 | 1.09450833 | 1.080111067 | 1.043249742 | 1.041086441 | 1.038530632 | 1.037374284 | 1.016734757 | 1.083214184 | 1.077180566 |
| Bus11 | 1.089611732 | 1.085065223 | 1.075246496 | 1.093679292 | 1.078520382 | 1.039398315 | 1.035908493 | 1.030797353 | 1.032190632 | 1.010342698 | 1.08155541 | 1.075547436 |
| Bus12 | 1.087943783 | 1.083097016 | 1.072548663 | 1.092296366 | 1.075864277 | 1.032946692 | 1.027218328 | 1.017769536 | 1.023490594 | 0.999593596 | 1.078785381 | 1.072820291 |
| Bus13 | 1.081562907 | 1.075563683 | 1.062209833 | 1.087008373 | 1.065686204 | 1.00805444 | 0.993550764 | 0.966875726 | 0.989782215 | 0.957767666 | 1.068168879 | 1.062368626 |
| Bus14 | 1.079327405 | 1.072923876 | 1.05858506 | 1.085156114 | 1.062117917 | 0.999301671 | 0.981690088 | 0.948874163 | 0.97790674 | 0.943002394 | 1.064446609 | 1.058704221 |
| Bus15 | 1.077566875 | 1.070844972 | 1.055730569 | 1.083697381 | 1.059307901 | 0.992409855 | 0.972351599 | 0.934700811 | 0.968556606 | 0.931377217 | 1.061515346 | 1.055818522 |
| Bus16 | 1.075966427 | 1.068954916 | 1.053134756 | 1.082371407 | 1.056752575 | 0.986134085 | 0.963840484 | 0.921759219 | 0.960034737 | 0.920771911 | 1.05884967 | 1.053194306 |
| Bus17 | 1.073826194 | 1.06642697 | 1.049661395 | 1.080598503 | 1.053333484 | 0.977716688 | 0.952407548 | 0.904318231 | 0.948587029 | 0.906502039 | 1.055282714 | 1.049682882 |
| Bus18 | 1.073295127 | 1.065799729 | 1.048799682 | 1.080158562 | 1.052485228 | 0.975629853 | 0.949574335 | 0.900000069 | 0.945750179 | 0.902967462 | 1.05439779 | 1.048811731 |
| Bus19 | 1.095148323 | 1.094612322 | 1.09374186 | 1.094816112 | 1.093534755 | 1.094104924 | 1.095831722 | 1.096850954 | 1.09689848 | 1.097125957 | 1.093663996 | 1.093015143 |


| Bus20 | 1.097308741 | 1.097135771 | 1.096792573 | 1.096642068 | 1.096645222 | 1.097233241 | 1.097961543 | 1.098812639 | 1.098865178 | 1.098857864 | 1.096477354 | 1.096590166 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus21 | 1.098123142 | 1.098027357 | 1.097814294 | 1.097515147 | 1.097695228 | 1.098104831 | 1.098518094 | 1.099137183 | 1.099193583 | 1.099171131 | 1.097528147 | 1.097711554 |
| Bus22 | 1.099999735 | 1.099999778 | 1.099999825 | 1.099736227 | 1.099999746 | 1.099999956 | 1.099999955 | 1.099999922 | 1.099999995 | 1.099999982 | 1.099999764 | 1.099999909 |
| Bus23 | 1.087276373 | 1.086476072 | 1.085076193 | 1.087076018 | 1.083096835 | 1.080978397 | 1.076014593 | 1.078379609 | 1.078352982 | 1.081391733 | 1.082001486 | 1.083125377 |
| Bus24 | 1.080072385 | 1.079266681 | 1.077857332 | 1.079606193 | 1.074535279 | 1.072132758 | 1.059129929 | 1.061123472 | 1.064093996 | 1.069467848 | 1.071694444 | 1.074830256 |
| Bus25 | 1.076480758 | 1.075672354 | 1.07425827 | 1.075881859 | 1.070265799 | 1.067721383 | 1.050697381 | 1.052504886 | 1.056976151 | 1.063518139 | 1.066552958 | 1.070693833 |
| Bus26 | 1.084012532 | 1.082272136 | 1.078351611 | 1.084934925 | 1.072209029 | 1.053019196 | 1.052712969 | 1.05630175 | 1.045915271 | 1.05433191 | 1.073264866 | 1.076242274 |
| Bus27 | 1.082944275 | 1.08127732 | 1.077451649 | 1.083762445 | 1.070141793 | 1.050286066 | 1.050447455 | 1.055565543 | 1.042493389 | 1.053625774 | 1.071257055 | 1.075110887 |
| Bus28 | 1.072814502 | 1.071131239 | 1.066771639 | 1.073147515 | 1.052781672 | 1.023416936 | 1.020254107 | 1.031024482 | 1.006302726 | 1.031729294 | 1.053400842 | 1.06290751 |
| Bus29 | 1.066415991 | 1.06472241 | 1.060025219 | 1.066442279 | 1.041808826 | 1.006416978 | 1.001144155 | 1.015501435 | 0.983380871 | 1.01788267 | 1.042113926 | 1.055197756 |
| Bus30 | 1.06348336 | 1.061785046 | 1.056933064 | 1.063369014 | 1.036778279 | 0.998620146 | 0.99237833 | 1.00838271 | 0.972863496 | 1.011533417 | 1.036939297 | 1.051663872 |
| Bus31 | 1.058682138 | 1.056976048 | 1.051870109 | 1.058337107 | 1.028530864 | 0.985812025 | 0.977968403 | 0.996694448 | 0.955550953 | 1.001114054 | 1.028454879 | 1.045875969 |
| Bus32 | 1.057617401 | 1.055909588 | 1.050747334 | 1.057221216 | 1.026701977 | 0.982971961 | 0.974773215 | 0.994102664 | 0.951712278 | 0.998803603 | 1.026573442 | 1.044592441 |
| Bus33 | 1.056978542 | 1.055269693 | 1.050073643 | 1.056551658 | 1.025604471 | 0.981267333 | 0.972855311 | 0.992547129 | 0.949407825 | 0.997416987 | 1.025444391 | 1.043822274 |


| Lambda P (UM/MWh) for Scenario A |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Bus1 | 119.8176088 | 102.1443386 | 93.81151115 | 77.12681297 | 80.04508283 | 95.64721489 | 221.8959265 | 283.9075297 | 176.7007487 | 162.0846295 | 238.2953293 | 298.5242203 |
| Bus2 | 119.9507547 | 102.2412956 | 93.8932903 | 77.18154907 | 80.10413621 | 95.73039555 | 222.3300117 | 284.6038743 | 176.9748977 | 162.3172 | 238.788388 | 299.2938893 |
| Bus3 | 120.7818953 | 102.8517694 | 94.41073919 | 77.51790081 | 80.46596497 | 96.2194544 | 225.0194657 | 289.0792994 | 178.6663646 | 163.5959686 | 241.8650481 | 304.3038142 |
| Bus4 | 120.9371328 | 102.964285 | 94.48369187 | 77.55014385 | 80.51192933 | 96.26182002 | 226.2432703 | 291.5383804 | 179.2201754 | 163.9682467 | 243.2683713 | 306.8829196 |
| Bus5 | 121.0868021 | 103.0749325 | 94.55457584 | 77.57952492 | 80.55545035 | 96.29380753 | 227.3748945 | 293.7956432 | 179.7041045 | 164.2908358 | 244.5280369 | 309.2256715 |
| Bus6 | 121.3827486 | 103.2987799 | 94.69600714 | 77.63364864 | 80.63964954 | 96.33645087 | 229.4977933 | 297.9851907 | 180.5432779 | 164.8434825 | 246.7965992 | 313.5113325 |
| Bus7 | 121.17028 | 103.1328089 | 94.55462345 | 77.54403215 | 80.54175386 | 96.23110355 | 229.3971153 | 298.284322 | 180.488945 | 164.7244285 | 246.9067553 | 313.8818664 |
| Bus8 | 119.1908381 | 101.5982534 | 93.24904292 | 76.71093755 | 79.63295622 | 95.21960957 | 227.7994535 | 299.5022375 | 179.5666996 | 163.3459561 | 246.9466132 | 315.5651207 |
| Bus9 | 117.9824369 | 100.6677652 | 92.45816844 | 76.20267187 | 79.07930729 | 94.5838966 | 226.4402266 | 299.3224065 | 178.7600751 | 162.3471873 | 246.3918523 | 315.5507333 |
| Bus10 | 118.2701567 | 100.8247906 | 92.57960746 | 76.30293207 | 79.18333377 | 94.87514941 | 230.3030507 | 308.4741734 | 181.1649173 | 163.98361 | 252.0454815 | 325.9400788 |
| Bus11 | 118.3186099 | 100.8512136 | 92.60003862 | 76.3197993 | 79.20083466 | 94.92421837 | 230.9613416 | 310.0505601 | 181.5735472 | 164.260904 | 253.0135036 | 327.7328921 |
| Bus12 | 118.3994537 | 100.8952796 | 92.63410955 | 76.34792547 | 79.23001754 | 95.00610559 | 232.067099 | 312.7150394 | 182.2587858 | 164.7251695 | 254.6439724 | 330.7664358 |


| Bus13 | 118.6720419 | 101.0436869 | 92.74882536 | 76.44261931 | 79.32826941 | 95.28241344 | 235.865441 | 322.0262304 | 184.6018034 | 166.3057831 | 260.286889 | 341.3988534 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus14 | 118.756154 | 101.0894208 | 92.78416519 | 76.47179009 | 79.3585363 | 95.36775148 | 237.0629514 | 325.0206195 | 185.3365804 | 166.7990114 | 262.081406 | 344.8299101 |
| Bus15 | 118.8296191 | 101.1293581 | 92.81502546 | 76.49726246 | 79.38496587 | 95.44229532 | 238.1119808 | 327.6514558 | 185.9797644 | 167.230456 | 263.6554011 | 347.8459895 |
| Bus16 | 118.8992531 | 101.1672017 | 92.84426648 | 76.52139759 | 79.41000798 | 95.51296538 | 239.1110585 | 330.1684874 | 186.5915889 | 167.6404088 | 265.1574092 | 350.7339261 |
| Bus17 | 118.9796619 | 101.2108563 | 92.87798894 | 76.54923093 | 79.43888733 | 95.59462977 | 240.2843739 | 333.1710815 | 187.3071219 | 168.1179837 | 266.9334262 | 354.188345 |
| Bus18 | 119.0024779 | 101.2232462 | 92.88756076 | 76.5571311 | 79.44708439 | 95.61779779 | 240.6160008 | 334.0169379 | 187.5095494 | 168.2532123 | 267.4346606 | 355.1609563 |
| Bus19 | 119.907051 | 102.208558 | 93.86498444 | 77.16551077 | 80.08726735 | 95.71098632 | 222.1516531 | 284.2342776 | 176.8633579 | 162.2797017 | 238.5626758 | 298.8631092 |
| Bus20 | 119.4242454 | 101.8510921 | 93.55869355 | 76.97989558 | 79.89007627 | 95.46127822 | 220.2069519 | 280.4450956 | 175.6603282 | 161.6575402 | 236.177173 | 294.513105 |
| Bus21 | 119.2707577 | 101.7383633 | 93.46273267 | 76.91895148 | 79.82491244 | 95.37392252 | 219.5950569 | 279.3078976 | 175.2846235 | 161.412979 | 235.4438378 | 293.2236865 |
| Bus22 | 118.9670062 | 101.5165966 | 93.27488745 | 76.79547545 | 79.69229226 | 95.18930935 | 218.3944025 | 277.1583049 | 174.5513027 | 160.8605917 | 234.0303617 | 290.8111038 |
| Bus23 | 121.3408441 | 103.266502 | 94.79040043 | 77.78213384 | 80.73650047 | 96.60236076 | 225.8175114 | 289.8122638 | 179.4305743 | 164.2360164 | 242.7301137 | 305.3382009 |
| Bus24 | 122.3972163 | 104.0421794 | 95.48378363 | 78.25837001 | 81.23087561 | 97.30667345 | 227.9628162 | 292.5766818 | 181.1310551 | 165.6547186 | 245.1369611 | 308.7158025 |
| Bus25 | 122.9303326 | 104.4330227 | 95.83307135 | 78.49789408 | 81.47952299 | 97.66145562 | 229.046612 | 293.9733208 | 181.9901023 | 166.3706266 | 246.353551 | 310.4256513 |
| Bus26 | 121.680419 | 103.5310233 | 94.88185984 | 77.74208858 | 80.7644622 | 96.4548352 | 230.0543687 | 298.5290219 | 180.759728 | 165.0743552 | 247.1280635 | 313.9797086 |
| Bus27 | 122.0246551 | 103.800742 | 95.09691289 | 77.86621588 | 80.90803709 | 96.58617389 | 230.6588276 | 299.0593248 | 180.9755472 | 165.3167176 | 247.4408435 | 314.4016402 |
| Bus28 | 123.7447898 | 105.1085759 | 96.15514615 | 78.50683195 | 81.63149902 | 97.38010852 | 234.8316399 | 304.5433101 | 182.9997015 | 167.1984952 | 250.9818866 | 319.9250027 |
| Bus29 | 124.8551135 | 105.9510084 | 96.83540524 | 78.91696856 | 82.09513949 | 97.88839874 | 237.5400687 | 308.1039877 | 184.3009778 | 168.4085757 | 253.2682597 | 323.5074396 |
| Bus30 | 125.4205861 | 106.3796523 | 97.18121919 | 79.1250904 | 82.33051576 | 98.14632722 | 238.9228166 | 309.9221127 | 184.9625116 | 169.0238232 | 254.4328307 | 325.3358141 |
| Bus31 | 126.2465306 | 107.0047958 | 97.68481412 | 79.42728408 | 82.67252951 | 98.52083786 | 240.9506491 | 312.5890993 | 185.9259303 | 169.9200127 | 256.1342011 | 328.0157545 |
| Bus32 | 126.4235393 | 107.1386459 | 97.79254066 | 79.49181123 | 82.7455919 | 98.60080644 | 241.3863167 | 313.1621734 | 186.1320273 | 170.1117514 | 256.4988715 | 328.5913363 |
| Bus33 | 126.5213986 | 107.212592 | 97.85201269 | 79.52738482 | 82.78588477 | 98.64489269 | 241.6276358 | 313.47964 | 186.2458092 | 170.2176166 | 256.7005003 | 328.9100752 |


| Lambda P (UM/MWh) for Scenario A |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Bus1 | 180.5413537 | 189.5583118 | 215.6330855 | 180.5402732 | 262.2038785 | 425.4779278 | 573.0839495 | 638.7937892 | 574.9556673 | 538.736493 | 286.2309527 | 231.392277 |
| Bus2 | 180.8332155 | 189.878349 | 216.0432804 | 180.8351241 | 262.8205903 | 427.0621292 | 575.9549889 | 643.7655287 | 577.8835115 | 541.2725015 | 286.9680085 | 231.8677725 |
| Bus3 | 182.6001769 | 191.8474707 | 218.6061458 | 182.4448615 | 266.7811112 | 438.0444681 | 596.1454706 | 676.8144086 | 599.160561 | 559.2932656 | 291.5927323 | 234.9389927 |
| Bus4 | 183.0972878 | 192.4634382 | 219.5870055 | 182.7927543 | 268.4536419 | 444.7320385 | 605.9064019 | 695.5097404 | 611.1897393 | 569.7851656 | 293.2725504 | 236.0525529 |
| Bus5 | 183.5431878 | 193.0164924 | 220.4727185 | 183.0949962 | 270.0353449 | 451.1339414 | 615.0217004 | 713.274165 | 622.7247828 | 579.554081 | 294.8368673 | 237.0646601 |
| Bus6 | 184.3464163 | 194.0139238 | 222.0820843 | 183.6149211 | 273.0895036 | 463.6751907 | 632.2950473 | 747.5868314 | 645.3489755 | 597.9780437 | 297.7983975 | 238.9201146 |


| Bus7 | 184.1430576 | 193.8424696 | 221.9864192 | 183.3229789 | 272.7107408 | 463.7552371 | 632.4791981 | 752.9319597 | 645.3819073 | 600.1147585 | 297.3079237 | 238.7541199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus8 | 181.9702075 | 191.8788098 | 220.4798251 | 180.4059101 | 268.5112257 | 461.3544837 | 628.5348053 | 791.344563 | 639.9257533 | 613.1539528 | 292.0007867 | 236.5538427 |
| Bus9 | 180.4829639 | 190.4676501 | 219.2017282 | 178.5125472 | 265.5673579 | 458.0751874 | 622.9014022 | 807.9041699 | 633.3301022 | 617.035989 | 288.3479823 | 234.8321395 |
| Bus10 | 182.2794659 | 192.7182301 | 222.800541 | 179.9778779 | 269.8456051 | 477.1812496 | 658.6573376 | 894.0903277 | 669.8698469 | 662.9087845 | 293.1819854 | 238.7290527 |
| Bus11 | 182.5838542 | 193.1001373 | 223.4135226 | 180.2257979 | 270.5741329 | 480.5003448 | 664.9406169 | 909.7895602 | 676.2922016 | 671.0651656 | 294.0055866 | 239.392904 |
| Bus12 | 183.093453 | 193.7400741 | 224.4428649 | 180.6405192 | 271.7973302 | 486.1388159 | 675.6892054 | 937.506416 | 687.2801055 | 685.1201351 | 295.3888339 | 240.5077603 |
| Bus13 | 184.8281276 | 195.9236168 | 227.9758242 | 182.0490787 | 275.9940002 | 506.1207714 | 714.5291922 | 1044.16387 | 726.9991058 | 736.9615091 | 300.1385887 | 244.3350878 |
| Bus14 | 185.3693302 | 196.6067335 | 229.0886167 | 182.4874079 | 277.3152461 | 512.6520329 | 727.515361 | 1082.747138 | 740.2847388 | 754.7164953 | 301.6354048 | 245.5409122 |
| Bus15 | 185.8427274 | 197.2044938 | 230.0633004 | 182.8706837 | 278.4724371 | 518.4049477 | 738.9970901 | 1118.475622 | 752.0320848 | 770.4819139 | 302.9465471 | 246.5971208 |
| Bus16 | 186.2925245 | 197.7728021 | 230.9913744 | 183.2346441 | 279.5741782 | 523.9302329 | 750.0858363 | 1155.540938 | 763.378542 | 785.8011685 | 304.1951345 | 247.6028804 |
| Bus17 | 186.8164398 | 198.4361849 | 232.0804945 | 183.6577234 | 280.8666407 | 530.6066692 | 763.7283358 | 1208.211281 | 777.3427651 | 805.0142718 | 305.6609742 | 248.783408 |
| Bus18 | 186.9647949 | 198.6239396 | 232.388375 | 183.7775813 | 281.2320324 | 532.4828069 | 767.5500199 | 1227.235645 | 781.2543623 | 810.3810142 | 306.0753118 | 249.1171131 |
| Bus19 | 180.7371884 | 189.7586432 | 215.8678538 | 180.8014257 | 262.5408996 | 426.0146174 | 573.9985889 | 641.0615961 | 575.6749845 | 539.3691108 | 286.674275 | 231.6361747 |
| Bus20 | 179.615216 | 188.425827 | 213.9929659 | 180.1535041 | 259.5712344 | 415.5988481 | 554.4424795 | 614.6120021 | 554.1227155 | 520.6797233 | 283.402454 | 229.2649255 |
| Bus21 | 179.2449852 | 188.0000519 | 213.411612 | 179.885674 | 258.6552427 | 412.5519691 | 548.6996587 | 606.9866369 | 547.9199291 | 515.2713866 | 282.357843 | 228.5537976 |
| Bus22 | 178.4929031 | 187.1549324 | 212.2833798 | 179.2651933 | 256.8856032 | 406.9223646 | 538.0726654 | 593.1048299 | 536.6386713 | 505.3866208 | 280.2872245 | 227.2097424 |
| Bus23 | 183.5292894 | 192.7967164 | 219.5968697 | 183.4272984 | 268.1810098 | 439.274618 | 601.6184053 | 682.4293434 | 602.8799919 | 561.6825223 | 293.5717349 | 236.2228359 |
| Bus24 | 185.4857981 | 194.8551179 | 221.9475946 | 185.4562548 | 271.6006138 | 445.0763178 | 617.0913636 | 700.3389608 | 615.8794584 | 571.7365484 | 298.0976713 | 239.1396795 |
| Bus25 | 186.475723 | 195.896619 | 223.1370482 | 186.4833761 | 273.3355631 | 448.0215914 | 625.0749697 | 709.5860649 | 622.5508983 | 576.8718917 | 300.4019383 | 240.6187698 |
| Bus26 | 184.7271824 | 194.3990047 | 222.5102367 | 184.0281445 | 274.1690932 | 466.3670493 | 635.7551801 | 749.2431289 | 650.3200689 | 599.5425816 | 298.9625672 | 239.4789402 |
| Bus27 | 185.1440092 | 194.8152833 | 222.9610951 | 184.4856544 | 275.389202 | 469.3755271 | 639.4356881 | 750.4228829 | 655.8828539 | 600.9123275 | 300.265952 | 240.0792446 |
| Bus28 | 187.8905081 | 197.7145773 | 226.4708851 | 187.3544305 | 282.5985081 | 489.2451004 | 670.1186662 | 779.1079129 | 694.5569027 | 621.3085283 | 308.3520689 | 244.4225499 |
| Bus29 | 189.6651512 | 199.5880731 | 228.7415813 | 189.2099233 | 287.3299834 | 502.5712576 | 690.8507101 | 798.2418651 | 721.0623241 | 634.8358167 | 313.6644223 | 247.2416409 |
| Bus30 | 190.569362 | 200.542679 | 229.8991842 | 190.155744 | 289.757481 | 509.4746452 | 701.6263755 | 808.1297812 | 734.9269257 | 641.8085226 | 316.3911794 | 248.6808859 |
| Bus31 | 191.8910596 | 201.9381072 | 231.5928181 | 191.5392619 | 293.3466048 | 519.8485175 | 717.9110405 | 822.9275119 | 756.1101448 | 652.1987132 | 320.4258183 | 250.7915733 |
| Bus32 | 192.1744435 | 202.2373086 | 231.9561537 | 191.8360328 | 294.1215946 | 522.1110694 | 721.475165 | 826.1467191 | 760.7778123 | 654.4530791 | 321.2974137 | 251.2450419 |
| Bus33 | 192.3311677 | 202.4027843 | 232.157182 | 192.0002169 | 294.5525189 | 523.3787054 | 723.4772648 | 827.9468909 | 763.4130333 | 655.7111887 | 321.7822268 | 251.4962211 |


| Active Power Losses (I^2*R) (Watt) for Scenario A |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Branch 1 | 0.64 | 0.47 | 0.39 | 0.26 | 0.28 | 0.39 | 2.01 | 3.19 | 1.26 | 1.09 | 2.26 | 3.52 |
| Branch 2 | 4.64 | 3.45 | 2.93 | 1.84 | 1.98 | 2.53 | 12.78 | 20.44 | 8.26 | 5.67 | 14.37 | 23.00 |
| Branch 3 | 0.23 | 0.17 | 0.09 | 0.03 | 0.05 | 0.03 | 3.36 | 7.80 | 1.13 | 0.61 | 3.78 | 7.61 |
| Branch 4 | 0.21 | 0.16 | 0.08 | 0.02 | 0.04 | 0.02 | 2.68 | 6.06 | 0.80 | 0.42 | 2.82 | 5.76 |
| Branch 5 | 0.38 | 0.30 | 0.14 | 0.03 | 0.07 | 0.02 | 4.26 | 9.28 | 1.09 | 0.56 | 4.10 | 8.53 |
| Branch 6 | 0.71 | 0.61 | 0.53 | 0.32 | 0.36 | 0.29 | 0.07 | 0.15 | 0.03 | 0.14 | 0.02 | 0.20 |
| Branch 7 | 6.89 | 5.78 | 5.03 | 3.10 | 3.41 | 2.94 | 1.62 | 0.21 | 0.85 | 2.10 | 0.07 | 0.37 |
| Branch 8 | 4.38 | 3.62 | 3.14 | 1.95 | 2.14 | 1.97 | 1.91 | 0.37 | 1.07 | 1.85 | 0.49 | 0.36 |
| Branch 9 | 0.31 | 0.13 | 0.09 | 0.09 | 0.09 | 0.48 | 13.74 | 40.76 | 8.73 | 5.03 | 24.01 | 46.75 |
| Branch 10 | 0.05 | 0.02 | 0.01 | 0.01 | 0.01 | 0.07 | 2.06 | 6.13 | 1.31 | 0.75 | 3.60 | 7.03 |
| Branch 11 | 0.07 | 0.03 | 0.02 | 0.02 | 0.02 | 0.11 | 3.02 | 9.01 | 1.91 | 1.10 | 5.29 | 10.34 |
| Branch 12 | 0.19 | 0.08 | 0.06 | 0.06 | 0.06 | 0.30 | 8.74 | 26.18 | 5.53 | 3.18 | 15.33 | 30.07 |
| Branch 13 | 0.05 | 0.02 | 0.01 | 0.01 | 0.01 | 0.08 | 2.24 | 6.74 | 1.42 | 0.82 | 3.94 | 7.75 |
| Branch 14 | 0.03 | 0.01 | 0.01 | 0.01 | 0.01 | 0.05 | 1.57 | 4.73 | 0.99 | 0.57 | 2.76 | 5.44 |
| Branch 15 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.04 | 1.12 | 3.38 | 0.71 | 0.41 | 1.97 | 3.88 |
| Branch 16 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.03 | 0.86 | 2.60 | 0.54 | 0.31 | 1.52 | 3.00 |
| Branch 17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.37 | 0.08 | 0.04 | 0.22 | 0.43 |
| Branch 18 | 0.04 | 0.03 | 0.03 | 0.01 | 0.01 | 0.01 | 0.13 | 0.32 | 0.09 | 0.02 | 0.18 | 0.39 |
| Branch 19 | 0.52 | 0.39 | 0.34 | 0.19 | 0.19 | 0.22 | 1.71 | 3.63 | 1.15 | 0.34 | 2.22 | 4.31 |
| Branch 20 | 0.19 | 0.14 | 0.12 | 0.07 | 0.08 | 0.10 | 0.64 | 1.23 | 0.42 | 0.21 | 0.79 | 1.42 |
| Branch 21 | 0.44 | 0.32 | 0.28 | 0.18 | 0.19 | 0.26 | 1.48 | 2.60 | 0.95 | 0.65 | 1.76 | 2.95 |
| Branch 22 | 2.21 | 1.68 | 1.68 | 1.21 | 1.18 | 1.65 | 1.32 | 0.73 | 1.90 | 1.60 | 1.35 | 1.26 |
| Branch 23 | 4.32 | 3.23 | 3.07 | 2.16 | 2.16 | 3.06 | 5.13 | 5.15 | 5.12 | 4.29 | 5.57 | 6.89 |
| Branch 24 | 1.08 | 0.81 | 0.77 | 0.54 | 0.54 | 0.76 | 1.28 | 1.29 | 1.28 | 1.07 | 1.39 | 1.72 |
| Branch 25 | 1.34 | 1.14 | 0.87 | 0.45 | 0.55 | 0.35 | 1.28 | 0.72 | 0.33 | 0.44 | 0.40 | 0.49 |
| Branch 26 | 1.28 | 1.09 | 0.83 | 0.42 | 0.52 | 0.31 | 1.10 | 0.55 | 0.25 | 0.36 | 0.30 | 0.35 |
| Branch 27 | 9.32 | 7.50 | 5.89 | 3.27 | 3.85 | 3.27 | 14.92 | 15.12 | 5.92 | 6.18 | 9.46 | 13.91 |
| Branch 28 | 4.93 | 3.97 | 3.12 | 1.73 | 2.04 | 1.73 | 7.91 | 8.01 | 3.13 | 3.27 | 5.01 | 7.37 |
| Branch 29 | 2.00 | 1.61 | 1.26 | 0.70 | 0.82 | 0.70 | 3.20 | 3.25 | 1.27 | 1.32 | 2.03 | 2.99 |


| Branch 30 | 2.16 | 1.74 | 1.36 | 0.76 | 0.89 | 0.76 | 3.47 | 3.52 | 1.37 | 1.43 | 2.20 | 3.24 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Branch 31 | 0.31 | 0.25 | 0.19 | 0.11 | 0.13 | 0.11 | 0.49 | 0.50 | 0.19 | 0.20 | 0.31 | 0.46 |
| Branch 32 | 0.08 | 0.07 | 0.05 | 0.03 | 0.03 | 0.03 | 0.14 | 0.14 | 0.05 | 0.06 | 0.09 | 0.13 |


| Active Power Losses (I^2*R) (Watt) for Scenario A |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Branch | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Branch 1 | 1.37 | 1.50 | 1.90 | 1.40 | 2.91 | 7.45 | 13.64 | 15.45 | 13.77 | 11.56 | 3.49 | 2.21 |
| Branch 2 | 8.58 | 9.63 | 12.42 | 7.67 | 19.22 | 48.70 | 85.91 | 104.40 | 90.43 | 71.93 | 21.88 | 15.34 |
| Branch 3 | 0.84 | 1.18 | 2.30 | 0.52 | 4.23 | 22.27 | 22.56 | 29.72 | 33.44 | 28.24 | 3.48 | 2.52 |
| Branch 4 | 0.63 | 0.89 | 1.74 | 0.38 | 3.52 | 18.56 | 17.47 | 22.20 | 27.42 | 21.50 | 2.79 | 1.93 |
| Branch 5 | 0.92 | 1.30 | 2.59 | 0.53 | 5.90 | 31.14 | 26.72 | 33.40 | 45.13 | 32.18 | 4.46 | 2.92 |
| Branch 6 | 0.33 | 0.22 | 0.06 | 0.57 | 0.53 | 0.02 | 0.11 | 4.25 | 0.19 | 1.24 | 0.75 | 0.15 |
| Branch 7 | 4.12 | 3.09 | 1.53 | 6.33 | 7.22 | 2.54 | 6.14 | 40.45 | 7.94 | 5.13 | 9.73 | 2.71 |
| Branch 8 | 3.27 | 2.69 | 1.80 | 4.59 | 6.03 | 4.52 | 9.53 | 31.27 | 11.27 | 3.32 | 7.88 | 2.76 |
| Branch 9 | 4.91 | 6.83 | 12.80 | 3.38 | 12.41 | 72.34 | 130.34 | 288.97 | 130.62 | 197.99 | 13.49 | 13.08 |
| Branch 10 | 0.73 | 1.02 | 1.92 | 0.51 | 1.86 | 10.90 | 19.71 | 43.98 | 19.75 | 30.04 | 2.02 | 1.96 |
| Branch 11 | 1.07 | 1.49 | 2.81 | 0.74 | 2.72 | 16.07 | 29.15 | 65.56 | 29.22 | 44.62 | 2.96 | 2.87 |
| Branch 12 | 3.10 | 4.32 | 8.13 | 2.13 | 7.88 | 46.84 | 85.37 | 193.69 | 85.58 | 131.27 | 8.57 | 8.31 |
| Branch 13 | 0.80 | 1.11 | 2.09 | 0.55 | 2.02 | 12.09 | 22.09 | 50.41 | 22.15 | 34.07 | 2.20 | 2.13 |
| Branch 14 | 0.56 | 0.78 | 1.46 | 0.38 | 1.42 | 8.50 | 15.57 | 35.72 | 15.61 | 24.08 | 1.54 | 1.49 |
| Branch 15 | 0.40 | 0.55 | 1.04 | 0.27 | 1.01 | 6.07 | 11.16 | 25.73 | 11.19 | 17.30 | 1.10 | 1.06 |
| Branch 16 | 0.30 | 0.42 | 0.80 | 0.21 | 0.78 | 4.69 | 8.64 | 20.03 | 8.66 | 13.44 | 0.85 | 0.82 |
| Branch 17 | 0.04 | 0.06 | 0.11 | 0.03 | 0.11 | 0.67 | 1.23 | 2.86 | 1.23 | 1.92 | 0.12 | 0.12 |
| Branch 18 | 0.06 | 0.09 | 0.14 | 0.01 | 0.22 | 1.02 | 2.02 | 3.04 | 2.51 | 2.12 | 0.20 | 0.21 |
| Branch 19 | 0.90 | 1.17 | 1.75 | 0.42 | 2.66 | 10.70 | 20.61 | 30.12 | 24.84 | 21.19 | 2.64 | 2.41 |
| Branch 20 | 0.37 | 0.45 | 0.63 | 0.26 | 0.95 | 3.34 | 6.36 | 8.97 | 7.38 | 6.35 | 1.01 | 0.81 |
| Branch 21 | 0.93 | 1.06 | 1.43 | 0.83 | 2.12 | 6.66 | 12.57 | 17.09 | 14.03 | 12.18 | 2.40 | 1.72 |
| Branch 22 | 2.67 | 2.53 | 2.13 | 2.99 | 2.83 | 1.01 | 8.62 | 8.05 | 4.28 | 2.34 | 4.68 | 3.06 |
| Branch 23 | 6.46 | 6.47 | 6.49 | 6.95 | 9.12 | 9.74 | 35.43 | 37.00 | 25.28 | 17.68 | 13.22 | 8.56 |
| Branch 24 | 1.62 | 1.62 | 1.62 | 1.74 | 2.28 | 2.44 | 8.91 | 9.31 | 6.35 | 4.44 | 3.31 | 2.14 |


| Branch 25 | 0.95 | 0.87 | 0.82 | 1.13 | 3.28 | 6.43 | 5.46 | 0.91 | 10.44 | 1.29 | 3.20 | 1.18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Branch 26 | 0.82 | 0.74 | 0.67 | 1.00 | 2.98 | 5.70 | 4.40 | 0.24 | 9.11 | 0.81 | 2.86 | 1.00 |
| Branch 27 | 10.29 | 10.33 | 11.44 | 11.30 | 30.12 | 71.81 | 90.53 | 59.98 | 129.66 | 47.81 | 31.86 | 14.92 |
| Branch 28 | 5.45 | 5.47 | 6.06 | 5.98 | 15.99 | 38.27 | 48.32 | 31.94 | 69.39 | 25.43 | 16.92 | 7.91 |
| Branch 29 | 2.21 | 2.21 | 2.45 | 2.42 | 6.49 | 15.58 | 19.69 | 12.99 | 28.32 | 10.33 | 6.87 | 3.20 |
| Branch 30 | 2.39 | 2.40 | 2.66 | 2.62 | 7.04 | 16.95 | 21.45 | 14.12 | 30.92 | 11.23 | 7.45 | 3.47 |
| Branch 31 | 0.34 | 0.34 | 0.38 | 0.37 | 1.00 | 2.41 | 3.05 | 2.00 | 4.39 | 1.59 | 1.06 | 0.49 |
| Branch 32 | 0.09 | 0.09 | 0.10 | 0.10 | 0.27 | 0.66 | 0.84 | 0.55 | 1.21 | 0.44 | 0.29 | 0.14 |


| Voltage Magnitude (pu) for Scenario B |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Bus1 | 1.0931 | 1.095307908 | 1.094418135 | 1.095916415 | 1.094013398 | 1.095481048 | 1.096634528 | 1.095291928 | 1.095162324 | 1.090965805 | 1.096481326 | 1.09370604 |
| Bus2 | 1.0918 | 1.093639803 | 1.092896789 | 1.094376287 | 1.092761595 | 1.093948995 | 1.095743593 | 1.094456342 | 1.094479743 | 1.090321512 | 1.095632057 | 1.092141884 |
| Bus3 | 1.0838 | 1.084396588 | 1.084326564 | 1.085937801 | 1.085629509 | 1.085600655 | 1.09085023 | 1.089792361 | 1.09057967 | 1.087168395 | 1.090976915 | 1.083487815 |
| Bus4 | 1.0793 | 1.078378098 | 1.078888961 | 1.080245166 | 1.081144856 | 1.080341854 | 1.089235972 | 1.088394454 | 1.089953102 | 1.086877389 | 1.089735807 | 1.07900089 |
| Bus5 | 1.0755 | 1.073176328 | 1.074198323 | 1.075375528 | 1.077275201 | 1.075882056 | 1.087746774 | 1.087084072 | 1.089442246 | 1.086658073 | 1.088658006 | 1.075118109 |
| Bus6 | 1.0685 | 1.063857835 | 1.065854275 | 1.066756563 | 1.070441283 | 1.068095506 | 1.084915887 | 1.084608309 | 1.088662048 | 1.086383559 | 1.086764634 | 1.068239317 |
| Bus7 | 1.0678 | 1.06267002 | 1.064659498 | 1.065310423 | 1.069304565 | 1.066754355 | 1.085713326 | 1.085314875 | 1.089180018 | 1.087103516 | 1.087368462 | 1.067774527 |
| Bus8 | 1.0642 | 1.053325696 | 1.055901752 | 1.05362861 | 1.061742087 | 1.056391024 | 1.094184476 | 1.094209093 | 1.095674681 | 1.094995133 | 1.094220642 | 1.066382826 |
| Bus9 | 1.0643 | 1.050847492 | 1.053491662 | 1.049747374 | 1.059535567 | 1.053162465 | 1.099843557 | 1.099999778 | 1.099999825 | 1.099999989 | 1.098966385 | 1.067823813 |
| Bus10 | 1.0433 | 1.02027932 | 1.025866764 | 1.019283641 | 1.037153562 | 1.024086971 | 1.094859565 | 1.096225157 | 1.096311083 | 1.097831082 | 1.093362677 | 1.046001628 |
| Bus11 | 1.0401 | 1.015666051 | 1.021697566 | 1.014686129 | 1.033775432 | 1.019698909 | 1.094107131 | 1.09565529 | 1.095754181 | 1.097503626 | 1.092516693 | 1.042707963 |
| Bus12 | 1.0348 | 1.007928794 | 1.014709819 | 1.006975428 | 1.028120225 | 1.012341942 | 1.092852114 | 1.094705006 | 1.094825532 | 1.096957744 | 1.09110547 | 1.037195023 |
| Bus13 | 1.0144 | 0.977996922 | 0.987717118 | 0.977147352 | 1.006330626 | 0.983902908 | 1.088054211 | 1.091073889 | 1.091277207 | 1.094873239 | 1.085709012 | 1.01596078 |
| Bus14 | 1.0072 | 0.967459432 | 0.97822072 | 0.966646575 | 0.998673492 | 0.973894459 | 1.086373762 | 1.089802359 | 1.090034686 | 1.094143487 | 1.083818722 | 1.00849992 |
| Bus15 | 1.0015 | 0.959162635 | 0.970743504 | 0.958378681 | 0.99264423 | 0.966014138 | 1.085050329 | 1.088800954 | 1.089056127 | 1.093568754 | 1.082330042 | 1.002625171 |
| Bus16 | 0.9964 | 0.951603282 | 0.963932988 | 0.950845719 | 0.987155455 | 0.958835397 | 1.08384739 | 1.087890809 | 1.088166751 | 1.093046461 | 1.080976833 | 0.997277426 |
| Bus17 | 0.9895 | 0.941454407 | 0.954794423 | 0.940732409 | 0.979797249 | 0.949200232 | 1.082239101 | 1.086674177 | 1.086977896 | 1.09234844 | 1.079167485 | 0.990109152 |
| Bus18 | 0.9878 | 0.938939007 | 0.952529077 | 0.938225814 | 0.977972747 | 0.946811963 | 1.08184 | 1.086372252 | 1.086682863 | 1.092175204 | 1.078718502 | 0.988331683 |
| Bus19 | 1.0922 | 1.093975251 | 1.093277151 | 1.094676478 | 1.093136231 | 1.094238422 | 1.095819248 | 1.094566808 | 1.094611845 | 1.090238582 | 1.095720812 | 1.092444973 |


| Bus20 | 1.0966 | 1.09777506 | 1.097375811 | 1.098092478 | 1.097107992 | 1.09769963 | 1.097573303 | 1.096625033 | 1.096573305 | 1.090730692 | 1.097540768 | 1.096385332 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus21 | 1.0978 | 1.098617812 | 1.098351119 | 1.098825591 | 1.098145513 | 1.098529708 | 1.098284246 | 1.097474098 | 1.09731479 | 1.09120975 | 1.0982683 | 1.097573331 |
| Bus22 | 1.1000 | 1.09999994 | 1.09999993 | 1.099999926 | 1.09999982 | 1.099999952 | 1.099999666 | 1.099488739 | 1.098989632 | 1.092690667 | 1.099999731 | 1.099999325 |
| Bus23 | 1.0830 | 1.084751593 | 1.084455464 | 1.086615001 | 1.085671508 | 1.085769999 | 1.088621423 | 1.087445073 | 1.087973068 | 1.084757464 | 1.088533748 | 1.082134543 |
| Bus24 | 1.0771 | 1.079644748 | 1.079479009 | 1.082436847 | 1.081489685 | 1.080799644 | 1.082217732 | 1.081034342 | 1.081565499 | 1.078859142 | 1.081865941 | 1.074696411 |
| Bus25 | 1.0742 | 1.077099581 | 1.07699888 | 1.080354856 | 1.079405863 | 1.078322561 | 1.079025543 | 1.077838636 | 1.078371373 | 1.075919119 | 1.078541942 | 1.070987876 |
| Bus26 | 1.0681 | 1.063993882 | 1.066024966 | 1.067338707 | 1.070606854 | 1.06866107 | 1.083432425 | 1.083081938 | 1.087843532 | 1.085462825 | 1.085727984 | 1.067641452 |
| Bus27 | 1.0679 | 1.064534536 | 1.066574602 | 1.068386356 | 1.071089983 | 1.069685221 | 1.081834211 | 1.081423922 | 1.087002391 | 1.084486922 | 1.084658495 | 1.06727016 |
| Bus28 | 1.0582 | 1.055723633 | 1.05877499 | 1.062571568 | 1.064800344 | 1.063877671 | 1.069710734 | 1.0692956 | 1.079354221 | 1.076625518 | 1.07503708 | 1.055477085 |
| Bus29 | 1.0520 | 1.050158744 | 1.053849255 | 1.058899935 | 1.060828725 | 1.060210614 | 1.062051564 | 1.061633363 | 1.074524248 | 1.071660789 | 1.068959947 | 1.048026761 |
| Bus30 | 1.0492 | 1.04760829 | 1.051591814 | 1.057217359 | 1.059008648 | 1.058530135 | 1.058540886 | 1.058121277 | 1.072310717 | 1.069385482 | 1.066174663 | 1.04461183 |
| Bus31 | 1.0445 | 1.043433657 | 1.047897499 | 1.054464831 | 1.056030928 | 1.055781045 | 1.052791157 | 1.052369233 | 1.068688459 | 1.065661977 | 1.061615117 | 1.039019093 |
| Bus32 | 1.0435 | 1.042507868 | 1.047078222 | 1.053854402 | 1.055370561 | 1.055171378 | 1.051516094 | 1.051093657 | 1.06788516 | 1.064836226 | 1.060603972 | 1.037778843 |
| Bus33 | 1.0429 | 1.04195239 | 1.046586661 | 1.05348816 | 1.054974355 | 1.054805594 | 1.050751008 | 1.050328263 | 1.067403188 | 1.064340781 | 1.059997273 | 1.037034648 |


| Voltage Magnitude (pu) for Scenario B |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Bus1 | 1.095949734 | 1.095409499 | 1.095711091 | 1.095692081 | 1.094690478 | 1.096293608 | 1.094927535 | 1.094541082 | 1.098248931 | 1.095153889 | 1.095020638 | 1.093902293 |
| Bus2 | 1.095023183 | 1.094440681 | 1.094772766 | 1.094765523 | 1.093330666 | 1.094591125 | 1.092660317 | 1.092404675 | 1.095905825 | 1.093424436 | 1.093530458 | 1.092719727 |
| Bus3 | 1.089882306 | 1.088987531 | 1.08948763 | 1.089862663 | 1.085673856 | 1.085216122 | 1.080247993 | 1.080652125 | 1.08309405 | 1.083987268 | 1.085370138 | 1.085844286 |
| Bus4 | 1.088453728 | 1.087296489 | 1.087937092 | 1.088712302 | 1.082516726 | 1.080462618 | 1.076311354 | 1.077599659 | 1.077531125 | 1.080643068 | 1.082504096 | 1.08338501 |
| Bus5 | 1.087219009 | 1.085832566 | 1.086577115 | 1.087724651 | 1.079629233 | 1.075951387 | 1.072734145 | 1.074965045 | 1.072291567 | 1.077624271 | 1.079937288 | 1.081231995 |
| Bus6 | 1.085062399 | 1.083269988 | 1.084153212 | 1.086072715 | 1.07416967 | 1.066985651 | 1.065966881 | 1.070265323 | 1.061835305 | 1.071967908 | 1.075196339 | 1.077394404 |
| Bus7 | 1.08563543 | 1.083721997 | 1.084744688 | 1.08664167 | 1.074928056 | 1.068259889 | 1.067602656 | 1.071775835 | 1.063812735 | 1.073132634 | 1.076106693 | 1.077764825 |
| Bus8 | 1.092419505 | 1.089639978 | 1.091700211 | 1.094594429 | 1.084021318 | 1.082072474 | 1.086856931 | 1.088371112 | 1.084278754 | 1.086114051 | 1.086649254 | 1.083422612 |
| Bus9 | 1.097233322 | 1.094056846 | 1.096622533 | 1.099999788 | 1.09064846 | 1.091659686 | 1.099999749 | 1.0999482 | 1.098221413 | 1.095272764 | 1.094202702 | 1.087999182 |
| Bus10 | 1.090611414 | 1.086244585 | 1.089956083 | 1.09450833 | 1.080111067 | 1.080213843 | 1.086371318 | 1.0856631 | 1.083868106 | 1.083093203 | 1.083214184 | 1.077180566 |
| Bus11 | 1.089611732 | 1.085065223 | 1.088949677 | 1.093679292 | 1.078520382 | 1.078486046 | 1.084314114 | 1.083506792 | 1.081701505 | 1.081254667 | 1.08155541 | 1.075547436 |
| Bus12 | 1.087943783 | 1.083097016 | 1.087270495 | 1.092296366 | 1.075864277 | 1.075600475 | 1.08087697 | 1.079903592 | 1.07808104 | 1.078183727 | 1.078785381 | 1.072820291 |
| Bus13 | 1.081562907 | 1.075563683 | 1.080846515 | 1.087008373 | 1.065686204 | 1.06453887 | 1.067689205 | 1.066074808 | 1.064185451 | 1.066408035 | 1.068168879 | 1.062368626 |


| Bus14 | 1.079327405 | 1.072923876 | 1.078595892 | 1.085156114 | 1.062117917 | 1.060660207 | 1.063063286 | 1.061223457 | 1.059310583 | 1.06227847 | 1.064446609 | 1.058704221 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus15 | 1.077566875 | 1.070844972 | 1.076823456 | 1.083697381 | 1.059307901 | 1.057605802 | 1.059420503 | 1.057403178 | 1.055471788 | 1.059026505 | 1.061515346 | 1.055818522 |
| Bus16 | 1.075966427 | 1.068954916 | 1.075212178 | 1.082371407 | 1.056752575 | 1.054828031 | 1.056107064 | 1.053928099 | 1.05197984 | 1.056068896 | 1.05884967 | 1.053194306 |
| Bus17 | 1.073826194 | 1.06642697 | 1.073057448 | 1.080598503 | 1.053333484 | 1.051110816 | 1.051671662 | 1.04927587 | 1.047304965 | 1.052110621 | 1.055282714 | 1.049682882 |
| Bus18 | 1.073295127 | 1.065799729 | 1.072522784 | 1.080158562 | 1.052485228 | 1.050188633 | 1.050571407 | 1.048121862 | 1.046145344 | 1.051128665 | 1.05439779 | 1.048811731 |
| Bus19 | 1.095148323 | 1.094612322 | 1.094924949 | 1.094816112 | 1.093534755 | 1.094686858 | 1.092759088 | 1.092547838 | 1.095904618 | 1.09353777 | 1.093663996 | 1.093015143 |
| Bus 20 | 1.097308741 | 1.097135771 | 1.097268543 | 1.096642068 | 1.096645222 | 1.097183611 | 1.095974987 | 1.095963281 | 1.097959258 | 1.096397639 | 1.096477354 | 1.096590166 |
| Bus21 | 1.098123142 | 1.098027357 | 1.098111478 | 1.097515147 | 1.097695228 | 1.098025418 | 1.097149505 | 1.097167887 | 1.098542595 | 1.097460415 | 1.097528147 | 1.097711554 |
| Bus22 | 1.099999735 | 1.099999778 | 1.099999729 | 1.099736227 | 1.099999746 | 1.099999936 | 1.099999946 | 1.09999975 | 1.099999902 | 1.099999708 | 1.099999764 | 1.099999909 |
| Bus23 | 1.087276373 | 1.086476072 | 1.086902656 | 1.087076018 | 1.083096835 | 1.083100798 | 1.074487653 | 1.074427024 | 1.07902068 | 1.080109688 | 1.082001486 | 1.083125377 |
| Bus24 | 1.080072385 | 1.079266681 | 1.079696146 | 1.079606193 | 1.074535279 | 1.074272883 | 1.057577931 | 1.057104526 | 1.064770845 | 1.068171216 | 1.071694444 | 1.074830256 |
| Bus25 | 1.076480758 | 1.075672354 | 1.076103258 | 1.075881859 | 1.070265799 | 1.069870373 | 1.049132795 | 1.048452594 | 1.05765759 | 1.062214198 | 1.066552958 | 1.070693833 |
| Bus26 | 1.084012532 | 1.082272136 | 1.083019332 | 1.084934925 | 1.072209029 | 1.063669262 | 1.062634859 | 1.067794233 | 1.057541876 | 1.069585121 | 1.073264866 | 1.076242274 |
| Bus27 | 1.082944275 | 1.08127732 | 1.08185302 | 1.083762445 | 1.070141793 | 1.060043773 | 1.05910599 | 1.065268545 | 1.052871827 | 1.06708434 | 1.071257055 | 1.075110887 |
| Bus28 | 1.072814502 | 1.071131239 | 1.071218485 | 1.073147515 | 1.052781672 | 1.033453737 | 1.029195732 | 1.040976814 | 1.017102944 | 1.045492012 | 1.053400842 | 1.06290751 |
| Bus29 | 1.066415991 | 1.06472241 | 1.064500842 | 1.066442279 | 1.041808826 | 1.016631286 | 1.010266035 | 1.025612214 | 0.994450258 | 1.031838507 | 1.042113926 | 1.055197756 |
| Bus30 | 1.06348336 | 1.061785046 | 1.061421887 | 1.063369014 | 1.036778279 | 1.008916038 | 1.001583097 | 1.018566288 | 0.984056768 | 1.025577953 | 1.036939297 | 1.051663872 |
| Bus31 | 1.058682138 | 1.056976048 | 1.056380627 | 1.058337107 | 1.028530864 | 0.996243314 | 0.987311011 | 1.006998666 | 0.966951056 | 1.015305342 | 1.028454879 | 1.045875969 |
| Bus32 | 1.057617401 | 1.055909588 | 1.055262663 | 1.057221216 | 1.026701977 | 0.993433265 | 0.984146379 | 1.004433625 | 0.963158227 | 1.013027423 | 1.026573442 | 1.044592441 |
| Bus33 | 1.056978542 | 1.055269693 | 1.05459186 | 1.056551658 | 1.025604471 | 0.991746668 | 0.982246836 | 1.002894156 | 0.960881333 | 1.011660348 | 1.025444391 | 1.043822274 |


| Lambda P (UM/MWh) for Scenario B |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Bus1 | 273.8056657 | 328.8511766 | 301.9717297 | 305.4257832 | 251.3565917 | 304.2644199 | 171.9343241 | 162.0864962 | 133.7183429 | 123.4477874 | 165.2800617 | 308.2120968 |
| Bus2 | 274.4455849 | 329.7537801 | 302.7340163 | 306.1949304 | 251.8846893 | 305.0324513 | 172.2017078 | 162.3266575 | 133.8802435 | 123.5888678 | 165.5251764 | 309.0304357 |
| Bus3 | 278.8068886 | 336.2980474 | 308.1764558 | 311.7864804 | 255.5181766 | 310.4481713 | 173.8162276 | 163.7032244 | 134.8287907 | 124.3014109 | 166.9856587 | 314.3943741 |
| Bus4 | 281.2818936 | 340.5973027 | 311.6607489 | 315.5719215 | 257.8127305 | 313.8952109 | 174.359922 | 164.0961253 | 134.9644608 | 124.3550654 | 167.3799968 | 317.2217853 |
| Bus5 | 283.5407541 | 344.5462497 | 314.8432754 | 319.0241572 | 259.8938265 | 317.0119742 | 174.8750928 | 164.4686121 | 135.0757093 | 124.3944695 | 167.7349242 | 319.7914861 |
| Bus6 | 287.6997452 | 351.8711286 | 320.7039445 | 325.36543 | 263.691998 | 322.6702528 | 175.8725478 | 165.1903536 | 135.2500704 | 124.4437135 | 168.3774302 | 324.4949424 |


| Bus7 | 288.2248702 | 353.2633459 | 321.8308186 | 326.8133063 | 264.4228843 | 323.9613183 | 175.6085522 | 164.8839056 | 135.0763519 | 124.2438849 | 168.1827025 | 324.9587244 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus8 | 291.4357598 | 363.2840971 | 329.8863624 | 337.603577 | 269.5890525 | 333.4227914 | 172.9814924 | 161.9406985 | 133.3648775 | 122.3592977 | 166.1624437 | 327.3264857 |
| Bus9 | 292.4006326 | 367.6387385 | 333.3473463 | 342.5774124 | 271.7669099 | 337.6658824 | 171.2821816 | 160.0944593 | 132.2656951 | 121.1954753 | 164.8098788 | 327.6196255 |
| Bus10 | 302.3355066 | 386.6756204 | 348.7449059 | 360.2699626 | 281.7003143 | 354.1699037 | 172.5566687 | 160.9936387 | 132.9915222 | 121.5847892 | 166.1921156 | 339.1772532 |
| Bus11 | 304.0512839 | 390.0073948 | 351.4287095 | 363.36615 | 283.4192208 | 357.0522635 | 172.7721638 | 161.1454452 | 133.1140485 | 121.6503882 | 166.4260113 | 341.1752766 |
| Bus12 | 306.9558692 | 395.6928745 | 355.9971079 | 368.6492964 | 286.3325058 | 361.964517 | 173.1325158 | 161.3990784 | 133.3187485 | 121.7598707 | 166.8173055 | 344.5596686 |
| Bus13 | 317.149817 | 416.0955221 | 372.2776928 | 387.6049591 | 296.5901845 | 379.529069 | 174.3552159 | 162.2576627 | 134.0115705 | 122.1293615 | 168.1466008 | 356.4570366 |
| Bus14 | 320.4444448 | 422.8622584 | 377.6338543 | 393.8905556 | 299.9178942 | 385.3301483 | 174.7352778 | 162.5238269 | 134.2263068 | 122.2435018 | 168.5603725 | 360.3095137 |
| Bus15 | 323.3412857 | 428.8368702 | 382.3566864 | 399.440168 | 302.8455392 | 390.4485653 | 175.0675529 | 162.7564395 | 134.4139693 | 122.3432074 | 168.9221883 | 363.6978525 |
| Bus16 | 326.1160545 | 434.5954549 | 386.8997744 | 404.7888623 | 305.6523075 | 395.3768449 | 175.3830039 | 162.977143 | 134.5920165 | 122.4377357 | 169.2657902 | 366.9448643 |
| Bus17 | 329.4391453 | 441.6348241 | 392.4177696 | 411.3261562 | 309.0237286 | 401.381194 | 175.7493691 | 163.2329282 | 134.7983341 | 122.5469869 | 169.6652865 | 370.8393652 |
| Bus18 | 330.3745559 | 443.6088787 | 393.9669198 | 413.1594624 | 309.9721943 | 403.0659662 | 175.8531815 | 163.3054429 | 134.8568269 | 122.57798 | 169.7784579 | 371.9352987 |
| Bus19 | 274.0069177 | 328.971749 | 302.1012048 | 305.5055237 | 251.4905069 | 304.4077749 | 172.1209893 | 162.2823824 | 133.8412354 | 123.5966166 | 165.453972 | 308.5561742 |
| Bus20 | 269.8108277 | 321.6360302 | 296.1664657 | 299.0470217 | 247.7543817 | 298.4660225 | 171.1398725 | 161.643511 | 133.3459402 | 123.4530175 | 164.5797329 | 303.7954552 |
| Bus21 | 268.6235142 | 319.5970713 | 294.5172557 | 297.2538391 | 246.7058282 | 296.793093 | 170.8079798 | 161.40685 | 133.1740319 | 123.3556218 | 164.2821971 | 302.3912832 |
| Bus22 | 266.4889315 | 315.9936474 | 291.6022478 | 294.0882535 | 244.8349388 | 293.8011931 | 170.1222858 | 160.8894254 | 132.8126486 | 123.0866634 | 163.6649376 | 299.7751128 |
| Bus23 | 279.3974644 | 336.4073406 | 308.3697271 | 311.6934933 | 255.669867 | 310.6312041 | 174.570977 | 164.4420232 | 135.4988417 | 124.8710625 | 167.7776233 | 315.4124145 |
| Bus24 | 281.8438976 | 338.9457424 | 310.6372387 | 313.6111016 | 257.2455749 | 312.9097334 | 176.2207341 | 165.9994909 | 136.7809128 | 125.9608948 | 169.4295165 | 318.9023512 |
| Bus25 | 283.0786128 | 340.2248334 | 311.7795394 | 314.5756028 | 258.0381216 | 314.0575572 | 177.0541275 | 166.7862848 | 137.4285752 | 126.5109129 | 170.2644226 | 320.6690756 |
| Bus26 | 287.9831777 | 351.8799117 | 320.6854462 | 325.0951816 | 263.6701998 | 322.4108086 | 176.3787387 | 165.6822289 | 135.4656434 | 124.663991 | 168.7178056 | 324.9476556 |
| Bus27 | 288.2008774 | 351.6936675 | 320.4993598 | 324.5960346 | 263.5335922 | 321.9280473 | 176.9558987 | 166.2473416 | 135.702859 | 124.9112933 | 169.0935646 | 325.3389516 |
| Bus28 | 292.383767 | 356.2977868 | 324.1976714 | 327.3710697 | 265.9669523 | 324.6734268 | 180.1153573 | 169.217984 | 137.2084864 | 126.3398485 | 171.4694741 | 331.064232 |
| Bus29 | 295.085328 | 359.2655301 | 326.5763964 | 329.1485044 | 267.527036 | 326.4318141 | 182.1653845 | 171.145535 | 138.176294 | 127.2585533 | 173.0029928 | 334.7777891 |
| Bus30 | 296.461548 | 360.7760254 | 327.7859407 | 330.0506328 | 268.3191846 | 327.3242633 | 183.211844 | 172.1294837 | 138.6682741 | 127.7256709 | 173.7839703 | 336.6731192 |
| Bus31 | 298.4725374 | 362.9800576 | 329.5480799 | 331.3609502 | 269.4705543 | 328.620494 | 184.7461358 | 173.5721446 | 139.3847 | 128.4061265 | 174.9246341 | 339.4512852 |
| Bus32 | 298.9036258 | 363.4521097 | 329.9251244 | 331.6407971 | 269.7165592 | 328.8973287 | 185.0757207 | 173.8820489 | 139.537951 | 128.5517142 | 175.1690835 | 340.0479746 |
| Bus33 | 299.1420007 | 363.7129583 | 330.1333184 | 331.7950996 | 269.8522464 | 329.0499687 | 185.2582593 | 174.0536886 | 139.6225542 | 128.6320999 | 175.3042245 | 340.3784069 |


| Lambda P (UM/MWh) for Scenario B |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Bus1 | 180.5413537 | 189.5583118 | 182.755187 | 180.5402732 | 262.2038785 | 321.6739992 | 426.5009901 | 404.3881529 | 434.9006314 | 329.6448906 | 286.2309527 | 231.392277 |
| Bus2 | 180.8332155 | 189.878349 | 183.0545352 | 180.8351241 | 262.8205903 | 322.6079896 | 428.1597186 | 405.8721183 | 436.612831 | 330.6257808 | 286.9680085 | 231.8677725 |
| Bus3 | 182.6001769 | 191.8474707 | 184.8970506 | 182.4448615 | 266.7811112 | 328.8633532 | 439.0636998 | 415.6084873 | 448.7485973 | 336.9041779 | 291.5927323 | 234.9389927 |
| Bus4 | 183.0972878 | 192.4634382 | 185.4449743 | 182.7927543 | 268.4536419 | 332.1372618 | 442.7439295 | 418.3439883 | 454.3939546 | 339.2340105 | 293.2725504 | 236.0525529 |
| Bus5 | 183.5431878 | 193.0164924 | 185.9425866 | 183.0949962 | 270.0353449 | 335.3471676 | 446.2365531 | 420.8461362 | 459.9380764 | 341.4256834 | 294.8368673 | 237.0646601 |
| Bus6 | 184.3464163 | 194.0139238 | 186.8543904 | 183.6149211 | 273.0895036 | 341.8248516 | 453.00734 | 425.4695853 | 471.1461807 | 345.6284248 | 297.7983975 | 238.9201146 |
| Bus7 | 184.1430576 | 193.8424696 | 186.6422188 | 183.3229789 | 272.7107408 | 341.0490267 | 451.4314256 | 424.3258677 | 469.541866 | 344.9148653 | 297.3079237 | 238.7541199 |
| Bus8 | 181.9702075 | 191.8788098 | 184.3828886 | 180.4059101 | 268.5112257 | 333.0148391 | 435.6805926 | 412.444543 | 453.4027824 | 337.3602033 | 292.0007867 | 236.5538427 |
| Bus9 | 180.4829639 | 190.4676501 | 182.8405264 | 178.5125472 | 265.5673579 | 327.6744571 | 425.5428851 | 404.5484449 | 442.9668247 | 332.2524124 | 288.3479823 | 234.8321395 |
| Bus10 | 182.2794659 | 192.7182301 | 184.6739957 | 179.9778779 | 269.8456051 | 333.4175276 | 434.4084594 | 413.3996516 | 452.722771 | 338.4406406 | 293.1819854 | 238.7290527 |
| Bus11 | 182.5838542 | 193.1001373 | 184.9846663 | 180.2257979 | 270.5741329 | 334.3966137 | 435.9238869 | 414.9138877 | 454.3919775 | 339.4965516 | 294.0055866 | 239.392904 |
| Bus12 | 183.093453 | 193.7400741 | 185.5048009 | 180.6405192 | 271.7973302 | 336.0415819 | 438.4738758 | 417.4631102 | 457.2022792 | 341.2715017 | 295.3888339 | 240.5077603 |
| Bus13 | 184.8281276 | 195.9236168 | 187.2755067 | 182.0490787 | 275.9940002 | 341.6955197 | 447.2754914 | 426.2738158 | 466.9171177 | 347.3807622 | 300.1385887 | 244.3350878 |
| Bus14 | 185.3693302 | 196.6067335 | 187.8280109 | 182.4874079 | 277.3152461 | 343.479275 | 450.0658409 | 429.0713447 | 470.0023779 | 349.3112845 | 301.6354048 | 245.5409122 |
| Bus15 | 185.8427274 | 197.2044938 | 188.311301 | 182.8706837 | 278.4724371 | 345.0420146 | 452.5121787 | 431.5245305 | 472.7079626 | 351.0030012 | 302.9465471 | 246.5971208 |
| Bus16 | 186.2925245 | 197.7728021 | 188.7705089 | 183.2346441 | 279.5741782 | 346.5305736 | 454.8449739 | 433.8646794 | 475.289006 | 352.6150068 | 304.1951345 | 247.6028804 |
| Bus17 | 186.8164398 | 198.4361849 | 189.3054315 | 183.6577234 | 280.8666407 | 348.279682 | 457.596602 | 436.6283256 | 478.3376521 | 354.5115796 | 305.6609742 | 248.783408 |
| Bus18 | 186.9647949 | 198.6239396 | 189.4569006 | 183.7775813 | 281.2320324 | 348.7739926 | 458.3735836 | 437.4084981 | 479.1982486 | 355.0474152 | 306.0753118 | 249.1171131 |
| Bus19 | 180.7371884 | 189.7586432 | 182.946335 | 180.8014257 | 262.5408996 | 322.1162086 | 427.383112 | 405.1668124 | 435.518551 | 330.2013573 | 286.674275 | 231.6361747 |
| Bus20 | 179.615216 | 188.425827 | 181.7272469 | 180.1535041 | 259.5712344 | 316.988469 | 419.0681263 | 397.6709243 | 424.509086 | 325.5614497 | 283.402454 | 229.2649255 |
| Bus21 | 179.2449852 | 188.0000519 | 181.3345632 | 179.885674 | 258.6552427 | 315.4288644 | 416.4903731 | 395.3596485 | 421.2564258 | 324.0999269 | 282.357843 | 228.5537976 |
| Bus22 | 178.4929031 | 187.1549324 | 180.5503963 | 179.2651933 | 256.8856032 | 312.451448 | 411.4989003 | 390.9014651 | 415.1998205 | 321.2330685 | 280.2872245 | 227.2097424 |
| Bus23 | 183.5292894 | 192.7967164 | 185.8317138 | 183.4272984 | 268.1810098 | 330.3579435 | 444.2391942 | 420.8620163 | 452.6240248 | 339.55667 | 293.5717349 | 236.2228359 |
| Bus24 | 185.4857981 | 194.8551179 | 187.8141605 | 185.4562548 | 271.6006138 | 334.7035718 | 455.6988356 | 431.9931302 | 462.3709862 | 345.6496592 | 298.0976713 | 239.1396795 |
| Bus25 | 186.475723 | 195.896619 | 188.817219 | 186.4833761 | 273.3355631 | 336.9095069 | 461.6122723 | 437.7418661 | 467.3730321 | 348.7619839 | 300.4019383 | 240.6187698 |
| Bus26 | 184.7271824 | 194.3990047 | 187.2709473 | 184.0281445 | 274.1690932 | 344.1625166 | 456.2077166 | 427.6643459 | 475.480348 | 347.3144793 | 298.9625672 | 239.4789402 |
| Bus27 | 185.1440092 | 194.8152833 | 187.7305573 | 184.4856544 | 275.389202 | 346.8927252 | 459.8814902 | 430.1102452 | 480.5675694 | 349.22095 | 300.265952 | 240.0792446 |
| Bus28 | 187.8905081 | 197.7145773 | 190.6606022 | 187.3544305 | 282.5985081 | 361.2712413 | 481.5322896 | 446.2164773 | 508.2371568 | 360.7453013 | 308.3520689 | 244.4225499 |
| Bus29 | 189.6651512 | 199.5880731 | 192.5558744 | 189.2099233 | 287.3299834 | 370.9033938 | 496.1440927 | 456.9487739 | 527.1669221 | 368.3788828 | 313.6644223 | 247.2416409 |


| Bus30 | 190.569362 | 200.542679 | 193.5220063 | 190.155744 | 289.757481 | 375.8905622 | 503.7346773 | 462.4923709 | 537.0608859 | 372.3114314 | 316.3911794 | 248.6808859 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus31 | 191.8910596 | 201.9381072 | 194.9353172 | 191.5392619 | 293.3466048 | 383.3782658 | 515.1955317 | 470.7821454 | 552.1566902 | 378.1658683 | 320.4258183 | 250.7915733 |
| Bus32 | 192.1744435 | 202.2373086 | 195.2384897 | 191.8360328 | 294.1215946 | 385.0104503 | 517.7024975 | 472.5846946 | 555.4801797 | 379.4353671 | 321.2974137 | 251.2450419 |
| Bus33 | 192.3311677 | 202.4027843 | 195.4062199 | 192.0002169 | 294.5525189 | 385.9245363 | 519.1101638 | 473.5923086 | 557.3553349 | 380.1435318 | 321.7822268 | 251.4962211 |


| Active Power Losses (1^2*R) (Watt) for Scenario B |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Branch 1 | 2.90 | 4.10 | 3.44 | 3.47 | 2.36 | 3.46 | 1.27 | 1.15 | 0.77 | 0.68 | 1.16 | 3.74 |
| Branch 2 | 20.59 | 28.51 | 24.27 | 23.80 | 16.56 | 23.14 | 7.82 | 6.96 | 4.87 | 3.20 | 7.04 | 24.44 |
| Branch 3 | 8.38 | 15.51 | 12.57 | 13.87 | 8.46 | 11.77 | 1.10 | 0.80 | 0.15 | 0.03 | 0.64 | 8.46 |
| Branch 4 | 6.41 | 11.78 | 9.49 | 10.35 | 6.36 | 8.64 | 0.93 | 0.69 | 0.10 | 0.02 | 0.48 | 6.40 |
| Branch 5 | 9.63 | 17.53 | 14.00 | 15.04 | 9.34 | 12.28 | 1.58 | 1.20 | 0.12 | 0.01 | 0.71 | 9.45 |
| Branch 6 | 0.57 | 2.51 | 2.08 | 3.30 | 1.41 | 2.70 | 0.59 | 0.77 | 0.38 | 0.61 | 0.36 | 0.30 |
| Branch 7 | 2.06 | 13.01 | 10.69 | 18.57 | 7.18 | 14.65 | 6.52 | 7.96 | 4.14 | 6.06 | 4.24 | 0.71 |
| Branch 8 | 0.42 | 4.01 | 3.21 | 6.27 | 2.07 | 4.72 | 4.65 | 5.38 | 2.91 | 3.94 | 3.22 | 0.35 |
| Branch 9 | 49.06 | 103.58 | 84.69 | 102.87 | 55.71 | 93.77 | 2.78 | 1.60 | 1.52 | 0.53 | 3.51 | 52.97 |
| Branch 10 | 7.38 | 15.64 | 12.78 | 15.54 | 8.39 | 14.15 | 0.42 | 0.24 | 0.23 | 0.08 | 0.53 | 7.97 |
| Branch 11 | 10.86 | 23.11 | 18.85 | 22.95 | 12.34 | 20.89 | 0.61 | 0.35 | 0.33 | 0.11 | 0.77 | 11.73 |
| Branch 12 | 31.58 | 67.57 | 55.02 | 67.10 | 35.93 | 61.04 | 1.75 | 1.01 | 0.96 | 0.33 | 2.22 | 34.14 |
| Branch 13 | 8.14 | 17.47 | 14.21 | 17.35 | 9.27 | 15.77 | 0.45 | 0.26 | 0.25 | 0.08 | 0.57 | 8.80 |
| Branch 14 | 5.72 | 12.30 | 10.00 | 12.21 | 6.51 | 11.10 | 0.31 | 0.18 | 0.17 | 0.06 | 0.40 | 6.18 |
| Branch 15 | 4.08 | 8.81 | 7.15 | 8.74 | 4.65 | 7.94 | 0.22 | 0.13 | 0.12 | 0.04 | 0.28 | 4.41 |
| Branch 16 | 3.15 | 6.81 | 5.53 | 6.77 | 3.59 | 6.14 | 0.17 | 0.10 | 0.09 | 0.03 | 0.22 | 3.41 |
| Branch 17 | 0.45 | 0.97 | 0.79 | 0.96 | 0.51 | 0.87 | 0.02 | 0.01 | 0.01 | 0.00 | 0.03 | 0.48 |
| Branch 18 | 0.48 | 0.93 | 0.75 | 0.84 | 0.45 | 0.70 | 0.05 | 0.02 | 0.03 | 0.00 | 0.04 | 0.43 |
| Branch 19 | 4.84 | 8.98 | 7.20 | 8.00 | 4.41 | 6.92 | 0.72 | 0.50 | 0.44 | 0.05 | 0.66 | 4.75 |
| Branch 20 | 1.45 | 2.56 | 2.06 | 2.28 | 1.29 | 2.03 | 0.32 | 0.25 | 0.20 | 0.07 | 0.29 | 1.55 |
| Branch 21 | 2.74 | 4.65 | 3.75 | 4.12 | 2.41 | 3.77 | 0.82 | 0.71 | 0.51 | 0.33 | 0.75 | 3.18 |
| Branch 22 | 0.55 | 0.22 | 0.20 | 0.21 | 0.13 | 0.21 | 1.96 | 2.10 | 2.55 | 2.16 | 2.33 | 1.16 |


| Branch 23 | 4.35 | 3.25 | 3.08 | 2.17 | 2.18 | 3.08 | 5.11 | 5.12 | 5.11 | 4.33 | 5.54 | 6.89 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Branch 24 | 1.09 | 0.81 | 0.77 | 0.54 | 0.54 | 0.77 | 1.28 | 1.28 | 1.28 | 1.08 | 1.39 | 1.72 |
| Branch 25 | 0.23 | 0.05 | 0.05 | 0.21 | 0.04 | 0.20 | 1.82 | 1.95 | 0.58 | 0.70 | 0.91 | 0.43 |
| Branch 26 | 0.17 | 0.17 | 0.17 | 0.49 | 0.13 | 0.47 | 1.69 | 1.84 | 0.51 | 0.64 | 0.80 | 0.30 |
| Branch 27 | 9.59 | 7.79 | 6.11 | 3.40 | 3.98 | 3.39 | 14.73 | 14.74 | 5.87 | 6.21 | 9.29 | 13.94 |
| Branch 28 | 5.08 | 4.12 | 3.23 | 1.80 | 2.10 | 1.79 | 7.80 | 7.81 | 3.11 | 3.28 | 4.92 | 7.38 |
| Branch 29 | 2.06 | 1.67 | 1.31 | 0.73 | 0.85 | 0.72 | 3.16 | 3.16 | 1.26 | 1.33 | 1.99 | 2.99 |
| Branch 30 | 2.23 | 1.81 | 1.42 | 0.79 | 0.92 | 0.78 | 3.43 | 3.43 | 1.36 | 1.44 | 2.16 | 3.24 |
| Branch 31 | 0.32 | 0.26 | 0.20 | 0.11 | 0.13 | 0.11 | 0.49 | 0.49 | 0.19 | 0.20 | 0.31 | 0.46 |
| Branch 32 | 0.09 | 0.07 | 0.06 | 0.03 | 0.04 | 0.03 | 0.13 | 0.13 | 0.05 | 0.06 | 0.08 | 0.13 |

Active Power Losses (।^2*R) (Watt) for Scenario B

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Branch 1 | 1.37 | 1.50 | 1.41 | 1.40 | 2.91 | 4.47 | 7.98 | 7.10 | 8.31 | 4.67 | 3.49 | 2.21 |
| Branch 2 | 8.58 | 9.63 | 9.07 | 7.67 | 19.22 | 29.43 | 51.38 | 45.97 | 56.14 | 29.48 | 21.88 | 15.34 |
| Branch 3 | 0.84 | 1.18 | 1.00 | 0.52 | 4.23 | 9.84 | 6.66 | 3.98 | 13.77 | 4.78 | 3.48 | 2.52 |
| Branch 4 | 0.63 | 0.89 | 0.77 | 0.38 | 3.52 | 8.77 | 5.50 | 3.01 | 12.16 | 3.90 | 2.79 | 1.93 |
| Branch 5 | 0.92 | 1.30 | 1.16 | 0.53 | 5.90 | 15.93 | 9.12 | 4.45 | 21.88 | 6.37 | 4.46 | 2.92 |
| Branch 6 | 0.33 | 0.22 | 0.35 | 0.57 | 0.53 | 1.40 | 3.00 | 1.99 | 3.18 | 1.18 | 0.75 | 0.15 |
| Branch 7 | 4.12 | 3.09 | 4.33 | 6.33 | 7.22 | 16.76 | 34.25 | 24.17 | 36.58 | 14.74 | 9.73 | 2.71 |
| Branch 8 | 3.27 | 2.69 | 3.42 | 4.59 | 6.03 | 12.81 | 25.14 | 18.61 | 27.00 | 11.60 | 7.88 | 2.76 |
| Branch 9 | 4.91 | 6.83 | 4.97 | 3.38 | 12.41 | 14.63 | 20.73 | 22.77 | 22.99 | 16.56 | 13.49 | 13.08 |
| Branch 10 | 0.73 | 1.02 | 0.74 | 0.51 | 1.86 | 2.19 | 3.11 | 3.42 | 3.45 | 2.48 | 2.02 | 1.96 |
| Branch 11 | 1.07 | 1.49 | 1.09 | 0.74 | 2.72 | 3.21 | 4.56 | 5.01 | 5.06 | 3.64 | 2.96 | 2.87 |
| Branch 12 | 3.10 | 4.32 | 3.14 | 2.13 | 7.88 | 9.30 | 13.21 | 14.53 | 14.67 | 10.54 | 8.57 | 8.31 |
| Branch 13 | 0.80 | 1.11 | 0.81 | 0.55 | 2.02 | 2.39 | 3.40 | 3.74 | 3.77 | 2.71 | 2.20 | 2.13 |
| Branch 14 | 0.56 | 0.78 | 0.56 | 0.38 | 1.42 | 1.67 | 2.38 | 2.62 | 2.64 | 1.90 | 1.54 | 1.49 |
| Branch 15 | 0.40 | 0.55 | 0.40 | 0.27 | 1.01 | 1.19 | 1.70 | 1.87 | 1.88 | 1.35 | 1.10 | 1.06 |
| Branch 16 | 0.30 | 0.42 | 0.31 | 0.21 | 0.78 | 0.92 | 1.31 | 1.44 | 1.45 | 1.04 | 0.85 | 0.82 |
| Branch 17 | 0.04 | 0.06 | 0.04 | 0.03 | 0.11 | 0.13 | 0.19 | 0.20 | 0.21 | 0.15 | 0.12 | 0.12 |


| Branch 18 | 0.06 | 0.09 | 0.08 | 0.01 | 0.22 | 0.41 | 0.60 | 0.55 | 1.09 | 0.31 | 0.20 | 0.21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Branch 19 | 0.90 | 1.17 | 1.04 | 0.42 | 2.66 | 4.65 | 7.06 | 6.47 | 11.31 | 3.81 | 2.64 | 2.41 |
| Branch 20 | 0.37 | 0.45 | 0.41 | 0.26 | 0.95 | 1.58 | 2.50 | 2.28 | 3.55 | 1.41 | 1.01 | 0.81 |
| Branch 21 | 0.93 | 1.06 | 0.98 | 0.83 | 2.12 | 3.40 | 5.60 | 5.07 | 7.12 | 3.26 | 2.40 | 1.72 |
| Branch 22 | 2.67 | 2.53 | 2.64 | 2.99 | 2.83 | 2.16 | 13.65 | 15.67 | 7.53 | 6.26 | 4.68 | 3.06 |
| Branch 23 | 6.46 | 6.47 | 6.47 | 6.95 | 9.12 | 9.70 | 35.53 | 37.29 | 25.25 | 17.73 | 13.22 | 8.56 |
| Branch 24 | 1.62 | 1.62 | 1.62 | 1.74 | 2.28 | 2.43 | 8.93 | 9.38 | 6.34 | 4.45 | 3.31 | 2.14 |
| Branch 25 | 0.95 | 0.87 | 1.10 | 1.13 | 3.28 | 9.26 | 9.61 | 5.30 | 15.70 | 4.85 | 3.20 | 1.18 |
| Branch 26 | 0.82 | 0.74 | 0.96 | 1.00 | 2.98 | 8.84 | 8.87 | 4.65 | 14.97 | 4.39 | 2.86 | 1.00 |
| Branch 27 | 10.29 | 10.33 | 11.34 | 11.30 | 30.12 | 70.34 | 88.86 | 58.78 | 126.71 | 46.50 | 31.86 | 14.92 |
| Branch 28 | 5.45 | 5.47 | 6.01 | 5.98 | 15.99 | 37.49 | 47.42 | 31.29 | 67.78 | 24.73 | 16.92 | 7.91 |
| Branch 29 | 2.21 | 2.21 | 2.43 | 2.42 | 6.49 | 15.25 | 19.32 | 12.72 | 27.66 | 10.05 | 6.87 | 3.20 |
| Branch 30 | 2.39 | 2.40 | 2.63 | 2.62 | 7.04 | 16.60 | 21.04 | 13.84 | 30.19 | 10.92 | 7.45 | 3.47 |
| Branch 31 | 0.34 | 0.34 | 0.37 | 0.37 | 1.00 | 2.36 | 2.99 | 1.96 | 4.29 | 1.55 | 1.06 | 0.49 |
| Branch 32 | 0.09 | 0.09 | 0.10 | 0.10 | 0.27 | 0.65 | 0.82 | 0.54 | 1.18 | 0.43 | 0.29 | 0.14 |

Voltage Magnitude (pu) for Scenario C

| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus1 | 1.0931 | 1.0953 | 1.094418135 | 1.095916415 | 1.094013398 | 1.095416601 | 1.096230419 | 1.092057095 | 1.090076094 | 1.081498917 | 1.09120341 | 1.09369152 |
| Bus2 | 1.0918 | 1.0936 | 1.092896789 | 1.094376287 | 1.092761595 | 1.093892335 | 1.095363507 | 1.091279039 | 1.08948429 | 1.080970253 | 1.090491716 | 1.092272584 |
| Bus3 | 1.0838 | 1.0844 | 1.084326564 | 1.085937801 | 1.085629509 | 1.085577992 | 1.090559063 | 1.086965535 | 1.086138083 | 1.078529956 | 1.086574372 | 1.084359806 |
| Bus4 | 1.0793 | 1.0784 | 1.078888961 | 1.080245166 | 1.081144856 | 1.080351142 | 1.089030663 | 1.085881987 | 1.086008314 | 1.078882221 | 1.086004469 | 1.080543977 |
| Bus5 | 1.0755 | 1.0732 | 1.074198323 | 1.075375528 | 1.077275201 | 1.07591808 | 1.087611541 | 1.084841803 | 1.085926031 | 1.079217916 | 1.085498287 | 1.077229292 |
| Bus6 | 1.0685 | 1.0639 | 1.065854275 | 1.066756563 | 1.070441283 | 1.068180361 | 1.084920582 | 1.08282969 | 1.085885601 | 1.079903269 | 1.084623962 | 1.071352713 |
| Bus7 | 1.0678 | 1.0627 | 1.064659498 | 1.065310423 | 1.069304565 | 1.066844336 | 1.085674261 | 1.083701246 | 1.086660961 | 1.080950477 | 1.085420678 | 1.071112426 |
| Bus8 | 1.0642 | 1.0533 | 1.055901752 | 1.05362861 | 1.061742087 | 1.056573013 | 1.094299353 | 1.093701656 | 1.094882869 | 1.09108755 | 1.094353164 | 1.071821139 |
| Bus9 | 1.0643 | 1.0508 | 1.053491662 | 1.049747374 | 1.059535567 | 1.053379359 | 1.099999848 | 1.099999989 | 1.099999989 | 1.097126842 | 1.099999989 | 1.074141895 |
| Bus10 | 1.0433 | 1.0203 | 1.025866764 | 1.019283641 | 1.037153562 | 1.024511687 | 1.095519315 | 1.097721003 | 1.098632147 | 1.097917585 | 1.097740043 | 1.056185032 |
| Bus11 | 1.0401 | 1.0157 | 1.021697566 | 1.014686129 | 1.033775432 | 1.02015498 | 1.094842883 | 1.097376928 | 1.09842563 | 1.098036976 | 1.097398842 | 1.053474631 |


| Bus12 | 1.0348 | 1.0079 | 1.014709819 | 1.006975428 | 1.028120225 | 1.012850933 | 1.093714742 | 1.096803329 | 1.098081411 | 1.098235893 | 1.096830038 | 1.048941775 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus13 | 1.0144 | 0.9780 | 0.987717118 | 0.977147352 | 1.006330626 | 0.984619398 | 1.089402774 | 1.094612889 | 1.096767405 | 1.098994573 | 1.094657922 | 1.03151409 |
| Bus14 | 1.0072 | 0.9675 | 0.97822072 | 0.966646575 | 0.998673492 | 0.974684442 | 1.087892648 | 1.093846037 | 1.096307452 | 1.099260049 | 1.093897489 | 1.025395531 |
| Bus15 | 1.0015 | 0.9592 | 0.970743504 | 0.958378681 | 0.99264423 | 0.966861979 | 1.086703346 | 1.093242086 | 1.095945202 | 1.099469138 | 1.093298591 | 1.020577554 |
| Bus16 | 0.9964 | 0.9516 | 0.963932988 | 0.950845719 | 0.987155455 | 0.959736099 | 1.085622368 | 1.092693236 | 1.095616024 | 1.099659107 | 1.092754335 | 1.016193383 |
| Bus17 | 0.9895 | 0.9415 | 0.954794423 | 0.940732409 | 0.979797249 | 0.95017225 | 1.084177237 | 1.091959712 | 1.09517614 | 1.099912893 | 1.092026953 | 1.010320477 |
| Bus18 | 0.9878 | 0.9389 | 0.952529077 | 0.938225814 | 0.977972747 | 0.947801632 | 1.083818616 | 1.091777666 | 1.095066966 | 1.099975885 | 1.091846431 | 1.008863939 |
| Bus19 | 1.0922 | 1.0940 | 1.093277151 | 1.094676478 | 1.093136231 | 1.094184618 | 1.095458542 | 1.091350285 | 1.08955477 | 1.080805417 | 1.090549972 | 1.092563239 |
| Bus20 | 1.0966 | 1.0978 | 1.097375811 | 1.098092478 | 1.097107992 | 1.097667649 | 1.097373495 | 1.093051533 | 1.090954087 | 1.080557499 | 1.09206426 | 1.096344628 |
| Bus21 | 1.0978 | 1.0986 | 1.098351119 | 1.098825591 | 1.098145513 | 1.098507936 | 1.09814975 | 1.093803687 | 1.091542615 | 1.080837467 | 1.092746797 | 1.097532733 |
| Bus22 | 1.1000 | 1.1000 | 1.09999993 | 1.099999926 | 1.09999982 | 1.099999953 | 1.099999375 | 1.095651372 | 1.092953303 | 1.081978622 | 1.09443859 | 1.099999849 |
| Bus23 | 1.0830 | 1.0848 | 1.084455464 | 1.086615001 | 1.085671508 | 1.085730356 | 1.088280054 | 1.084478357 | 1.083312421 | 1.075831674 | 1.083814346 | 1.082686838 |
| Bus24 | 1.0771 | 1.0796 | 1.079479009 | 1.082436847 | 1.081489685 | 1.080759817 | 1.081874322 | 1.078049805 | 1.076876837 | 1.069883678 | 1.077117014 | 1.075252572 |
| Bus25 | 1.0742 | 1.0771 | 1.07699888 | 1.080354856 | 1.079405863 | 1.078282642 | 1.078681113 | 1.074845196 | 1.073668715 | 1.066918844 | 1.073778263 | 1.07154597 |
| Bus26 | 1.0681 | 1.0640 | 1.066024966 | 1.067338707 | 1.070606854 | 1.068737439 | 1.083419265 | 1.081225114 | 1.08494892 | 1.078828797 | 1.083431084 | 1.07058562 |
| Bus27 | 1.0679 | 1.0645 | 1.066574602 | 1.068386356 | 1.071089983 | 1.069749691 | 1.081796109 | 1.079458483 | 1.083943205 | 1.077639654 | 1.082143807 | 1.069976231 |
| Bus28 | 1.0582 | 1.0557 | 1.05877499 | 1.062571568 | 1.064800344 | 1.0639425 | 1.069672182 | 1.067306896 | 1.076272738 | 1.06972659 | 1.072499097 | 1.058214543 |
| Bus29 | 1.0520 | 1.0502 | 1.053849255 | 1.058899935 | 1.060828725 | 1.06027567 | 1.062012728 | 1.059629931 | 1.071428666 | 1.064729192 | 1.066407226 | 1.050784089 |
| Bus30 | 1.0492 | 1.0476 | 1.051591814 | 1.057217359 | 1.059008648 | 1.058595295 | 1.058501919 | 1.056111089 | 1.06920867 | 1.062438906 | 1.063615184 | 1.047378273 |
| Bus31 | 1.0445 | 1.0434 | 1.047897499 | 1.054464831 | 1.056030928 | 1.055846375 | 1.052751975 | 1.050347931 | 1.065575805 | 1.058690817 | 1.059044535 | 1.041800527 |
| Bus32 | 1.0435 | 1.0425 | 1.047078222 | 1.053854402 | 1.055370561 | 1.055236746 | 1.051476864 | 1.04906989 | 1.064770154 | 1.057859614 | 1.058030927 | 1.040563601 |
| Bus33 | 1.0429 | 1.0420 | 1.046586661 | 1.05348816 | 1.054974355 | 1.054870985 | 1.050711749 | 1.048303016 | 1.06428677 | 1.057360898 | 1.057422751 | 1.039821402 |


| Voltage Magnitude (pu) for Scenario C |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Bus1 | 1.092945783 | 1.095976199 | 1.095740389 | 1.093395844 | 1.094855063 | 1.078219812 | 1.076534387 | 1.075393422 | 1.08637085 | 1.076522135 | 1.095020638 | 1.093902293 |
| Bus2 | 1.092152821 | 1.09511846 | 1.094880693 | 1.092510052 | 1.093522803 | 1.077046282 | 1.074779259 | 1.073763221 | 1.08457802 | 1.075297019 | 1.093530458 | 1.092719727 |
| Bus3 | 1.087705275 | 1.090290291 | 1.090004459 | 1.087855461 | 1.086021596 | 1.070199041 | 1.064628307 | 1.064323616 | 1.073750852 | 1.068396816 | 1.085370138 | 1.085844286 |
| Bus4 | 1.086906454 | 1.089150911 | 1.088820209 | 1.086928961 | 1.083001939 | 1.067803954 | 1.06290184 | 1.063526794 | 1.070161838 | 1.067427325 | 1.082504096 | 1.08338501 |
| Bus5 | 1.086206186 | 1.088158829 | 1.087771084 | 1.086133948 | 1.080231673 | 1.065249476 | 1.061134594 | 1.062757021 | 1.06648746 | 1.066398062 | 1.079937288 | 1.081231995 |


| Bus6 | 1.085009322 | 1.086421946 | 1.085903952 | 1.084813461 | 1.074974403 | 1.059774894 | 1.057630501 | 1.061418407 | 1.058975203 | 1.06427093 | 1.075196339 | 1.077394404 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus7 | 1.085727249 | 1.087093775 | 1.086583364 | 1.085498682 | 1.075794171 | 1.061339331 | 1.059213159 | 1.063014932 | 1.060172787 | 1.065915849 | 1.076106693 | 1.077764825 |
| Bus8 | 1.094389696 | 1.09477964 | 1.094621254 | 1.094235345 | 1.085335891 | 1.082033121 | 1.084464498 | 1.085879566 | 1.084755633 | 1.086065237 | 1.086649254 | 1.083422612 |
| Bus9 | 1.0999998 | 1.09997308 | 1.099999822 | 1.099999793 | 1.092160368 | 1.094468242 | 1.099999806 | 1.099999709 | 1.099999742 | 1.098269884 | 1.094202702 | 1.087999182 |
| Bus10 | 1.096583464 | 1.094908295 | 1.095206902 | 1.095582014 | 1.082332435 | 1.096003994 | 1.099198266 | 1.098451278 | 1.098410059 | 1.098749924 | 1.083214184 | 1.077180566 |
| Bus11 | 1.096067685 | 1.094143665 | 1.094483312 | 1.094915054 | 1.08084883 | 1.096235874 | 1.099077248 | 1.098217497 | 1.09817005 | 1.098822403 | 1.08155541 | 1.075547436 |
| Bus12 | 1.095207656 | 1.092868285 | 1.09327645 | 1.093802727 | 1.078371884 | 1.096622154 | 1.09887556 | 1.097827822 | 1.09776999 | 1.098943166 | 1.078785381 | 1.072820291 |
| Bus13 | 1.091921889 | 1.087992377 | 1.088663005 | 1.089551307 | 1.068883249 | 1.098095009 | 1.098105818 | 1.09634018 | 1.09624268 | 1.099403824 | 1.068168879 | 1.062368626 |
| Bus14 | 1.09077136 | 1.086284583 | 1.087047213 | 1.088062403 | 1.065557094 | 1.098610324 | 1.097836403 | 1.095819433 | 1.095708043 | 1.099565024 | 1.064446609 | 1.058704221 |
| Bus15 | 1.089865247 | 1.084939617 | 1.085774698 | 1.086889813 | 1.062937736 | 1.099016193 | 1.097624216 | 1.095409303 | 1.095286974 | 1.099691985 | 1.061515346 | 1.055818522 |
| Bus16 | 1.089041732 | 1.083717096 | 1.084618058 | 1.085824029 | 1.060555929 | 1.099384926 | 1.097431409 | 1.095036613 | 1.094904342 | 1.099807339 | 1.05884967 | 1.053194306 |
| Bus17 | 1.087940956 | 1.082082611 | 1.08307171 | 1.084399223 | 1.057369345 | 1.09987748 | 1.097173778 | 1.094538569 | 1.094393011 | 1.09996145 | 1.055282714 | 1.049682882 |
| Bus18 | 1.087667778 | 1.08167701 | 1.082687977 | 1.084045646 | 1.056578748 | 1.09999974 | 1.097109835 | 1.094414961 | 1.094266105 | 1.099999702 | 1.05439779 | 1.048811731 |
| Bus19 | 1.092262417 | 1.095247289 | 1.09502331 | 1.092533041 | 1.093715345 | 1.077187308 | 1.075016171 | 1.074000906 | 1.084994553 | 1.07537722 | 1.093663996 | 1.093015143 |
| Bus20 | 1.094246532 | 1.097361927 | 1.097257991 | 1.094108283 | 1.096716051 | 1.079913508 | 1.079275729 | 1.078083912 | 1.090474941 | 1.077801289 | 1.096477354 | 1.096590166 |
| Bus21 | 1.095057179 | 1.098158632 | 1.098095896 | 1.094913759 | 1.097739155 | 1.081050691 | 1.081012222 | 1.079721586 | 1.092427849 | 1.07892655 | 1.097528147 | 1.097711554 |
| Bus22 | 1.096970853 | 1.099982221 | 1.09999925 | 1.097018996 | 1.099999732 | 1.083765523 | 1.085107711 | 1.083550042 | 1.096677837 | 1.081753723 | 1.099999764 | 1.099999909 |
| Bus23 | 1.08480014 | 1.087538328 | 1.087250537 | 1.084968289 | 1.083384025 | 1.066839214 | 1.057571257 | 1.056804008 | 1.068408612 | 1.063286747 | 1.082001486 | 1.083125377 |
| Bus24 | 1.077579407 | 1.080336107 | 1.080046375 | 1.077483677 | 1.074824788 | 1.057873641 | 1.040378764 | 1.039179167 | 1.054011931 | 1.051153441 | 1.071694444 | 1.074830256 |
| Bus25 | 1.073979411 | 1.076745363 | 1.07645466 | 1.073751952 | 1.070556468 | 1.053402268 | 1.031791541 | 1.030375277 | 1.046825003 | 1.045098794 | 1.066552958 | 1.070693833 |
| Bus26 | 1.083815035 | 1.085297006 | 1.084687494 | 1.083620699 | 1.072980841 | 1.055834221 | 1.053704438 | 1.05836487 | 1.054143506 | 1.061283964 | 1.073264866 | 1.076242274 |
| Bus27 | 1.08254479 | 1.084123225 | 1.083405001 | 1.082371797 | 1.07086706 | 1.051344506 | 1.049356333 | 1.055033983 | 1.048728507 | 1.057945297 | 1.071257055 | 1.075110887 |
| Bus28 | 1.072411114 | 1.074004953 | 1.072786406 | 1.071742596 | 1.053519624 | 1.024505952 | 1.019126901 | 1.030479106 | 1.012792531 | 1.036147411 | 1.053400842 | 1.06290751 |
| Bus29 | 1.066010134 | 1.067613719 | 1.066078851 | 1.065028328 | 1.042554822 | 1.007525442 | 0.999993969 | 1.014947279 | 0.99003322 | 1.022363394 | 1.042113926 | 1.055197756 |
| Bus30 | 1.06307637 | 1.064684426 | 1.063004523 | 1.061950921 | 1.037527967 | 0.999737549 | 0.991217579 | 1.00782452 | 0.979590659 | 1.016042896 | 1.036939297 | 1.051663872 |
| Bus31 | 1.058273287 | 1.059888688 | 1.057970867 | 1.056912206 | 1.029286644 | 0.986944265 | 0.976790075 | 0.996129571 | 0.962403031 | 1.005671114 | 1.028454879 | 1.045875969 |
| Bus32 | 1.057208138 | 1.058825168 | 1.056854589 | 1.055794806 | 1.027459108 | 0.98410749 | 0.973590992 | 0.993536305 | 0.958592044 | 1.003371211 | 1.026573442 | 1.044592441 |
| Bus33 | 1.05656903 | 1.058187038 | 1.056184798 | 1.055124342 | 1.026362413 | 0.982404838 | 0.971670747 | 0.99197988 | 0.956304236 | 1.001990931 | 1.025444391 | 1.043822274 |


| Lambda P (UM/MWh) for Scenario C |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Bus1 | 273.8056657 | 328.8511766 | 301.9717297 | 305.4257832 | 251.3565917 | 302.8280639 | 167.7399246 | 149.2424557 | 113.6300277 | 97.67609301 | 136.7987962 | 279.7074766 |
| Bus2 | 274.4455849 | 329.7537801 | 302.7340163 | 306.1949304 | 251.8846893 | 303.589104 | 167.9954345 | 149.4496221 | 113.7506205 | 97.76922428 | 136.9725534 | 280.3867148 |
| Bus3 | 278.8068886 | 336.2980474 | 308.1764558 | 311.7864804 | 255.5181766 | 308.9475449 | 169.5078003 | 150.6275088 | 114.4479408 | 98.21244677 | 137.9536064 | 284.7519268 |
| Bus4 | 281.2818936 | 340.5973027 | 311.6607489 | 315.5719215 | 257.8127305 | 312.3512879 | 169.9865674 | 150.9067904 | 114.4634149 | 98.14272126 | 138.0801465 | 286.8855489 |
| Bus5 | 283.5407541 | 344.5462497 | 314.8432754 | 319.0241572 | 259.8938265 | 315.4280559 | 170.4402835 | 151.1771021 | 114.4699256 | 98.07500295 | 138.1929317 | 288.8223263 |
| Bus6 | 287.6997452 | 351.8711286 | 320.7039445 | 325.36543 | 263.691998 | 321.0117473 | 171.31895 | 151.7140795 | 114.4632726 | 97.93999199 | 138.3944383 | 292.3632958 |
| Bus7 | 288.2248702 | 353.2633459 | 321.8308186 | 326.8133063 | 264.4228843 | 322.2826547 | 171.0364825 | 151.3919122 | 114.2682634 | 97.72806887 | 138.1375965 | 292.5723016 |
| Bus8 | 291.4357598 | 363.2840971 | 329.8863624 | 337.603577 | 269.5890525 | 331.5863208 | 168.2714299 | 148.3842446 | 112.4584908 | 95.83246294 | 135.7270929 | 293.0839292 |
| Bus9 | 292.4006326 | 367.6387385 | 333.3473463 | 342.5774124 | 271.7669099 | 335.7512438 | 166.5076743 | 146.5462794 | 111.3576073 | 94.72089223 | 134.2460062 | 292.5532807 |
| Bus10 | 302.3355066 | 386.6756204 | 348.7449059 | 360.2699626 | 281.7003143 | 352.0289493 | 167.6198413 | 147.0408223 | 111.5825673 | 94.56413909 | 134.6953549 | 300.8919944 |
| Bus11 | 304.0512839 | 390.0073948 | 351.4287095 | 363.36615 | 283.4192208 | 354.870959 | 167.8077718 | 147.1241686 | 111.6204383 | 94.53685861 | 134.7710802 | 302.3260189 |
| Bus12 | 306.9558692 | 395.6928745 | 355.9971079 | 368.6492964 | 286.3325058 | 359.713595 | 168.121917 | 147.2632893 | 111.6836138 | 94.489357 | 134.8974743 | 304.7475739 |
| Bus13 | 317.149817 | 416.0955221 | 372.2776928 | 387.6049591 | 296.5901845 | 377.0207217 | 169.1867932 | 147.7329289 | 111.896515 | 94.32070835 | 135.324124 | 313.1876941 |
| Bus14 | 320.4444448 | 422.8622584 | 377.6338543 | 393.8905556 | 299.9178942 | 382.7335147 | 169.517426 | 147.8780438 | 111.9621691 | 94.26498644 | 135.4559487 | 315.8937315 |
| Bus15 | 323.3412857 | 428.8368702 | 382.3566864 | 399.440168 | 302.8455392 | 387.773558 | 169.8064425 | 148.0048128 | 112.019508 | 94.2111558 | 135.571106 | 318.2701284 |
| Bus16 | 326.1160545 | 434.5954549 | 386.8997744 | 404.7888623 | 305.6523075 | 392.6256982 | 170.080757 | 148.1250068 | 112.0738495 | 94.15199904 | 135.680289 | 320.5421185 |
| Bus17 | 329.4391453 | 441.6348241 | 392.4177696 | 411.3261562 | 309.0237286 | 398.5345999 | 170.3990658 | 148.2639487 | 112.1365692 | 94.06514216 | 135.8064976 | 323.2459147 |
| Bus18 | 330.3745559 | 443.6088787 | 393.9669198 | 413.1594624 | 309.9721943 | 400.1927221 | 170.4892796 | 148.3033631 | 112.154368 | 94.0244603 | 135.8423001 | 324.0079679 |
| Bus19 | 274.0069177 | 328.971749 | 302.1012048 | 305.5055237 | 251.4905069 | 302.9730633 | 167.9300283 | 149.4192291 | 113.729909 | 97.78992538 | 136.9485668 | 280.0321191 |
| Bus20 | 269.8108277 | 321.6360302 | 296.1664657 | 299.0470217 | 247.7543817 | 297.1115331 | 167.095051 | 148.9245662 | 113.4216381 | 97.80697213 | 136.5432069 | 276.4043147 |
| Bus21 | 268.6235142 | 319.5970713 | 294.5172557 | 297.2538391 | 246.7058282 | 295.4607015 | 166.804447 | 148.7316116 | 113.3057281 | 97.76451905 | 136.3827616 | 275.3173921 |
| Bus22 | 266.4889315 | 315.9936474 | 291.6022478 | 294.0882535 | 244.8349388 | 292.5074586 | 166.1927439 | 148.297449 | 113.0500146 | 97.60977081 | 136.0189043 | 273.2654113 |
| Bus23 | 279.3974644 | 336.4073406 | 308.3697271 | 311.6934933 | 255.669867 | 309.1370617 | 170.2559373 | 151.342669 | 115.0592511 | 98.71197543 | 138.6780718 | 285.8083989 |
| Bus24 | 281.8438976 | 338.9457424 | 310.6372387 | 313.6111016 | 257.2455749 | 311.4047996 | 171.8659462 | 152.7840464 | 116.1574903 | 99.58812788 | 140.0556028 | 288.9675025 |
| Bus25 | 283.0786128 | 340.2248334 | 311.7795394 | 314.5756028 | 258.0381216 | 312.547188 | 172.6792668 | 153.5122456 | 116.7123473 | 100.030392 | 140.7519203 | 290.5667226 |
| Bus26 | 287.9831777 | 351.8799117 | 320.6854462 | 325.0951816 | 263.6701998 | 320.7585302 | 171.8191621 | 152.1861322 | 114.6681768 | 98.13927878 | 138.7130548 | 292.8553038 |
| Bus27 | 288.2008774 | 351.6936675 | 320.4993598 | 324.5960346 | 263.5335922 | 320.2850966 | 172.3913904 | 152.7333665 | 114.9002046 | 98.36975618 | 139.0761681 | 293.3270932 |
| Bus28 | 292.383767 | 356.2977868 | 324.1976714 | 327.3710697 | 265.9669523 | 323.0161276 | 175.4695818 | 155.4729705 | 116.182456 | 99.50961657 | 141.0397558 | 298.4615702 |
| Bus29 | 295.085328 | 359.2655301 | 326.5763964 | 329.1485044 | 267.527036 | 324.7653222 | 177.466882 | 157.2507778 | 117.0067541 | 100.2428163 | 142.3072644 | 301.7914847 |


| Bus30 | 296.461548 | 360.7760254 | 327.7859407 | 330.0506328 | 268.3191846 | 325.6531052 | 178.4864272 | 158.158326 | 117.4257987 | 100.6156482 | 142.9527978 | 303.4909128 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus31 | 298.4725374 | 362.9800576 | 329.5480799 | 331.3609502 | 269.4705543 | 326.9425572 | 179.98126 | 159.4890613 | 118.0360555 | 101.1588398 | 143.8957025 | 305.9816902 |
| Bus32 | 298.9036258 | 363.4521097 | 329.9251244 | 331.6407971 | 269.7165592 | 327.217944 | 180.3023689 | 159.7749345 | 118.1666011 | 101.2750699 | 144.0977799 | 306.5166235 |
| Bus33 | 299.1420007 | 363.7129583 | 330.1333184 | 331.7950996 | 269.8522464 | 327.3697856 | 180.4802132 | 159.9332699 | 118.2386719 | 101.3392506 | 144.2094996 | 306.8128435 |


| Lambda P (UM/MWh) for Scenario C |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Bus1 | 153.5417332 | 166.719263 | 167.1234277 | 171.3991634 | 256.5401778 | 215.6526618 | 324.2556837 | 302.2805007 | 335.2806308 | 225.8623201 | 286.2309527 | 231.392277 |
| Bus2 | 153.7583521 | 166.9703169 | 167.3764887 | 171.6678023 | 257.131873 | 216.1141484 | 325.2908297 | 303.1778274 | 336.3638792 | 226.3665118 | 286.9680085 | 231.8677725 |
| Bus3 | 155.0050038 | 168.4856192 | 168.8963136 | 173.1228571 | 260.9258087 | 218.8634349 | 331.4376489 | 308.5103292 | 343.0692603 | 229.275011 | 291.5927323 | 234.9389927 |
| Bus4 | 155.2069017 | 168.8424641 | 169.2578147 | 173.3852017 | 262.4922336 | 219.7876915 | 332.2853781 | 308.7684635 | 345.1449838 | 229.6156391 | 293.2725504 | 236.0525529 |
| Bus5 | 155.3850863 | 169.1630369 | 169.5858641 | 173.6122049 | 263.9770354 | 220.7761554 | 333.1539741 | 309.0070649 | 347.2866425 | 229.9780355 | 294.8368673 | 237.0646601 |
| Bus6 | 155.6989231 | 169.741874 | 170.1864866 | 174.0004182 | 266.8525393 | 222.9629711 | 334.9921851 | 309.4546567 | 351.8599255 | 230.7741875 | 297.7983975 | 238.9201146 |
| Bus7 | 155.4210952 | 169.5057798 | 169.9274637 | 173.6904649 | 266.4500872 | 221.8133491 | 332.8292861 | 307.7251708 | 349.5146616 | 229.6732393 | 297.3079237 | 238.7541199 |
| Bus8 | 152.7609503 | 167.1342724 | 167.3605078 | 170.6761248 | 262.1058972 | 211.6295631 | 313.5099489 | 292.1647331 | 328.4758234 | 219.8536358 | 292.0007867 | 236.5538427 |
| Bus9 | 151.096542 | 165.5877538 | 165.7054806 | 168.7645027 | 259.118117 | 205.7402235 | 302.2744045 | 283.0421283 | 316.1878499 | 214.1358583 | 288.3479823 | 234.8321395 |
| Bus10 | 151.8639117 | 166.8397998 | 166.8904492 | 169.8757838 | 262.9981771 | 204.458174 | 302.6262922 | 283.6874279 | 316.9298063 | 213.3327264 | 293.1819854 | 238.7290527 |
| Bus11 | 151.993407 | 167.0515245 | 167.0907594 | 170.0635495 | 263.6582918 | 204.2259497 | 302.6856002 | 283.7961341 | 317.0547632 | 213.1841841 | 294.0055866 | 239.392904 |
| Bus12 | 152.2097084 | 167.4055969 | 167.4256727 | 170.377405 | 264.7660369 | 203.8025926 | 302.7846785 | 283.9775763 | 317.2632727 | 212.9074797 | 295.3888339 | 240.5077603 |
| Bus13 | 152.9414142 | 168.6071916 | 168.5616326 | 171.4411686 | 268.5610936 | 202.219696 | 303.1187625 | 284.589483 | 317.9663289 | 211.8474421 | 300.1385887 | 244.3350878 |
| Bus14 | 153.1680658 | 168.9807614 | 168.9145813 | 171.7714097 | 269.7539032 | 201.6643815 | 303.2217146 | 284.7782795 | 318.1832498 | 211.4657047 | 301.6354048 | 245.5409122 |
| Bus15 | 153.3661252 | 169.3073701 | 169.2231341 | 172.0600783 | 270.7983562 | 201.0872599 | 303.3116505 | 284.9431895 | 318.372715 | 211.0588976 | 302.9465471 | 246.5971208 |
| Bus16 | 153.5540116 | 169.6174548 | 169.516036 | 172.334054 | 271.7923863 | 200.3952675 | 303.3968923 | 285.0995055 | 318.5522997 | 210.5577383 | 304.1951345 | 247.6028804 |
| Bus17 | 153.771628 | 169.977639 | 169.8560986 | 172.6519348 | 272.9569602 | 199.2660686 | 303.4952115 | 285.27999 | 318.7596556 | 209.714903 | 305.6609742 | 248.783408 |
| Bus18 | 153.833331 | 170.0796967 | 169.9524656 | 172.7420297 | 273.2862931 | 198.6628882 | 303.5231223 | 285.3312059 | 318.8184947 | 209.2518233 | 306.0753118 | 249.1171131 |
| Bus19 | 153.7172952 | 166.8936535 | 167.3030093 | 171.6444401 | 256.8683709 | 216.0361196 | 325.102757 | 303.005526 | 336.0717076 | 226.3060039 | 286.674275 | 231.6361747 |
| Bus20 | 153.1337315 | 165.9833277 | 166.421265 | 171.106885 | 254.0555224 | 214.8898191 | 322.4372056 | 300.6251173 | 332.6376926 | 225.216869 | 283.402454 | 229.2649255 |
| Bus21 | 152.9190586 | 165.6798271 | 166.1253593 | 170.8731916 | 253.1843439 | 214.4620968 | 321.4608766 | 299.7636921 | 331.5033221 | 224.7766572 | 282.357843 | 228.5537976 |
| Bus22 | 152.4515557 | 165.0588526 | 165.5171534 | 170.3187501 | 251.4959144 | 213.5236455 | 319.3439021 | 297.9086403 | 329.1987851 | 223.7687783 | 280.2872245 | 227.2097424 |
| Bus23 | 155.8660544 | 169.3804656 | 169.7927221 | 174.0846982 | 262.3188053 | 220.2885676 | 336.0842827 | 313.1123833 | 346.6954289 | 231.5537334 | 293.5717349 | 236.2228359 |


| Bus24 | 157.5354273 | 171.1852343 | 171.6028807 | 176.0179779 | 265.6618385 | 223.2778639 | 345.0495495 | 321.68861 | 354.3173446 | 235.8464816 | 298.0976713 | 239.1396795 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bus25 | 158.3801268 | 172.0983787 | 172.5187582 | 176.9967211 | 267.3579239 | 224.7961219 | 349.6810747 | 326.1232279 | 358.2311182 | 238.0409325 | 300.4019383 | 240.6187698 |
| Bus26 | 156.0610505 | 170.1139726 | 170.5903292 | 174.4081559 | 267.9213217 | 224.7738081 | 337.7956139 | 311.4303084 | 355.5416307 | 232.1741079 | 298.9625672 | 239.4789402 |
| Bus27 | 156.4700401 | 170.5282594 | 171.0436588 | 174.8641495 | 269.1335372 | 226.9550323 | 341.1207966 | 313.736934 | 359.9673476 | 233.8298088 | 300.265952 | 240.0792446 |
| Bus28 | 158.7929714 | 173.052159 | 173.7052495 | 177.5906389 | 276.1687451 | 236.5404328 | 357.5289023 | 325.7431797 | 380.8903432 | 241.6947285 | 308.3520689 | 244.4225499 |
| Bus29 | 160.2939512 | 174.6828959 | 175.4267627 | 179.3542047 | 280.7857183 | 242.9683623 | 368.6173091 | 333.7522458 | 395.2145845 | 246.908799 | 313.6644223 | 247.2416409 |
| Bus30 | 161.0587313 | 175.5137674 | 176.3042943 | 180.2531895 | 283.1544151 | 246.2980508 | 374.3810007 | 337.891225 | 402.7037599 | 249.5959236 | 316.3911794 | 248.6808859 |
| Bus31 | 162.1766338 | 176.7282232 | 177.5879349 | 181.5682529 | 286.6564651 | 251.3011624 | 383.0923619 | 344.0856735 | 414.1366595 | 253.5988363 | 320.4258183 | 250.7915733 |
| Bus32 | 162.416324 | 176.988608 | 177.8632836 | 181.8503474 | 287.4126352 | 252.3922807 | 384.9991007 | 345.4332957 | 416.6545713 | 254.4671854 | 321.2974137 | 251.2450419 |
| Bus33 | 162.5488843 | 177.1326104 | 178.0156168 | 182.006415 | 287.8330874 | 253.0035721 | 386.0702436 | 346.1868978 | 418.0755638 | 254.9517212 | 321.7822268 | 251.4962211 |


| Active Power Losses (1^2*R) (Watt) for Scenario C |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Branch 1 | 2.90 | 4.10 | 3.44 | 3.47 | 2.36 | 3.43 | 1.22 | 1.00 | 0.58 | 0.46 | 0.84 | 3.11 |
| Branch 2 | 20.59 | 28.51 | 24.27 | 23.80 | 16.56 | 22.94 | 7.47 | 5.98 | 3.61 | 1.95 | 4.93 | 20.33 |
| Branch 3 | 8.38 | 15.51 | 12.57 | 13.87 | 8.46 | 11.62 | 0.97 | 0.48 | 0.01 | 0.06 | 0.13 | 6.09 |
| Branch 4 | 6.41 | 11.78 | 9.49 | 10.35 | 6.36 | 8.53 | 0.83 | 0.43 | 0.00 | 0.05 | 0.10 | 4.62 |
| Branch 5 | 9.63 | 17.53 | 14.00 | 15.04 | 9.34 | 12.12 | 1.42 | 0.79 | 0.00 | 0.09 | 0.15 | 6.87 |
| Branch 6 | 0.57 | 2.51 | 2.08 | 3.30 | 1.41 | 2.65 | 0.66 | 1.02 | 0.69 | 1.12 | 0.80 | 0.07 |
| Branch 7 | 2.06 | 13.01 | 10.69 | 18.57 | 7.18 | 14.36 | 7.07 | 9.99 | 6.59 | 9.97 | 7.86 | 0.08 |
| Branch 8 | 0.42 | 4.01 | 3.21 | 6.27 | 2.07 | 4.61 | 4.93 | 6.41 | 4.16 | 5.86 | 5.08 | 0.55 |
| Branch 9 | 49.06 | 103.58 | 84.69 | 102.87 | 55.71 | 92.44 | 2.25 | 0.58 | 0.21 | 0.07 | 0.57 | 35.92 |
| Branch 10 | 7.38 | 15.64 | 12.78 | 15.54 | 8.39 | 13.95 | 0.34 | 0.09 | 0.03 | 0.01 | 0.09 | 5.40 |
| Branch 11 | 10.86 | 23.11 | 18.85 | 22.95 | 12.34 | 20.59 | 0.49 | 0.13 | 0.05 | 0.02 | 0.12 | 7.93 |
| Branch 12 | 31.58 | 67.57 | 55.02 | 67.10 | 35.93 | 60.15 | 1.42 | 0.37 | 0.13 | 0.04 | 0.36 | 23.03 |
| Branch 13 | 8.14 | 17.47 | 14.21 | 17.35 | 9.27 | 15.54 | 0.36 | 0.09 | 0.03 | 0.01 | 0.09 | 5.93 |
| Branch 14 | 5.72 | 12.30 | 10.00 | 12.21 | 6.51 | 10.94 | 0.25 | 0.07 | 0.02 | 0.01 | 0.06 | 4.16 |
| Branch 15 | 4.08 | 8.81 | 7.15 | 8.74 | 4.65 | 7.83 | 0.18 | 0.05 | 0.02 | 0.01 | 0.05 | 2.97 |
| Branch 16 | 3.15 | 6.81 | 5.53 | 6.77 | 3.59 | 6.05 | 0.14 | 0.04 | 0.01 | 0.00 | 0.04 | 2.29 |


| Branch 17 | 0.45 | 0.97 | 0.79 | 0.96 | 0.51 | 0.86 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.33 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Branch 18 | 0.48 | 0.93 | 0.75 | 0.84 | 0.45 | 0.69 | 0.04 | 0.01 | 0.01 | 0.02 | 0.01 | 0.31 |
| Branch 19 | 4.84 | 8.98 | 7.20 | 8.00 | 4.41 | 6.82 | 0.63 | 0.35 | 0.24 | 0.00 | 0.28 | 3.53 |
| Branch 20 | 1.45 | 2.56 | 2.06 | 2.28 | 1.29 | 2.00 | 0.29 | 0.20 | 0.12 | 0.02 | 0.16 | 1.19 |
| Branch 21 | 2.74 | 4.65 | 3.75 | 4.12 | 2.41 | 3.72 | 0.77 | 0.59 | 0.35 | 0.18 | 0.49 | 2.52 |
| Branch 22 | 0.55 | 0.22 | 0.20 | 0.21 | 0.13 | 0.21 | 2.02 | 2.32 | 2.92 | 2.62 | 2.83 | 1.47 |
| Branch 23 | 4.35 | 3.25 | 3.08 | 2.17 | 2.18 | 3.08 | 5.11 | 5.15 | 5.16 | 4.41 | 5.58 | 6.88 |
| Branch 24 | 1.09 | 0.81 | 0.77 | 0.54 | 0.54 | 0.77 | 1.28 | 1.29 | 1.29 | 1.10 | 1.40 | 1.72 |
| Branch 25 | 0.23 | 0.05 | 0.05 | 0.21 | 0.04 | 0.20 | 1.88 | 2.13 | 0.72 | 0.92 | 1.18 | 0.62 |
| Branch 26 | 0.17 | 0.17 | 0.17 | 0.49 | 0.13 | 0.46 | 1.75 | 2.04 | 0.66 | 0.88 | 1.10 | 0.46 |
| Branch 27 | 9.59 | 7.79 | 6.11 | 3.40 | 3.98 | 3.39 | 14.73 | 14.80 | 5.91 | 6.29 | 9.33 | 13.86 |
| Branch 28 | 5.08 | 4.12 | 3.23 | 1.80 | 2.10 | 1.79 | 7.80 | 7.84 | 3.13 | 3.33 | 4.94 | 7.34 |
| Branch 29 | 2.06 | 1.67 | 1.31 | 0.73 | 0.85 | 0.72 | 3.16 | 3.18 | 1.26 | 1.35 | 2.00 | 2.98 |
| Branch 30 | 2.23 | 1.81 | 1.42 | 0.79 | 0.92 | 0.78 | 3.43 | 3.44 | 1.37 | 1.46 | 2.17 | 3.22 |
| Branch 31 | 0.32 | 0.26 | 0.20 | 0.11 | 0.13 | 0.11 | 0.49 | 0.49 | 0.19 | 0.21 | 0.31 | 0.46 |
| Branch 32 | 0.09 | 0.07 | 0.06 | 0.03 | 0.04 | 0.03 | 0.13 | 0.13 | 0.05 | 0.06 | 0.08 | 0.13 |


| Active Power Losses (1^2*R) (Watt) for Scenario C |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Branch 1 | 1.04 | 1.19 | 1.20 | 1.28 | 2.80 | 2.31 | 5.13 | 4.42 | 5.35 | 2.51 | 3.49 | 2.21 |
| Branch 2 | 6.34 | 7.55 | 7.67 | 6.93 | 18.46 | 15.13 | 33.10 | 28.65 | 37.50 | 15.38 | 21.88 | 15.34 |
| Branch 3 | 0.25 | 0.53 | 0.57 | 0.34 | 3.88 | 2.39 | 1.15 | 0.23 | 5.21 | 0.37 | 3.48 | 2.52 |
| Branch 4 | 0.19 | 0.40 | 0.45 | 0.24 | 3.24 | 2.60 | 1.15 | 0.20 | 5.26 | 0.40 | 2.79 | 1.93 |
| Branch 5 | 0.28 | 0.59 | 0.68 | 0.34 | 5.47 | 5.83 | 2.40 | 0.37 | 10.96 | 0.88 | 4.46 | 2.92 |
| Branch 6 | 0.73 | 0.49 | 0.56 | 0.72 | 0.62 | 5.27 | 7.96 | 6.21 | 8.00 | 4.74 | 0.75 | 0.15 |
| Branch 7 | 7.45 | 5.49 | 6.15 | 7.61 | 8.01 | 47.26 | 73.34 | 57.80 | 74.28 | 43.09 | 9.73 | 2.71 |
| Branch 8 | 5.00 | 3.98 | 4.38 | 5.26 | 6.46 | 27.88 | 44.50 | 35.42 | 45.42 | 25.76 | 7.88 | 2.76 |
| Branch 9 | 1.31 | 2.87 | 2.57 | 2.19 | 10.79 | 0.26 | 0.07 | 0.27 | 0.28 | 0.03 | 13.49 | 13.08 |
| Branch 10 | 0.20 | 0.43 | 0.38 | 0.33 | 1.62 | 0.04 | 0.01 | 0.04 | 0.04 | 0.00 | 2.02 | 1.96 |
| Branch 11 | 0.29 | 0.63 | 0.56 | 0.48 | 2.37 | 0.06 | 0.02 | 0.06 | 0.06 | 0.01 | 2.96 | 2.87 |


| Branch 12 | 0.82 | 1.81 | 1.62 | 1.38 | 6.85 | 0.17 | 0.05 | 0.17 | 0.18 | 0.02 | 8.57 | 8.31 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Branch 13 | 0.21 | 0.46 | 0.42 | 0.35 | 1.76 | 0.04 | 0.01 | 0.04 | 0.05 | 0.00 | 2.20 | 2.13 |
| Branch 14 | 0.15 | 0.33 | 0.29 | 0.25 | 1.23 | 0.03 | 0.01 | 0.03 | 0.03 | 0.00 | 1.54 | 1.49 |
| Branch 15 | 0.10 | 0.23 | 0.21 | 0.18 | 0.88 | 0.02 | 0.01 | 0.02 | 0.02 | 0.00 | 1.10 | 1.06 |
| Branch 16 | 0.08 | 0.18 | 0.16 | 0.14 | 0.67 | 0.02 | 0.00 | 0.02 | 0.02 | 0.00 | 0.85 | 0.82 |
| Branch 17 | 0.01 | 0.03 | 0.02 | 0.02 | 0.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.12 |
| Branch 18 | 0.02 | 0.05 | 0.05 | 0.01 | 0.20 | 0.04 | 0.10 | 0.10 | 0.23 | 0.03 | 0.20 | 0.21 |
| Branch 19 | 0.47 | 0.75 | 0.74 | 0.32 | 2.49 | 0.89 | 2.14 | 1.96 | 3.39 | 0.73 | 2.64 | 2.41 |
| Branch 20 | 0.23 | 0.31 | 0.32 | 0.22 | 0.90 | 0.46 | 1.06 | 0.95 | 1.37 | 0.44 | 1.01 | 0.81 |
| Branch 21 | 0.64 | 0.79 | 0.79 | 0.73 | 2.02 | 1.29 | 2.94 | 2.58 | 3.33 | 1.35 | 2.40 | 1.72 |
| Branch 22 | 3.17 | 2.91 | 2.91 | 3.17 | 2.93 | 4.18 | 18.60 | 21.01 | 10.88 | 9.59 | 4.68 | 3.06 |
| Branch 23 | 6.49 | 6.46 | 6.46 | 6.97 | 9.12 | 10.00 | 36.73 | 38.59 | 25.77 | 18.31 | 13.22 | 8.56 |
| Branch 24 | 1.62 | 1.62 | 1.62 | 1.75 | 2.28 | 2.51 | 9.24 | 9.71 | 6.47 | 4.59 | 3.31 | 2.14 |
| Branch 25 | 1.20 | 1.07 | 1.25 | 1.22 | 3.37 | 13.00 | 13.45 | 8.10 | 20.54 | 7.47 | 3.20 | 1.18 |
| Branch 26 | 1.10 | 0.95 | 1.13 | 1.10 | 3.09 | 13.19 | 13.28 | 7.80 | 20.69 | 7.36 | 2.86 | 1.00 |
| Branch 27 | 10.30 | 10.27 | 11.31 | 11.33 | 30.08 | 71.65 | 90.74 | 60.04 | 127.88 | 47.38 | 31.86 | 14.92 |
| Branch 28 | 5.45 | 5.44 | 5.99 | 6.00 | 15.97 | 38.19 | 48.43 | 31.97 | 68.42 | 25.20 | 16.92 | 7.91 |
| Branch 29 | 2.21 | 2.20 | 2.42 | 2.43 | 6.48 | 15.54 | 19.73 | 13.00 | 27.92 | 10.24 | 6.87 | 3.20 |
| Branch 30 | 2.39 | 2.38 | 2.63 | 2.63 | 7.03 | 16.91 | 21.50 | 14.14 | 30.48 | 11.13 | 7.45 | 3.47 |
| Branch 31 | 0.34 | 0.34 | 0.37 | 0.37 | 1.00 | 2.40 | 3.05 | 2.01 | 4.33 | 1.58 | 1.06 | 0.49 |
| Branch 32 | 0.09 | 0.09 | 0.10 | 0.10 | 0.27 | 0.66 | 0.84 | 0.55 | 1.19 | 0.43 | 0.29 | 0.14 |

